Planetary theory of the Indian Classical Astronomy

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I. Introduction

The history of Indian astronomy can roughly be summarised as follows. [For an overview of Indian astronomy, see Ôhashi (1998) in Japanese or more detailed Ôhashi (2009) in English.]

(i) Indus valley civilisation period.
(ii) Vedic period. (ca.1500 BC – ca.500 BC).
(iii) *Vedāṅga* astronomy period. (From sometime between the 6th and 4th centuries BC up to sometime between the 2nd and 5th centuries AD?).
(iv) Period of the introduction of Greek astrology and astronomy.
(v) Classical Siddhānta period (Classical Hindu astronomy period). (From the end of the 5th century up to the 12th century AD).
(vi) Coexistent period of the Hindu astronomy and Islamic astronomy. (From the 13/14th century up to the 18/19th century AD).
(vii) Modern period (Coexistent period of the modern astronomy and traditional astronomy). (From the 18/19th century onwards).

This paper is a continuation of my paper presented at the 7th International Conference on Oriental Astronomy (Tokyo, 2010) [See Ôhashi (2011).], in which the *Vedāṅga* astronomy was mainly discussed. In this paper, I would like to discuss some topics in the Classical Hindu astronomy period. In this period, geometrical models were used for planetary orbits, but Indian models are somewhat different from Greek models. I shall discuss some special features of the Classical Hindu astronomy.

II. Classical Hindu Astronomy

(II.1) Classical Hindu Astronomy period

In the 2nd (?) or 3rd century AD, Greek horoscopic astrology was introduced into India, and around the 4th (?) century AD, Greek mathematical astronomy seems to have been

introduced into India. In the Classical Hindu Astronomy period (from the end of the 5th century to the 12th century AD), Indian astronomy did not receive apparent foreign influence, and developed individually. In the Classical Hindu Astronomy, the eccentric model and the epicyclic model have been used, but they are somewhat different from Greek models.

The Classical Hindu Astronomy period produced several famous astronomers, such as, Āryabhaṭa (b.476 AD), Varāhamihira (6th century AD), Bhāskara I (fl.629 AD), Brahmagupta (b.598 AD), Lalla (ca.8th or 9th century AD), Vaṭeśvara (b.880 AD), Mañjula (fl.932 AD), Śrīpati (fl.1039/1056 AD), Bhāskara II (b.1114 AD) etc. And also the anonymous *Sūrya-siddhānta* (ca.10th or 11th century AD) is a very popular Sanskrit astronomical text of this period. Some of these works are still considered to be authoritative by modern traditional Hindu calendar makers etc. The period during which these classical astronomical works were composed can be called Classical Siddhānta period or Classical Hindu Astronomy period. The "Siddhānta" is the fundamental treatise of mathematical astronomy in Sanskrit.

(II.2) Planetary models in the Classical Hindu Astronomy

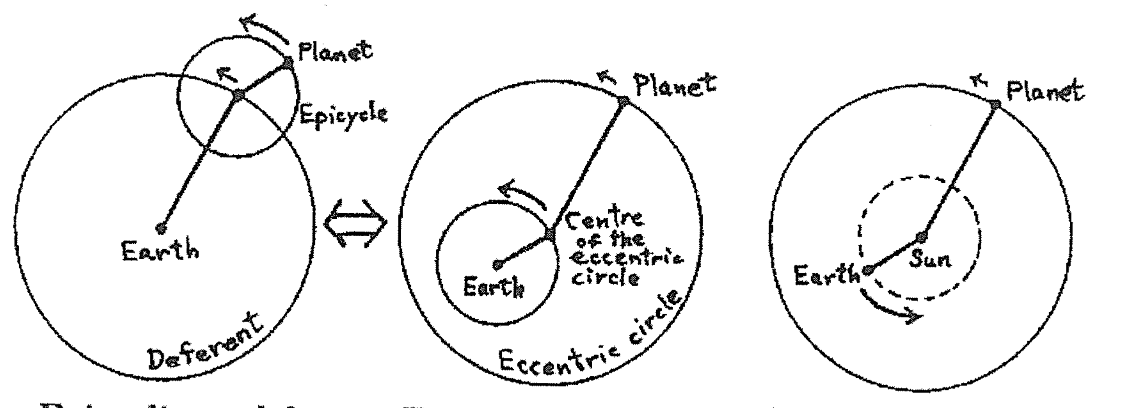
(II.2.1) The *manda*-correction and the *śīghra*-correction

In the Classical Hindu Astronomy, geocentric epicyclic and eccentric systems are used. The *Mahābhāskarīya* of Bhāskara I (7th century) treats the epicyclic and eccentric systems as mathematically equivalent models for both of the *manda*-correction and the *śīghra*-correction. [For its edited text with English translation, see Shukla (1960).]

Firstly, mean (*madhya*) planet, which is supposed to rotate constantly around the earth, is calculated, and then, corrections are applied to the mean planet in order to obtain the true (*sphuṭa*) planet. One correction is the *manda*-correction, which corresponds to our equation of centre. The other is the *śīghra*-correction, which corresponds to the annual parallax in the case of outer planets, and the planet's own revolution in the case of inner planets. Firstly, the *manda*-correction is applied to the mean planet, which corresponds to the planet's own mean revolution in the case of outer planets, and the sun's mean revolution in the case of inner planets. The result is called "*manda-sphuṭa* planet", which is the mean planet corrected by the equation of centre only. Then, the *śīghra*-correction is applied to the "*manda-sphuṭa* planet", and the true planet is obtained. In the actual calculation, some special methods are used in the classical texts, some of which will be discussed below.

We know that a Greek astronomer Apollonius (3rd century BC) has shown the equivalence of the epiciclic model and eccentric model in order to explain the planetary motion. (See the Almagest (XII.1) of Ptolemy. [For its English translation, see Toomer (1998).]) In India also, they are treated to be equivalent. (See Fig.1.) The possibility of the relationship between Apollonius and Indian astronomy is open to the future research.

It may be noted here that the "equant model" of Ptolemy was not used in India. If the "equant model" is used, it will become inconvenient to use epicyclic model and eccentric model similarly. [The "equant model" is that a heavenly body does not move along its eccentric orbit in uniform speed, but moves on its eccentric orbit around a point called "equant" with a constant angular velocity. It is a good approximation of the Kepler motion, when the earth and the "equan" correspond to the foci.] In the Classical Hindu Astronomy, a heavenly body basically moves along its orbit in uniform speed, and both the epicyclic model and the eccentric model could be used freely.



Epicyclic model　Eccentric model　(Heliocentric model)

Fig.1

(II.2.2) The size of the epicycles in the *Āryabhaṭīya*

One interesting feature of the *Āryabhaṭīya* (AD499) of Āryabhaṭa is that the size of the epicycles of the planets changes in different anomalistic quadrants. [For its edited text with English translation, see Shukla and Sarma (1976).] This is quite different from simple geometrical model. The modern *Sūrya-siddhānta* (ca. 10 ~ 11th century AD) etc. also use similar method. [For its English translation, see Burgess (1935).]

According to the *Āryabhaṭīya*, the *manda*-epicycles of the Mars, Jupiter and Saturn are small in the 1st and 3rd quadrants, and are large in the 2nd and 4th quadrants. The *manda*-epicycles of the Mercury and Venus, and the *śīghra*-epicycles of the five planets are large in the 1st and 3rd quadrants, and are small in the 2nd and 4th quadrants.

According to the interpretation of Bhāskara I, their size given in the *Āryabhaṭīya* is the value at the beginning of each quadrant, and the size changes linearly. However, there are other Hindu astronomers who interpret that the size is the value at the end of

each quadrant.

In the case of the *manda*-epicycles of the Mars, Jupiter and Saturn, if the interpretation of Bhāskara I is correct, it can be said that the change of the distance of the planets becomes somewhat similar to that of the "equant model". However, the change of the *śīghra*-epicycles is not understandable. Further researches are necessary.

In the Greek models, which is purely geometrical, the size of epicycle does not change, and when apparent size of an epicycle had to be changed, the distance of the epicycle in the same size was changed by certain mechanical model. On the contrary, Indian models seem to be based on certain natural philosophy, in which certain power influences heavenly bodies and produces certain physical effects. In India, the size of epicycles themselves could be changed. Therefore, Indian models should not be understood as simple geometrical models. The Indian natural philosophy behind Indian models should be investigated further.

(II.2.3) Indian method of the *manda*-correction

The *manda*-correction is not based on a simple eccentric model, but a special modification is applied. Its result is that the equation of centre in this method becomes a simple sine function of anomaly. Its accuracy is slightly less than the simple eccentric model, but their errors (in the opposite direction) are not so different.

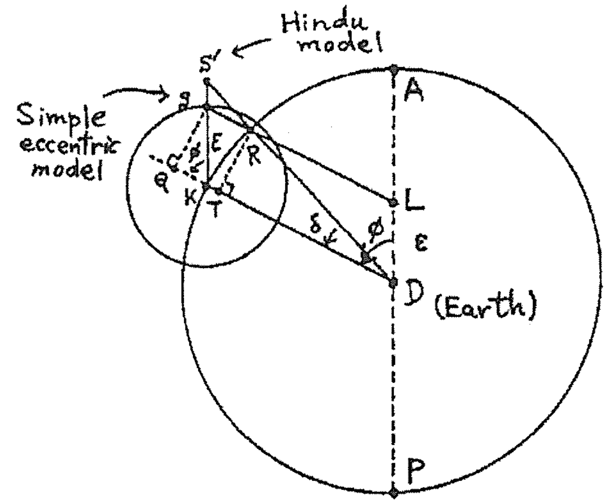


Fig.2

Equation of centre in the Classical Hindu Astronomy

In Fig.2, the point S corresponds to the heavenly body in the simple eccentric model (In the figure, SK//AP, where A corresponds to the apogee, and P corresponds to the perigee.). In the figure, S[1] corresponds to the heavenly body in the Hindu model. Here, KS = DL = $\varepsilon = 2e$ (e is eccentricity in modern sense), SL//KD, and the point S[1] is at the cross point of the straight lines KS and DR. Now, RT = SQ = $\varepsilon \cdot \sin\varphi$. Therefore, $\delta \approx \sin\delta$ = RT = $\varepsilon \cdot \sin\varphi$. So, we can say that the Hindu equation of centre is a simple sine function of anomaly. This Hindu method is well explained in the *Mahābhāskarīya* (IV) of Bhāskara I (7th century).

As I have shown in the Appendix 1, the equation of centre of the simple eccentric model and that of the true Kepler motion are:

$$\theta \approx 2e\sin\varphi + 2e^2\sin 2\varphi \quad \text{(simple eccentric model)}$$

$$\theta \approx 2e\sin\varphi + \frac{5}{4}e^2\sin 2\varphi \quad \text{(true Kepler motion)}$$

$$\theta \approx 2e\sin\varphi \quad \text{(Indian method)}$$

It is clear from the above equations that the second term of the simple eccentric model is too large. The accuracy of the Indian method is not so different from that of the simple eccentric model (in the opposite direction).

This Indian method is quite different from Greek method. The Ptolemaic "equant model" (which was used for the deferents of planets in his theory), which is closer to the true Kepler motion, has not been used in the Classical Hindu astronomy. The possibility of the relationship between Pre-Ptolemaic Greek Astronomy and Classical Hindu Astronomy is still open to the future research. Besides the apparent movement of planets, the distance of planets was considered in India in their own way. The Indian method is more physical or philosophical than Greek pure geometrical method.

(II.2.4) The method in the *Brāhma-sphuṭa-siddhānta*

The method in the *Brāhma-sphuṭa-siddhānta* (AD 628) of Brahmagupta is very interesting. [For its Sanskrit text, see Dvivedī (1902).] Its process, except for the Mars, is as follows. Firstly, the amount of the *manda*-correction is calculated from the mean planet, and is applied to the mean planet. The result is the *manda-sphuṭa* planet. Secondly, the amount of the *śīghra*-correction is calculated from the *manda-sphuṭa* planet, and is applied to the *manda-sphuṭa* planet. The result is the true planet after the first approximation. From the result, the amount of the *manda*-correction is calculated, and applied to the original mean planet. From this result, the amount of the *śīghra*-correction is calculated and applied. The result is the true planet after the second approximation. This process is repeated until a constant value is obtained.

In the case of the Mars, the above mentioned process is not used. The method for the Mars is as follows. Firstly, a half of the amount of the *manda*-correction and a half of the *śīghra*-correction are applied, and the once corrected Mars is obtained. From the result, the amount of the *manda*-correction is calculated, and its whole amount is

applied to the original mean Mars. From this result, the amount of the *śīghra*-correction is calculated, and its whole amount is applied. The result is the true Mars. This method is the ordinary method used by other Indian astronomers in those days for five planets.

It may be mentioned here that Brahmagupta did not use the above mentioned successive approximation for the calculation of true planets in his *Khaṇḍa-khādyaka* (AD 665), but used the ordinary method, like the case of Mars in his earlier work, for five planets. [For its edited text with English translation, see Chatterjee (1970).]

The above mentioned process of the *Brāhma-sphuṭa-siddhānta*, except for the case of the Mars, means that the amount of the equation of centre is a function of the position of true planet, and not of the position of mean planet. This fact shows that the Indian model of planetary motion is not a simple imitation of the Greek geometrical model, and further investigation of the Indian way of thinking is needed. At present, I suspect that Indian astronomers in those days considered that the inequality of the *manda*-correction is produced by a kind of physical force originated to the apogee, and this force is equilibrated with the displacement of planetary position due to the inequality. If so, the amount of the inequality should be a function of the actual position of the true planet (instead of the mean position as in the Greek model). Ancient Greek astronomers tried to make planetary models by geometrical combinations of circles, due to the Greek (Platonic and Aristotelian) natural philosophy. However, ancient Indian astronomers seem to have been free from that kind of preconception. So, I suppose that ancient Indian astronomers tried to consider certain physical reason behind the model.

(II.2.5) The lunar theory in the *Laghu-mānasa*

Mañjula was an astronomer of the 10th century AD. His name is sometimes spelled Muñjāla, but Kripa Shankar Shukla pointed out that Mañjula is his real name.

In the ancient Indian planetary theory before Mañjula, only the equation of centre was considered. Mañjula considered another inequality "evection" for the first time in India.

Mañjula composed the *Laghu-mānasa* (932 AD). [For its edited text with English translation, see Shukla (1990).] This is a *karaṇa* work (handy practical work of astronomy) of mathematical astronomy. It is a small but very important work. It contains the second correction for the moon. [The first correction is the equation of centre.] According to Yallaya's commentary (1482 AD) on the *Laghu-mānasa*, this correction is originated from Vaṭeśvara's work (early 10th century). The statement of Yallaya has not been confirmed by Vaṭeśvara's extant works.

According to K.S.Shukla's commentary in his edition of the *Laghu-mānasa*, Mañjula's second correction for the moon can be expressed as follows:

144'26" cos(S–U) sin(M–S) -------(A)

where S, M, and U are the true longitudes of the sun, moon, and the moon's apogee (*mandocca*) respectively. Shukla pointed out that this is a combination of the "deficit of the equation of centre" and the "evection".

We can understand this expression as follows. From the equation (8) in my Appendix 3, the eccentricity looks $E \approx e - \frac{15}{8} me \cdot \cos 2\xi$, and if it is obtained from the observation of lunar eclipses (like Hipparchus etc.), when ξ is 180°, the eccentricity looks $E \approx e - \frac{15}{8} me$, and the equation of centre looks $2(e - \frac{15}{8} me)\sin\varphi$. Here, $2\times\frac{15}{8} me \cdot \sin\varphi$ corresponds to the "deficit of the equation of centre". Then:

"deficit of the equation of centre" = $\frac{15}{4} me \sin\varphi$ -------(B)

"evection" = $\frac{15}{4} me \sin(2\xi-\varphi)$ -------(C)

Mañjula's second correction corresponds to the sum of (B) and (C).

$$(B)+(C) = \frac{15}{4} me \{\sin\varphi + \sin(2\xi-\varphi)\}$$
$$= \frac{15}{4} me \times 2\{\sin\xi \cdot \cos(\varphi-\xi)\}$$
$$= \frac{15}{2} me \{\sin(JI-☼) \cdot \cos(☼-A)\} \quad \text{-------(D)}$$

If we use the notation of the expression (A), the expression (D) can be expressed as:

$$(B)+(C) = \frac{15}{2} me \{\sin(M-S) \cdot \cos(S-U)\}$$

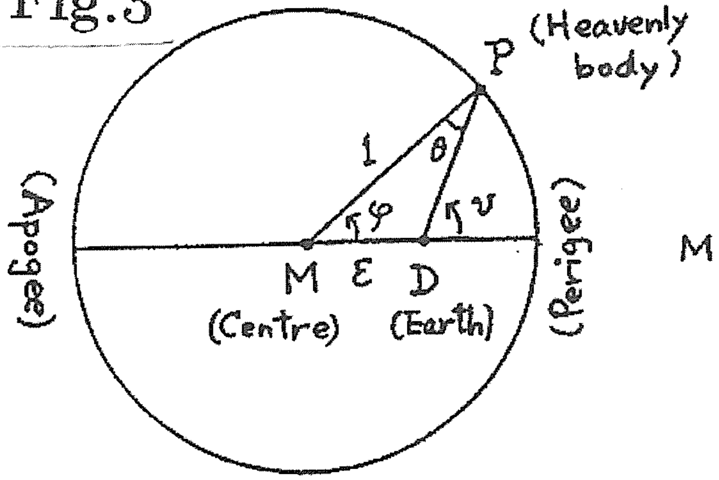
This expression corresponds to the expression (A). From this fact, we know that Mañjula's second correction is justified. Mañjula's way of thinking might have been different from Ptolemy, and we should investigate his method further.

III. Conclusion

After receiving the influence of Greek astronomy, the Classical Hindu astronomy was created, and the eccentric model and the epicyclic model were used there. However, Indian methods are not simple imitations of Greek geometrical models, and there are several differences. It seems to me that ancient Hindu astronomers tried to consider certain physics (or natural philosophy), which is different from Greek philosophy, behind the model. Further investigation of the special feature of the Classical Hindu astronomy is needed. We should investigate their way of thinking (which may be called their natural philosophy) behind their theory.

Appendix 1, The accuracy of the Simple eccentric model

Fig.3



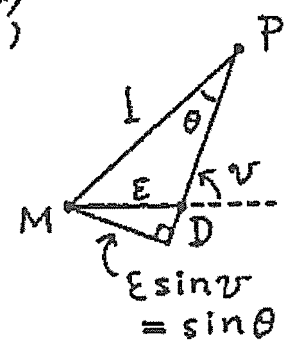


Fig.4

Simple Eccentric Model

Fig.3 shows an eccentric circle whose radius is 1, and a heavenly body P moves along the circle with a constant speed. Let this model be called "Simple eccentric model". In this figure, let

φ: Mean anomaly,

v: True anomaly,

θ (equation of centre) ≡ $v - \varphi$

MD ≡ ε (eccentric distance).

From Fig.4,

$$\sin\theta = \varepsilon \sin v = \varepsilon \sin(\varphi+\theta) = \varepsilon \sin\varphi\cos\theta + \varepsilon \cos\varphi\sin\theta$$

Therefore,

$$\sin\theta(1-\varepsilon\cos\varphi) = \varepsilon\sin\varphi\cos\theta$$

$$\tan\theta = \frac{\varepsilon\sin\varphi}{1-\varepsilon\cos\varphi} \approx \varepsilon\sin\varphi(1+\varepsilon\cos\varphi)$$

$$= \varepsilon\sin\varphi + \varepsilon^2\sin\varphi\cos\varphi = \varepsilon\sin\varphi + \tfrac{1}{2}\varepsilon^2\sin 2\varphi$$

Now, let ε = 2*e*. As θ is small, tanθ ≈ θ approximately. Therefore, we get the following expression as the **equation of center according to the "Simple eccentric model"**.

$$\theta \approx 2e\sin\varphi + 2e^2\sin 2\varphi \quad \text{———(1)}$$

Here, *e* corresponds to the eccentricity in modern sense.

In the **true Kepler motion** (according to modern astronomy), the equation of centre is expressed as:

$$\theta \approx 2e\sin\varphi + \frac{5}{4}e^2\sin 2\varphi \quad \text{———(2)}$$

Therefore, if $\varepsilon = 2e$, the first terms of the equations (1) and (2) are the same, and their second terms differs slightly. It means that these equations give the same result at the apogee, perigee, and the points which are at the distance of 90° from them, but give slightly different results at other points.

Appendix 2, The composition of the equation of centre and the evection of the moon

In this section, I shall show the composition of the equation of centre and the evection of the moon. It will be shown that Ptolemy's theory and Mañjula's theory are justified as far as lunar longitude is concerned. [I have consulted Araki (1956) pp.259-260 and Araki (1980) pp.259-260.]

Let:

Л: lunar mean longitude

☼: solar mean longitude

A: mean longitude of lunar apogee (which revolves once in about 9 years to the direct direction)

And also:

$\Phi \equiv$ Л $-$ A (lunar mean anomaly)

$\xi \equiv$ Л $-$ ☼ (mean angular distance between the moon and the sun)

If we neglect higher term than e^2 (e is eccentricity), the modern equation of center and evection can be expressed as follows:

$$\text{Equation of centre} = 2e\sin\varphi \qquad \text{——— (1)}$$

$$\text{Evection} = \tfrac{15}{4}me\sin(2\xi-\varphi) \qquad \text{——— (2)}$$

Here, $m \equiv n'/n$

(n' is the sun's mean angular velocity, n is the moon's mean angular velocity, and e and m are small values at the order of 10^{-2} or so.)

Let us express the composition of the equation of centre and evection as follows:

$$\text{Equation of centre + evection} \approx 2E\sin(\varphi+\delta) \qquad \text{——— (3)}$$

We can define E and δ as follows:

$$E\sin\delta \equiv \tfrac{15}{8}me\sin 2\xi \qquad \text{——— (4)}$$

$$E\cos\delta \equiv e - \tfrac{15}{8}me\cos 2\xi \qquad \text{——— (5)}$$

The reason why we can define like this is that we can obtain the following equation from the equation (3):

$$\text{Equation of centre + evection} \approx 2E(\sin\varphi\cos\delta + \cos\varphi\sin\delta)$$

If we substitute the equations (4) and (5) here, the result is the same as the sum of the equations (1) and (2).

The sum of the squares of the equations (4) and (5) is:

$$E^2 = (e - \tfrac{15}{8}me\cos 2\xi)^2 + (\tfrac{15}{8}me\sin 2\xi)^2$$
$$= e^2 - \tfrac{15}{4}me^2\cos 2\xi + \tfrac{15}{8}m^2e^2$$

Neglecting higher terms of m and e, and using Taylor series $(1+x)^{1/2} \approx 1+\tfrac{1}{2}x$, we get:

$$E \approx e - \tfrac{15}{8}me\cos 2\xi \qquad \text{——— (6)}$$

Dividing the equation (4) by the equation (5), we get :

$$\tan\delta = \frac{\tfrac{15}{8}me\sin 2\xi}{e - \tfrac{15}{8}me\cos 2\xi} \approx \tfrac{15}{8}m\sin 2\xi$$

As δ is small, using $\tan\delta \approx \delta$, we get :

$$\delta \approx \tfrac{15}{8}m\sin 2\xi \qquad \text{——— (7)}$$

From the equations (3), (6) and (7), we get :

Equation of centre + evection $\approx 2E\sin(\varphi+\delta)$ ——— (8)

Here, $E \approx e - \tfrac{15}{8}me\cos 2\xi$, $\delta \approx \tfrac{15}{8}m\sin 2\xi$

From the equation (8), we know that : E is smallest at the time of new moon and full moon, and is largest at the time of half moon ; δ is positive in the 1st and 3rd quadrants, and the perigee looks as if going back, and is negative in the 2nd and 4th quadrants, and the perigee looks as if moving ahead. These results agree with Ptolemy's theory as well as Mañjula's theory as far as lunar longitude is concerned.

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