History of the Mathematical Sciences II

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With best compliments,

SL Singh

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Contents

1	Lagi	ange's strategy for solving algebraic equation	1				
Aı	ijing	QU					
	1.1	Introduction	1				
	1.2	Lagrange's strategy: simple case	2				
		1.2.1 Examples	5,				
		1.2.2 The solution of quadratic equations	5,				
		1.2.3 The solution of cubic equations \dots	6				
		1.2.4 The solution of quartic equations	8,				
	1.3	Lagrange's strategy: general case	9				
	1.4	Some remarks	13;				
		1.4.1 Lagrange resolvent	14				
		1.4.2 Lagrange's attitude	16				
2	Evo	olution of the idea of abstract groups	19				
N	Iingli	Deng, Xianfen Wang					
	2.1	Introduction	20				
	2.2	2.2 From symmetry to a group					
	2.3	From a concrete group to an abstract group	22				
	2.4	Concretization of an abstract group	25				
3	Sel	ection principles: history and recent development	29				
L	jubiš	a D.R. Kočinac					
	3.1	Historical background	29				
		3.1.1 Motivation for S_1	30				
		3.1.2 Motivation for S_{fin}	30				
		3.1.3 Motivation for U_{fin}	31				

		3.1.4 The Menger conjecture	31				
		The state of the s	31				
		•	31				
	a a	G.I.O REMEDIE WITH CERES INCIDED OF INSCRIPTIONS CO	33				
	3.2	Recent innovations	,,,				
4	Kar	amata's proofs of Pappus- Pascal and Desargues theorems 3	39				
$\mathbf{A}\mathbf{l}$	eksaı	ndar M. Nikolić					
	4.1	Introduction	40				
	4.2	Proof of the Pappus-Pascal proposition	41				
	4.3	Proof of the Desargues Theorem	43				
	4.4	Conclusion	48				
5	Art	n's theory of braids and algebraization of topology	51				
B.	S. 1	adav (adav					
	5.1	Artin's theory of braids	52				
	5.2	Stone-Weierstrass Theorem	57				
		5.2.1 The Stone-Weierstrass Theorem	58				
		5.2.2 The Double Commutant Theorem	58				
6	Tur	ning an algebraic identity into an infinite series	61				
K	. Ma	hesh, R. Venketeswara Pai and K. Ramasubramanian					
	6.1	Introduction	62				
	6.2	The work Kriyākramakarī and its authorship	63				
	6.3	The infinite series for the circumference of a circle cited in Kriyākramakari	64				
	6.4						
		6.4.1 Conversion of an expression into infinite series	66				
		· · · · · · · · · · · · · · · · · · ·	70				
	6.5	Conclusion	73				
	6.6						
		W. A					

Yı

31	/			reece and India	03
	Vı		Ôhash		
31	1,			- action	83
33				ary models in Ancient Greece, Islamic World, and Europe	84
39		,	7.2.1	Apollonius	84
			7.2.2	Hipparchus	84
40			7.2.3	Ptolemy	85
41			7.2.4	Ptolemy's lunar theory (1) (equation of center)	86
43			7.2.5	Ptolemy's lunar theory (2) (evection)	87
48			7.2.6	Ptolemy's planetary theory	88
10			7.2.7	Islamic World	88
51			7.2.8	Europe	88
		7.3	Planeta	ary models in traditional India	89
52			7.3.1	The manda-correction and the śīghra-correction	89
57			7.3.2	The size of the epicycles in the Āryabhatīya	89
58			7.3.3	Indian method of the manda-correction	90
58			7.3.4	The method in the Brāhma-sphuta-siddhānta	91
			7.3.5	The lunar theory in the Laghu-mānasa	92
61			7.3.6	The Bīja-upanaya controversy	93
		7.4	Conclu	ısion	93
62					
63	8			roadening the horizon of ancient and Medieval Indian	
64		ma	themat	ical sciences	103
64	V	. Mac	lhukar	Mallayya	
66		8.1	Introd	uction	104
70		8.2	A peep	into potential source materials	105
73		8.3	A glan	ce at the mathematical treasure trove	123
73		8.4	Deside	eratum	129

9 Indian Elements in al-Bīrūnī's Mathematics	133
Mohammad Bagheri	
9.1 Introduction	133
9.2 Al-Bīrūnī and Indian culture	
9.3 Conclusion	
10 Divisibility and primality in Vedic literature	141
Subhash Kak	
10.1 Introduction	141
10.2 The mathematics of the Vedic times	143
10.3 Number divisors	145
10.3.1 Divisors of sixty and the 6-day week	147
10.3.2 The division of the year into 10,800 muhūrtas	147
10.4 Astronomical and non-astronomical numbers	148
10.5 A prime number from ideal physiology	149
10.6 Conclusions	149
11 Cultural-cognitive co-evolutionary archaeological approach to Artificial Memory Systems: A 5,200-year-old binary information system and its support to Leibnitz	153
Honghai Deng	
11.1 Introduction	154
11.2 Method and materials	155
11.3 Hole patterns	166
11.4 6-Bit binary coding of data	168
11.5 Table 1: The number of holes on a stone knife and the corresponding length of the sun- shadow	171
11.6 Data transformation into information	
11.7 Table 2: The binary-coded data input and information output of the	172
hexagram balls	172

133	11.8 Conclusion and discussion	1/8
133	12 Toward a cross-cultural history of mathematics between the Chinese, and Arabic mathematical theories of music: the puzzle of the Indian	185
134	case	
138	Tito M. Tonietti	186
	12.1 The Chinese case	189
141	12.2 The Arabic case	193
	12.3 The Indian puzzle	197
141	12.4 Cultural differences	
143	control of the contro	
145	13 Introduction of geometry and physics in theoretical music in the Renaissance	205
147	Oscar João Abdounur	
147	13.1 Origins of the relationship between mathematics and music	206
148 149	13.2 The context of the relationship between mathematics and music in late Middle Ages and Renaissance	207
149	13.3 Scales and Temperament: from commensurable magnitudes to irrational numbers	209
	13.4 Equal division of the tone	212
	13.5 Theories of ratio in mathematical contexts	214
153	13.6 The mathematical basis of Renaissance theoretical music: from arithmetic to geometry	215
154	13.7 Consonance: from a numerical dogmatism to an experimental conception	218
155	13.8 Conclusion	219
166		
168	14 Paradoxes in hydrodynamics: an historical overview	223
	Sunil Datta	
171	14.1 Introduction	223
172	14.2 Listing of Paradoxes	225
	14.3 Description and resolution	225
172		

Chapter 7

Mathematical structure of the eccentric and epicyclic models in ancient Greece and India

Yukio Ôhashi

Abstract

The eccentric and epicyclic models were created in Ancient Greece, and further developed in India, Islamic World and Europe. The accuracy of their mathematical structures is examined in this paper. And also, special features of Indian models are discussed.

Keywords: Eccentric model, Epicyclic model, Greek astronomy, Indian astronomy, Geocentric system.

7.1 Introduction

The eccentric model (in the geocentric theory) is that the center of the orbit of a heavenly body is different from the earth. The epicyclic model is that a planet tevolves along a small epicycle and that the center of the epicycle revolves along a large deferent. (This deferent can be an eccentric circle.)

The eccentric and epicyclic models were created in Ancient Greece sometime around

the 3^{rd} century BC, and were introduced into India sometime around the 4^{th} century AD or so. Indian models were not simple imitation of Greek models, but have special features. I would like to discuss the mathematical structure of some of these models

7.2 Planetary models in Ancient Greece, Islamic World, and Europe

7.2.1 Apollonius

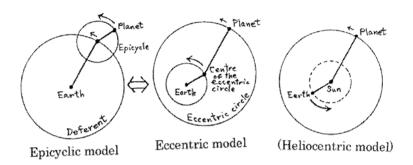


Figure 1: Geocentric Models and Heliocentric Model

We do not know who invented the eccentric and epicyclic models, but the first as tronomer who mathematically treated these models seems to be Apollonius (ca. end of the 3^{rd} century BC). Ptolemy wrote in his Almagest (XII.1) that Apollonius explained the retrograde motion of planets by the epicyclic model as well as the eccentric model which is mathematically equivalent to the epicyclic model. (See Fig.1.)

7.2.2 Hipparchus

The equation of center of the sun and moon was explained by Hipparchus $(2^{nd}$ century BC) using the eccentric model as well as the epicyclic model which is mathematically equivalent to the eccentric model. Ptolemy explained the method of Hipparchus in his Almagest (III \sim IV). The model of Hipparchus is that the sun (or moon) revolves along an eccentric circle with a constant speed. Let this model be called "Simple eccentric model".

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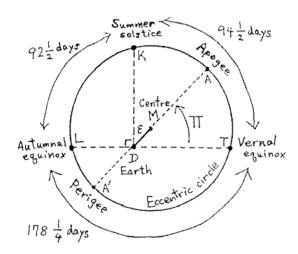


Figure 2: Simple Eccentric Model of the Solar Orbit

According to the Almagest (III.4), the solar theory of Hipparchus is as follows. (See Fig.2.) Hipparchus based on the observational data that the period from the vernal equinox to the summer solstice is $94^{1}/_{2}$ days, and that the period from the summer solstice to the autumnal equinox is $92^{1}/_{2}$ days, and determined the eccentric distance ε and the longitude of the apogee Π of the eccentric circle (radius = 1). If the length of a year is assumed to be $365^{1}/_{4}$ days, the period from the autumnal equinox to the vernal equinox becomes $178^{1}/_{4}$ days. From these data, ε and Π can be determined by plane geometry. In this model, the eccentric distance ε is double the modern eccentricity. (See Appendix 1.)

According to the *Almagest* (IV), Hipparchus determined the orbit of the moon (with certain error) from the observational data of three lunar eclipses. Hipparchus's method must have been similar to the method of Ptolemy as shown in Figure 3.

7.2.3 Ptolemy

Ptolemy (2^{nd} century AD), in his *Almagest*, also used the "Simple eccentric model" for the solar motion, but newly introduced the second inequality "evection" for the lunar motion, and also introduced the "Equant model" for the planetary motion.

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7.2.4 Ptolemy's lunar theory (1) (equation of center)

According to the Almagest (IV.1), observational data of the lunar eclipses should be used to determine the equation of the center of the moon, because the moon is at the opposite direction to the sun, and its position can be determined without parallax from the (already known) solar position. The Almagest (IV.6) explains the method to determine the equation of center using the epicyclic model. For this purpose, the observational data of the lunar longitude at the time of maximum obscuration of three lunar eclipses as well as the (already known) length of the anomalistic month and that of the tropical month are used. From these data, the mean movement of the lunar apogee and that of the lunar longitude can be obtained.

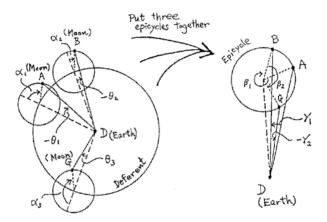


Figure 3: Determination of the Lunar Equation of Centre

The three points $A,\,B,\,G$ in the left side of Figure 3 are the positions of the moon at the time of maximum obscuration of lunar eclipses. From the period between two eclipses and the length of the tropical month, the mean change of lunar longitude during this period is calculated. From this value and the actually observed change of lunar longitude during this period, the following values are estimated.

$$Y_1 = \theta_2 - \theta_1, \quad Y_2 = \theta_3 - \theta_2$$

From the mean movement of the lunar apogee during this period, which can be calculated from the length of the anomalistic month and that of the tropical month, the following values are estimated.

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$$\beta_1 = \alpha_2 - \alpha_1, \quad \beta_2 = \alpha_3 - \alpha_2$$

Putting these three epicycles together like the right side of Figure 3, we can determine the position of the earth D from the above four values using plane geometry. By this method, the radius of the epicycle (with reference to the radius of the deferent) and the longitude of the lunar apogee at a certain point of time are obtained. Actually, the equation of center determined by this method is slightly smaller than the modern equation of center, due to the effect of evection at the time of lunar eclipse (i.e. full moon).

7.2.5 Ptolemy's lunar theory (2) (evection)

As the model as above does not coincide with the actual observations except for the new moon and full moon, Ptolemy introduced "evection" for the first time in his Almagest (V).

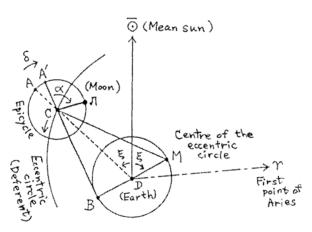


Figure 4: Ptolemy's Lunar Model with "Evection"

In Figure 4, the center M of the eccentric circle revolves along a small circle (MB). In the figure, ξ is the angular distance between the mean moon and the mean sun. By this method, the epicycle comes near to the earth at the time of half moon, and the eccentricity looks increased.

There is another effect also. Let B be the opposite point of M, and A^1 be a cross point of the epicycle and the straight line BC. The moon revolves once in an anomalistic

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month from A^1 (not from A). In the figure, α is the mean anomaly. Due to the effect of δ (= \angle AC A^1), the longitude of the apogee looks changed except for the time of new moon, full moon, and half moon. (See Appendix 3.)

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7.2.6 Ptolemy's planetary theory

For planets, Ptolemy used the epicyclic model, where an epicycle is revolving along a deferent. The deferent is an eccentric circle. He determined the distance of the deferent of Venus (from the earth) by the observation of the greatest elongation which indicates the apparent radius of the epicycle. From this distance, eccentricity is estimated. Then, from the difference between the actual observation and the prediction according to the "Simple eccentric model" of the deferent when Venus is at the right angled point from the apogee, he introduced a point which was later called "equant". He also used the "Equant model" for other planets. (See Appendix 2.)

Ptolemy used a very complicated model for Mercury, which I shall not discuss here.

7.2.7 Islamic World

In the Islamic world, some geometrical models which give similar results to the "Equant model" were devised. One of them was the "Double epicycle model" of $a\underline{sh}$ - \underline{Sh} \overline{atir} (14^{th} century AD). (See Appendix 4.)

7.2.8 Europe

Copernicus (1473 – 1543) also used a model which is equivalent to the "Double epicycle model" for planets. It is not known whether he was directly influenced by the model of ash-Shātir or not. Copernicus, however, still used a model which is basically similar to the "Simple eccentric model" for the earth. He mentioned something like double epicycle (or its variation) for the earth in his *De Revolutionibus* (III), but it was for the secular variation of the eccentricity and the longitude of perihelion, and was not for the effect due to the "Equant model". In the Copernican model, the earth was still different from other planets.

Tycho Brahe (1546 – 1601) also used a model which is equivalent to the "Double epicycle model" for planets, but used the "Simple eccentric model" for the sun.

Kepler (1571 - 1630) used elliptic orbit for all planets including the earth, and the earth became an ordinary planet truly.

7.3 Planetary models in traditional India

7.3.1 The manda-correction and the śighra-correction

In the Hindu Classical Astronomy, geocentric epicyclic and eccentric systems are used. The $Mah\bar{a}bh\bar{a}skariya$ of Bhāskara I (7^{th} century) treats the epicyclic and eccentric systems as mathematically equivalent models for both of the manda-correction and the \acute{sighra} -correction.

Firstly, mean (madhya) planet, which is supposed to rotate constantly around the earth, is calculated, and then, corrections are applied to the mean planet in order to obtain the true (sphuta) planet. One correction is the manda-correction, which corresponds to our equation of center. The other is the \acute{sighra} -correction, which corresponds to the annual parallax in the case of outer planets, and the planet's own revolution in the case of inner planets. Firstly, the manda-correction is applied to the mean planet, which corresponds to the planet's own mean revolution in the case of outer planets, and the sun's mean revolution in the case of inner planets. The result is called "manda-sphuta planet", which is the mean planet corrected by the equation of center only. Then, the \acute{sighra} -correction is applied to the "manda-sphuta planet", and the true planet is obtained. In the actual calculation, some special methods are used in the classical texts.

7.3.2 The size of the epicycles in the Āryabhaṭīya

One interesting feature of the $\bar{A}ryabhat\bar{i}ya$ (AD499) of $\bar{A}ryabhata$ is that the size of the epicycles of the planets changes in different anomalistic quadrants. This is quite different from the simple geometrical model. The modern $Surya\text{-}siddh\bar{a}nta$ (ca. 10 \sim 11th century AD) etc. also use a similar method.

According to the $\bar{A}ryabhat\bar{i}ya$, the manda-epicycles of Mars, Jupiter and Saturn are small in the 1^{st} and 3^{rd} quadrants, and are large in the 2^{nd} and 4^{th} quadrants. The manda-epicycles of the Mercury and Venus, and the $\acute{sig}hra$ -epicycles of the five planets are large in the 1^{st} and 3^{rd} quadrants, and are small in the 2^{nd} and 4^{th} quadrants.

According to the interpretation of Bhāskara I, their size given in the $\bar{A}ryabhat\bar{i}ya$ is the value at the beginning of each quadrant, and the size changes linearly. However, there are other Hindu astronomers who interpret that the size is the value at the end of each quadrant.

In the case of the *manda*-epicycles of the Mars, Jupiter and Saturn, if the interpretation of Bhāskara I is correct, it can be said that the change of the distance of

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the planets becomes somewhat similar to that of the "equant" model. However, t_{he} change of the \tilde{sighra} -epicycles is not understandable. Further researches are necessary,

7.3.3 Indian method of the manda-correction

The manda-correction is not based on a simple geometrical model, but a special modification is applied. Its result is that the equation of center in this method becomes a simple sine function of anomaly. Its accuracy is slightly less than the simple eccentric model, but their errors (in the opposite direction) are not so different.

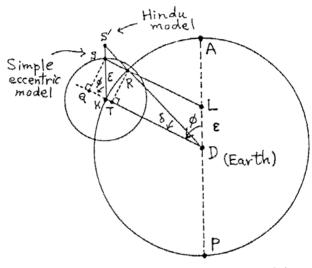


Figure 5: Equation of Centre in the Hindu Classical Astronomy

In Figure 5, the point S corresponds to the heavenly body in the simple eccentric model (In the figure, SK||AP, where AS corresponds to the apogee, and P corresponds to the perigee.). In the Figure 5, S^1 corresponds to the heavenly body in the Hindu model. Here, $KS = DL = \varepsilon = 2e$ (e is eccentricity in modern sense), SL||KD, and the point S^1 is at the cross point of the straight lines KS and DR. Now, $RT = SQ = \varepsilon \cdot \sin \phi$. Therefore, $\delta \approx \sin \delta = RT = \varepsilon \cdot \sin \phi$. So, we can say that the Hindu equation of center is a simple sine function of anomaly. This Hindu method is well explained in the $Mah\bar{a}bh\bar{a}skar\bar{i}ya$ (IV) of Bhāskara I (7^{th} century).

As I have shown in Appendix 1, the equation of center of the simple eccentric model and that of the true Kepler motion are:

 $\theta \approx 2$ $\theta \approx 2$

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a special s method he simple rent. $\theta \approx 2e \sin \phi + 2e^2 \sin 2\phi$ (simple eccentric model) $\theta \approx 2e \sin \varphi + \frac{5}{4}e^2 \sin 2\varphi$ (true Kepler motion)

It is clear from the above equations that the second term of the simple eccentric model is too large. The Indian method can be expressed as $\theta \approx 2e \sin \phi$, and its inaccuracy is not so different from that of the simple eccentric model (in the opposite direction).

This Indian method is quite different from the Greek method.

7.3.4 The method in the Brāhma-sphuṭa-siddhānta

The method in the $Br\bar{a}hma$ -sphuta-siddh $\bar{a}nta$ (AD 628) of Brahmagupta is very interesting. Its process, except for Mars, is as follows. Firstly, the amount of the manda-correction is calculated from the mean planet, and is applied to the mean planet. The result is the manda-sphuta planet. Secondly, the amount of the $s\bar{i}ghra$ -correction is calculated from the manda-sphuta planet, and is applied to the manda-sphuta planet. The result is the true planet after the first approximation. From the result, the amount of the manda-correction is calculated, and applied to the original mean planet. From this result, the amount of the $s\bar{i}ghra$ -correction is calculated and applied. The result is the true planet after the second approximation. This process is repeated until a constant value is obtained.

In the case of Mars, the above mentioned process is not used. The method for Mars is as follows. Firstly, a half of the amount of the manda-correction and a half of the $\hat{sig}hra$ -correction are applied, and the once corrected Mars is obtained. From the result, the amount of the manda-correction is calculated, and its whole amount is applied to the original mean Mars. From this result, the amount of the $\hat{sig}hra$ -correction is calculated, and its whole amount is applied. The result is the true Mars. This method is the ordinary method used by other Indian astronomers in those days for five planets.

The above mentioned process of the Brāhma-sphuṭa-siddhānta, except for the case of Mars, means that the amount of the equation of center is a function of the true anomaly of the planet, and not of the mean anomaly. This fact shows that the Indian model of planetary motion is not a simple imitation of the Greek geometrical model, and further investigation of the Indian model is needed. At present, I suspect that Indian astronomers in those days considered that the inequality of the manda-correction is produced by a kind of physical force originated to the apogee, and this force is equilibrated with the displacement of planetary position due to the inequality. If so, the amount of the inequality should be a function of the actual position of the

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true planet (instead of the mean position like the Greek model).

It may be mentioned here that Brahmagupta did not use the above mentioned successive approximation for the calculation of true planets in his $Khanda-kh\bar{a}dyaka$ (AD 665), but used the ordinary method, like the case of Mars in his earlier work, for five planets.

7.3.5 The lunar theory in the Laghu-mānasa

Mañjula was an astronomer of the 10^{th} century AD. His name is sometimes spelled Muñjala, but Kripa Shankar Shukla pointed out that Mañjula is his real name.

Mañjula composed the Laghu- $m\bar{a}nasa$ (932 AD). This is a karaṇa work (handy practical work of astronomy) of mathematical astronomy. It is a small but very important work. It contains the second correction for the moon. According to Yallaya's commentary (1482 AD) on the Laghu- $m\bar{a}nasa$, this correction originated from Vaṭeśvara's work (early 10^{th} century). The statement of Yallaya has not been confirmed by Vateśvara's extant works.

According to K. S. Shukla's commentary in his edition of the Laghu- $m\bar{a}nasa$, $Ma\tilde{n}$ jula's second correction for the moon can be expressed as follows:

$$144'26'' \cos(S-U) \sin(M-S)$$
 (A)

where S, M, and U are the true longitudes of the sun, moon, and the moon's apogee (mandocca) respectively. Shukla pointed out that this is a combination of the "deficit of the equation of center" and the "evection".

We can understand this expression as follows. From the equation (8) in my Appendix 3, the eccentricity looks $E\approx e^{-\frac{15}{8}}me$. $\cos2\xi$, and if it is obtained from the observation of lunar eclipses (like Hipparchus etc.), when ξ is 180; the eccentricity appears as $E\approx e^{-\frac{15}{8}}me$, and the equation of center appear as $2(e^{-\frac{15}{8}}me)\sin\phi$. Here, $2\times\frac{15}{8}me$. $\sin\phi$ corresponds to the "deficit of the equation of center". Then:

"deficit of the equation of center"
$$=\frac{15}{4} me \sin \phi$$
 (B)

"evection" =
$$\frac{15}{4}$$
 me $\sin(2\xi - \phi)$ (C)

Mañjula's second correction corresponds to the sum of (B) and (C).

$$\begin{array}{rcl} (B) + (C) & = & \frac{15}{4} me \{ \sin \phi + \sin(2\xi - \phi) \} \\ & = & \frac{15}{4} me \times 2 \{ \sin \xi . \cos(\phi - \xi) \} \end{array}$$

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$$(B) + (C) = \frac{15}{2} me \{ \sin(\Lambda - \Xi) \cdot \cos(\Xi - A) \}.$$
 (D)

If we use the notation of the expression (A), the expression (D) can be expressed as:

$$(B) + (C) = \frac{15}{2} me \{ sin(M - S) \cdot \cos(S - U) \}.$$

This expression corresponds to the expression (A). From this fact, we know that Mañjula's second correction is justified. Mañjula's way of thinking might have been different from Ptolemy, and we should investigate his method further.

7.3.6 The Bija-upanaya controversy

There is a small work $B\bar{i}ja$ -upanaya (= $B\bar{i}jopanaya$), where the lunar inequality corresponding to the "variation" is mentioned and attributed to Bhāskara II, but its authorship is controversial. Ghosh maintained that it is Bhāskara II's own work, and published the following edition:

Ghosh, Ekendranath (ed.): Bhaskariya-Bijopanaya, Motilal Banarsidass, Lahore, 1926.

This work mentions two corrections, an improvement of the equation of center in Hindu Classical Astronomy, and an inequality corresponding to the variation. Although the text itself mentions Bhāskara as its author, Kuppanna Sastry, criticizing Ghosh, pointed out several reasons that it cannot be the work of Bhāskara II, some of which are as follows. The first correction produces an error which is unusual for the otherwise accurate system of Bhāskara II. The $B\bar{ij}a$ -upanaya was not known to astronomers who followed Bhāskara II. In this work, the corrections are given in the form of tabular values, although this kind of calculation is usually given in the form of equations. And also, the style of this work is not Bhāskara II's.

These arguments of Kuppanna Sastry are understandable, and the $B\bar{i}ja$ -upanaya will not be a work of Bhāskara II, but the real author of this $B\bar{i}ja$ -upanaya is still to be investigated.

7.4 Conclusion

We have seen that eccentric and epicyclic models in Indian Classical Astronomy have several origins. Greek models are evidently geometrical, but Indian models may not

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be purely geometrical. More research on the Indian models is needed.

Appendix

Appendix 1: The accuracy of the simple eccentric model

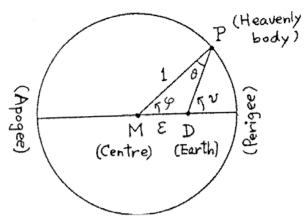


Figure 6: Simple Eccentric Model

Figure 6 shows an eccentric circle whose radius is 1, and a heavenly body P moves along the circle with a constant speed. Let this model be called "Simple eccentric model". In this figure, let

 ϕ : Mean anomaly,

v: True anomaly,

 θ (equation of center) $\equiv v - \phi$

 $MD \equiv \varepsilon$ (eccentric distance).

From Figure 7,

 $\sin\theta = \varepsilon\sin v = \varepsilon\sin(\phi + \theta) = \varepsilon\sin\phi \,\cos\theta + \varepsilon\cos\phi\sin\theta$

Therefore,

 $\sin\theta(1-\varepsilon\cos\phi) = \varepsilon\sin\phi\cos\theta$

 $\tan\theta = \frac{\varepsilon\sin\varphi}{1 - \varepsilon\cos\varphi} \approx \varepsilon\sin\phi(1 + \varepsilon\cos\phi)$

 $=\varepsilon\sin\phi+\varepsilon^2\sin\phi\cos\phi=\varepsilon\sin\phi+\tfrac{1}{2}\varepsilon^2\sin2\phi$

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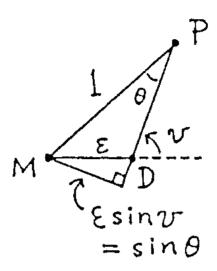


Figure 7: A part of the Figure 6

Now, let $\varepsilon = 2e$. As θ is small, $\tan \theta \approx \theta$ approximately. Therefore, we get the following expression as the equation of center according to the "Simple eccentric model".

$$\theta \approx 2e\sin\phi e^2\sin2\phi\tag{1}$$

Here, e corresponds to the eccentricity in the modern sense.

In the **true Kepler motion** (according to modern astronomy), the equation of center is expressed as:

$$\theta \approx 2e\sin\varphi + \frac{5}{4}e^2\sin2\varphi \tag{2}$$

Therefore, if $\varepsilon = 2e$, the first terms of the equations (1) and (2) are the same, and their second terms differs slightly. It means that these equations give the same result at the apogee, perigee, and the points which are at the distance of 90° from them, but give slightly different results at other points.

Appendix 2: The accuracy of the "Equant" model

Figure 8 shows an eccentric circle whose radius is 1, and a heavenly body P revolves around a point E ("equant") with a constant angular velocity. Let this model be called "Equant model".

P moves eccentric

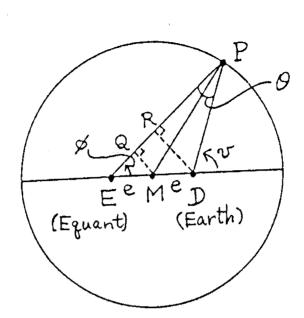


Figure 8: Equant Model

Let MD=ME=e. Then, as $QM=e\sin\phi$, and PM=1, we get

$$PQ = \sqrt{1 - e^2 \sin^2 \varphi}.$$

And also, from this equation and $RQ = QE = e \cos \phi$,

$$PR = PQ - RQ = \sqrt{1 - e^2 \sin^2 \varphi} - e \cos \varphi.$$

And also, as $RD = 2e \sin \phi$, we have

$$\tan\theta = \frac{RD}{PR} = \frac{2e\sin\varphi}{\sqrt{1-e^2\sin^2\varphi} - e\cos\varphi}.$$

Multiply its denominator and numerator by $\sqrt{1-e^2\sin^2\varphi}+e\cos\varphi$, and also, use Taylor series $(1+x)^{1/2}\approx 1+1/2x$, where x is small. Then, we get:

$$\tan\theta \approx \frac{2e\sin\varphi(1-\frac{1}{2}e^2\sin^2\varphi+e\cos\varphi)}{1-e^2}.$$

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Here, if terms higher than e^3 is ignored, we get:

$$\tan \theta \approx 2e \sin \phi + 2e^2 \sin \phi \cos \phi$$

Then, as θ is small, using $\tan \theta \approx \theta$, we get the following expression as the equation of center according to the "Equant model".

$$\theta \approx 2e\sin\phi + e^2\sin2\phi \tag{3}$$

From this equation, we know that the second term of the "Equant model" is closer to that of the true Kepler motion (equation (2)) than that of the "Simple eccentric model" (equation (1)). And also, it is evident that the distance to the heavenly body in the "Equant model" is close to the actual distance, but that of the "Simple eccentric model" is not so.

Appendix 3: The composition of the equation of center and the evection of the moon

In this section, I shall show that Ptolemy's model (regarding the longitude of the moon) is justified with respect to the composition of the modern equation of center and the evection of the moon. (Of course, the distance of the moon in Ptolemy's model is not justified.) (I have consulted Araki (1956) p.259-260 and Araki (1980) p.259-260.)

Let:

 Λ : lunar mean longitude

 Ξ : solar mean longitude

A: mean longitude of lunar apogee (which revolves once in about 9 years to the direct direction).

And also:

 $\Phi \equiv \Lambda - A$ (lunar mean anomaly)

 $\xi \equiv \Lambda - \Xi$

(mean angular distance between the moon and the sun).

If we neglect higher term than e^2 (e is eccentricity), the modern equation of center and evection can be expressed as follows:

Equation of center =
$$2e \sin \phi$$
 (1)

97

also, use

Evection =
$$\frac{15}{4} me \sin(2\xi - \phi)$$
 (2)

Here, $m \equiv n'/n$;

(n' is the sun's mean angular velocity, n is the moon's mean angular velocity, and ℓ and m are small values at the order of 10^{-2} or so.)

Let us express the composition of the equation of center and evection as follows:

Equation of center + evection
$$\approx 2E\sin(\phi + \delta)$$
. (3)

We can define E and δ as follows:

$$E\sin\delta \equiv \frac{15}{8} \ me \ \sin 2\xi. \tag{4}$$

$$E\cos\delta \equiv e - \frac{15}{8} \ me \ \cos 2\xi. \tag{5}$$

The reason why we can define like this is that we can obtain the following equation from the equation (3):

Equation of center + evection $\approx 2E(\sin\phi\cos\delta + \cos\phi\sin\delta)$.

If we substitute the equations (4) and (5) here, the result is the same as the sum of the equations (1) and (2).

The sum of the squares of the equations (4) and (5) is:

$$E^{2} = (e - \frac{15}{8}me\cos 2\xi)^{2} + (\frac{15}{8}me\sin 2\xi)^{2}$$
$$= e^{2} - \frac{15}{4}me^{2}\cos 2\xi + \frac{15}{8}m^{2}e^{2}.$$

Neglecting higher terms of m and e, and using Taylor series $(1+x)^{1/2} \approx 1+1/2x$, we get:

$$E \approx e - \frac{15}{8} me \cos 2\xi. \tag{6}$$

Dividing the equation (4) by the equation (5), we get:

$$\tan \delta = \frac{\frac{15}{8}me\sin 2\xi}{e - \frac{15}{8}me\cos 2\xi} \approx \frac{15}{8}m\sin 2\xi.$$

CHAPTER 7: MATHEMATICAL STRUCTURE OF THE ECCENTRIC AND EPICYCLIC MODELS

As δ is small, using $\tan \delta \approx \delta$, we get:

$$\delta \approx \frac{15}{8} m \sin 2\xi. \tag{7}$$

From the equations (3), (6) and (7), we get:

Equation of center + evection
$$\approx 2E\sin(\phi + \delta)$$
; (8)

here,
$$E \approx e - \frac{15}{8} me \cos 2\xi$$
, $\delta \approx \frac{15}{8} m \sin 2\xi$.

From the equation (8), we know that: E is smallest at the time of new moon and full moon, and is largest at the time of half moon; δ is positive in the 1st and 3rd quadrants, and the perigee looks as if going back, and is negative in the 2^{nd} and 4th quadrants, and the perigee looks as if moving ahead. These results agree with Ptolemy's model.

We should note that Ptolemy's model is only good for the lunar longitude. As regards the distance of the moon from the earth (or apparent diameter of the moon), Ptolemy's model is far from the truth.

Appendix 4: Double epicycle model

The "Double epicycle model" is a device to give an almost similar result to the "Equant model" by a combination of uniform circular motion only. It will be clear from Fig. 9.

In the "Double epicycle model", the first (larger) epicycle is on the deferent, and is revolving eastwards. There is a second (smaller) epicycle on the first epicycle, and the second epicycle is revolving westwards with respect to the line between the center of the deferent and the center of the first epicycle, and so the second epicycle is always at the direction of the apogee on the first epicycle. The heavenly body is on the second epicycle, and revolves eastwards. The heavenly body's speed is double the speed of the center of the epicycles, and is in the direction of the perigee at the apogee and perigee.

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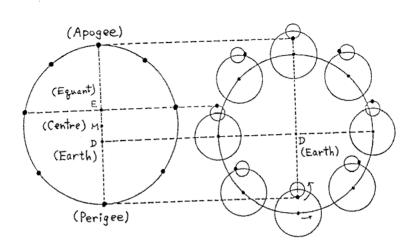


Figure 9: Equant Model and Double Epicycle Model

References

(A) Greece, Islamic World, and Europe

(A.1) General

Neugebauer, O.: A History of Ancient Mathematical Astronomy, 3 pts., Springer-Verlag, Berlin, 1975.

van der Waerden, B. L.: Die Astronomie der Greichen, Wissenschaftliche Buchgesellschaft, Darmstadt, 1988.

Evans, James: The History and Practice of Ancient Astronomy, Oxford University Press, New York, 1998.

Jacobsen, Theodor S.: Planetary Systems from the Ancient Greeks to Kepler, Department of Astronomy, University of Washington, Seattle, 1999.

Aaboe, Asger: Episodes from the Early History of Astronomy, Springer, New York, 2001.

(A.2) Ptolemy

Pedersen, Olaf: A Survey of the Almagest, Odense University Press, Odense, 1974.

Toomer, G.J. (tr.): *Ptolemy's Almagest*, (originally published in 1984 in London), Princeton University Press, Princeton, 1998.

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(A.3) Islamic World

Kennedy, E.S. et al.: Studies in the Islamic Exact Sciences, American University of Beirut, Beirut, 1983.

(A.4) Copernicus

Copernicus, Nicholas: On the Revolutions, translation and commentary by Edward Rosen, The John Hopkins University Press, Baltimore, 1992.

(A.5) Tycho Brahe

Thoren, Victor E.: The Lord of Uraniborg, A Biography of Tycho Brahe, Cambridge University Press, Cambridge, 1990.

(A.6) Kepler

Kepler, Johannes: New Astronomy, translated by William H. Donahue, Cambridge University Press, Cambridge, 1992.

(B) Indian Classical Astronomy

Shukla and Sarma (ed. and tr.): Aryabhatiya of Aryabhata, Critically edited with Introduction, English Translation, Notes, Comments and Indexes, critically edited by Kripa Shankar Shukla in collaboration with K.V. Sarma, (Āryabhatīya Critical Edition Series Pt.1), Indian National Science Academy, New Delhi, 1976.

Shukla, Kripa Shankar (ed. and tr.): Maha-bhaskariya, (Hindu Astronomical and Mathematical Texts Series No.3), Department of Mathematics and Astronomy, Lucknow University, Lucknow, 1960.

Shukla, Kripa Shankar (ed. and tr.): Laghu-bhaskariya, (Hindu astronomical and Mathematical Texts Series No.4), Department of Mathematics and Astronomy, Lucknow University, Lucknow, 1963.

Dvivedi, Sudhakara (ed. with his own commentary): Brāhmasphutasiddhānta and Dhyanaqrahopadesadhyaya by Brahmagupta, (originally published in The Pandit, NS 23-24, 1901-1902), reprinted: Medical Hall Press, Benares, 1902.

Chatterjee, Bina (ed. and tr.): The Khandakhādyaka (an astronomical treatise) of Brahmagupta with the commentary of Bhattotpala, 2 vols., published by the author. New Delhi, 1970.

Shukla, Kripa Shankar (ed. and tr.): A Critical Study of the Laghumanasa of Mañjula, (originally published in the Indian Journal of History of Science, 25, Supplement, 1990, i-x, 1-200, errata), Indian National Science Academy, New Delhi, 1990.

Burgess, Ebenezer (tr.): The Sûrya Siddhânta, a text-book of Hindu astronomy, (orig-

Springer-

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pler, De-

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Yukio Ôhashi

inally published in the Journal of the American Oriental Society, $\mathbf{6}(2)$, 1860, 141 \sim 498), Reprint edited by Phanindralal Gangooly with an introduction by Prabodhchandra Sengupta, Calcutta, 1935; reprinted: Motilal Banarsidass, Delhi, 1989.

Sastry, T.S. Kuppanna: "The Bījopanaya: Is it a work of Bhāskaracarya?", Journal of the Oriental Institute (Baroda), 8, 1958-59, 399-409, reprinted in his Collected Papers on Jyotisha, Kendriya Sanskrit Vidyapeetha, Tirupati Series No.52, Kendriya Sanskrit Vidyapeetha, Tirupati, 1989.

Shukla, Kripa Shankar: "The Evection and the Deficit of the Equation of the Center of the Moon in Hindu Astronomy", *Proceedings of the Benares Mathematical Society*, New Series, **7**(2), 1945, p.9-28.

(C) Modern celestial mechanics

Godfray, Hugh : An Elementary Treatise on the Lunar Theory, 2^{nd} ed., Macmillan, Cambridge, 1859.

Brown, Ernest W.: An Introductory Treatise on the Lunar Theory, (originally published in 1896 by Cambridge University Press), reprinted: Dover, New York, 1960.

Araki, Toschima: Gendai Tenmongaku Jiten (Lexique de l'Astronomie des Temps Modernes, in Japanese), Kooseisha, Tokyo, 1956

Araki Toschima, $Tentai\ Rikigaku$ (Mécanique céleste, in Japanese), Kooseisha, Tokyo, 1980.

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Yukio Ôhashi was born in 1955 in Tokyo. Studied the history of Indian Astronomy at Lucknow University from 1983 to 1987, and obtained Ph.D. in 1992.

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