Guo Shoujing and Shoushi-li

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Introduction

Guō Shǒujìng (郭守敬) (Guō is surname) is a Chinese water conservancy engineer and astronomer in the Yuán (元) dynasty. Guō Shǒujìng (in Pinyin system of Romanization) was spelled "Kuo Shou-Ching" by J. Needham. Guō Sǒujìng's public name was Ruòsī (若思).

Guō Shǒujìng was born in 1231 at Xíngtái (邢台) in Shùndé (順徳) (now in Héběi (河北) province), and died in 1316 AD at Dàdū (大都) (now Beijing).

In his youth, Guō Shǒujìng studied under his grandfather Guō Róng (郭榮) who was versed in Chinese classics, mathematics, and water conservancy, and then under Liú Bǐngzhōng (劉秉忠) (1216-1274 AD) who was well versed in philosophy, geography, astronomy, and astrology. Liú Bǐngzhōng was a friend of Guō Róng. Among Liú Bǐngzhōng's disciples was Wáng Xún (王恂) (1235-1281 or 82 AD) who later made the Shòushí calendar (授時暦) with Guō Shǒujìng.

In 1262 AD, Guō Shǒujìng met Khubilai Khan (Shìzǔ, 世祖) (reign 1260-1294 AD), and was appointed to be a water conservancy engineer. In 1271, Khubilai Khan made the Chinese name "Yuán" (元) of the Mongolian dynasty. In 1276, Khubilai Khan ordered to make a new calendar. At that time, the Revised Dàmíng calendar (重修大明暦) of Zhào Zhīwēi (趙知微) of the previous Jīn (金) dynasty (1115-1234 AD) was still used, but its error had grown up, and more accurate calendar for the new Yuán dynasty (1271-1368 AD) was needed. Although the Yuán dynasty already had a national observatory "Sītiān-tái" (司天臺), a new department for the compilation of a new calendar was established, and Wáng Xún, Guō Shǒujìng etc. took in charge. Wáng Xún was in charge of calculation, and Guō Shǒujìng was in charge of observation. In 1278 (according to the "Shìzǔ-jì" (世祖紀) and "Bǎiguān-zhì" (百官志) in the *Yuán-shǐ* (元史)) or 1279 (according to the "Guō-Shǒujìng-zhuàn" (郭守敬傳) in the *Yuán-shǐ* (元史)), the department was developed into the "Tàishǐ-yuàn" (太史院) (Institute of chronology (and astronomy)). The institute was constructed in Dàdū (大都) (now Beijing), and Wáng Xún was appointed to be its director, and Guō Shǒujìng its deputy director. Their work was supervised by Xǔ Héng (許衡) (1208-1281). In 1280, the Shòushí calendar (授時暦) was established by them, and was officially used since 1281. Shortly after, Xǔ Héng expired in 1281, Wáng Xún expired in 1281 (according to the "Wáng-Xún-zhuàn" (王恂傳) in the *Yuán-shǐ*) or 1282 (according to the "Guō-Shǒujìng-zhuàn" (郭守敬傳) in the *Yuán-shǐ*), and Guō Shǒujìng continued to compile monographs of the Shòushí calendar. In 1283, the *Shòushílìyì* (授時暦議）(Theoretical exposition of the Shòushí calendar) was composed by Lǐ Qiān (李謙) (1223-1302). In 1286, Guō Shǒujìng was appointed to be the director of the Institute of chronology (and astronomy), and completed the monographs of the Shòushí calendar.

Guō Shǒujìng further continued his work of astronomy as well as water conservancy. In 1294 AD, he was appointed to be the "Zhī-tàishǐyuàn-shì" (知太史院事) (Governor of the Institute of chronology (and astronomy)).

Astronomical contribution

(i) Introduction

At the time of Pre-Yuán Mongol (1206-1271 AD) and Yuán (1271-1368 AD) dynasty, huge area was ruled by Mongols, and Islamic astronomy was introduced into China. A Khitan politician and astronomer Yēlǜ Chǔcái (耶律楚材) (1190-1244 AD) was an early contributor to the introduction of Islamic astronomy. And also, seven "Western (Islamic) astronomical instruments" were made in China by a Persian astronomer Jamālud-Dīn in 1267 AD. In 1271, Huíhuí-sītiān-tái (回回司天臺) (Islamic astronomical observatory) was established at Shàngdū (上都) (in Inner Mongolia), and Jamālud-Dīn was appointed to be its director. Guō Shǒujìng (郭守敬) basically followed Chinese traditional astronomy, but he might have received certain influence of the newly introduced Islamic astronomy.

(ii) Observational astronomy

Guō Shǒujìng created 17 new astronomical instruments. Among them, 13 instruments are for the Institute of chronology (and astronomy), and 4 are for traveling observers.

Among the instruments for the institute, the most important ones are the "jiǎnyí" (簡儀) (simplified instrument) and the "gāobiǎo" (高表) (high gnomon) along with the "jǐngfú" (景符) (or possibly pronounced as "yǐngfú") (tally for shadow).

The "jiǎnyí" is a simplified version of the previous complicated armillary sphere, and is a device to observe equatorial system of coordinates. To this instrument is attached a device to observe alt-azimuth system of coordinates of heavenly bodies. The former consists of the equatorial circle and an hour circle, and the latter consists of the horizontal circle and a vertical circle. The original "jiǎnyí" is not extant, but its reproduction made in the 15th century AD is preserved in the Purple Mountain Observatory (紫金山天文臺) in Nánjīng (南京). (See Fig.1.) Although J. Needham supposed that the "jiǎnyí" is a simplified version of the Western "torquetum", I think that it is quite doubtful. The "torquetum" is an instrument to determine ecliptical coordinates, but the "jiǎnyí" is not. Even if Guō Shǒujìng was given a hint by the "torquetum", the "jiǎnyí" is basically a successor of the Chinese traditional armillary sphere, where the equatorial system of coordinates is the base and the ecliptic is only additional.

Fig.1. The "jianyi" designed by Guo Shoujin (Reconstructed at the time of Ming dynasty (now in Purple Mountain Observatory, Nanjing))

The "gāobiǎo" is a high gnomon. The gnomon was used in China since ancient period to observe the sun's midday shadow, and determine the winter solstice, which is the fundamental point of time in Chinese classical calendars. Guō Shǒujìng improved it and made it five times higher than the previous traditional gnomons. A huge gnomon constructed by Guō Shǒujìng etc. exists in Gàochéng, Dēngfēng (登封) city, Hénán (河南) province, and is called Guānxīng-tái (觀星臺) (Observatory to watch stars). (See Fig.2.)

Fig.2. The "gaobiao" in the Guanxingtai (觀星台) Dengfeng (登封) in Henan (河南) province

The main difficulty of the observation of gnomon-shadow is that the sun is not a point source, and the shadow's penumbra produces ambiguity of shadow-length. Guō Shǒujìng overcame this difficulty by using "jǐngfú", which is a kind of pinhole camera. The image of the sun is projected through a pinhole, and the pinhole is adjusted so that the shadow of the horizontal bar which is in the window at the top of the gnomon-wall exactly passes through the centre of the image of the sun. Then, the position of the shadow of the bar indicates the length of the exact gnomon-shadow when the height of the bar is considered to be the height of gnomon. (See Fig.3.)

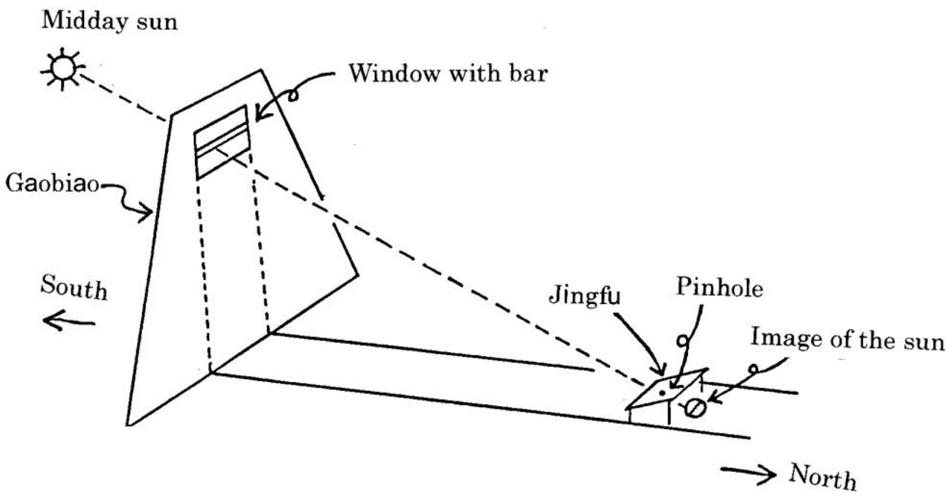

Fig.3. The principle of "jingfu"

Guō Shǒujìng and his colleagues observed the gnomon-shadow by the "gāobiǎo" around the winter and summer solstices, and determined the time of solstices by the method devised by Zǔ Chōngzhī (祖沖之) (429-500 AD). This determination led them to use the fairly accurate length of a tropical year 365.2425 days in the Shòushí calendar. Actually, this value had already been used in the Tǒngtiān calendar (統天暦) (1198 AD) of Yáng Zhōngfǔ (楊忠輔), and it was again confirmed by Guō Shǒujìng.

The method devised by Zǔ Chōngzhī (郭守敬) is as follows. (See Fig.4.)

For the determination of the time of winter solstice, three observations (A, B, and C in the figure) of the midday gnomon-shadow (a, b, and c) are used. Here, $b > a > c$, and the period BC is one day. In ancient China, one day was divided into 100 *"ke"* (刻). Let an imaginary gnomon-length at D (between B and C) be equal to a. The point E is the midpoint of AB, and F the midpoint of AD, that is the time of winter solstice. Then, EF is a half of BD. Now, by linear interpolation:

$$BD = \frac{100 \times (b-a)}{b-c} \; ke, \quad \text{hence } EF = \frac{100 \times (b-a)}{2 \times (b-c)} \; ke.$$

As the time E is already known, the time F of winter solstice is obtained from this equation.

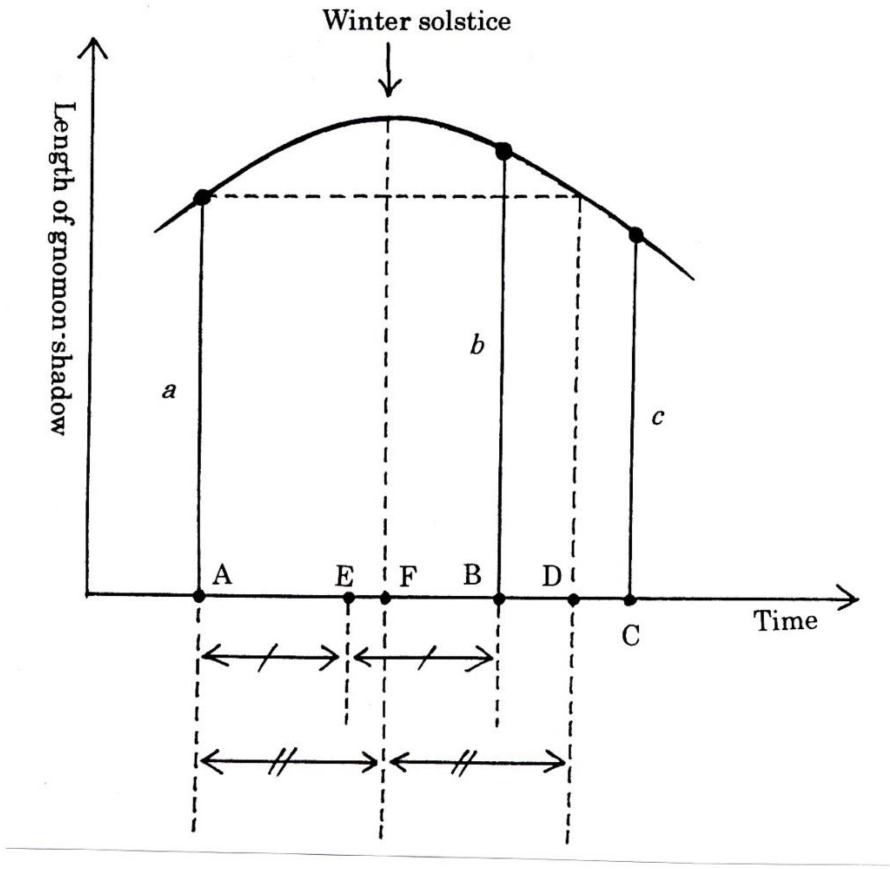

Fig.4. Zu Chongzhi's method

Guō Shǒujìng and his colleagues also determined the point of the winter solstice on the celestial sphere, the time when the moon passes its perigee, the time when the moon passes its nodes, the right ascensional distances of lunar mansions, the time of sunrise and sunset at Dàdū (now Beijing), etc.

They also conducted astronomical observations at 27 different places, and observed the altitude of the celestial North Pole, the length of gnomon-shadow at solstices, the length of daytime and nighttime, etc.

Another famous observation is that of the obliquity of ecliptic. Guō Shǒujìng's value was quoted by Pierre-Simon Laplace (1749-1827) in his *L'exposition du système du monde* (1796) in order to show that the obliquity of ecliptic is diminishing.

(iii) Theoretical astronomy

Guō Shǒujìng (郭守敬) and his colleagues compiled the Shòushí calendar (授時暦) (1280 AD), which is the finest Chinese inherent calendar. They incorporated several superior devices of their predecessors. Besides the accurate astronomical constants, there are some significance in this calendar as follows.

Almost all Chinese classical calendars used grand epoch when the sun, moon, and planets are assumed to be in conjunction. One exception is the Fútiān calendar (符天暦) of Cáo Shìwěi (曹士蔿), a privately made calendar in the 8th century, which did not use grand epoch. The Shòushí calendar also abandoned the artificial grand epoch, and used contemporary epoch with certain initial condition obtained by observations.

Almost all Chinese classical calendars used fraction with different denominators. One exception is the Fútiān calendar (8th century) which used 10,000 as the denominator. The Shòushí calendar also used 10,000 as the denominator. Although it was not the first calendar to use this denominator, it was certainly one step to approach decimal fraction.

The Shòushí calendar succeeded the method of the Tǒngtiān calendar (1198 AD) of Yáng Zhōngfǔ that the length of a tropical year gradually diminishes. It is true that the length diminishes, but the values of the Tǒngtiān calendar and the Shòushí calendar are too large. This idea that the length diminishes was abandoned in 1385 AD in the Dàtǒng calendar (大統暦) (1368 AD) of the Míng (明) dynasty (1368-1644 AD) which almost completely followed the Shòushí calendar otherwise.

The Shòushí calendar also used some new mathematical devices, such as the third order interpolation, and a mathematical method to transform spherical coordinates, where the method devised by Shěn Kuò (沈括) (1031-1095 AD), an encyclopaedic scientist in the Northern Sòng (北宋) dynasty, was used.

(iv) The Shòushí calendar and foreign lands

Although the Shòushí calendar is basically made in Chinese traditional style, one possibility of Indian and Islamic astronomies' influence was recently pointed out by Qū Ānjīng (曲安京). All Chinese calendars before Shòushí calendar used numerical methods to calculate the time of contacts of eclipses, but the Shòushí calendar used geometrical model to calculate it, which is similar to Indian and Islamic methods which had been introduced into China. This topic should be studied further.

The Shòushí calendar was introduced into Vietnam and Korea, and used there. It was not officially used in Japan, but was well studied in the early Edo period in the 17th century, particularly by Shibukawa Harumi (渋川春海), Seki Takakazu (関孝和), Takebe Katahiro (建部賢弘) etc., and played an important role for the development of calendrical astronomy in Japan.

References and historiography

(i) Guō Shǒujìng and his colleagues' work

The Shòushí calendar is recorded in the chapter of calendar in the *Yuán-shǐ* (Official history of the Yuán dynasty)(1370 AD)(Vols.52-55) edited by Sòng Lián (1310-1381) et.al., and also in the *Xīn-Yuán-shǐ* (New official history of the Yuán dynasty)(1920 AD)(Vols.36-40) of Kē Shàomín (1850-1932).

(ii) Classical biographies and related sources

There is a memorial writing for Guō Shǒujìng written by Qí Lǚqiān (d.1329 AD), a successor of Guō Shǒujìng, the "Zhī-tàishǐyuàn-shì Guō-gōng xíngzhuàng" (Memorial writing for the governor of the Institute of chronology (and astronomy) Mr. Guō), which is included in the *Yuán-wén-lèi* (Collected works of the Yuán dynasty) (Vol.50) edited by Sū Tiānjué (included in the *Wényuāngé Sìkù-quánshū* (Imperial collection of Chinese classics compiled in the 18th century), reprinted by Taiwan Commercial Press (1983-86), Vol.1367, pp.647-655).

There are also inscriptions for Guō Shōujìng's instruments written by Yáo Suì and Yáng Huán, which are recorded in the *Yuán-wén-lèi* (Vol.17) (included in the *Wényuāngé Sìkù-quánshū* (Imperial collection of Chinese classics compiled in the 18th century), reprinted by Taiwan Commercial Press (1983-86), Vol.1367, pp.207-211).

The official biography of Guō Shǒujìng is included in the *Yuán-shǐ* (Official history of the Yuán dynasty)(1370 AD)(Vol.164), and also in the *Xīn-Yuán-shǐ* (New official history of the Yuán dynasty)(1920 AD)(Vol.171).

Some classical accounts of Guō Shǒujìng are collected in the *Chóurén-zhuàn* (Biographies of astronomers)(1799)(Vol.25) of Ruǎn Yuán (1764-1846) (republished by Shìjiè-shūjú, Taipei, Taiwan, 1962).

(iii) Modern biographies

There is a detailed biography of Guō Shǒujìng written in English as follows.

Chan, H.L. and P.Y. Ho : "Kuo Shou-ching (1231-1316)", in Rachewitz, Igor de et.al. (eds.): *In the Service of the Khan*, Asiatische Forschungen Band 121, Harrassowitz Verlag, Wiesbaden, Germany, 1993, pp.282-335.

The followings are some biographies of Guō Shǒujìng written in Chinese.

Lǐ Dí : *Guō Shǒujìng* (in Chinese), Shànghǎi-rénmín-chūbǎnshè (People's publishing house of Shanghai), Shanghai, 1966.

Pān Nài and Xiàng Yīng : *Guō Shǒujìng* (in Chinese), Shànghǎi-rénmín-chūbǎnshè (People's

publishing house of Shanghai), Shanghai, 1980

Chén Měidōng : "Guō Shǒujìng", in Dù Shírán (ed.): *Zhōngguó gǔdài kēxué-jiā zhuànjì* (Biographies of scientists in ancient China, in Chinese), Vol.2, Kēxué-chūbǎnshè (Science publishing house), Beijing, China, 1993, pp.667-681.

Chén Měidōng : "Guō Shǒujìng", in Jīn Qiūpéng (ed.): *Zhōngguó kēxué jìshù shǐ, Rénwù-juàn* (A History of Science and Technology in China, Biographical Volume, in Chinese), Kēxué-chūbǎnshè (Science publishing houde), Beijing, China, 1998, pp.464-483.

(iv) Other references

There is a paper on the spherical astronomy of Guō Shǒujìng written in French as follows.

Gauchet, L.: "Note sur la trigonométrie sphérique de Kouo Cheou-king", *T'oung Pao*, **18**, 1917, 151-174.

For the social background of the Shòushí calendar, the following book written in Japanese may be consulted.

Yamada Keiji : *Juji-reki no michi* (Way of the Shǒushí calendar), Misuzu Shobo, Tokyo, Japan, 1980.

The followings are informative works on the Chinese classical calendars, including the Shǒushí calendar.

Qū Ānjīng, Jǐ Zhìgāng, and Wáng Róngbīn : *Zhōngguó gǔdài shùlǐ tiānwénxué tànxī* (Researches on the mathematical astronomy in ancient China, in Chinese), Xīběi-dàxué-chūbǎnshè (Northwest University Press), Xī'ān, China, 1994.

Chén Měidōng : *Gǔlì xīntàn* (New researches on the old calendars, in Chinese), Liáoníng-jiaòyù-chūbǎnshè (Educational publishing house of Liáoníng), Shěnyáng, 1995.

For the possible influence of Indian and Islamic astronomy on the Shòushí calendar, see the following paper.

Qū Ānjīng : "Zhōngguó gǔdài lìfǎ yǔ Yìndù Ālābó-de guānxì --- yǐ rìyuèshí qǐqì suànfǎ wéilì" (A Comparison Study of the Models of Eclipse Phase among Chinese, Indian and Islamic Astronomy, in Chinese), *Zìrán biànzhèngfǎ tōngxùn (Journal of Dialectics of Nature)*, **22**(3), 2000, 58-68. (Its Japanese version translated by me is in the *Sūgakushi Kenkyū* (Journal of History of Mathematics, Japan), No.164, 2000, 1-25.)

For Chinese astronomy in general, including the contribution of Guō Shǒujìng, the following work may be consulted.

Needham, Joseph (with the collaboration of Wang Ling): *Science and Civilisation in China*, Volume 3, Mathematics and the Sciences of the Heavens and the Earth, Cambridge University Press, London, UK, 1959.