TokyoTech (Tokyo Institute of Technology),　HMA (History of Mathematics and Astronomy)
Lecture note 5: (2019)
(Mathematics and astronomy in traditional Korea and Japan.)
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Mathematics and astronomy in traditional Korea

(1) Three Kingdoms period (三国時代)

(the 1st century BCE or CE – 660 CE)

----- Kogryo (高句麗), Paekche (百済) and Silla (新羅)

(From 金両基（監修）『図説・韓国の歴史』、河出書房新社, 2002, with my notes)

Koguryo (高句麗):

There are many tomb murals. The sun, moon and stars are painted in some of them.

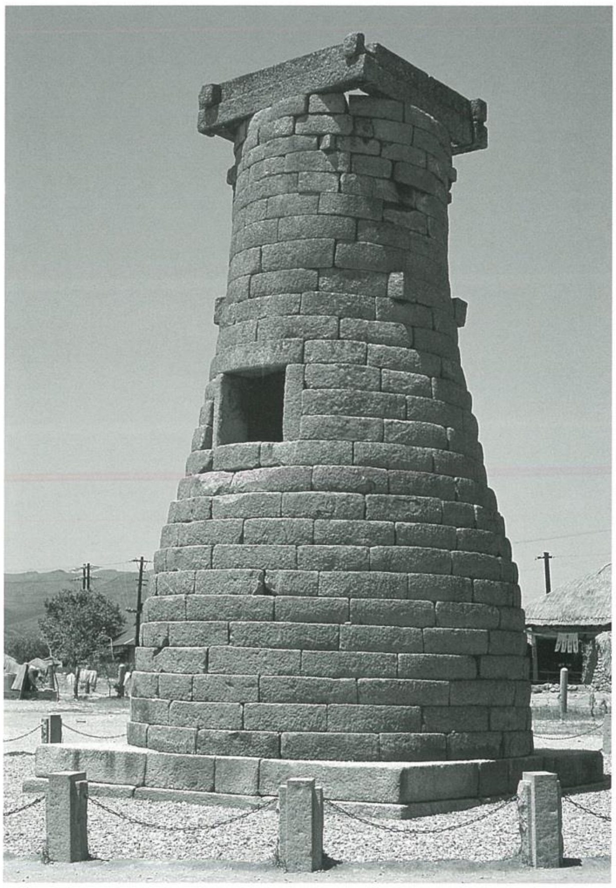
It is supposed that they influenced to Japanese tomb murals.

Paekche (百済):

Cinese *Yuanjia* calendar (元嘉暦) was used in Paekche. Calendar and astronomy were introduced to Japan from Paekche in 554 and 602 CE.

Silla (新羅):

The "Ch'omsongdae" (瞻星臺), which means Star Viewing Platform, was built in Kyongju (慶州), the capital of Silla in 647 CE.



The Ch'omsŏngdae Observatory, 647. Height 9.1 m. The oldest extant astronomical observatory in the world. The granite brick construction lends additional grace to the structure's elegant lines. Kyŏngju (慶州), capital of Silla (新羅).

(From Jeon (2011), p.10)

(2) Unified Silla (新羅) period (668 – 900 CE)

(From 金両基（監修）『図説・韓国の歴史』、河出書房新社, 2002, with my notes)

According to the *History of the Three Kingdoms* (*Samguksagi*, 三國史記), the national school was established in 682 CE, and mathematics was also taught there.

Chinese works of mathematics, *Jiuzhang suanshu* (九章算術), *Zhuishu* (綴術) etc., were used as text books.

In 718 CE, the office of water clock (漏刻典) was established.

(3) Koryo (高麗) Dynasty period (918 – 1392 CE)

(From 金両基（監修）『図説・韓国の歴史』、河出書房新社, 2002, with my notes)

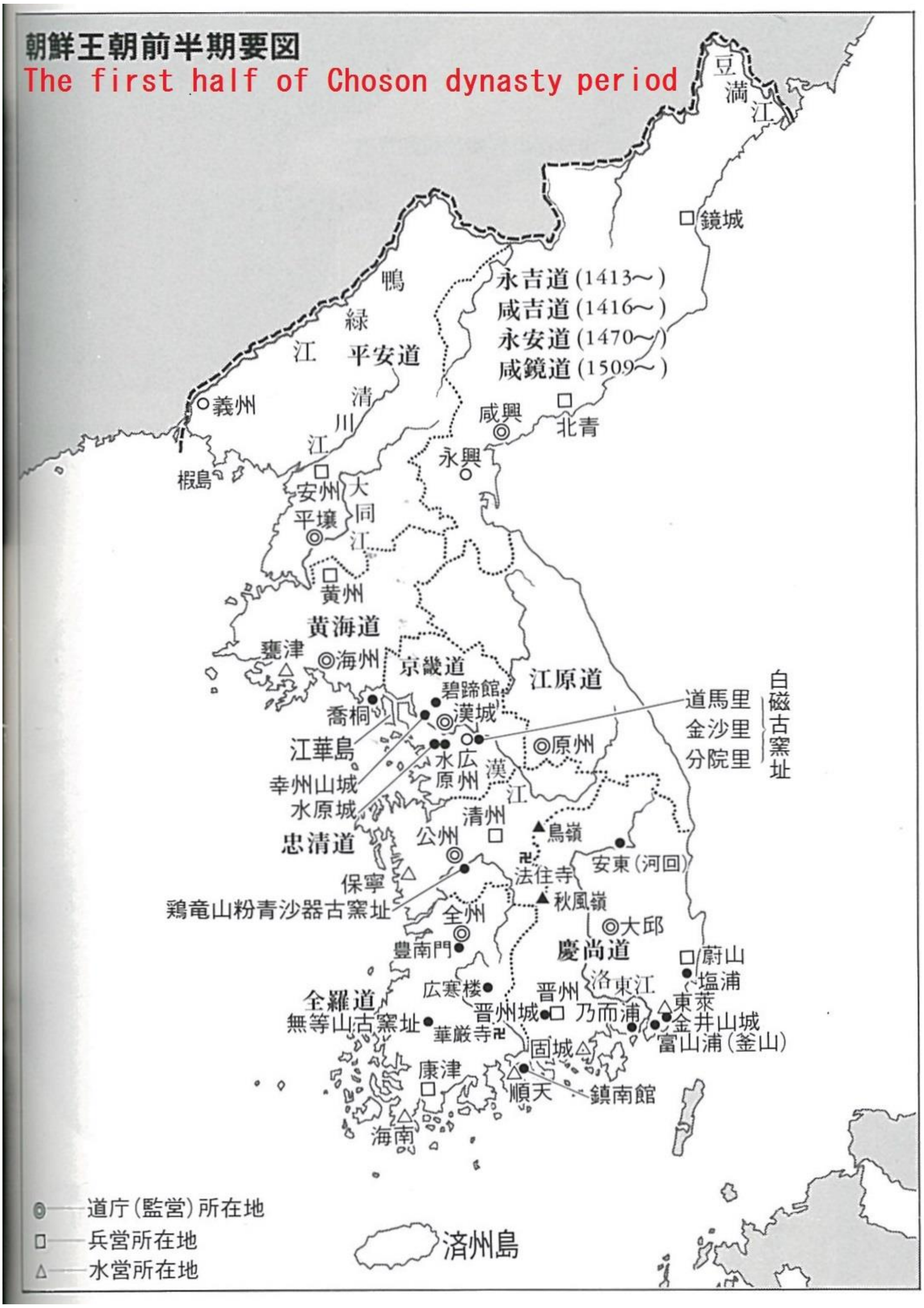
In the 12th century, the system of education was established, and mathematics was also taught.

In 1136 CE, the system of impereal examination (科擧) was established. Mathematics was also included in the examination, and Chinese works of mathematics, *Jiuzhang suanshu* (九章算術), *Zhuishu* (綴術) etc., were used as text books.

Chinese *Xuanming* calendar (宣明暦) was used from the beginning of this dynasty, and *Shoushi* calendar (授時暦) was also introduced in 1281 CE. There was the bureau of astronomy (太史局 or 書雲観).

The *History of Koryo Dynasty* (*Koryosa*, 高麗史) has sections of astronomy and calendar.

(4) Choson Dynasty (朝鮮王朝) period (1392-1897 CE)



(From 金両基（監修）『図説・韓国の歴史』、河出書房新社, 2002, with my notes)

(4.1) The first half of the Choson Dyasty period

A star map called *Ch'onsang yolch'a punyajido* (天象列次分野之圖) was made in the early Choson Dynasty.

The star map inscribed on a stone in 1395 CE (now preserved in the National Palace Museum of Korea, Seoul.)

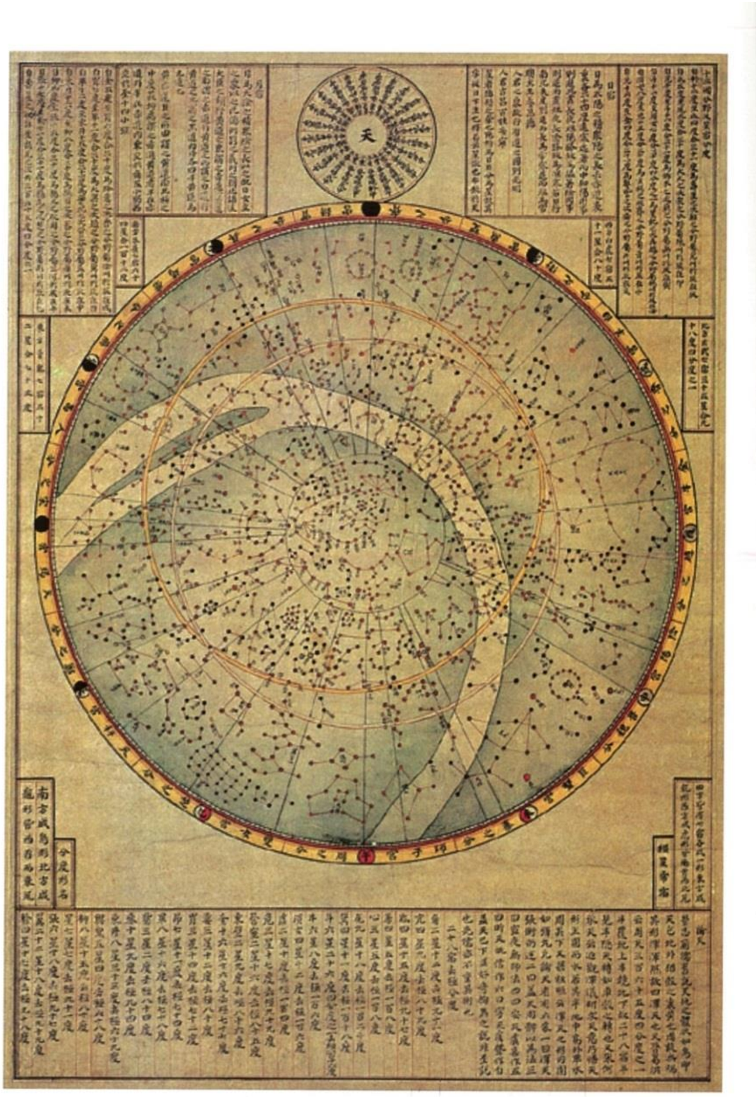
022　天象列次分野之図刻石 星座を刻んだ石

朝鮮、1395年 | 黒大理石 | 211.0×123.0cm、厚さ12.0cm | 国宝第228号

高句麗の天文知識に基づき、1395年(太祖4年)に完成された天文図で、中国の南宋代につくられた〈淳祐天文図〉(1241年)に次いで世界で2番目に古いものである。'星座12種類を配列順に並べた星宿図'という意味で'天象列次分野之図'と呼ばれた。片面に刻まれた文字数は2,932字であり、星座は1,467個に至る。周囲に28宿の名称や赤道宿度が記されている。この天文図に刻まれた権近(クォン・グン)(1352～1409年)の文章によると、高句麗にも刻石天文図があったが戦いで失われ、大同江という川の中に沈んでいった。この刻石天文図は、表面と裏面に星座が逆さまに刻まれ、表面には下段、裏面は上段に'天象列次分野之図'と刻まれていたという。1687年(粛宗13年)に作られた1395年製作の天象列次分野之図刻石の復刻品(宝物第837号)も現存している。

(From 国立古宮博物館ガイド、ソウル、国立古宮博物館, 2008,　p.36)

A colour copy of the star map:



Chosŏn star chart, 16th century. 120 × 88 cm. Color copy of the 122.5 × 211 × 12 cm celestial stele entitled *Chart of the Constellations and the Regions They Govern* (*Chŏnsang yŏlch'a punyajido* 天象列次分野之圖), on which 1,467 stars were inscribed in 1395. The carved stele is now designated National Treasure Number 228 and preserved in Tŏksugung Palace Exhibition Center. Color copy from the author's collection.

(From Jeon (2011), p.26)

At the time of King Sejong (世宗) (reign 1419 – 1450), mathematics and astronomy highly developed.

In this period, some Chinese mathematical works were considered to be important, namely:
Yang Hui suanfa **(楊輝算法) of Yang Hui (楊輝) (the 13th century),** ***Suanxue qimeng*** **(算学啓蒙) (ca.1299 CE) of Zhu Shijie (朱世傑),** ***Xiangming suanfa*** **(詳明算法) (1373 CE) of An Zhizhai (安止齋).**

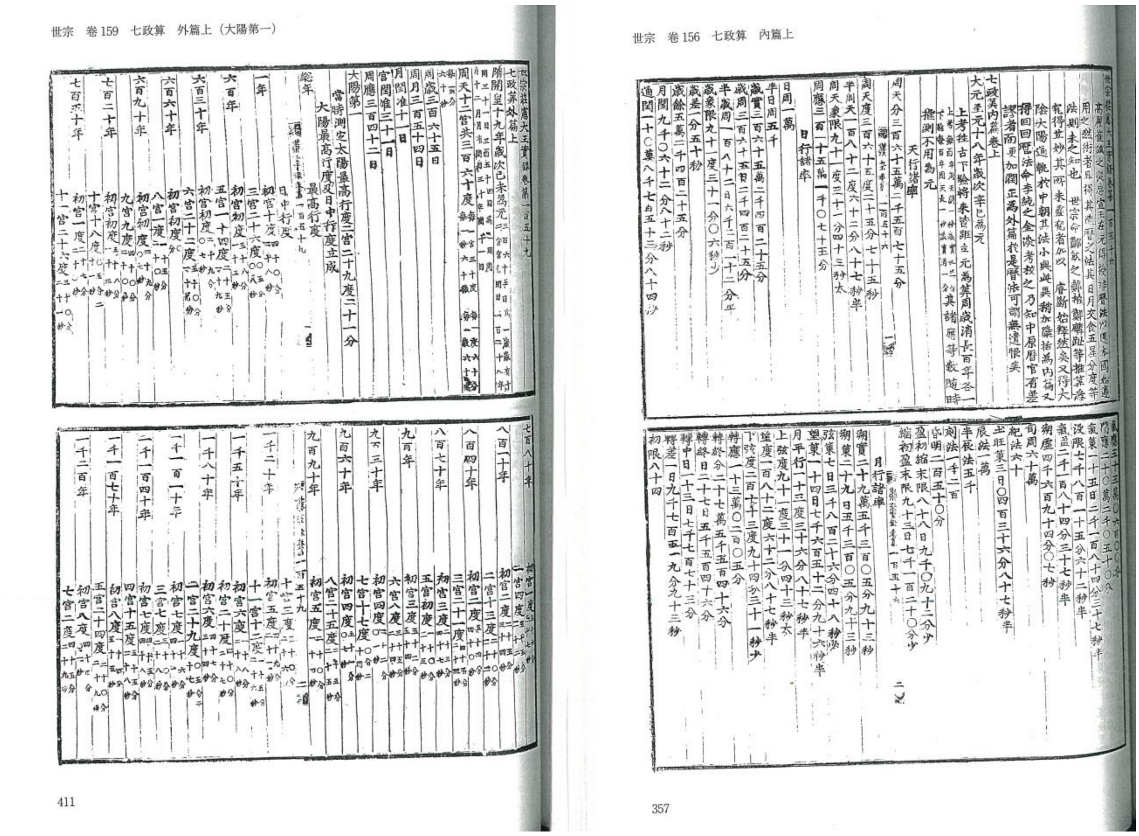
Two treatises of mathematical astronomy were composed:

(A) *Ch'ilchongsan naep'yon* (七政算内篇), which is based on the *Shoushi* calendar (授時暦), and

(B) *Ch'ilchongsan woep'yon* (七政算外篇), which is based on the *Huihui* calendar (回回暦法), which is a Chinese version of Islamic mathematical astronomy.

They are included in the *Veritable Records of the Choson Dynasty* (朝鮮王朝實録, *Chosonwangjo sillok*).

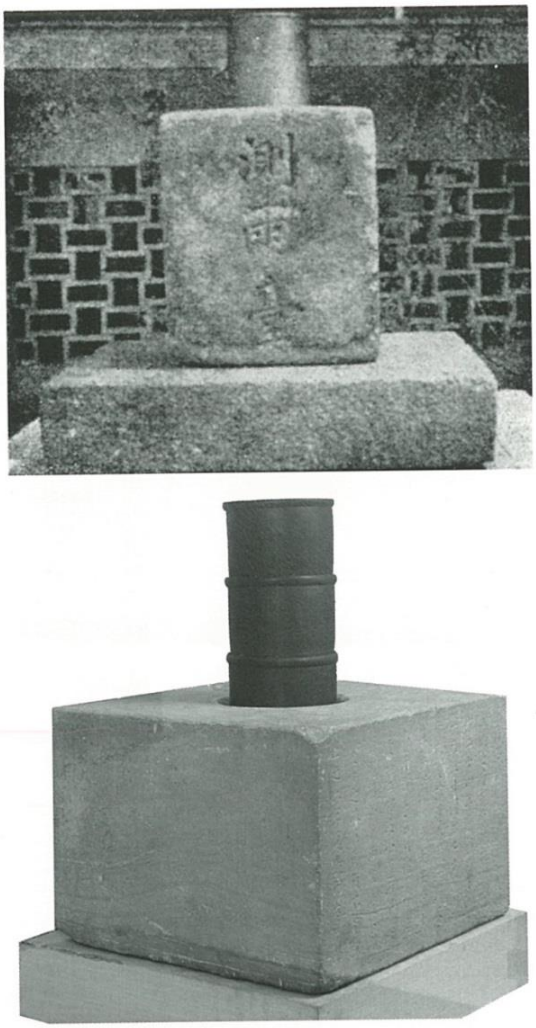
The followings are the first page of the *Ch'ilchongsan naep'yon* (七政算内篇) (right) and the first page of the *Ch'ilchongsan woep'yon* (七政算外篇) (left).



(From 『李朝実録』第十一冊，東京，学習院東洋文化研究所，1957.)

Several instruments were created in this period.

Rain gauge:



Upper: Rain gauge and rain gauge stand (測雨器 and 測雨臺), produced in 1770. 43 × 37 × 37 cm. Below: Rain gauge and rain gauge stand. Replica of the 1837 Kŭmyŏng rain gauge on its original 1782 marble stand. Tŏksugung Royal Palace Museum. This marble stand is inscribed on all four sides with information regarding the history and manufacture of the rain gauge. Both were made according to specifications for the world's first scientific rain gauge, invented in 1441 (Sejong 23). Rain gauges built to these specifications and installed in various locations allowed the accurate measurement of rainfall nationwide.

(From Jeon (2011), p.29.)

Hemispherical sundial:

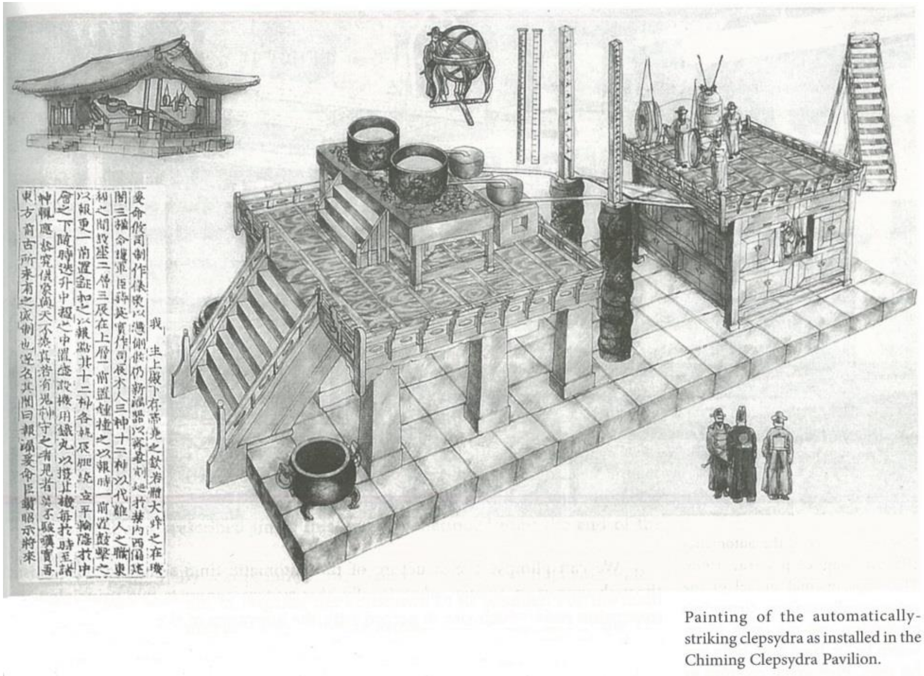
Bronze scaphe sundial, 18th century. Inner diameter 24.1 cm. First manufactured and installed as a public sundial in Sejong 19 (1437). This model continued as the representative sundial of the Chosŏn dynasty over a 500-year period. An elegant design with accurate divisions of hours and flawless functionality — everything required of a sundial. Sungshin Women's University Museum.

(From Jeon (2011), p.34)

Automatically-striking water clock:

Automatically-striking clepsydra (自擊漏). Three reservoirs and two inflow vessels, now under preservation, 1536. Main reservoir (播水壺): outer diameter 93.5 cm × height 70 cm. Inflow vessels (受水筒): outer diameter 37 cm × height 199 cm. This characteristic Chosŏn period water clock with an automatic time signal apparatus was originally constructed by Chang Yŏng-sil (蔣英實) in 1434, then revised and reconstructed in 1536. Tŏksugung Palace.

(From Jeon (2011), p.30.)   The remains of the water clock in Toksugung Palace (德壽宮).



(From Jeon (2011), p.111.)　A painting of the water clock

A reconstruction of the automatically-striking water clock in the National Palace Museum of Korea

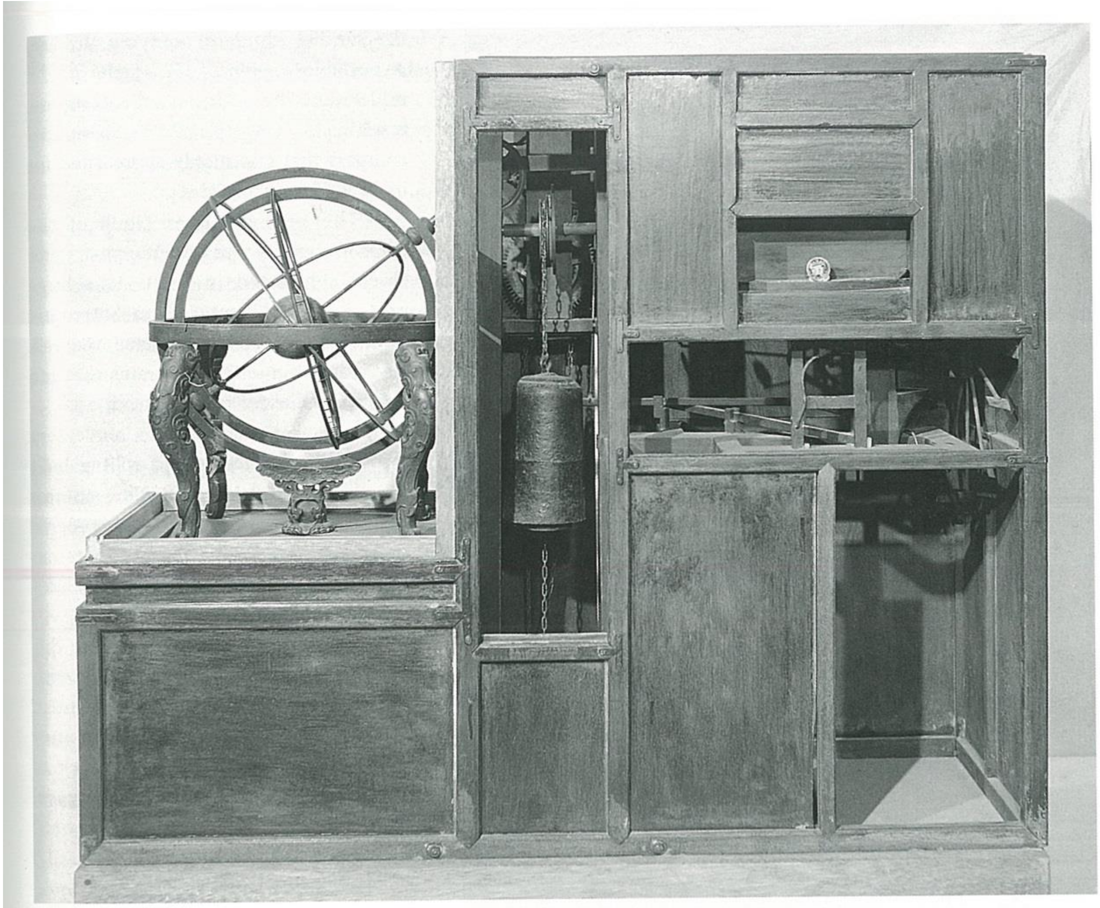
(From 国立古宮博物館ガイド、ソウル、国立古宮博物館, 2008,　pp.122 – 123)

(4.2) The second half of the Choson Dynasty period

In 1660 CE, the *Suanxue qimeng* (算学啓蒙) was reprinted. Several original mathematical works were also composed.

From 1653 CE, Chinese *Shixian* calendar (時憲暦), which is based on European astronomy, was used in Korea.

The armillary clock (渾天時計) constructed by Song Yi-yong (宋以頴) in 1669 CE is an interesting clock.



Armillary clock (渾天時計). Constructed by Song I-yŏng (宋以穎) in 1669, this new type of clock incorporated technology from both East Asian armillary clocks and Western mechanical clocks. The armillary sphere measures 40 cm in diameter. Korea University Museum.

(From Jeon (2011), p.31)

[For Korean astronomical instruments, see Needham et al., (1986).]

Several original mathematical astronomical works were composed in the second half of the Choson Dynasty period. For example:

***Kusuryak* (九數略) of Ch'oe Sok-chong (崔錫鼎) (1646 – 1715), which is a systematic treatise of mathematics based on Chinese natural philosophy, and**

***Kuiljip* (九一集) of Hong Jong-ha (洪正夏) (b.1684), which is a practical mathematical work with a detailed description of algebra using counting rods.**

Song Chu-dok (成周悳) wrote the *Soun'gwanji* (書雲観志, *Record of the Bureau of Astronomy*) (1818), which is an important work on Korean astronomy.

In the late Choson Dynasty period, Nam Byong-ch'ol (南秉哲) 1817 1863) and Nam Byong-gil (南秉吉) (1820 – 1869) (brothers), and Yi Sang-hyok (李尚赫) (b.1810) studied both traditional mathematics and Western mathematics.

Practical learning "Silhak" (實學):

Yi Ik (李瀷) (1682 – 1764) developed the "practical learning", and studied Western science and technology through Chinese texts.

Kim Song-mun (金錫文), Hong Dae-yong (洪大容) (1731 – 1783) and Pak Chi-won (朴趾源) (1737 – 1805) expressed the rotating earth theory.

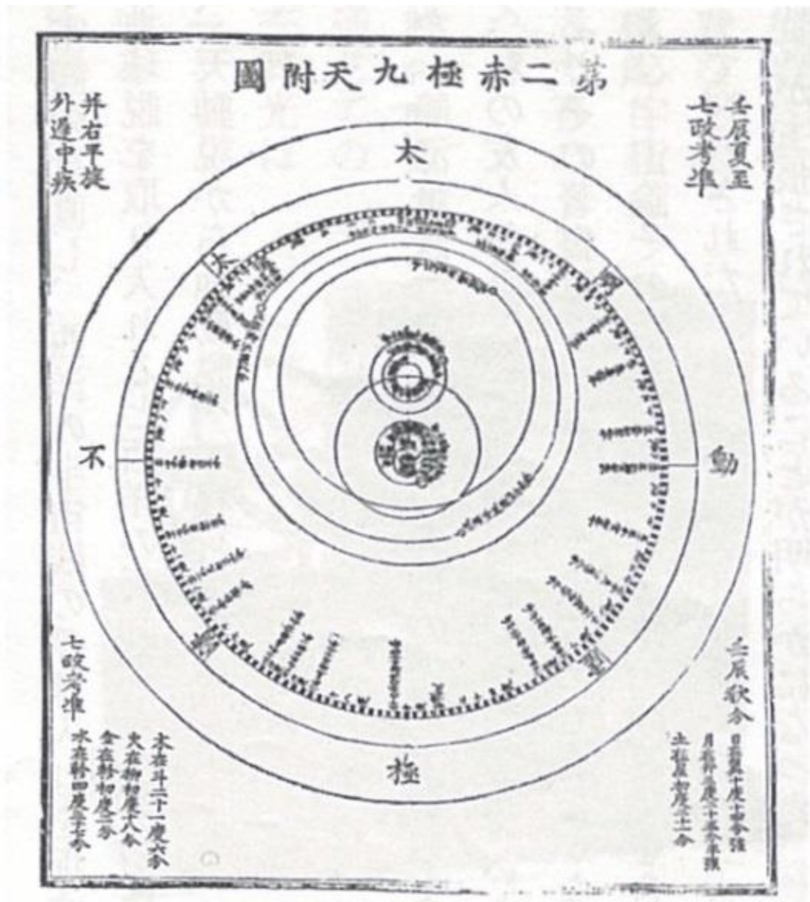


図4-10 『易学二十四図解』赤道図

Kim Song-mun's planetary model (From Im (2012), p.256)

Chong Yag-yong (丁若鏞) (1762 – 1836) developed new technology etc.

Ch'oe Han-gi (崔漢綺) (1803 – 1879) studied Western astronomy, geography, Newtonian mechanics etc. through Chinese texts.

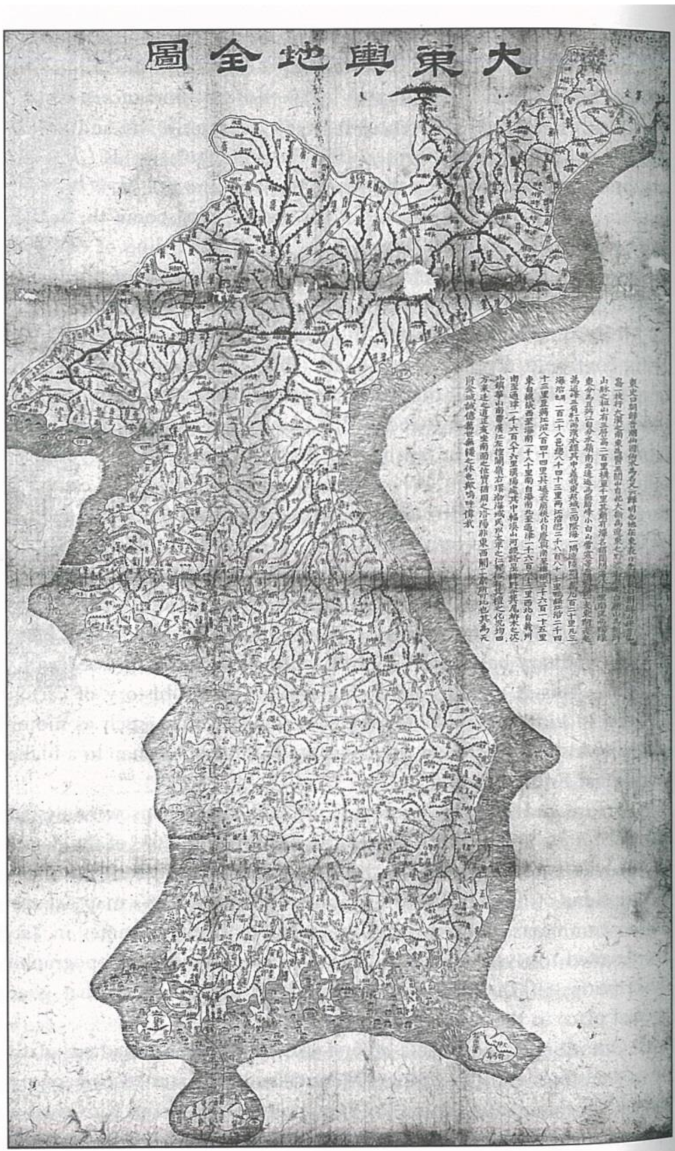
Ch'oe Han-gi's terrestrial sphere:

Ch'oe Han-gi's (崔漢綺) terrestrial sphere, 27.7 × 26.8 × 26.8 cm. A precious specimen of the globes that have been preserved intact, another of which is Song Yi-yŏng's armillary sphere.

(From Jeon (2011), p.155.)

Kim Chong-ho (金正浩) (19th century) made very precise maps of Korea.

The *Complete Map of the Great Eastern Kingdom* (*Taedong yojijondo*) (1880s) of Kim Chong-ho

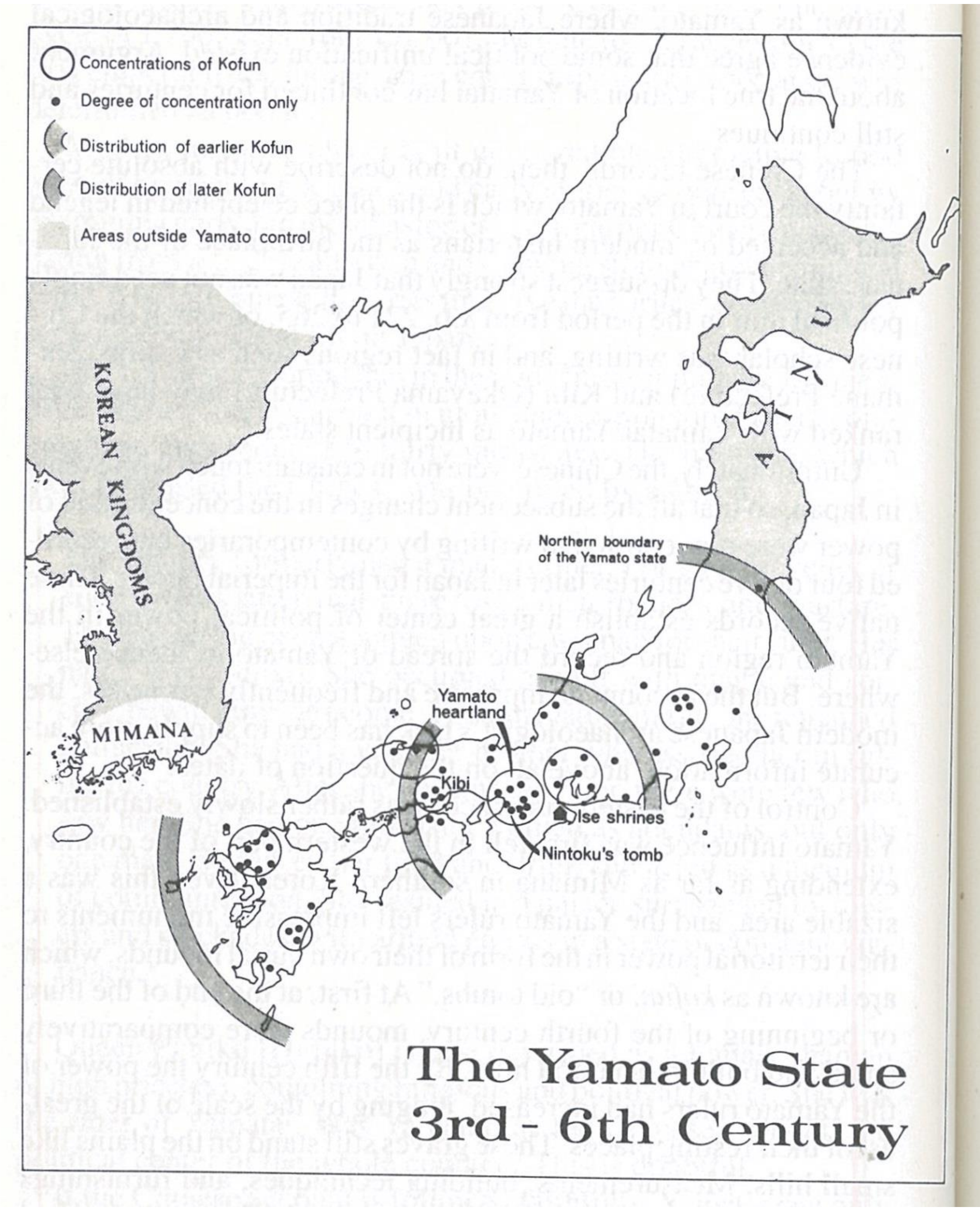


Complete Map of the Great Eastern Kingdom (*Taedong yŏjijŏndo*), 1880s, woodblock print by Kim Chŏng-ho, 114.3 × 64.8 cm. Sungshin Women's University Museum.

(From Jeon (2011), p.324.)

Mathematics and astronomy in traditional Japan

Kofun (old tombs) period (古墳時代) (mid-3rd century – 7th century CE) and Asuka period (飛鳥時代) (593 – 710 CE):



(From Mason and Caiger: A History of Japan, Revised ed., Tokyo, Tuttle Publishing, 1997.)

Introduction of calendar and astronomy from Korean peninsula

In 553 CE, Japan requested to Paekche to send specialists of calendar etc., and they came to Japan in 554 CE.

In 602 CE, Kwanruk (觀勒, Kanroku in Japanese), a Buddhist monk of Paekche, came to Japan with books on calendar, astronomy, etc.

It seems that Chinese *Yuanjia* calendar (元嘉暦, "Genka-reki" in Japanese) was used since 604 CE.

In 660 CE, Nakano'ōeno'ōji (中大兄皇子), who later became Emperor Tenji (天智天皇), made a water clock.

In 675 CE, "senseidai" (占星臺), which might have been a kind of observatory, was made.

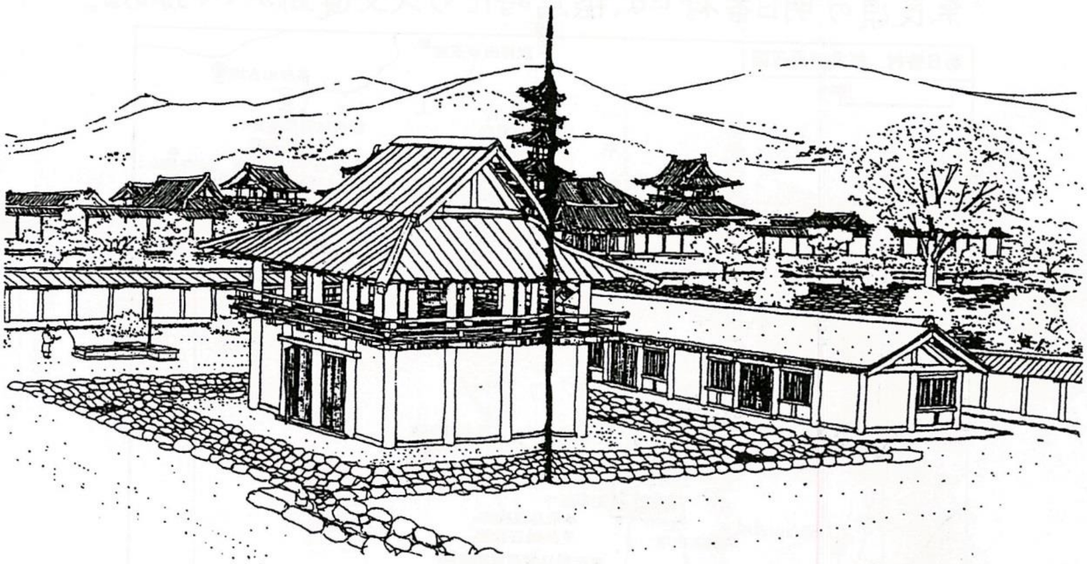
From ca.692, "Gihō-reki" (儀鳳暦), which is probably the same as Chinese *Linde* calendar (麟徳暦), was also used with the *Yuanjia* calendar.

Astronomical remains in Asuka (飛鳥) area

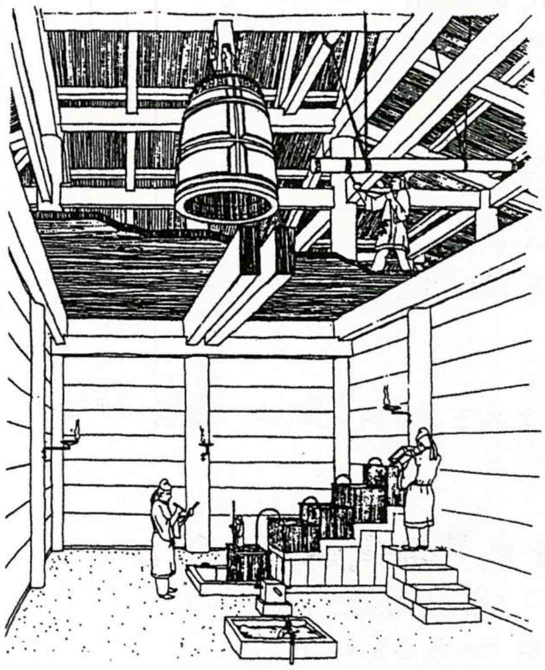
There are some astronomical remains in Asuka village (明日香村) of Nara prefecture (奈良県).

Mizuochi remains (水落遺跡, Mizuochi-iseki)

The Mizuochi remains (水落遺跡) is presumed to be the remain of water clock. We have seen above that the water clock was first made in 660 CE.



水時計建物の復原　北西からみたところ。水時計建物は高さ十数メートル、二階は開放で、報時用の鐘鼓を置き、板葺きか。時刻調整に必要な圭表もあっただろう。周囲には付属建物がたち、一郭を塀で囲む。東方に飛鳥寺の大伽藍。

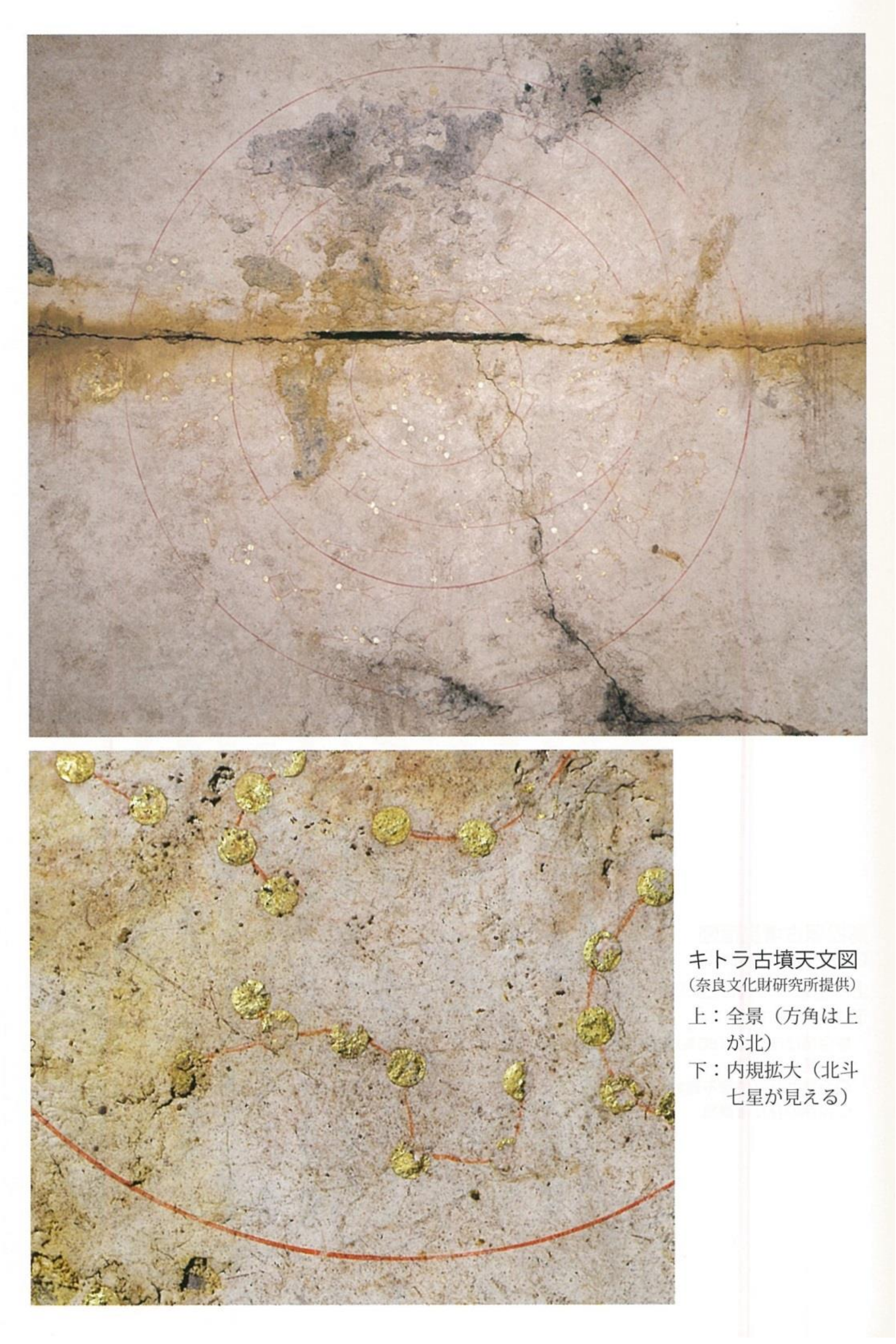


水時計建物内部の復原　一階内室は五・五メートル四方。水時計を置くと、意外に狭い。内室では、ラッパ型銅管でくみ揚げた水をこしたり、水時計を操作して時間を測ったり、また、装置の保守管理などの仕事を行なったのだろう。

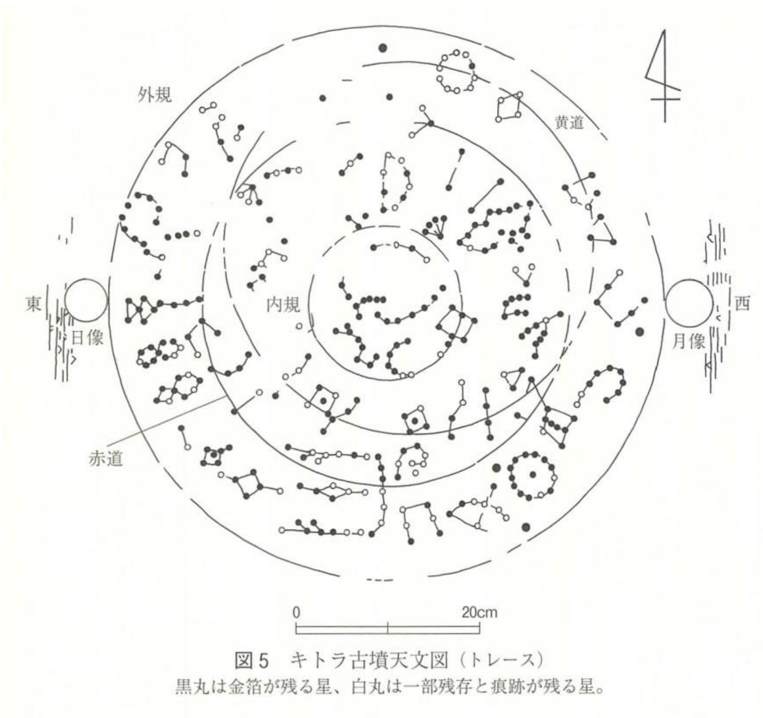
(From Kanō and Kinoshits (1985).)

Kitora Tomb (キトラ古墳, Kitora-kofun)

A circular star map is drawn on the ceiling of the chamber of the Kitora Tomb (the end of 7th century or the beginning of 8th century CE).



The star map of Kitora Tomb (From Izumi (2018))

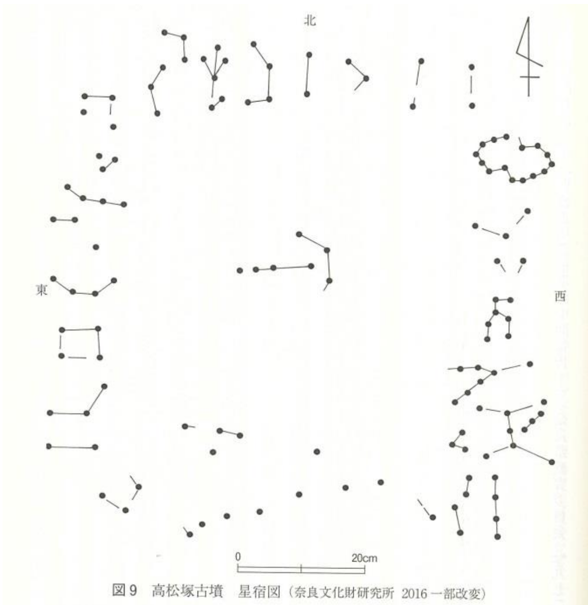


A tracing of the star map of Kitora Tomb (From Izumi (2018), p.90.)

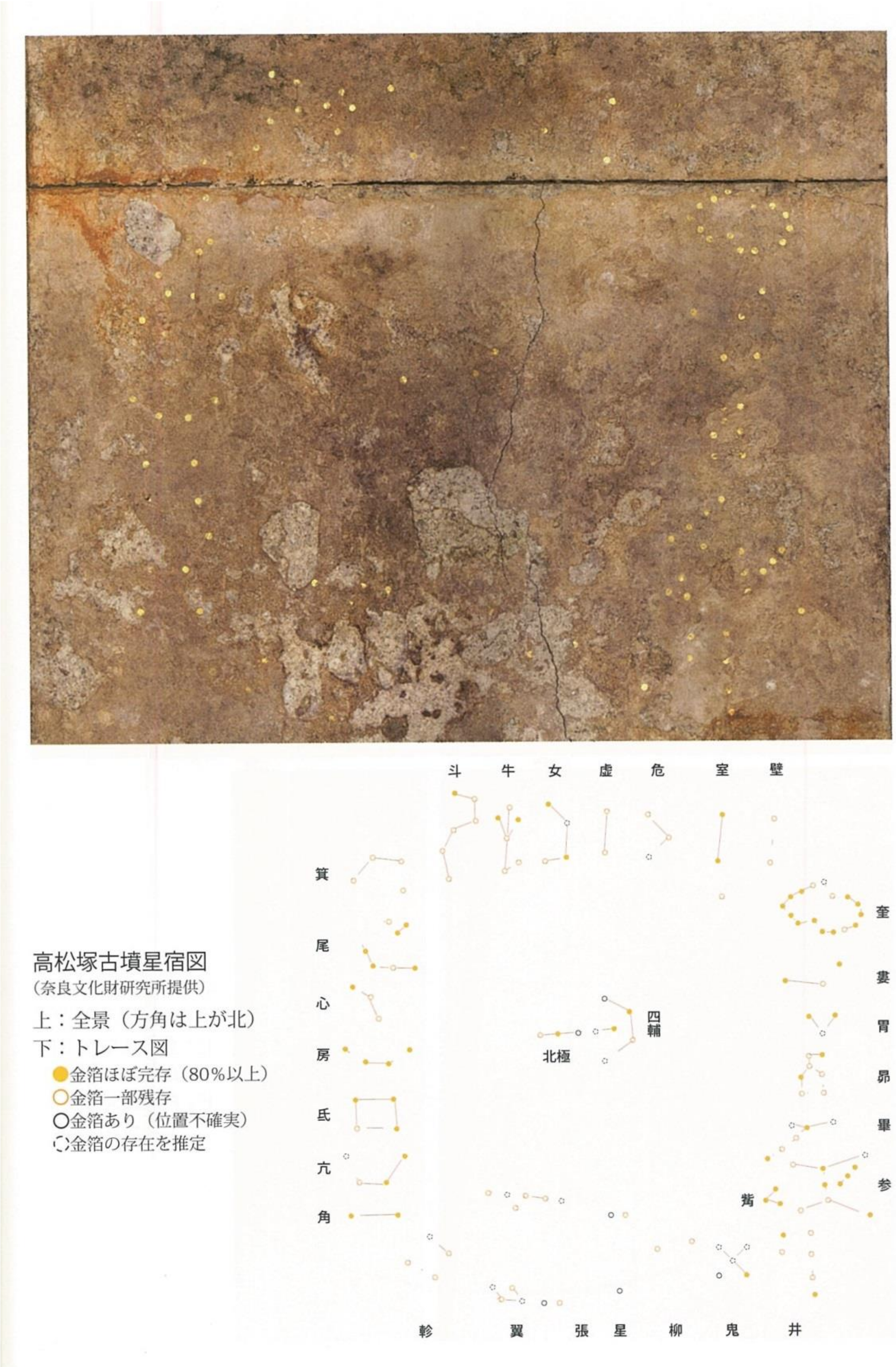
The constellations described here are Chinese traditional constellations.

Takamatsuzuka Tomb (高松塚古墳, Takamatsuzuka-kofun)

A simplified star map is drawn on the ceiling of the chamber of the Takamatsuzuka Tomb (the end of 7th century or the beginning of 8th century CE).



A tracing of the star map of Takamatsuzuka Tomb (From Izumi (2018), p.109.)

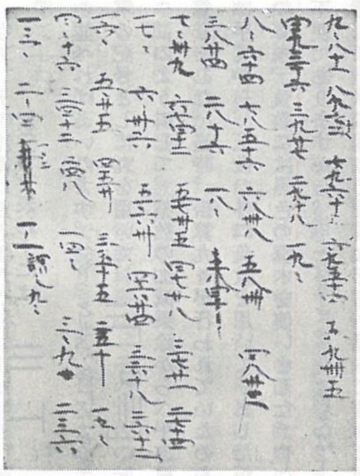


The star map of Takamatsuzuka Tomb (From Izumi (2018))

Astronomy under “ritsu-ryō system” (律令制)

Under the political system based on “ritsu-ryō” (律令), a kind of legal codes established in the early 8th century, “on’yō-ryō” (or “onmyō-ryō”) (陰陽寮), bureau of astronomy, calendar, water clock, and astrology, was established.

And also, mathematics was taught at “daigaku-ryō” (大学寮), a kind of national school. At that time, Chinese counting rods were used for calculation. Chinese multiplication table was also known.



第２図　千年前の掛算九々

これは平安朝の天禄元年(西暦 970)に書かれた,「口遊」という写本の一頁です.「口遊」は貴族の子弟の教科書で, 色々のことが載っており, 数学書ではありません. 今この図は, 山田孝雄博士の解説のある, 古典保存会の複製本から, 複写したものです. この九々は, 今日の九々とは反対に, 九九八十一から始まっていますが, これは中国の古い時代の九々の形なのです.

A multiplication table in the *Kuchizusami* (口遊) (970 CE). (From Ogura (1964), p.6.)

Buddhist astrology in the Heian (平安) period

Besides the “onmyō-dō” (陰陽道), which is a kind of astrology and divination based on Chinese natural philosophy, the “sukuyō-dō” (宿曜道) was also popular in the Heian (平安) period.

The “sukuyō-dō” (宿曜道) is a kind of Indian horoscopic astrology based on the Buddhist astrological text *Xiuyao-jing* (宿曜經, *Sukuyō-kyō* in Japanese), which was compiled in China.

Introduction of European astronomy by Jesuit missionaries

In 1543, three Portuguese arrived at Tanegashima (種子島) of Japan. They are the first Europeans visited Japan. Then, in 1549, Francis Xavier, a Jesuit missionary, visited Japan.

In 1593, Pedro Gomez wrote the "*Compendium*" in Latin, a kind of text book taught at Collegio (Jesuit college), and its first part "de Sphaera" is on the geocentric astronomy.

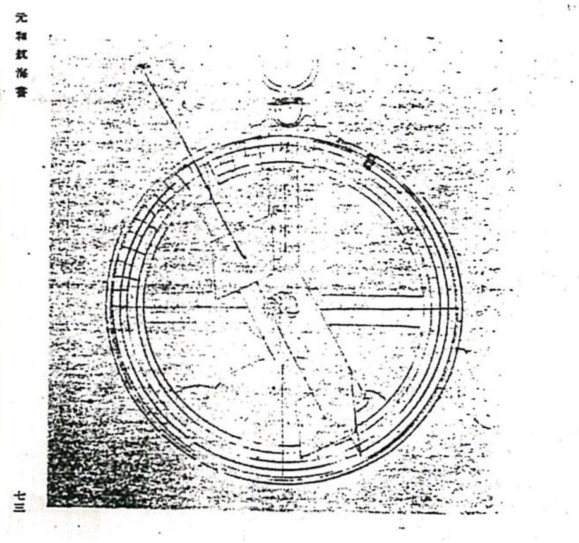
There are some Japanese works based on the Japanese version of this work such as:

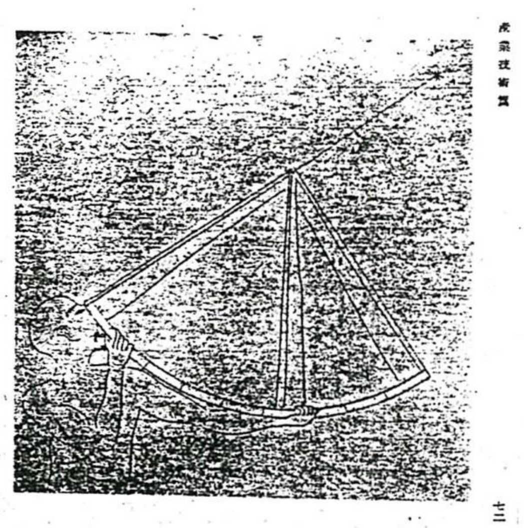
***Nigi ryakusetsu* (二儀略説) (published in 『近世科学思想（下）』(日本思想体系 63), 岩波書店、1971), and**

***Kenkon bensetsu* (乾坤弁説) (published in 『文明源流叢書』第二, 国書刊行会, 1914).**

And also, European navigation was introduced into Japan, and it is recorded in the *Genna kōkaisho* (元和航海書) (published in 『日本科学古典全書』第 12 巻、朝日新聞社, 1943).

Astrolabe and quadrant in the *Genna kōkaisho*:





The observer's latitude can be known by astronomical observations using these instruments.

Development of astronomy in the Edo period

Shibukawa Harumi (澁川春海) and his Jyōkyō-reki (貞享暦)

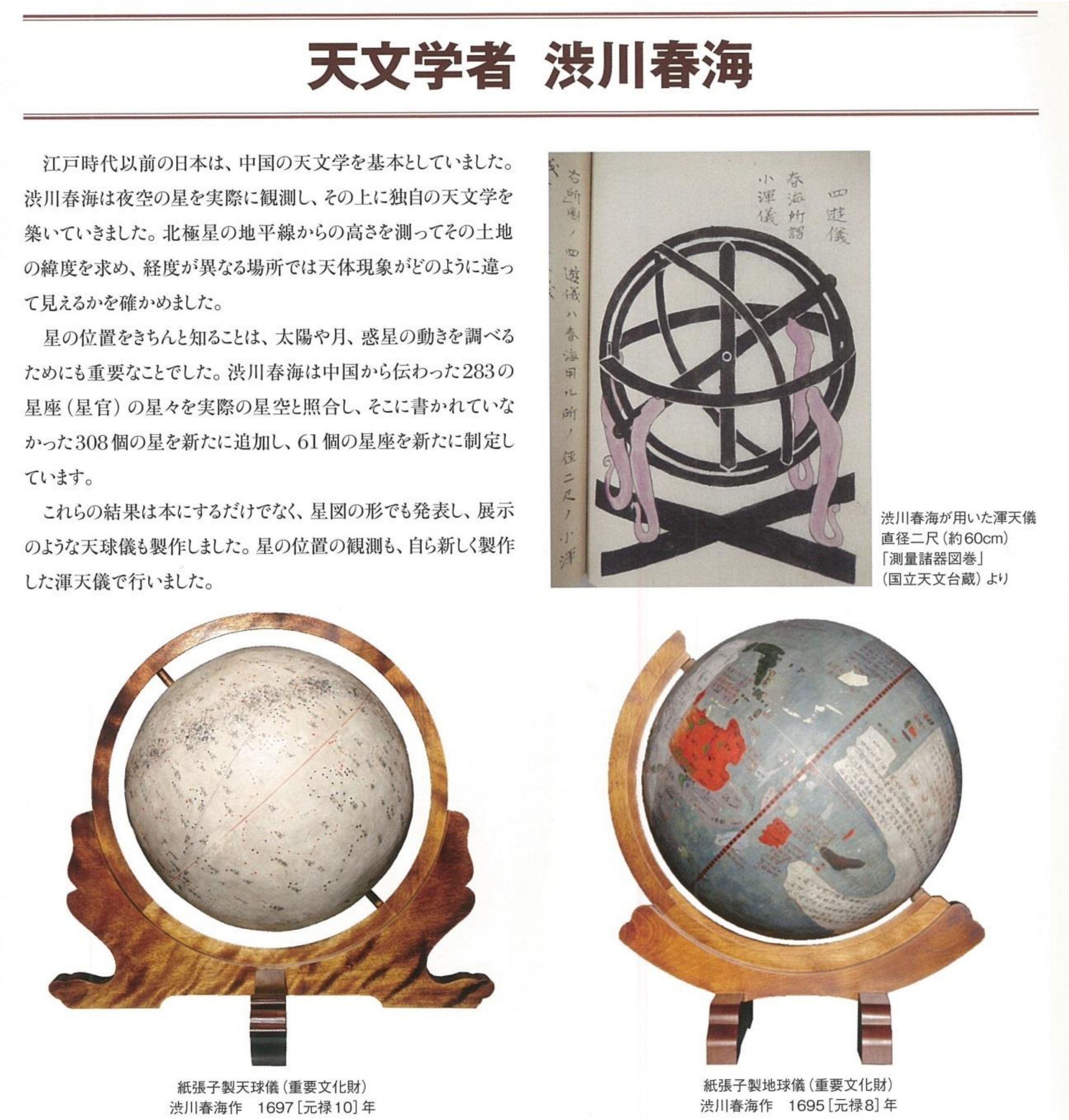
The *Xuanming* calendar (宣明暦, *Senmyō-reki* in Japanese) was used from AD 862 to 1684 in Japan.

By the beginning of the Edo (江戸) period (AD 1603 ~ 1867), some Japanese scholars noticed the inaccuracy of the old *Xuanming-li*, and tried to study the more accurate *Shoushi* calendar (授時暦, *Juji-reki* in Japanese), an excellent Chinese traditional calendrical system which was made by GUO Shoujing (郭守敬) etc. at the time of the Yuan (元) dynasty of China, and was used in China from AD 1281 (but has never been used officially in Japan).

In AD 1683, SHIBUKAWA Harumi (澁川春海, AD 1639 – 1715), who studied the *Shoushi-li*, proposed a new calendrical system, which was named *Jōkyō-reki* (貞享暦) in the next year, and was officially used in Japan from AD 1685. It was the first theory of calendrical system produced in Japan.

SHIBUKAWA Harumi considered the longitudinal difference between China and Japan, and the movement of the apogee of the solar orbit.

He was appointed to be the first “tenmon-kata” (天文方), the Shogunal astronomer, in 1684.



天文学者 渋川春海

江戸時代以前の日本は、中国の天文学を基本としていました。渋川春海は夜空の星を実際に観測し、その上に独自の天文学を築いていきました。北極星の地平線からの高さを測ってその土地の緯度を求め、経度が異なる場所では天体現象がどのように違って見えるかを確かめました。

星の位置をきちんと知ることは、太陽や月、惑星の動きを調べるためにも重要なことでした。渋川春海は中国から伝わった283の星座（星官）の星々を実際の星空と照合し、そこに書かれていなかった308個の星を新たに追加し、61個の星座を新たに制定しています。

これらの結果は本にするだけでなく、星図の形でも発表し、展示のような天球儀も製作しました。星の位置の観測も、自ら新しく製作した渾天儀で行いました。

渋川春海が用いた渾天儀
直径二尺（約60cm）
「測量諸器図巻」
（国立天文台蔵）より

紙張子製天球儀（重要文化財）
渋川春海作　1697［元禄10］年

紙張子製地球儀（重要文化財）
渋川春海作　1695［元禄8］年

(From 『渋川春海と江戸時代の天文学者たち』、国立科学博物館, 2016, p.3)

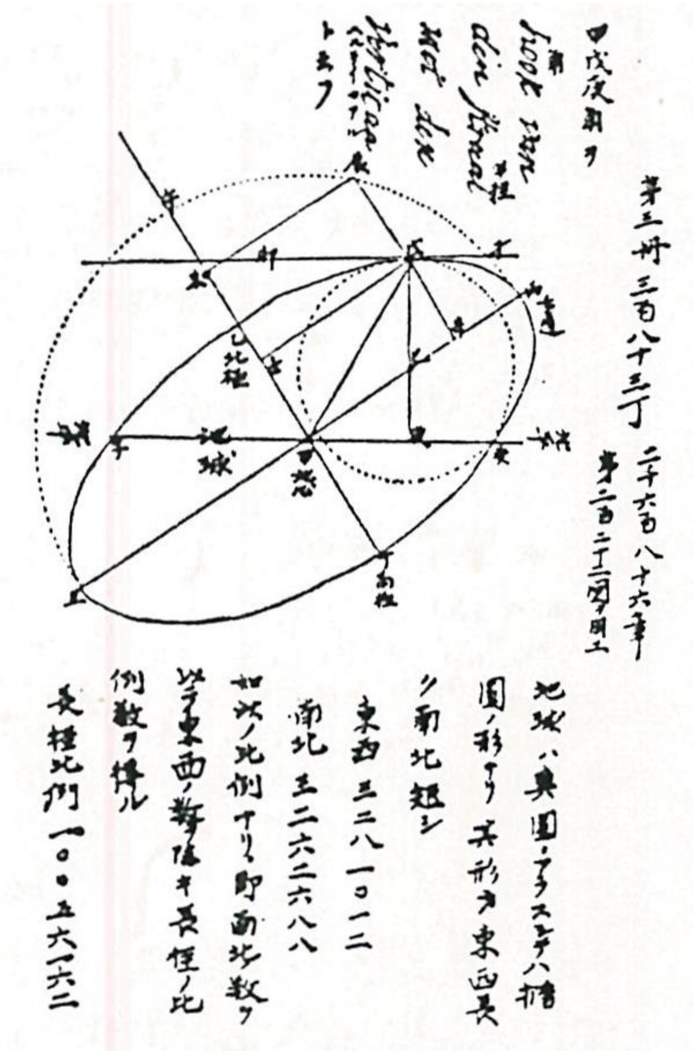
Development of astronomy in tenmonkata (天文方)

When SHIBUKAWA Harumi made his *Jōkyō-reki* (貞享暦), Chinese astronomy had developed further after receiving Western astronomy. (The *Shixian* calendar (時憲暦) had been used from 1645.) The office of "tenmon-kata" (天文方) started by SHIBUKAWA later became a centre of the study of Western astronomy.

More systematic adoption of Western elements in Japanese traditional calendar was done by TAKAHASHI Yoshitoki (高橋至時, AD 1764 – 1804), who became a "tenmon-kata", and his colleague HAZAMA Shigetomi (間重富, AD 1756 – 1816), and they made the calendrical system *Kansei-reki* (寛政暦).

TAKAHASHI Yoshitoki and HAZAMA Shigetomi were disciples of ASADA Gōryū (麻田剛立) (1784 – 1799) who studied Chinese texts of Western astronomy in Osaka (大阪) as an amateur astronomer.

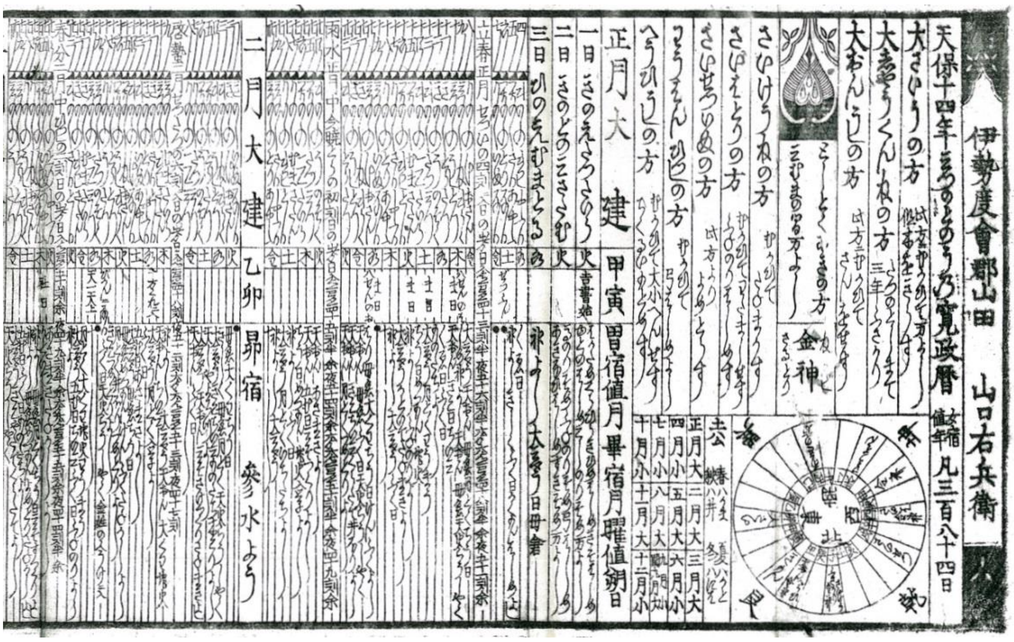
TAKAHASHI Yoshitoki slso studied a Dutch translation of the *Astronomie* of Lalande, a treatise of astronomy originally in French.



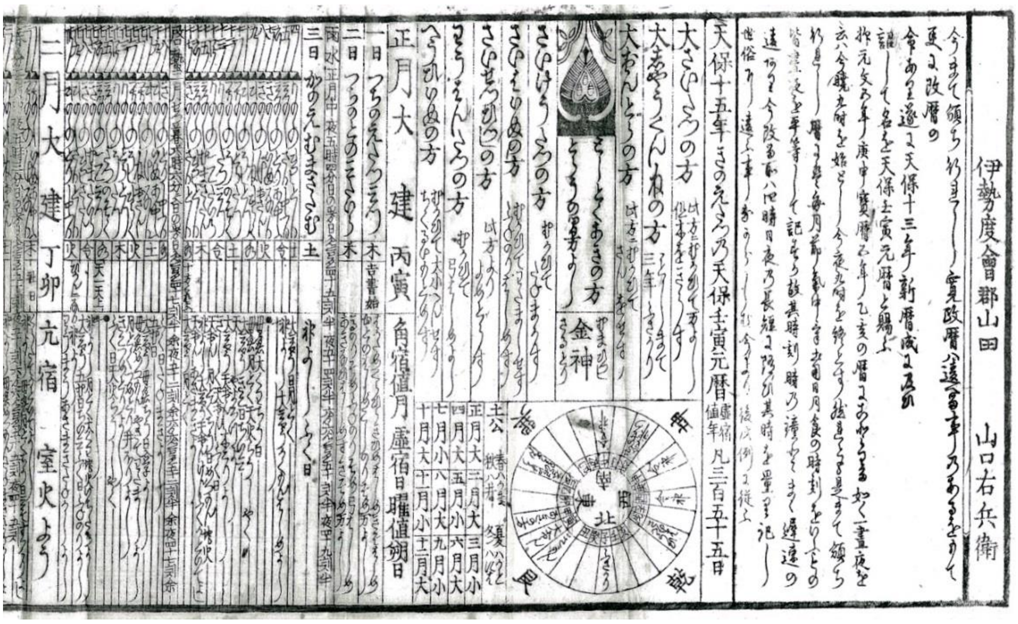
第 13 図　高橋至時稿『西洋人ラランデ暦書管見』の一部．羽間文庫蔵．

Takahashi Yoshitoki's note on the book by Lalande (From Nakayama (1972))

An example of the *Kansei-reki* (1843, the lasy year of the *Kansei-reki*) :



An example of the *Tenpō-reki* (1844, the first year of the *Tenpō-reki*) :



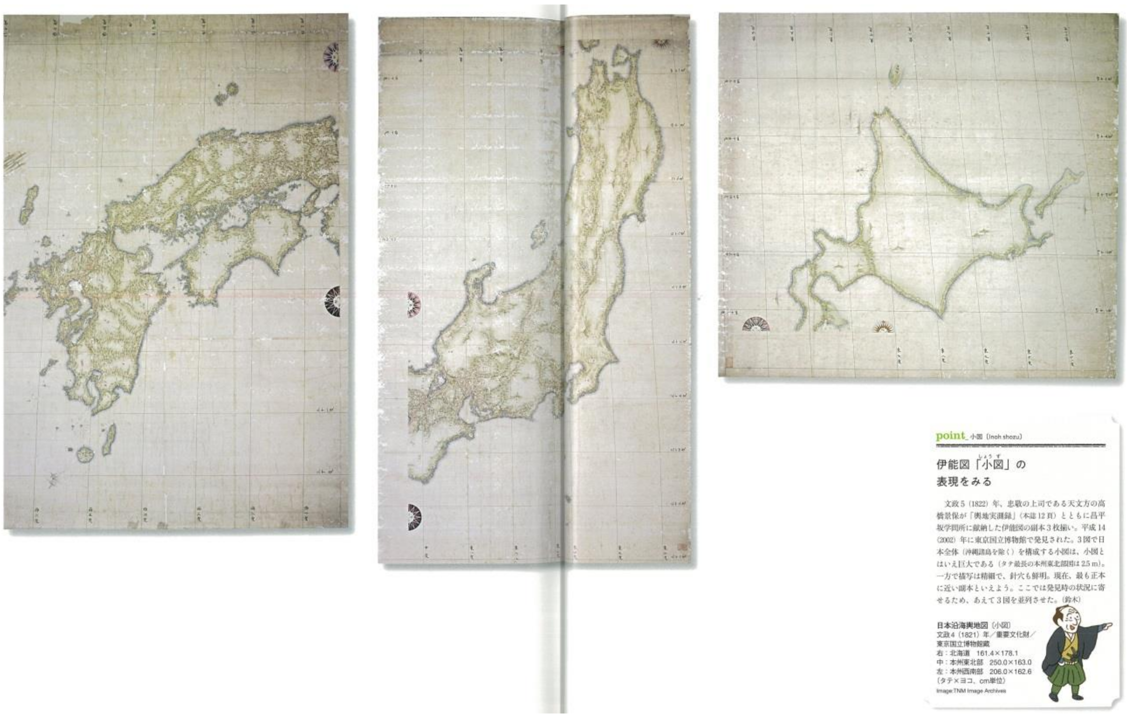
Western astronomy was more fully adopted by SHIBUKAWA Kagesuke (渋川景佑, AD 1787 – 1856), the second son of TAKAHASHI Yoshitoki. SHIBUKAWA Kagesuke became an adopted son of the SHIBUKAWA family, became a "tenmon-kata", and made the calendrical system *Tenpō-reki* (天保暦), which is the last traditional calendar in Japan.

Surveying of Inō Tadataka (伊能忠敬)

Inō Tadataka (伊能忠敬) (1745 – 1818), a disciple of Takahashi Yoshitoki, surveyed almost all areas of Japan, and made very accurate maps. He utilized astronomical observations also for surveying.

He made "small maps" (3 sheets), "middle maps" (8 sheets), and "large maps" (214 sheets), and some other maps.

The "small maps" of Inō Tadataka:



（From 星埜由尚（監修）『別冊太陽　日本のこころ 261，伊能忠敬』、平凡社, 2018, pp.14-15.）

A detailed map of Edo (present Tokyo) of INŌ Tadataka:

(From 星埜由尚（監修）『別冊太陽　日本のこころ 261，伊能忠敬』、平凡社, 2018, p.97)

(From 星埜由尚（監修）『別冊太陽　日本のこころ 261，伊能忠敬』、平凡社，2018, pp.108-109.)

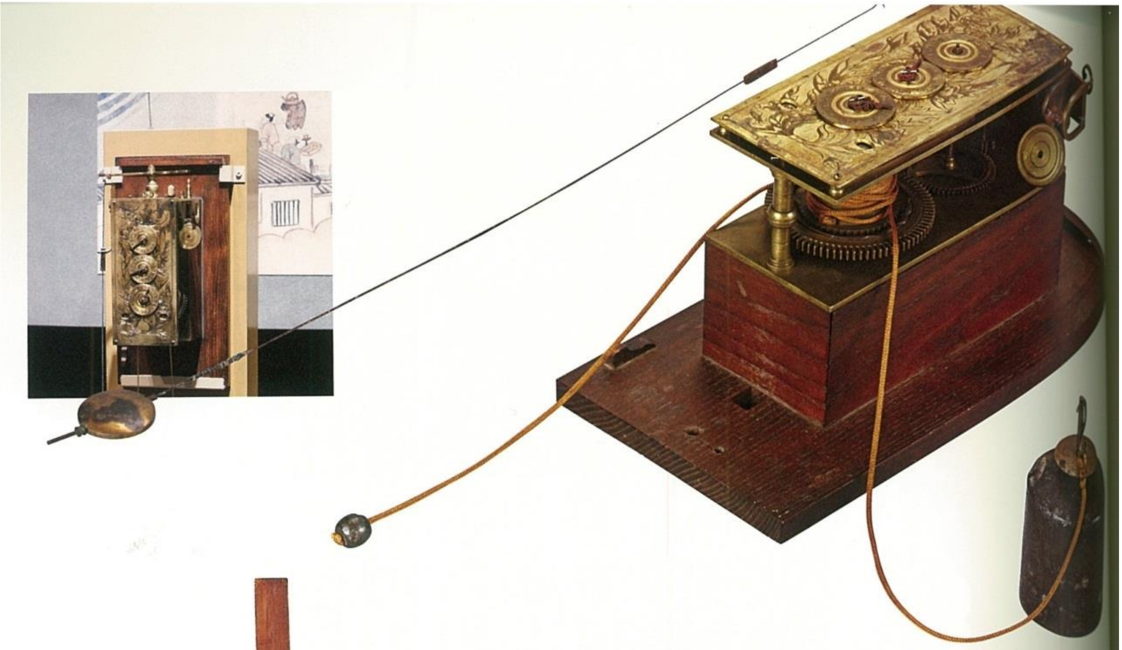
Surveying of Inō Tadataka at night

夜中測量之図
112頁から掲載の《浦島測量之図》（宮尾昌弘蔵・呉市入船山記念館保管）部分
夜、天体観測をしているようす。画面右、緋毛氈に坐し、手には灯り、そして頭上の「子午線儀」を目視している翁が忠敬と思われる。画面中央に据えられているのは「中象限儀」（本誌103頁）。

(From 星埜由尚（監修）『別冊太陽　日本のこころ 261，伊能忠敬』、平凡社, 2018, p116)

At night, the altitude of stars on the meridian (north-south line) was observed.　The time of transit (passage of the meridian) of stars is observed by the meridian instrument, and then the altitude is observed by the quadrant.　From this observation, the latitude is known.　INŌ Tadataka also tried to know longitude from astronomical observation, but it was not successful.

Some instruments:



垂揺球儀
振り子時計。日月蝕や木星の衛星の蝕などの時間を計測し、その数[illegible]経度を求めようとした。左写真は使用時の状態を示すもの。最大寸法[illegible]×14.8×29.1、本体盤面17.5×7.9、重錘径5.7、重錘高11.1、振り子[illegible]58.6cm／国宝／伊能忠敬記念館蔵

Pendulum clock

Middle-sized quadrant

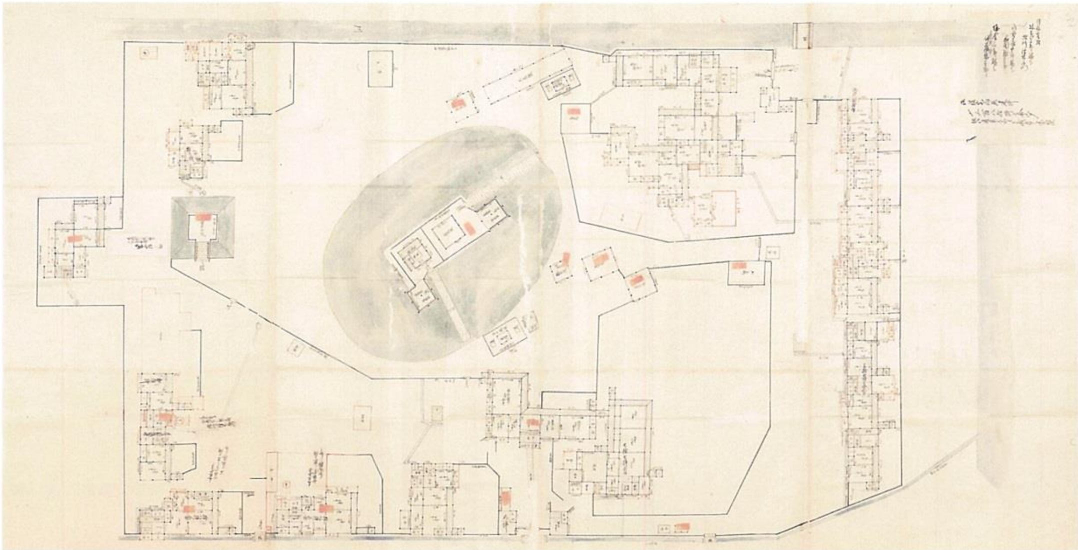
中象限儀
天体観測用の器具。恒星の高度を観測し、その土地の緯度を測定。写真の台は仮のもので、実際の使用にあたっては、もっと大がかりな架台を必要とした。測量地に持ち運び観測しているようすが本誌116頁に。本体長121.3、総高197.2、望遠鏡長120.0、幅5.9cm／国宝／伊能忠敬記念館蔵

[illegible]いる。

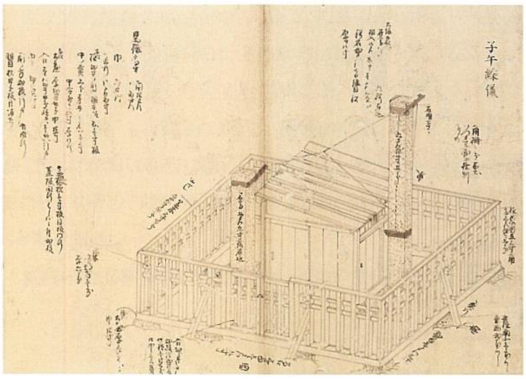
一方、伊能忠敬の測量は、道線法による測線が地図の骨格となっており、測量の原点となるものは設定されなかった。測線の精度は不安定であり、緯度の求められた測点があるとは言え、経度の測定には失敗しており、位置の基準となる三角点は設定されていない。伊能忠敬の測量技術はシーボルト事件もあり、後世に継承されることがなかった。伊能測量と近現代測量とは技術的には断絶があり、伊能測量は、近世における測量技術のひとつの頂点であったといえる。（星埜）

（From 星埜由尚（監修）『別冊太陽　日本のこころ 261, 伊能忠敬』、平凡社, 2018, p103）

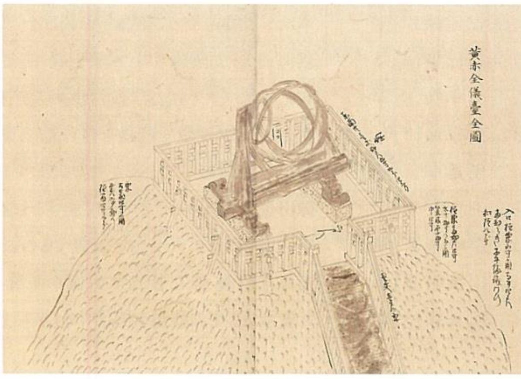
An observatory of Tenmonkata (天文方) in Asakusa (浅草):



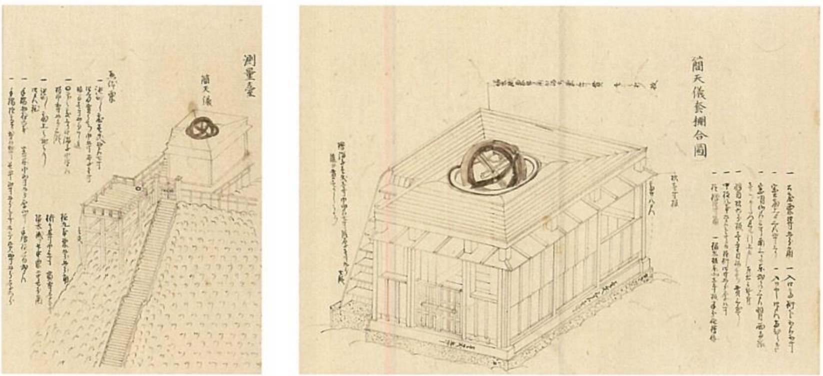
浅草天文台配置図 「順立帳」1868［明治2］年（東京都公文書館蔵、重要文化財）より



浅草天文台の子午線儀
南北に正確に張った細いひもを用いて天体の子午線通過時刻を測る装置。



浅草天文台の黄赤全儀
内側の環の回転軸を差し替えることによって天体の黄道座標と赤道座標の両方を測定可能にした装置。

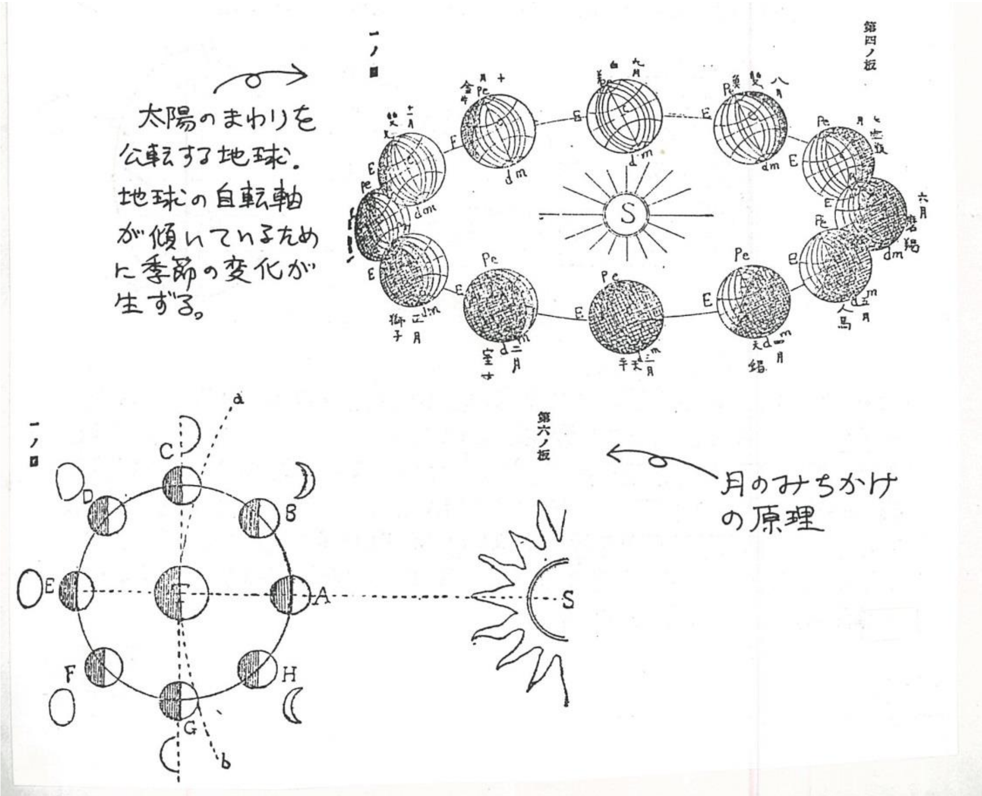


浅草天文台の簡天儀
天文台中央の築山（測量台）の上におかれた主力観測装置。天体の位置を測定するために用いられた。

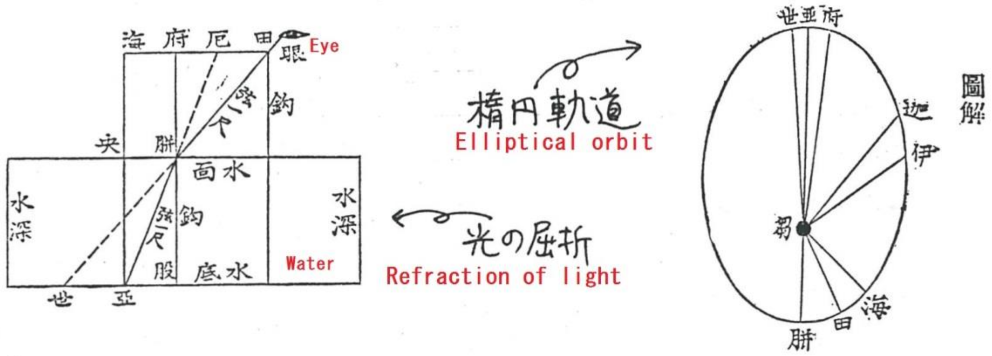
（From 『渋川春海と江戸時代の天文学者たち』、国立科学博物館, 2016, p.11）

Introduction of Western astronomy by civilians

MOTOKI Ryōei (本木良永) (1735 – 1794), a Japanese interpreter of Dutch, translated some Dutch books, in which heliocentric theory is explained. The following is from his book (『星術本原太陽窮理了解新制天地二球用法記』(1792) published in 三枝博音編『日本哲学全書』Vol.8, (1936)).



SHIZUKI Tadao (志筑忠雄) (1760 – 1806) wrote the *Rekishō shinsho* (暦象新書) (1798 – 1802) in which Newtonian mechanics etc. are explained. The following is from this book (published in 『文明源流叢書』Vol.2 (1914)).



Wasan (和算)

--- Japanese traditional mathematics

Introduction of “soroban” (East Asian abacus)

By the end of the 16th century, Chinese abacus, which is called “soroban” in Japanese, was introduced to Japan.

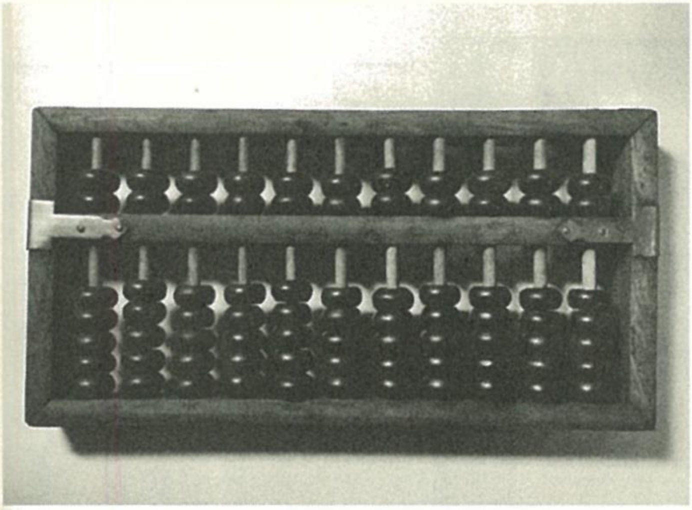


図10 中国ソロバン 日本のソロバンと違って、珠は丸みを帯びている。Chinese abacus ″suanpan″ (算盤)

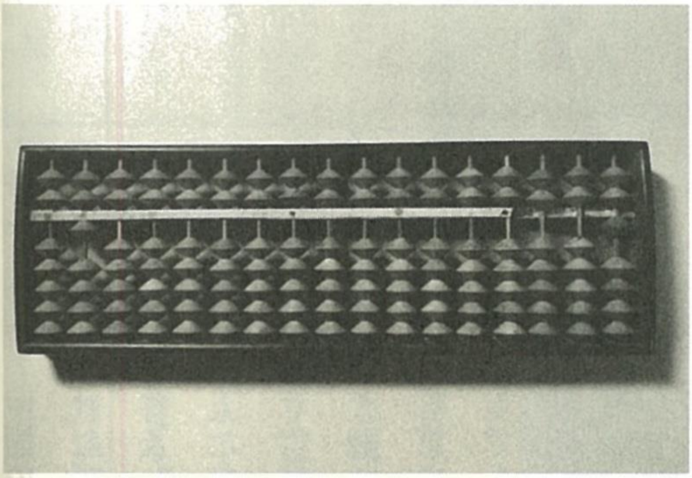


図11 日本のソロバン 速く珠を動かすことができるように改良されている。Japanese abacus ″soroban″

(From Ueno (2017), p.20)

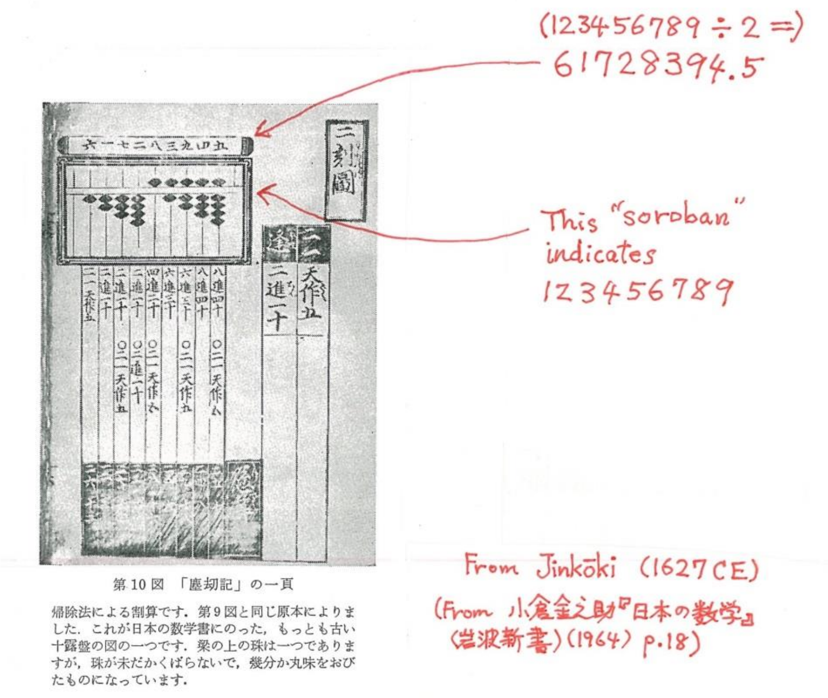
From the early 17th century, several books on mathematics were published.

Probably, the oldest is the *Sanyōki* (算用記), and the second is the *Warizansho* (割算書) (1622 CE) of Mōri Shigeyoshi (毛利重能).

Jinkōki (塵劫記) of Yoshida Mitsuyoshi (吉田光由)

In 1627, Yoshida Mitsuyoshi (吉田光由) (1598 – 1672) published the *Jinkōki* (塵劫記). It became a bestseller, and some revised editions were published by him. Besides the method to use "soroban", several interesting exercises with beautiful illustrations are given in this book.

He was largely influenced by the *Suanfa tongzong* (算法統宗) (1592 CE) of Cheng Dawei (程大位).



Some examples of the exercises in the *Jinkōki*:

35. Estimate the Height of a Tree

The height is 7^{ken} 5.

[a: Chapter 35
b: Estimation of the Height of a Tree with Paper
c: height 7^{ken} and a half]

⟨Process⟩ Take a sheet of paper. First fold it to make a square. Then, fold it again by putting a pair of the diagonal corners together. You will get a shape of a rectangular isosceles. Dangle a stone by a string[49] from the rectangular corner of the folded paper, and hold the paper such that one side of the triangle stands plumb. This preperation being done, move yourself to the place where you can look up at the top of the tree through the hypothenuse.

Now suppose you are standing 7^{ken} from the base of the tree. Add 3^{shaku} of your height to this. Then you know the height of the tree as 7^{ken} 5 [50].

[49] There is a special way in Japan to make a string from a sheet of paper by twisting it.
[50] Here, 1^{ken} is counted as 6^{shaku} .

(From Wasan Institute (2000), p.143 (left) and p.40 of its “Part IV, Facsimile”.)

(The text is 『新編塵劫記』(1641).)

“Nezumizan” (鼠算):

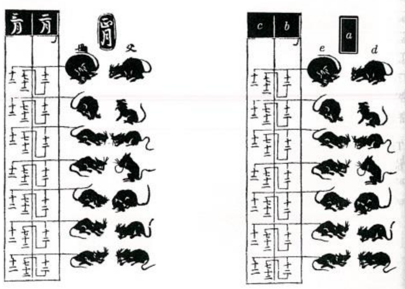
37. Rat’s Family

1. A breeding pair of rats produced 12 baby rats in January. Counting together with the parents, there were 14 rats at that time. Now a pair of baby rats itself grew up and, one month later, it bred 12 baby rats. Then the total number of rats amounted to 98 in February. In this way, once a month, the parents, their children, their grandchildren, their great-grandchildren, and so forth bred 12 children each, and augmented their number.

We now ask how many rats would there be 12 months later.

The number is 27,682,574,402.

⟨Process⟩ The number of outset is 2. Multiply by 7 to the power of 12. Then the total number of rats is what is shown above.



[a: January b: February c: March
d: Father e: Mother]

January	father mother 12 children
February	12 12 12 12 12 12 12
March	72 72 72 72 72 72 72 12 12 12 12 12 12 12
April	Newly born rats 4,116. Counted together with parents 4,802.
May	Newly born rats 28,812. Counted together with parents 33,614.
June	Newly born rats 201,684. Counted together with parents 235,298.
July	Newly born rats 1,411,788. Counted together with parents 1,647,086.
August	Newly born rats 9,882,516. Counted together with parents 11,529,602.
September	Newly born rats 69,177,612. Counted together with parents 80,707,214.
October	Newly born rats 484,243,284. Counted together with parents 564,950,498.
November	Newly born rats 3,389,702,988. Counted together with parents 3,954,653,486.
December	Newly born rats 23,727,920,916. Counted together with parents 27,682,574,402

(From Wasan Institute (2000), pp.146-147)

Development of mathematics using "soroban"

By this time, Imamura Chishō (今村知商) published the *Jugairoku* (竪亥録) (1639), which is a collection of mathematical formulae written in Classical Chinese.

Isomura Yoshinori (磯村吉徳) (d.1710) published the *Sanpōketsugishō* (算法闕疑抄) (1661), which is a high-level work on mathematics using "soroban". There appended 100 very difficult (challenging) problems without key. Isomura Toshinori later published their solution by himself.

And also, Imamura Shigekiyo (今村茂清) (d.1695) calculated the circular constant π as 3.1415926・・・ and published the *Sanso* (算俎) (1663).

Handing down of problems without key (遺題継承)

In 1641, Yoshida Mitsuyoshi (吉田光由) published his last edition of the *Jinkōki* (塵劫記) with some (challenging) problems without key.

In 1653, Enami Tomosumi (榎並和澄) published the Sanryōroku (参両録), in which the solutions of the problems in the *Jinkōki* are given, and new (challenging) problems without key are added. From this time, successive "handing down of (challenging) problems without key" (遺題継承, "idai-keishō") started, which lasted until the time of Seki Takakazu (關孝和). (Seki did not give problems without key.)

The *Sanpōketsugishō* (算法闕疑抄) (1661) of Isomura Yoshinori (磯村吉徳) is also in a line of "idai-keishō".

During this period, mathematics rapidly progressed in Japan. And (challenging) problems without key which are very difficult (or impossible) to solve using "soroban" appeared. So, Chinese method of algebra using counting rods "tianyuan-shu" (天元術) ("tengen-jutsu" in Japanese) was introduced.

Tengen-jutsu (天元術)

“Tengen-jutsu” (天元術) is a Chinese method of algebra using counting rods. The tengen-jutsu was used by Sawaguchi Kazuyuki (澤口一之) in his *Kokonsanpōki* (古今算法記) (1671).

Japanese “tengen-jutsu” is based on the *Suanxue qimeng* (算学啓蒙) (ca.1299 CE) of Zhu Shijie (朱世傑).

Counting rods “sangi” (算木):

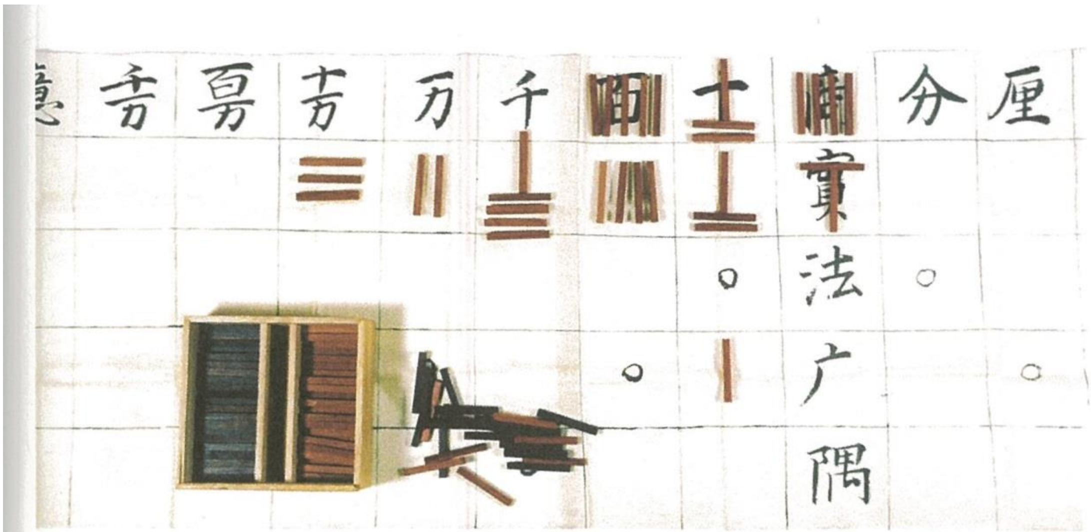
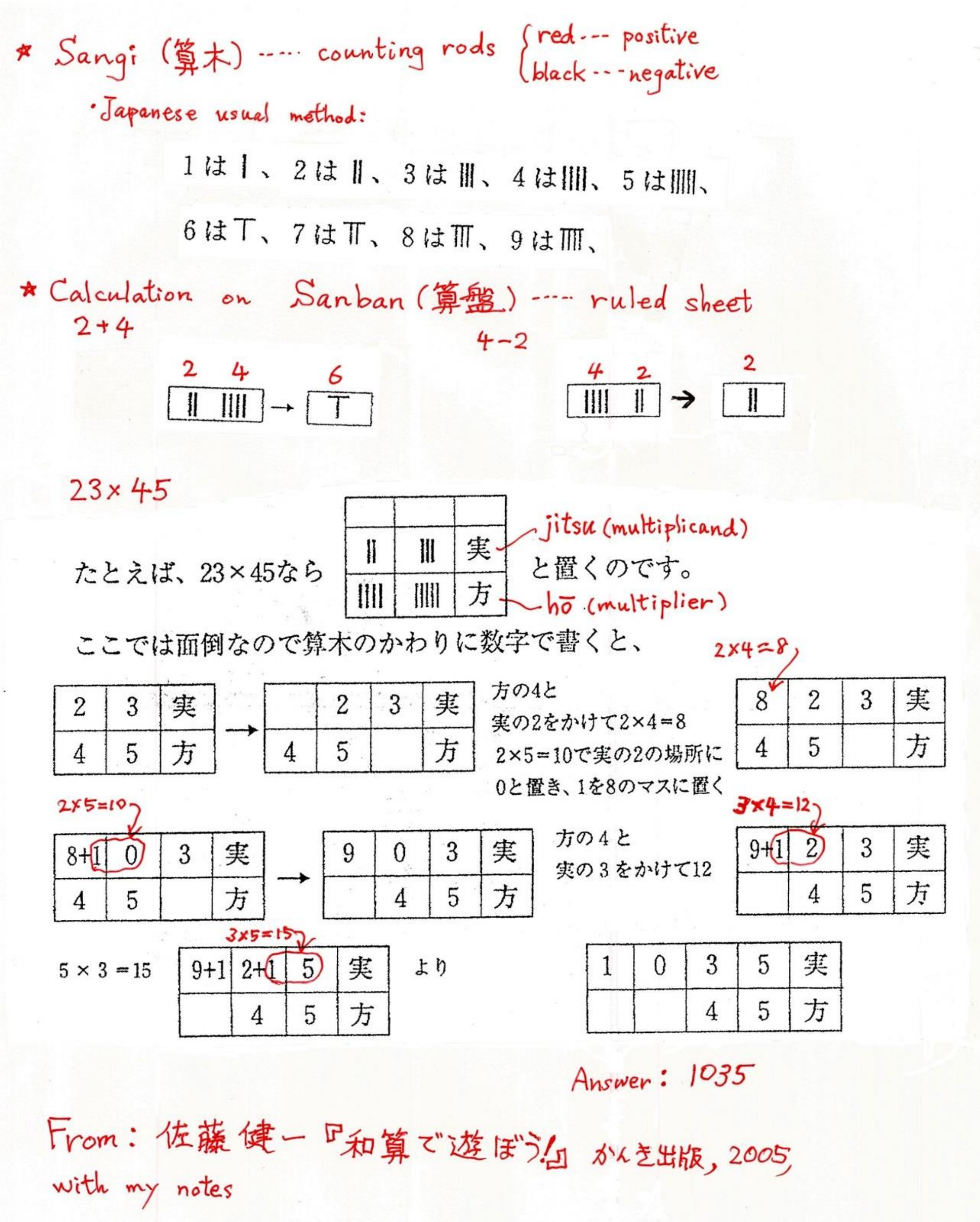


Plate 3. ***Sangi.*** *Sangi*, calculating rods, were placed on a ruled sheet of paper to form numerals and, with a prescribed series of operations, intricate arithmetic calculations could be performed. *Sangi* gradually lost out to the Japanese abacus, the *soroban*, although professional mathematicians used them well into the nineteenth century because they were better suited to complex calculations. (Photo: Fukagawa Hidetoshi.)

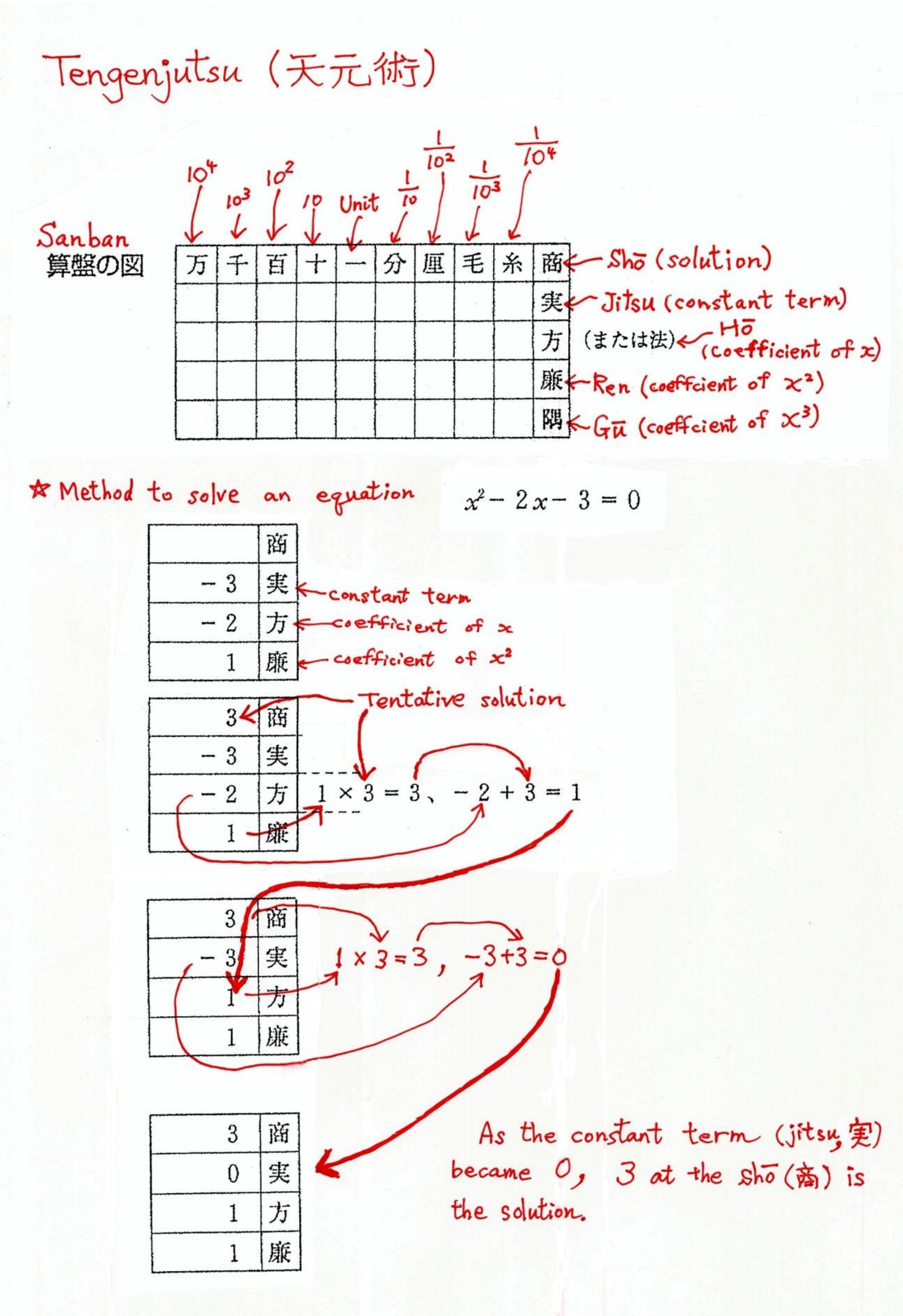
(From Fukagawa and Rothman (2008))

Method to use counting rods:

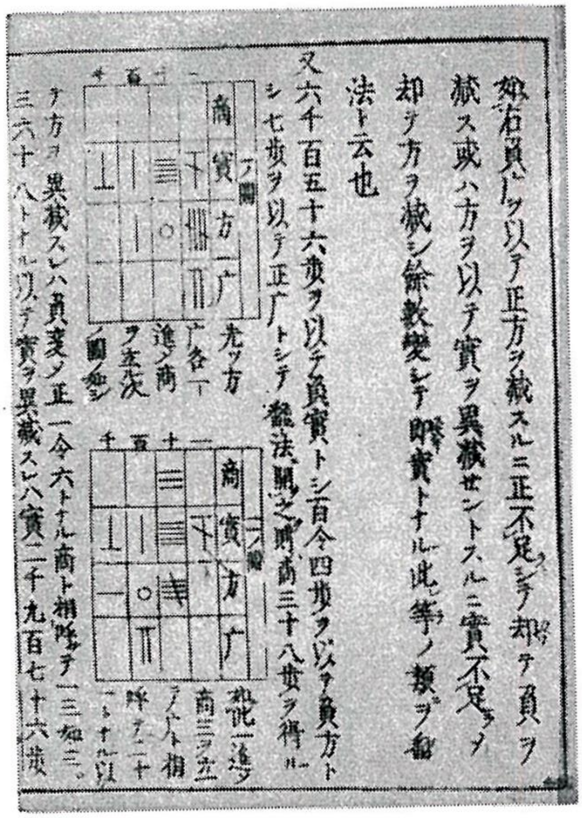


In order to express numbers by counting rods, “vertical form” and “horizontal form” are used formally (see my lecture note on China), but only “vertical form” was usually used in Japan.

An example of solving equation by "tengen-jutsu":



An example of a work on “tengen-jutsu”:



如右負ヲ以テ正方ヲ減スルニ正不足シテ却テ負ヲ減ス或ハ方ヲ以テ實ヲ異減セントスルニ實不足ヲメ却テ方ヲ減シ餘數變シテ贈實トナル此等ノ類ヲ翻法ト云也

又六千百五十六歩ヲ以テ負實トシ百令四歩ヲ以テ負方トシ七歩ヲ以テ正廣トシテ翻法ス開之則商三十八歩ヲ得ル

先ツ方廣各一進ノ商ヲ立次ノ圖ノ如シ

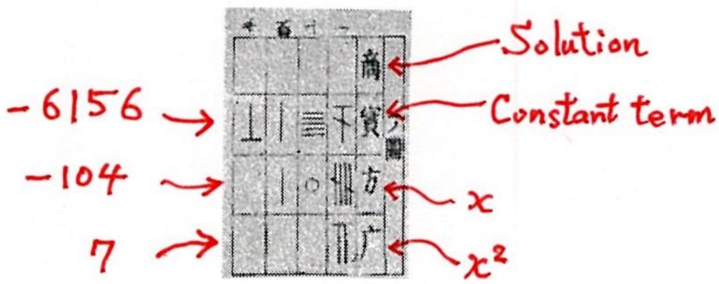
稍此一進ノ商三ヲ立テ廣ト相呼テ二十一トナル

ナ方ヲ異減スレハ負變ノ正一令六トナル商ト相呼テ一三如三三六十八トナル以テ實ヲ異減スレハ實二千九百七十六歩

第 24 図　方程式の解法

天元術による解法で，ホルナーの方法と同じもの．図は $7x^2-104x-6156=0$ を解いて，$x=38$ を得る計算の第一歩を示しています．「算法天元録」(元禄 10 年 1697)の一頁です．

An example of Tengenjutsu
(From the Sanpōtengenroku (1697CE))
(From 小倉 (1964) p.33)



$7x^2-104x-6156=0$

Seki Takakazu (關孝和)

Seki Takakazu (關孝和) solved the problems given in the *Kokonsanpōki* (古今算法記) (1671) of Sawaguchi Kazuyuki (澤口一之), and wrote the *Hatsubisanpō* (發微算法) in 1674.

A problem in the *Kokonsanpōki*:

第 26 図　「古今算法記」の遺題

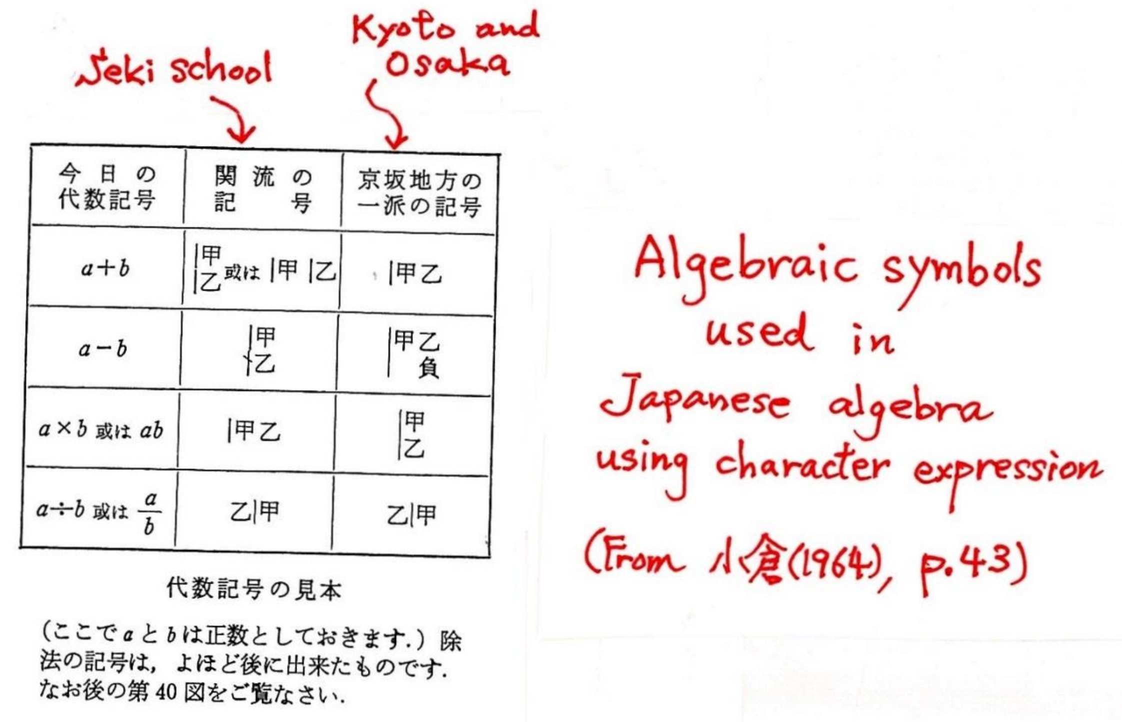
この遺題(すなわち好)の解答が，すなわち関孝和の「発微算法」なので，沢口の提出した問題は，日本数学史の上に，重要な地位を占めるわけです．(なお後の第 34 図をご覧なさい．)
今ここに第一番の問題の意味を説明しておきましょう．

大円の中に，図のように，中円と(二つの)小円があって，互いに切している．大円の内部にあって，中円と小円の外にある三つの(空な)部分の面積は 120 歩(平方寸)である．また小円の直径は中円の直径よりも 5 寸短い．大円，中円，小円の直径はそれぞれ幾何か．

(From Ogura (1964), p.35)

In his *Hatsubisanpō* (發微算法), Seki Takakazu (關孝和) used newly devised algebraic symbols. This method is called "bōsho-hō" (傍書法).

Algebraic symbols used in "wasan":



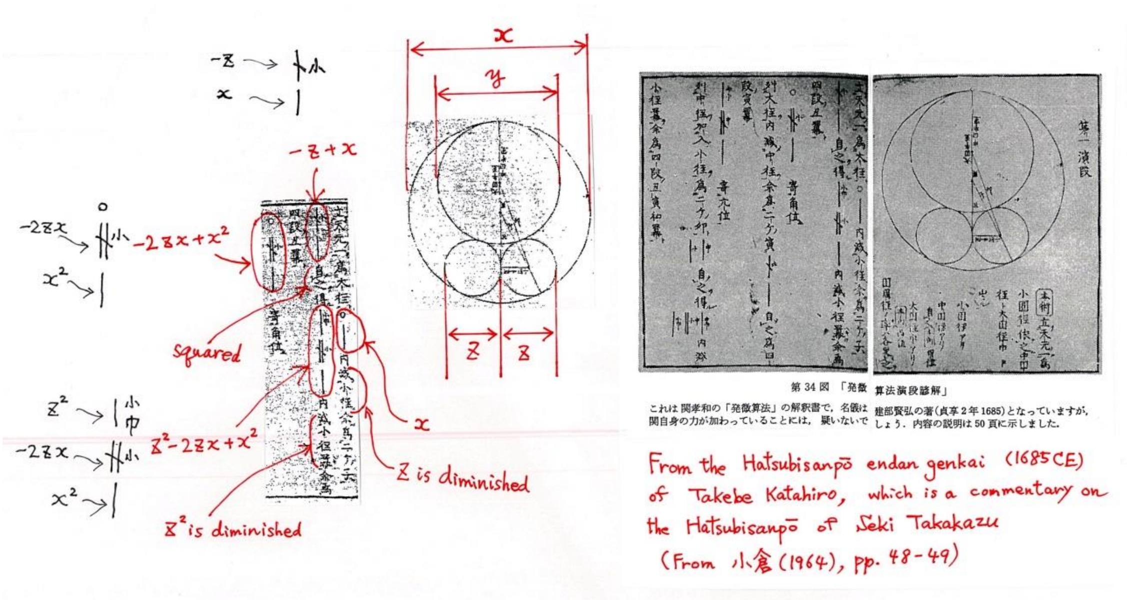
今日の代数記号	関流の記号	京坂地方の一派の記号
$a+b$	|甲 |乙 或は |甲 |乙	|甲乙
$a-b$	|甲 |乙	|甲乙 負
$a\times b$ 或は ab	|甲乙	|甲 |乙
$a\div b$ 或は $\frac{a}{b}$	乙|甲	乙|甲

代数記号の見本

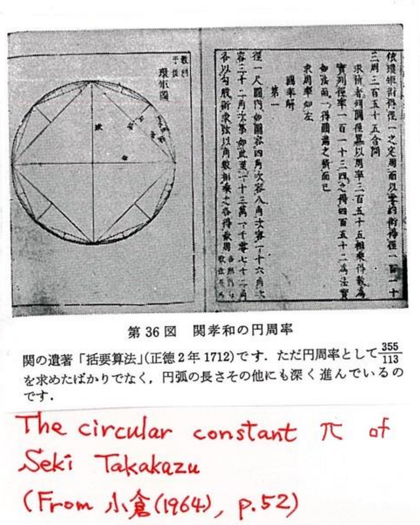
(ここでaとbは正数としておきます.) 除法の記号は，よほど後に出来たものです. なお後の第 40 図をご覧なさい.

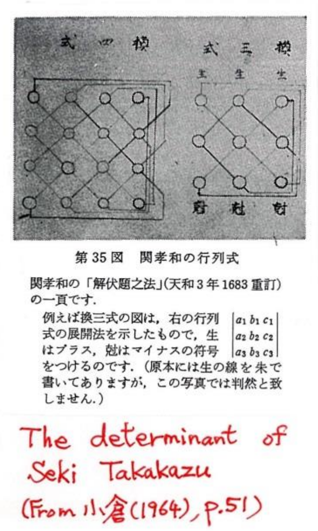
Seki started a new algebra, in which "bōsho-hō" is used, to solve multiple variable equations of higher degree. This new algebra was called "endan-jutsu" (演段術), and was later called "tenzan-jutsu" (點竄術). Seki studied determinants, the circular constant π etc.

Seki's "endan-jutsu":



Seki's determinant and circular constant π :





Seki's mathematics was succeeded by disciples, and the "Seki school" (關流) was created, and mathematics was transmitted from masters to disciples.

Schools of “wasan”

Seki school (關流, Seki-ryū):

One of the highest disciple of Seki was Takebe Katahiro (建部賢弘) (1664 - 1739). His elder brother Kata’akira (賢明) (1661 – 1716) was also a disciple of Seki and great mathematitian. Takebe Katahoro’s disciple Nakane Genkei (中根元圭) (1662 – 1733) was also a great mathematician and astronomer. Takebe Katahoro and Nakane Genkei were advisors of Shōgun Yoshimune (吉宗).

Succeeding Seki’s work, Takebe Katahoro developed the theory concerning circle, which was called “enri” (圓理) later.

Some other mathematicians of Seki school are, Araki Murahide (荒木村英) (1640 – 1718), Matsunaga Yoshisuke (松永良弼) (d.1744) (Araki’s disciple), Kurushima Yoshihiro (久留島義太) (d.1758) (originally a self-educated mathematician and later studied from Nakane); Matsunaga’s disciple Yamaji Nushizumi (山路主住) (1704 – 1773) who established the

system of Seki school (certificate was given for a disciple who completed the course of Seki school), Yamaji's disciples Arima Yoriyuki (有馬頼徸) (1714 – 1783), Ajima Naonobu (安島直圓) (1732 – 1798) and Fujita Sadasuke (藤田貞資) (1734 – 1807); and Ajima's disciple Kusaka Makoto (日下誠) (1764 – 1839); Kusaka's disciples Wada Yasushi (和田寧) (1787 – 1840) and Uchida Itsumi (or Izumi) (内田五観) (1805 – 1882), etc. Mathematics was highly developed in the Seki school.

There were some other schools of Japanese mathematics.

A notable school is "Saijō school" (最上流, Saijō-ryū) founded by Aida Yasuaki (or Yasuakira) (會田安明) (1747 – 1817). There was a controversy between Saijō school and Seki school.

Western mathematics "Yōsan" (洋算)

By the end of the Edo period, western mathematics "Yōsan" (洋算) was becoming popular.

Sangaku (算額)

"Sangaku" are wooden tablets on which mathematical problems (usually with beautiful geometrical figures) dedicated to and displayed in shrines and temples. There were many mathematics lovers, many of whom were common people, and there are many "sangaku" all over Japan.

Examples of "sangaku":

Plate 4. ***Sangaku*** **of the Sozūme shrine**. Dedicated in 1861 by a group of mathematics lovers to the Sozūme shrine of Okayama city, this *sangaku* depicts a teacher sitting before his pupils, who include two women and a child learning to do calculations on the *soroban*. On the right, people are discussing—we presume—how to solve high-degree equations. On the left side of the tablet, three problems are inscribed:

1. Find the side of the square having an area 85,000 square units, in other words, solve the equation $x^2 - 85{,}000 = 0$. (*Answer:* $x = 291{:}5$)
2. Find the diameter $2r$ of circle inscribed in a triangle with sides 10, 17 and 21. (*Answer:* $2r = 7$)
3. Find the side x of a cube having the volume 1,881,676,371,789,154,860,897,069 cubic units, or solve the equation $x^3 - 1{,}881{,}676{,}371{,}789{,}154{,}860{,}897{,}069 = 0$. (*Answer:* $x = 123{,}456{,}789$)

The tablet measures 170 cm by 93 cm. (© Asahi Shinbun.)

(From Fukagawa and Rothman (2008))

Plate 5. ***Sangaku*** **of the Katayamahiko shrine.** One of the most beautiful *sangaku*, this dragon-framed tablet was dedicated by Irie Shinjun in 1873 to the Katayamahiko shrine of Murahisagun Okayama city. We have used Irie's inscription on the tablet as the preface to chapter 5. The *sangaku* measures 162 cm by 88 cm.

(From Fukagawa and Rothman (2008))

"Sangaku" (算額) in Shibuya (渋谷)

There are three "sangaku" in the Konnoh Hachimangu Shrine (金王八幡宮) (3-5-12, Shibuya, Shibuya-ku, Tokyo).

Three "sangaku" exhibited in Konnoh Hachimangu Shrine:

One "sangaku":

The place of Konnoh Hachimangu Shrine:

References

References of the history of Korean science:

Jeon Sang-Woon: *Science and Technology in Korea*, Cambridge, Mas., M.I.T. Press, 1974, (There are Korean version (1966, 1976), and Japanese version (1978).) (Japanese version: 全相雲『韓国科学技術史』、高麗書林、1978).

Jeon: 全相運 (許東粲訳, 宮島一彦, 武田時昌校訂):『韓国科学史 : 技術的伝統の再照明』(in Japanese)、日本評論社, 2005. (Japanese translation of 한국과학사 of 전상운, 2000.)

Jeon Sang-woon (Translated by Robert Carrubba and Lee Sung Kyu): *A History of Korean Science and Technology*, Singapore, NUS Press, 2011. (English translation of 한국과학사 of 전상운, 2000.)

Im: 任正赫『朝鮮古代科学技術史研究 —古朝鮮から高麗時代までの諸問題—』、皓星社, 2014.

Im: 任正赫『朝鮮科学技術史研究 —李朝時代の諸問題—』、皓星社, 2001.

Im: 任正赫『朝鮮近代科学技術史研究 —開化期・植民地期の諸問題—』、皓星社, 2010.

Im: 任正赫『朝鮮科学史における近世』、思文閣出版, 2011.

Im: 任正赫『朝鮮の科学と技術』、明石書店、1993.

Im: 任正赫『朝鮮科学文化史へのアプローチ』、明石書店、1995.

Im: 任正赫『エピソードと遺跡をめぐる朝鮮科学史』、皓星社, 2012.

Kim: 金容雲・金容局『韓国数学史』、槙書店、1978. (There is a Korean version published by Sallim Publishing, 2009.)

Kawahara: 川原秀城『朝鮮数学史』、東京大学出版会、2010.

Needham, Joseph, Lu Gwei-Djen, John H. Combridge, John S, Major: *The Hall of Heavenly Records*, Korean astronomical instruments and clocks 1380 – 1780, Cambridge, Cambridge University Press, 1986. (There is a Korean translation published by Sallim Publishing, 2010.)

----- There are many more references in Korean language.

References of the history of Japanese astronomy:

Nakayama, Shigeru: *A History of Japanese Astronomy*, Cambridge, Mass., Harvard University Press, 1969.

Nakayama: 中山茂『日本の天文学』(岩波新書)、1972.

Hirose: 広瀬秀雄『日本人の天文観』(NHK ブックス)、日本放送出版協会, 1972.

Watanabe: 渡辺敏夫『近世日本天文学史』、全２冊、恒星社厚生閣, 1986 – 1987.

Nakamura: 中村士、『江戸の天文学者星空を翔ける』、技術評論社、2008.

Nakamura: 中村士（監修)、『江戸の天文学』、角川学芸出版、2012.

Kazu: 嘉数次人、『天文学者たちの江戸時代』、ちくま新書、筑摩書房、2016.

Kanō and Kinoshita: 狩野久、木下正史、『飛鳥藤原の都』、岩波書店, 1985.

Izumi: 泉武、『キトラ・高松塚古墳の星宿図』、同成社, 2018.

Nakamura: 中村士『古代の星空を読み解く，キトラ古墳天文図とアジアの星図』、東京大学出版会, 2018.

Hosoi: 細井浩志、『日本史を学ぶための〈古代の暦〉入門』、吉川弘文館、2014.

Hiraoka: 平岡隆二、『南蛮系宇宙論の原典的研究』、花書院、2013.

Watanabe: 渡辺敏夫、『近世日本科学史と麻田剛立』、雄山閣, 1983.

Ôtani: 大谷亮吉、『伊能忠敬』、岩波書店, 1917.

Bessatsu Taiyō: 星埜由尚（監修)『別冊太陽　日本のこころ 261，伊能忠敬』、平凡社, 2018.

----- And many more in Japanese language.

References of the history of Japanese mathematics:

Mikami, Yoshio: *The Development of Mathematics in China and Japan*, originally published at Leipzig in 1913, Second edition: New York, Chelsea Publishing Company, 1974.

Smith, David Eugen and Yoshio Mikami: *A History of Japanese Mathematics*, Chicago, The Open Court Publishing Company, 1914.

Mikami: 三上義夫『文化史上より見たる日本の数学』、創元社, 1974; 岩波文庫, 1999.

Ogura: 小倉金之助『日本の数学』、岩波新書, 1940, 2nd ed.: 1964.

Fujiwara: (藤原松三郎著), 日本学士院編『明治前日本数学史』、全５巻、岩波書店, 1954 – 1960.

Hirayama: 平山諦『和算の歴史』、(originally published in 1961)、ちくま学芸文庫、筑摩書房, 2008.

Hirayama: 平山諦『学術を中心とした和算史上の人々』、(originally published in 1965)、ちくま学芸文庫、筑摩書房, 2007.

Ōya: 大矢真一『和算以前』、中公新書, 1980.

Ōya: 大矢真一『和算入門』、日本評論社, 1987.

Shimodaira: 下平和夫『日本人の数学・和算』、(originally published in 1972)、講談社・学術文庫, 2011.

Shimodaira: 下平和夫『数学書を中心とした和算の歴史』(上)（下)、富士短期大学出版会, 1965 – 1970.

Sato: 佐藤健一『和算を楽しむ』、ちくまプリマー文庫、筑摩書房, 2006.

Ueno: 上野健爾『和算への誘い』、平凡社、2017.

Wasan Institute(和算研究所塵劫記委員会): Jinkōki, (English translation), Wasan Institute (和算研究所), 2000.

Jinkōki iinkai: 塵劫記委員会『塵劫記』(rendered into modern Japanese), 和算研究所, 2000.

Horiuchi, Annick, (Translated from the French by Silke Wimmer-Zaiger): *Japanese Mathematics in the Edo Period (1600 – 1868)*, (A study of the works of Seki Takakazu (? – 1708) and Takebe Katahiro (1664 – 1739), Basel, Birkhäuser, 2010.

Sato: 佐藤賢一『近世日本数学史』、(関孝和の実像を求めて)、東京大学出版会, 2005.

Ueno et al: 上野健爾、小川束、小林龍彦、佐藤賢一『関孝和論序説』、岩波書店, 2008.

Ogawa and Morimoto: 小川束、森本光生『江戸時代の数学最前線』、(和算から見た行列式)、技術評論社, 2014.

Ogawa et al.: 小川束、佐藤賢一、竹之内脩、森本光生『建部賢弘の数学』、共立出版, 2008.

Takenouchi Osamu and Morimoto Mitsuo (tr. and comm.): *Selected Mathematical Works of Takebe Katahiro*, Tokyo, Wasan Institute, c.2004.

Fukagawa Hidetoshi and Tony Rothman: *Sacred Mathematics, Japanese Temple Geometry*, Princeton, Princeton University Press, 2008, (Japanese version: 深川英俊、トニー・ロスマン『聖なる数学：和算』、森北出版, 2010)

----- And many more in Japanese language.

(Note: There are two Satō Ken'ichi who are specialists of "wasan". The elder Sato Kenichi (佐藤健一) is the Chief Director of Wasan Institute, and the younger Sato Ken'ichi (佐藤賢一) is a professor of The University of Electro-Communications. Their Chinese characters of "ken" are different. Be careful not to confuse them.)

Additional readings:

(Japan 1): ÔHASHI, Yukio: "Solar Eclipses and SHIBUKAWA Harumi", in SÔMA and TANIKAWA: *Proceedings of the Second Symposium on "Historical Records and Modern Science"* (第 2 回「歴史的記録と現代科学」研究会集録), Tokyo, Notional Astronomical Observatory of Japan, 2009, pp.31 – 44.

(Japan 2): ÔHASHI: 大橋由紀夫：「日本暦法史への招待 ―宣明暦と貞享暦を中心として―」、『数学史研究』、通巻 185 号, 2005, PP.43 – 63.

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