TokyoTech (Tokyo Institute of Technology),　HMA (History of Mathematics and Astronomy)
Lecture note 2: (2019)
(Mathematics and astronomy in ancient Greece and Rome.)
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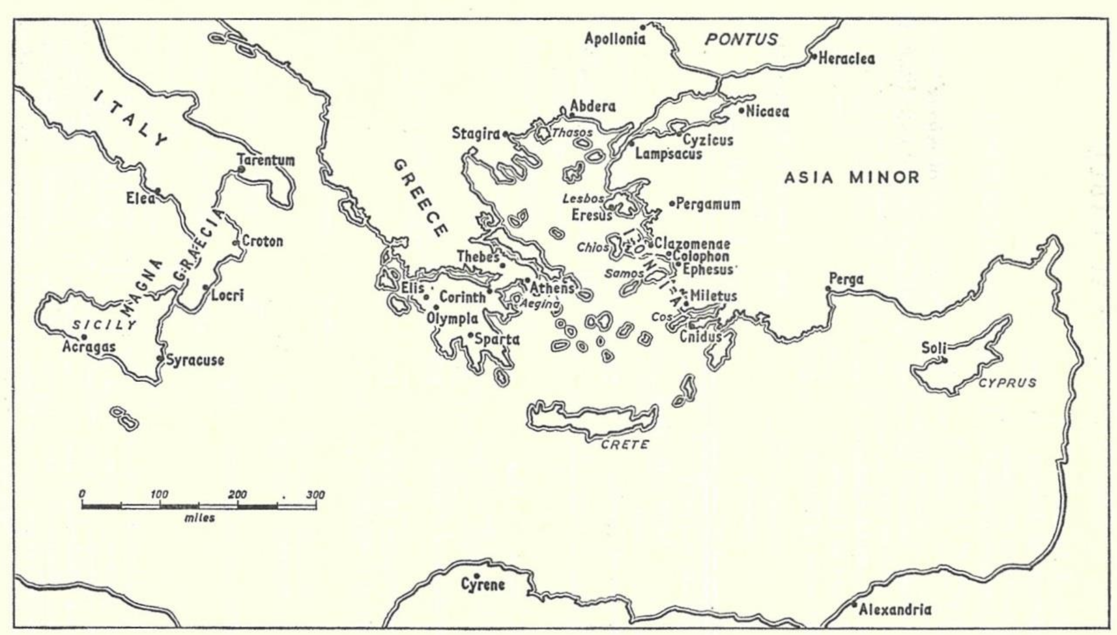
A map of the Greek world in the 5th and 4th centuries BCE



The Greek world in the fifth and fourth centuries B.C.

(From Lloyd: *Early Greek Science: Thales to Aristotle*, London, 1970.)

Mathematics and astronomy in ancient Greece

Chronological Summary

General History	Philosophers and Historians	Mathematicians and Astronomers
610 B.C. The beginning of the New Babylonian empire 540 B.C. The beginning of the Persian empire	Milesian school: Thales Anaximander Anaximenes	585 B.C. Thales 550 B.C. Pythagoras Anaximander
500 B.C. Ionian revolt 480 B.C. Persian wars 450 B.C. Pericles 420 B.C. Peloponnesian war	Heraclitus The Eleatics 450 B.C. Anaxagoras Herodotus 430 B.C. Atomists 410 B.C. Thucydides	500 B.C. Hippasus 500 B.C.–350 B.C. Pythagoreans Anaxagoras Oenopides 430 B.C. Hippocrates Democritus Theodorus
370 B.C. Epaminondas	Socrates †399 Plato Heraclides of Pontus Aristotle Eudemus	390 B.C. Archytas Theaetetus 370 B.C. Eudoxus Callippus Hicetas 350 B.C. Menaechmus Dinostratus Autolycus
333 B.C. Alexander the Great Hellenism	Stoics	300 B.C. Euclid 280 B.C. Aristarchus 250 B.C. Archimedes 240 B.C. Eratosthenes Nicomedes 210 B.C. Apollonius 150 B.C. Hipparchus
60 B.C. Julius Caesar 1 A.D. Augustus 400 A.D. Migration	Neo-Pythagoreans Neo-Platonists: Proclus	60 A.D. Heron 100 A.D. Menelaus 150 A.D. Ptolemy 250 A.D. Diophantus 320 A.D. Pappus

(From van der Waerden: *Science Awakening I*, Groningen, 1961, p.82.)

I. Greek Calendars: Luni-solar calendars.

(A) Around the second half of the 6th century BCE or so, "Eight-year cycle": There are 3 intercalary months in 8 years. (8 years = 2922 days, 1 year = 365.25 days, but 1 synodic month = 29.51515 days (inaccurate))

(B) 432 BCE, Metonic cycle: There are 7 intercalary months in 19 years. (19 years = 6940 days, 1 year = 365.2632 days, 1 synodic month = 29.53191 days)

(C) 330 BCE, Callippic cycle: There are 28 intercalary months in 76 years. (76 years = 27759 days, 1 year = 365.25 days, 1 synodic month = 29.53085 days)

(D) The 2nd century BCE, Hipparchic cycle: There are 112 intercalary months in 304 years. (Here, 304 years = 111035 days, 1 year = 365.2467 days, 1 synodic month = 29.53059 days)

(See Heath: *Greek Astronomy*, 1932, pp.xvi-xvii and 136-142.)

The Metonic, Callippic, and Hipparchic cycles are multiples of Metonic cycle. As the number of days in a cycle was considered to be integer, long period was necessary in order to make it accurate.

II. Early natural philosophers (before Socrates)

Milesian school:

Thales, ----- (Material cause of things --- "water".)

Anaximander, ----- (Material cause of things --- "boundless".)

Anaximenes. ----- (Material cause of things --- "air".)

Pythagoras, (and Pythagoreans).

----- (Principle of all things --- "numbers".)

Heraclitus. ----- (Everything is subject to change.)

(All things are an equal exchange for "fire".)

The Eleatics:

Parmenides, ----- (Denied that change can occur at all.)

Zeno. ----- ("Zeno's paradoxes")

Anaxagoras. ---- (In everything there is a portion of everything.)

Empedocles. ----- (Four elements: earth, water, air and fire.)

Atomists: Leucippus,

Democritus.

----- (The atoms and the void alone are real.).

(For more detail, see Lloyd: *Early Greek Science: Thales to Aristotle*, London, 1970, and references listed in this book. The most authentic collection of their fragments was made by Diels and Kranz, but a convenient concise reference book is Mansfield and Primavesi (2012) in Greek and German. There are some Japanese translations of their fragments: Yamamoto (1958), Uchiyama (1996-1998), Kusakabe (2000-2001).)

III. The Age of Socrates, Plato, Aristotle

Plato (427 – 347 BCE) mentioned his cosmology in his *Timaeus* etc.

Eudoxus of Cnidus (4th century BCE)

Eudoxus was a student and colleague of Plato.

Eudoxus's theory of concentric spheres:

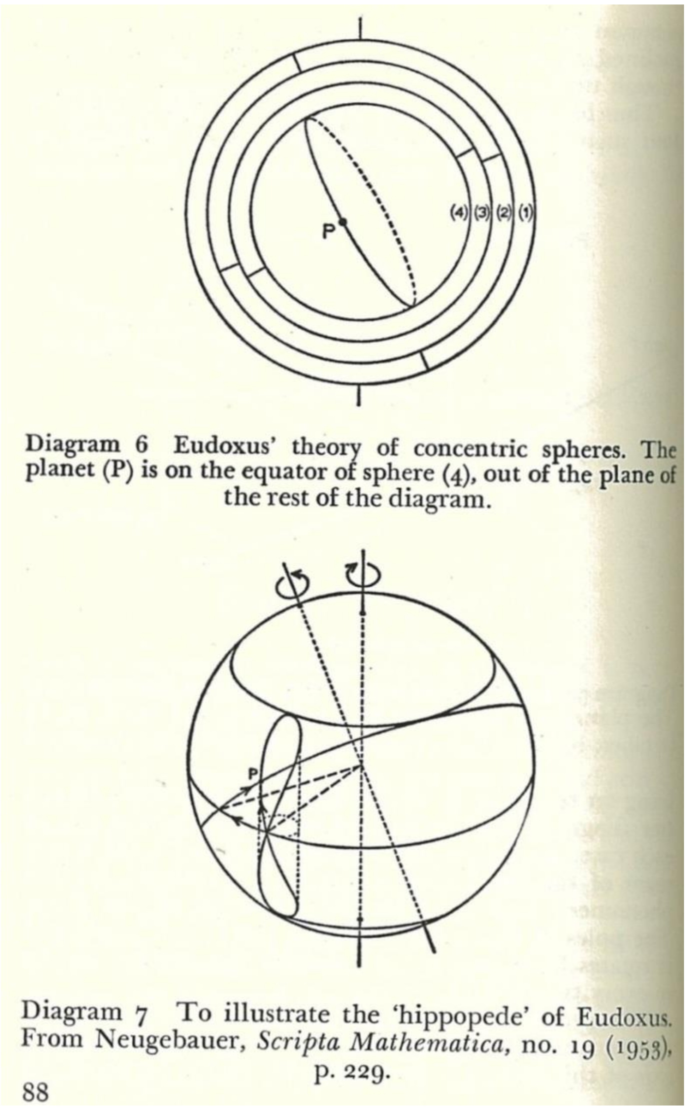


Diagram 6 Eudoxus' theory of concentric spheres. The planet (P) is on the equator of sphere (4), out of the plane of the rest of the diagram.

Diagram 7 To illustrate the 'hippopede' of Eudoxus. From Neugebauer, *Scripta Mathematica*, no. 19 (1953), p. 229.

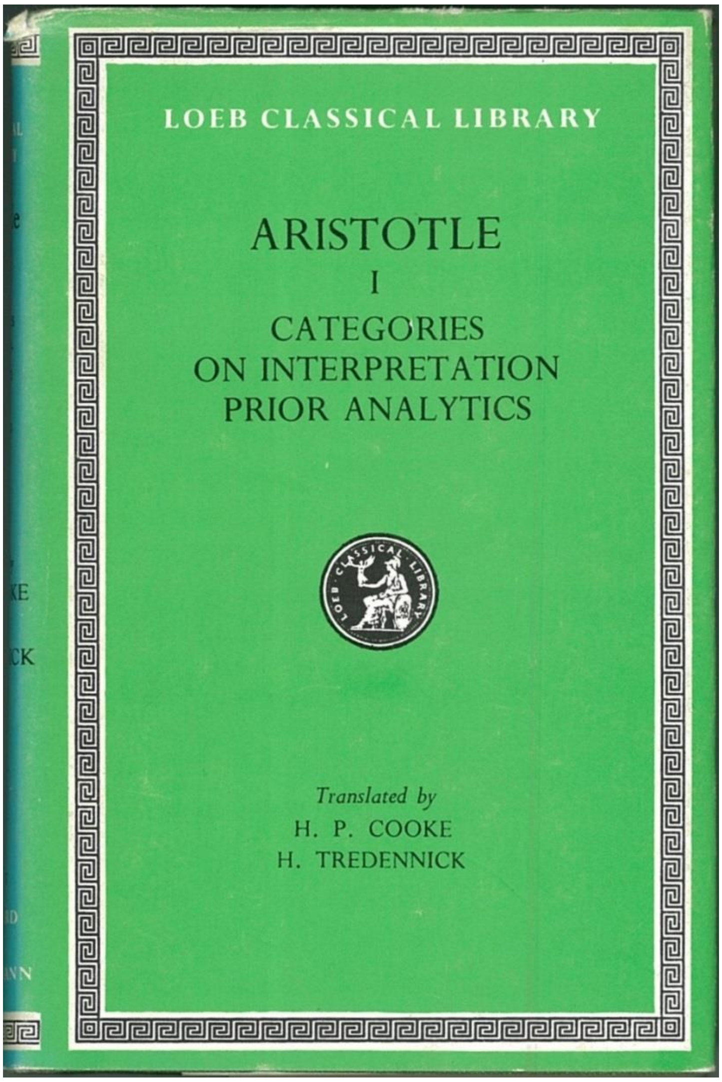
88

(From Lloyd: *Early Greek Science: Thales to Aristotle*, London, 1970, p.88.)

Aristotle (384 – 322 BCE):

Aristotle was a student of Plato. He left several works on formal logic, metaphysics, natural philosophy (including cosmology etc.), zoology, and several branches of humanities and social sciences.

Formal logic:



(*Aristotle* (in 23 volumes) *I*, (Loeb Classical Library), (Greek text and English translation), Cambridge, Mass., Harvard University Press, and London, William Heineman Ltd, 1938.)

If you want to read Greek or Latin texts with English translation, the "Loeb Classical Library" is a convenient series.

25 b

οὐκ ἀντιστρέφει, ἡ δὲ ἐν μέρει ἀντιστρέφει. τοῦτο δὲ ἔσται φανερὸν ὅταν περὶ τοῦ ἐνδεχομένου λέγωμεν.

Νῦν δὲ τοσοῦτον ἡμῖν ἔστω πρὸς τοῖς εἰρημένοις δῆλον, ὅτι τὸ ἐνδέχεσθαι μηδενὶ ἢ τινὶ μὴ ὑπάρχειν καταφατικὸν ἔχει τὸ σχῆμα· τὸ γὰρ ἐνδέχεται τῷ ἔστιν ὁμοίως τάττεται, τὸ δὲ ἔστιν, οἷς ἂν προσκατηγορῆται, κατάφασιν ἀεὶ ποιεῖ καὶ πάντως, οἷον τὸ ἔστιν οὐκ ἀγαθόν ἢ ἔστιν οὐ λευκόν ἢ ἁπλῶς τὸ ἔστιν οὐ τοῦτο. δειχθήσεται δὲ καὶ τοῦτο διὰ τῶν ἑπομένων. κατὰ δὲ τὰς ἀντιστροφὰς ὁμοίως ἕξουσι ταῖς ἄλλαις.

IV. Διωρισμένων δὲ τούτων λέγομεν ἤδη διὰ τίνων καὶ πότε καὶ πῶς γίγνεται πᾶς συλλογισμός· ὕστερον δὲ λεκτέον περὶ ἀποδείξεως. πρότερον δὲ περὶ συλλογισμοῦ λεκτέον ἢ περὶ ἀποδείξεως διὰ τὸ καθόλου μᾶλλον εἶναι τὸν συλλογισμόν· ἡ μὲν γὰρ ἀπόδειξις συλλογισμός τις, ὁ συλλογισμὸς δὲ οὐ πᾶς ἀπόδειξις.

Ὅταν οὖν ὅροι τρεῖς οὕτως ἔχωσι πρὸς ἀλλήλους ὥστε τὸν ἔσχατον ἐν ὅλῳ εἶναι τῷ μέσῳ καὶ τὸν μέσον ἐν ὅλῳ τῷ πρώτῳ ἢ εἶναι ἢ μὴ εἶναι, ἀνάγκη τῶν ἄκρων εἶναι συλλογισμὸν τέλειον. καλῶ δὲ μέσον μὲν ὃ καὶ αὐτὸ ἐν ἄλλῳ καὶ ἄλλο ἐν τούτῳ ἐστίν, ὃ καὶ τῇ θέσει γίγνεται μέσον· ἄκρα δὲ τὸ αὐτό τε ἐν ἄλλῳ ὂν καὶ ἐν ᾧ ἄλλο ἐστίν. εἰ γὰρ τὸ Α κατὰ παντὸς τοῦ Β καὶ τὸ Β κατὰ παντὸς τοῦ Γ, ἀνάγκη τὸ Α κατὰ παντὸς τοῦ Γ κατηγορεῖσθαι· πρότερον γὰρ εἴρηται πῶς

[a] Chs. xiii. ff. [b] Ch. xlvi.
[c] In the *Posterior Analytics*.
[d] 24 b 28.

convert, whereas the particular negative does. This will become clear when we discuss the possible.[a]

For the present we may regard this much as clear, in addition to what we have already said: that the statement 'it is possible for A to apply to no B' or 'not to apply to some B' is affirmative in form; for the expression 'is possible' corresponds to 'is,' and the word 'is,' to whatever terms it is attached in predication, has always and without exception the effect of affirmation: *e.g.*, 'is not good' or 'is not white' or in general 'is not X.' This also will be proved later.[b] In respect of conversion these premisses will be governed by the same conditions as other affirmatives.

Figures and moods of syllogism.

IV. Having drawn these distinctions we can now state by what means, and when, and how every syllogism is effected. Afterwards we must deal with demonstration.[c] The reason why we must deal with the syllogism before we deal with demonstration is that the syllogism is more universal; for demonstration is a kind of syllogism, but not every syllogism is a demonstration.

The First Figure.

When three terms are so related to one another that the last is wholly contained in the middle and the middle is wholly contained in or excluded from the first, the extremes must admit of perfect syllogism.

Middle term.

By 'middle term' I mean that which both is contained in another and contains another in itself, and which is the middle by its position also; and by

Extreme terms.

'extremes' (*a*) that which is contained in another, and (*b*) that in which another is contained.

(1) Both premisses universal. Barbara.

For if A is predicated of all B, and B of all C, A must necessarily be predicated of all C. We have already explained[d] what we mean by saying that one term

25 b

τὸ κατὰ παντὸς λέγομεν. ὁμοίως δὲ καὶ εἰ τὸ

26 a

μὲν Α κατὰ μηδενὸς τοῦ Β τὸ δὲ Β κατὰ παντὸς τοῦ Γ, ὅτι τὸ Α οὐδενὶ τῷ Γ ὑπάρξει.

Εἰ δὲ τὸ μὲν πρῶτον παντὶ τῷ μέσῳ ὑπάρχει, τὸ δὲ μέσον μηδενὶ τῷ ἐσχάτῳ ὑπάρχει, οὐκ ἔσται συλλογισμὸς τῶν ἄκρων· οὐδὲν γὰρ ἀναγκαῖον συμβαίνει τῷ ταῦτα εἶναι· καὶ γὰρ παντὶ καὶ μηδενὶ ἐνδέχεται τὸ πρῶτον τῷ ἐσχάτῳ ὑπάρχειν, ὥστε οὔτε τὸ κατὰ μέρος οὔτε τὸ καθόλου γίγνεται ἀναγκαῖον· μηδενὸς δὲ ὄντος ἀναγκαίου διὰ τούτων οὐκ ἔσται συλλογισμός. ὅροι τοῦ παντὶ ὑπάρχειν ζῷον—ἄνθρωπος—ἵππος, τοῦ μηδενὶ ζῷον—ἄνθρωπος—λίθος.

Οὐδ᾽ ὅταν μήτε τὸ πρῶτον τῷ μέσῳ μήτε τὸ μέσον τῷ ἐσχάτῳ μηδενὶ ὑπάρχῃ, οὐδ᾽ οὕτως ἔσται συλλογισμός. ὅροι τοῦ ὑπάρχειν ἐπιστήμη—γραμμή—ἰατρική, τοῦ μὴ ὑπάρχειν ἐπιστήμη—γραμμή—μονάς.

Καθόλου μὲν οὖν ὄντων τῶν ὅρων δῆλον ἐν τούτῳ τῷ σχήματι πότε ἔσται καὶ πότε οὐκ ἔσται συλλογισμός, καὶ ὅτι ὄντος τε συλλογισμοῦ τοὺς ὅρους ἀναγκαῖον ἔχειν ὡς εἴπομεν, ἄν θ᾽ οὕτως ἔχωσιν, ὅτι ἔσται συλλογισμός.

Εἰ δ᾽ ὁ μὲν καθόλου τῶν ὅρων ὁ δ᾽ ἐν μέρει πρὸς τὸν ἕτερον, ὅταν μὲν τὸ καθόλου τεθῇ πρὸς τὸ μεῖζον ἄκρον ἢ κατηγορικὸν ἢ στερητικόν, τὸ δὲ ἐν μέρει πρὸς τὸ ἔλαττον κατηγορικόν, ἀνάγκη συλλογισμὸν εἶναι τέλειον, ὅταν δὲ πρὸς τὸ ἔλαττον ἢ καὶ ἄλλως πως ἔχωσιν οἱ ὅροι,

Celarent.

is predicated of all of another. Similarly too if A is predicated of none of B, and B of all of C, it follows that A will apply to no C.

AE-

If, however, the first term applies to all the middle, and the middle to none of the last, the extremes cannot admit of syllogism; for no conclusion follows necessarily from the fact that they are such, since it is possible for the first term to apply either to all or to none of the last, and so neither a particular nor a universal conclusion necessarily follows; and if no necessary conclusion follows from the premisses there can be no syllogism. The positive relation of the extremes may be illustrated by the terms animal—man—horse; the negative relation by animal—man—stone.

EE-

Again, when the first applies to none of the middle, and the middle to none of the last, here too there can be no syllogism. The positive relation of the extremes may be illustrated by the terms science—line—medicine; the negative relation by science—line—unit.

Thus if the terms are in a universal relation it is clear, so far as this figure is concerned, when there will be a syllogism and when there will not. It is clear also that if there is a syllogism the terms must be related as we have said; and that if they are so related, there will be a syllogism.

(2) One universal and one particular premiss.

If, however, one of the ⟨extreme⟩ terms is in a universal and the other in a particular relation to the remaining term, when the universal statement, whether affirmative or negative, refers to the major term, and the particular statement is affirmative and refers to the minor term, there must be a perfect syllogism; but when the universal statement refers to the minor term, or the terms are related in any

(From *Aristotle I*, (Loeb Classical Library), 1938, pp. 208-211.)

Syllogism (in the *Prior Analytics* of Aristotle):

"If A is predicated of all B, and B of all C, A must necessarily be predicated of all C."

Example: A: living thing.

B: animal.

C: horse.

"If all animals are living things, and all horses are animals, all horses must necessarily be living things.

Aristotelian natural philosophy

Geocentric cosmology. Theory of concentric spheres.

(A) Everything on the earth:

Four elements (earth, water, air and fire).

Move upwards and downwards.

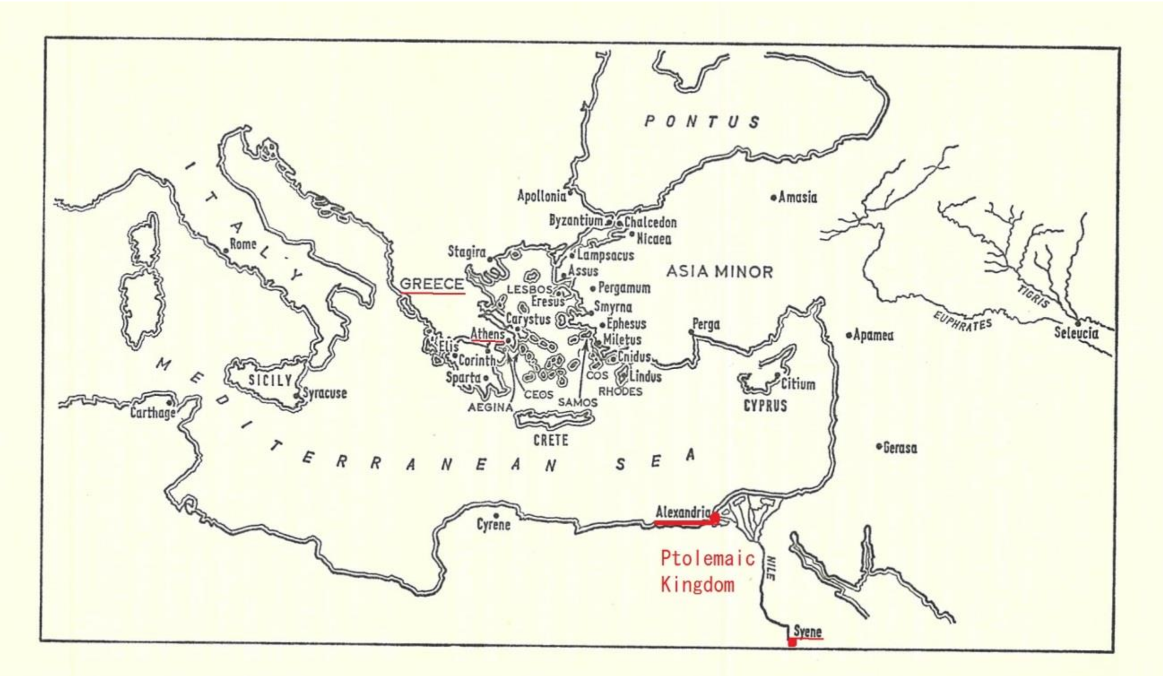
(B) Heavenly bodies:

The fifth element "ether" (or "aether").

Eternal, unvarying circular movements.

IV. Alexandrian Age
(The Age of Hellenistic Science)

From the death of Alexander the Great (323 BCE) to the Roman conquest of Ptolemaic Dynasty in Egypt (30 BCE).



(From Lloyd: *Greek Science after Aristotle*, London, 1973, with my additions.)

Alexandria in Egypt (Ptolemaic Kingdom) was a centre of Greek science. The Library and Museum were established in Alexandria.

Euclid of Alexandria (fl. around 300 BCE)

Euclid wrote some works on mathematics on mathematics and astronomy including the famous *Elements*.

Some works of Euclid:

***Elements*, --- on axiomatic geometry.**

----- For its English translation, see Heath: *Euclid, the thirteen books of The Elements*, 3 vols, 1956. There are Japanese translations of the *Elements* also. (one translation was published in 1971.).

***Data*, --- on the given information in geometrical problems.**

----- There are some English translations. One translation is included in Simson (1938). A Japanese translation by Ken Saito is included in 『エウクレイデス全集』Vol.4, 2008.

***Optics*, --- on the geometry of vision.**

----- See Burton (1945). A Japanese translation by Ken'ichi Takahashi is included in 『エウクレイデス全集』Vol.4, 2008.

***Catoptrics*, --- on the phenomena of reflecting light by mirrors.**

----- A Japanese translation by Ken'ichi Takahashi is included in 『エウクレイデス全集』Vol.4, 2008.

***Phaenomena*, --- on spherical astronomy.**

----- See Berggren and Thomas (1996).

And some other works.

Now Japanese translation of the complete works of Euclid is being published since 2008.

The *Elements* ----- Euclidian geometry (Axiomatic system)

BOOK I.

DEFINITIONS.

1. A **point** is that which has no part.
2. A **line** is breadthless length.
3. The extremities of a line are points.
4. A **straight line** is a line which lies evenly with the points on itself.
5. A **surface** is that which has length and breadth only.
6. The extremities of a surface are lines.
7. A **plane surface** is a surface which lies evenly with the straight lines on itself.
8. A **plane angle** is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
9. And when the lines containing the angle are straight, the angle is called **rectilineal.**
10. When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is **right,** and the straight line standing on the other is called a **perpendicular** to that on which it stands.
11. An **obtuse angle** is an angle greater than a right angle.
12. An **acute angle** is an angle less than a right angle.
13. A **boundary** is that which is an extremity of anything.
14. A **figure** is that which is contained by any boundary or boundaries.
15. A **circle** is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another;
16. And the point is called the **centre** of the circle.
17. A **diameter** of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line also bisects the circle.
18. A **semicircle** is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.
19. **Rectilineal figures** are those which are contained by straight lines, **trilateral** figures being those contained by three, **quadrilateral** those contained by four, and **multi-lateral** those contained by more than four straight lines.
20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle** that which has two of its sides alone equal, and a **scalene triangle** that which has its three sides unequal.
21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, an **obtuse-angled triangle** that which has an obtuse angle, and an **acute-angled triangle** that which has its three angles acute.
22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** that which is right-angled but not equilateral; a **rhombus** that which is equilateral but not right-angled; and a **rhomboid** that which has its opposite sides and angles equal to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called **trapezia.**
23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

POSTULATES.

Let the following be postulated:

1. To draw a straight line from any point to any point.

2. To produce a finite straight line continuously in a straight line.

3. To describe a circle with any centre and distance.

4. That all right angles are equal to one another.

5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

COMMON NOTIONS.

1. Things which are equal to the same thing are also equal to one another.

2. If equals be added to equals, the wholes are equal.

3. If equals be subtracted from equals, the remainders are equal.

[7] 4. Things which coincide with one another are equal to one another.

[8] 5. The whole is greater than the part.

(From Heath: *Euclid The Elements*, 1956, Vol.1. The above quotation is from pp.153-155, and the following quotation is from pp.241-242.) (There are Japanese translations of the *Elements* also. Now Japanese translation of the complete works of Euclid including *Elements* is being published.)

“Postulates” are now usually called “Axioms”.

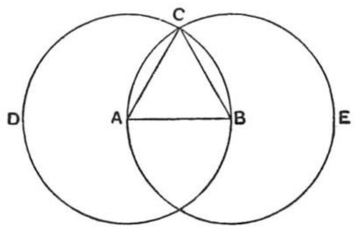
The axiomatic system is now a standard system of the basic theories of mathematics and physics.

Proposition 1.

On a given finite straight line to construct an equilateral triangle.

Let *AB* be the given finite straight line.

Thus it is required to construct an equilateral triangle on the straight line *AB*.



With centre *A* and distance *AB* let the circle *BCD* be described; [Post. 3]

again, with centre *B* and distance *BA* let the circle *ACE* be described; [Post. 3]

and from the point *C*, in which the circles cut one another, to the points *A*, *B* let the straight lines *CA*, *CB* be joined. [Post. 1]

Now, since the point *A* is the centre of the circle *CDB*,

AC is equal to *AB*. [Def. 15]

Again, since the point *B* is the centre of the circle *CAE*,

BC is equal to *BA*. [Def. 15]

But *CA* was also proved equal to *AB*;

therefore each of the straight lines *CA*, *CB* is equal to *AB*.

And things which are equal to the same thing are also equal to one another; [*C. N.* 1]

therefore *CA* is also equal to *CB*.

Therefore the three straight lines *CA*, *AB*, *BC* are equal to one another.

Therefore the triangle *ABC* is equilateral; and it has been constructed on the given finite straight line *AB*.

(Being) what it was required to do.

1. **On a given finite straight line.** The Greek usage differs from ours in that the definite article is employed in such a phrase as this where we have the indefinite. ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης, "on *the* given finite straight line," i.e. the finite straight line which we choose to take.

-3. **Let AB be the given finite straight line.** To be strictly literal we should have to translate in the reverse order "let the given finite straight line be the (straight line) *AB*"; but this order is inconvenient in other cases where there is more than one datum, e.g. in the *setting-out* of I. 2, "let the given point be *A*, and the given straight line *BC*," the awkwardness arising from the omission of the verb in the second clause. Hence I have, for clearness' sake, adopted the other order throughout the book.

8. **let the circle BCD be described.** Two things are here to be noted, (1) the elegant and practically universal use of the perfect passive imperative in constructions, γεγράφθω meaning of course "let it *have been* described" or "suppose it described," (2) the impossibility of expressing shortly in a translation the force of the words in their original order. κύκλος γεγράφθω ὁ ΒΓΔ means literally "let a circle have been described, the (circle, namely, which I denote by) *BCD*." Similarly we have lower down "let straight lines, (namely) the (straight lines) *CA*, *CB*, be joined," ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓΑ, ΓΒ. There seems to be no practicable alternative, in English, but to translate as I have done in the text.

13. **from the point C....** Euclid is careful to adhere to the phraseology of Postulate 1 except that he speaks of "joining" (ἐπεζεύχθωσαν) instead of "drawing" (γράφειν). He does not allow himself to use the shortened expression "let the straight line *FC* be joined" (without mention of the points *F*, *C*) until I. 5.

20. **each of the straight lines CA, CB,** ἑκατέρα τῶν ΓΑ, ΓΒ and 24. **the three straight lines CA, AB, BC,** αἱ τρεῖς αἱ ΓΑ, ΑΒ, ΒΓ. I have, here and in all similar expressions, inserted the words "straight lines" which are not in the Greek. The possession of the inflected definite article enables the Greek to omit the words, but this is not possible in English, and it would scarcely be English to write "each of *CA*, *CB*" or "the three *CA*, *AB*, *BC*."

Aristarchus of Samos (ca. 310 – 230 BCE)

Aristarchus' determination of the sizes and distances of the Sun and Moon

ARISTARCHUS OF SAMOS

ARISTARCHUS, *On the Sizes and Distances of the Sun and Moon.*

(Hypotheses.)

1. *That the moon receives its light from the sun.*

2. *That the earth is in the relation of a point and centre to the sphere in which the moon moves.*

3. *That, when the moon appears to us halved, the great circle which divides the dark and the bright portions of the moon is in the direction of our eye.*

4. *That, when the moon appears to us halved, its distance from the sun is then less than a quadrant by one-thirtieth part of a quadrant.*[1]

[1] i.e. $90° - 3°$ or $87°$. The true value is $89° 50'$.

5. *That the breadth of the earth's shadow is that of two moons.*

6. *That the moon subtends one-fifteenth part of a sign of the zodiac.*[1]

(Given these hypotheses) it is proved that:

1. *The distance of the sun from the earth is greater than eighteen times, but less than twenty times, the distance of the moon from the earth*: this follows from the hypothesis about the halved moon.

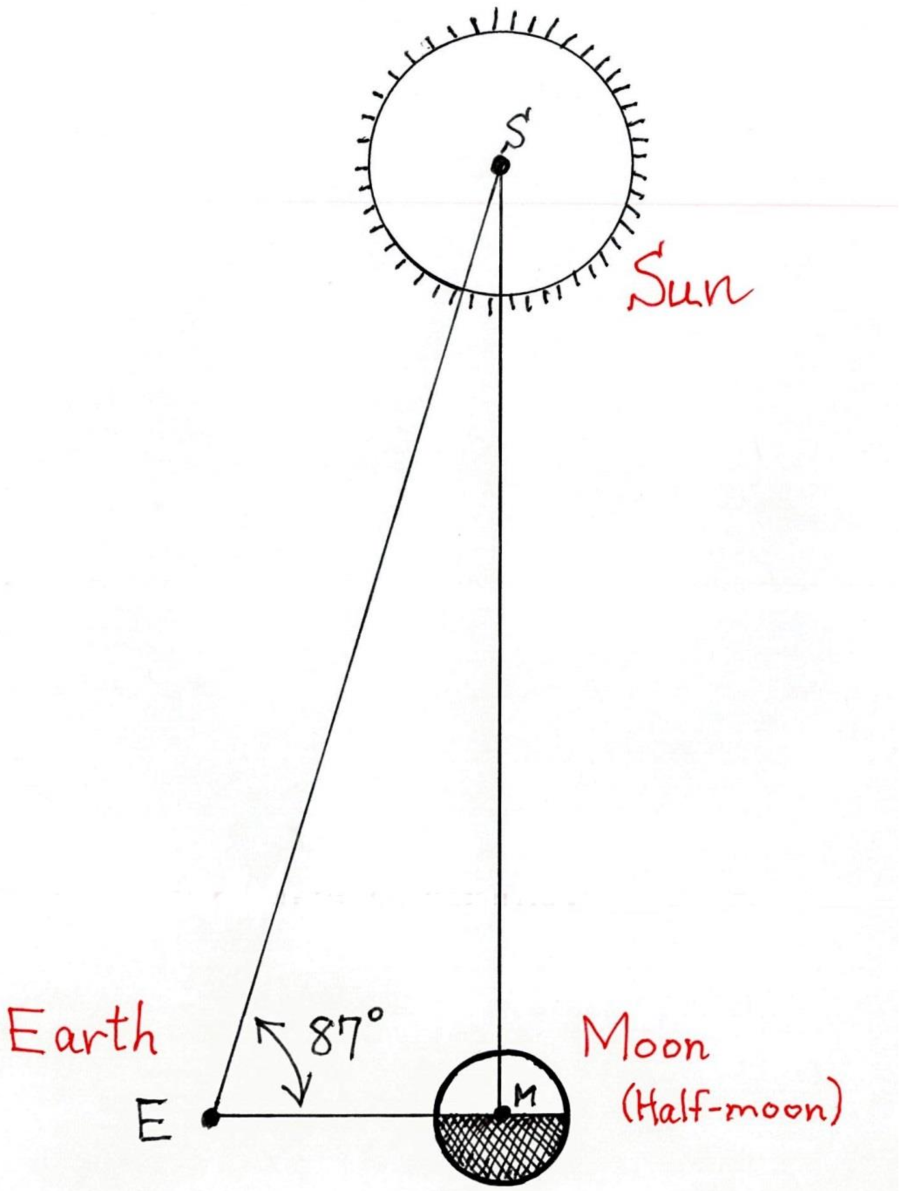
2. *The diameter of the sun has the same ratio as aforesaid to the diameter of the moon.*

3. *The diameter of the sun has to the diameter of the earth a ratio greater than that which* 19 *has to* 3, *but less than that which* 43 *has to* 6: this follows from the ratio thus discovered between the distances, the hypothesis about the shadow, and the hypothesis that the moon subtends one-fifteenth part of a sign of the zodiac.

[1] According to Archimedes, Aristarchus "discovered that the sun appeared to be about 1/720th part of the circle of the zodiac": that is, Aristarchus discovered (evidently at a date later than that of the treatise) the much more correct value of $\frac{1}{2}^{\circ}$ for the angular diameter of the sun or moon (for he maintained that both had the same angular diameter).

(From Heath: *Greek Astronomy*, 1932, pp.100 – 101.)

(Also see Japanese translation of the work of Aristarchus by Kusayama in Tamura (1972).)



Aristarchus' heliocentric system

The heliocentric system : Copernicus anticipated

ARCHIMEDES, *Psammites* (*Sand-reckoner*), c. I, I–IO.

There are some, King Gelon, who think that the number of the sand is infinite in multitude; and I mean by the sand not only that which exists about Syracuse and the rest of Sicily, but also that which is found in every region, whether inhabited or uninhabited. Again, there are some who, without regarding it as infinite, yet think that no number has been named which is great enough to exceed its multitude. And it is clear that they who hold this view, if they imagined a mass made up of sand as large in size as the mass of the earth, including in it all the seas and the hollows of the earth filled up to a height equal to that of the highest mountain, would be many times further still from recognizing that any number could be expressed which exceeded the multitude of the sand so taken. But I will try to show you, by means of geometrical proofs, which you will be able to follow, that, of the numbers named by me and given in the work which I sent to Zeuxippus, some exceed, not only the number of the mass of sand equal in size to the earth filled up in the way described, but also that of a mass equal in size to the universe. Now you are aware that "universe" is the name given by most astronomers to the sphere the centre of which is the centre of the earth, and the radius of which is equal to the straight line between the centre of the sun and the centre of the earth; this you have seen in the treatises written by astronomers.

But Aristarchus of Samos brought out a book consisting of certain hypotheses, in which the premisses lead to the conclusion that the universe is many times greater than that now so called. His hypotheses are that the fixed stars and the sun remain motionless, that the earth revolves about the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of the fixed stars, situated about the same centre as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the centre of the sphere bears to its surface.

Now it is easy to see that this is impossible; for, since the centre of the sphere has no magnitude, we cannot conceive it to bear any ratio whatever to the surface of the sphere. We must, however, take Aristarchus to mean this: since we conceive the earth to be, as it were, the centre of the universe, the ratio which the earth bears to what we describe as the "universe" is the same as the ratio which the sphere containing the circle in which he supposes the earth to revolve bears to the sphere of the fixed stars. For he adapts the proofs of the phenomena to a hypothesis of this kind, and in particular he appears to suppose the size of the sphere in which he represents the earth as moving to be equal to what we call the "universe."

I say then, that, even if a sphere were made up of sand to a size as great as Aristarchus supposes the sphere of the fixed stars to be, I shall still be able to prove that, of the numbers named in the "Principles," some exceed in multitude the number of the sand which is equal in size to the sphere referred to, provided that the following assumptions be made:

1. The perimeter of the earth is about 3,000,000 stades and not greater.[1]

It is true that some have tried, as you are, of course, aware, to prove that the said perimeter is about 300,000 stades. But I go farther and, putting the size of the earth at ten times the size that my predecessors thought it, I suppose its perimeter to be about 3,000,000 stades and not greater.

2. The diameter of the earth is greater than the diameter of the moon, and the diameter of the sun is greater than the diameter of the earth.

In this assumption I follow most of the earlier astronomers.

3. The diameter of the sun is about 30 times the diameter of the moon and not greater.

It is true that, of the earlier astronomers, Eudoxus declared it to be about nine times as great, and Phidias, my father, twelve times, while *Aristarchus tried to prove that the diameter of the sun is greater than 18 times, but less than 20 times, the diameter of the moon.* But I go even further than Aristarchus, in order that the truth of my proposition may be established beyond dispute, and I suppose the diameter of the sun to be about 30 times that of the moon and not greater.

4. The diameter of the sun is greater than the side of the chiliagon (a regular polygon of 1000 sides) inscribed in the greatest circle in the sphere of the universe.

I make this assumption because Aristarchus discovered that the sun appeared to be about $\frac{1}{720}$th part of the circle of the zodiac, and I myself tried, by a method which I

[1] Archimedes obviously here takes an extreme figure in order that he may be on the safe side.

will now describe, to find experimentally (by means of a mechanical contrivance), the angle subtended by the sun and having its vertex at the eye.

[In the end Archimedes finds, by sheer calculation, that the number of grains of sand that would be contained in a sphere of the size attributed to the universe is less than the number which we should express as 10^{63}.]

(From Heath: *Greek Astronomy*, 1932, pp.105 – 108.)

Also see Heath: *Aristarchus of Samos*, Oxford, The Clarendon Press, 1913, and/or Japanese translation of the works of Archimedes by Mita in Tamura (1972).

Archimedes of Syracuse (287? – 212 BCE)

An example from Archimedes' *The Method*

Heath (History of Greek mathematics II, p. 20) has the following to say about the general character of the works of Archimedes:

> The treatises are, without exception, monuments of mathematical exposition; the gradual revelation of the plan of attack, the masterly ordering of the propositions, the stern elimination of everything not immediately relevant to the purpose, the finish of the whole, are so impressive in their perfection as to create a feeling akin to awe in the mind of the reader. As Plutarch said, "It is not possible to find in geometry more difficult and troublesome questions or proofs set out in simpler and clearer propositions". There is at the same time a certain mystery veiling the way in which he arrived at his results. For it is clear that they were not *discovered* by the steps which lead up to them in the finished treatises. If the geometrical treatises stood alone, Archimedes might seem, as Wallis said, "as it were of set purpose to have covered up the traces of his investigation, as if he had grudged posterity the secret of his method of inquiry, while he wished to extort from them assent to his results". And indeed (again in the words of Wallis) "not only Archimedes but nearly all the ancients so hid from posterity their method of Analysis (though it is clear that they had one) that more modern mathematicians found it easier to invent a new Analysis than to seek out the old". A partial exception is now furnished by *The Method* of Archimedes, so happily discovered by Heiberg. In this book Archimedes tells us how he discovered certain theorems in quadrature and cubature, namely by the use of mechanics, weighing elements of a figure against elements of another simpler figure the mensuration of which was already known. At the same time he is careful to insist on the difference between (1) the means which may be sufficient to suggest the truth of theorems, although not furnishing scientific proofs of them, and (2) the rigorous demonstrations of them by orthodox geometrical methods which must follow before they can be finally accepted as established:
>
> "Certain things", he says, "first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge."

We begin with a discussion of this work from which we can best acquire an understanding of Archimedes' line of thought.

The "Method".

In 1906, the Danish philologist Heiberg, who had already provided us with so many excellent text-editions (i.a. of Euclid and of Archimedes), went to Constantinople to study a papyrus from the library of the San Sepulchri monastery in Jerusalem. It was a so-called "palimpsest", originally covered with Greek letters, obviously a mathematical text with diagrams, afterwards scraped off by monks and written upon anew. Heiberg succeeded in restoring and deciphering nearly the whole of the old text. It contained parts of various known works of Archimedes, but in addition the extremely important work "Method", which had been believed to be lost. We shall now briefly indicate the contents of this work.

1. *Area of the parabolic segment.*

The first example of Archimedes is at the same time best suited to explain his mechanical method. Let $AB\Gamma$ be a parabolic segment, bounded by a straight line $A\Gamma$ and a parabola $AB\Gamma$. Through Δ, the midpoint of $A\Gamma$, a line ΔBE is drawn, parallel to the axis of the parabola. Now Archimedes states that the segment $AB\Gamma$ is $1\frac{1}{3}$ times as large as the triangle $AB\Gamma$.

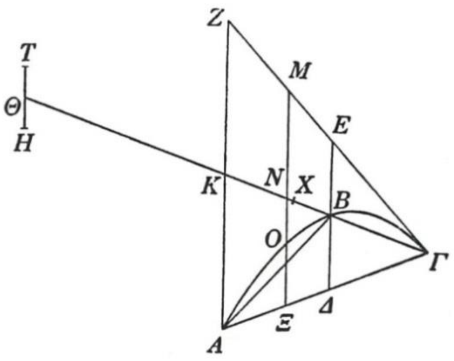


Fig. 71.

Draw AZ, parallel to ΔB, to its point of intersection Z with the tangent line ΓEZ. Extend ΓB beyond its intersection K with AZ and make $K\Theta = K\Gamma$. Now consider $\Gamma\Theta$ as a lever with fulcrum at K. Draw also $M\Xi$, parallel to $E\Delta$ through an arbitrary point O of the parabola.

It follows from properties of the parabola, which Archimedes assumes as known that $B\Delta = BE$, so that $N\Xi = NM$ and $KA = KZ$, and furthermore

(1) $$\Xi M : \Xi O = A\Gamma : A\Xi = K\Gamma : KN = K\Theta : KN.$$

Now we suspend, at the other end Θ of the lever, a line segment $TH = \Xi O$. From the law of the lever, which was first obtained by Archimedes himself in his treatise on the equilibrium of plane figures, it follows then that this segment TH will be in equilibrium with the segment $M\Xi$, placed where it is. For, the proportionality (1) states exactly that the weights of the two segments are inversely proportional to their lever arms. The same conclusion holds for all segments drawn in the triangle $A\Gamma Z$, parallel to ΔE. In the positions which they occupy, they are in equilibrium with their sections within the parabola, if transferred to the point Θ.

Thus far everything is completely rigorous. Now comes the crux of the method: *because the triangle $A\Gamma Z$ consists of all lines (like ΞM), which can be drawn in the triangle, and because the parabolic segment $AB\Gamma$ consists of all lines, like ΞO, within the parabola, therefore the triangle $A\Gamma Z$, placed where it is, will be in equilibrium with the parabolic segment, placed with its centroid at Θ, so that K is their common center of gravity.*

Now we are practically at home. For the centroid of the triangle $A\Gamma Z$ is at the point X, such that $KX = \frac{1}{3}K\Gamma$. Since now the lever arm $K\Theta$ of the parabolic segment is three times as long as the arm KX to the centroid of the triangle, and since the triangle is in equilibrium with the segment, the weight of the triangle must be equal to three times that of the segment. But triangle $A\Gamma Z$ is twice as large as $A\Gamma K$, and hence four times as large as triangle $AB\Gamma$; hence the parabolic segment is equal to $^4/_3$ of triangle $AB\Gamma$.

Archimedes remarks himself that this argument does not give a rigorous proof of the proposition; nevertheless it carries conviction.

To conceive of a parabolic segment or of a triangle as the sum of infinitely many line segments, is closely akin to the idea of Leibniz, who thought of the integral $\int y\,dx$ as the sum of infinitely many terms $y\,dx$. But, in contrast with Leibniz, Archimedes is fully aware that this conception is, as a matter of fact, incorrect and that the heuristic derivation should be supplemented by a rigorous proof.

(From van der Waerden: *Science Awakening I*, 1961, pp.212 – 214.)

Also see Heath: *The* Method *of Archimedes*, Cambridge, 1912 and/or Japanese translation of the works of Archimedes by Mita in Tamura (1972).

In the above discussion, some propositions in the "Quadrature of the parabola" and the "On the equilibrium of planes" of Archimedes are used. Let us see the beginning of the "Quadrature of the parabola".

QUADRATURE OF THE PARABOLA.

"ARCHIMEDES to Dositheus greeting.

"When I heard that Conon, who was my friend in his lifetime, was dead, but that you were acquainted with Conon and withal versed in geometry, while I grieved for the loss not only of a friend but of an admirable mathematician, I set myself the task of communicating to you, as I had intended to send to Conon, a certain geometrical theorem which had not been investigated before but has now been investigated by me, and which I first discovered by means of mechanics and then exhibited by means of geometry. Now some of the earlier geometers tried to prove it possible to find a rectilineal area equal to a given circle and a given segment of a circle; and after that they endeavoured to square the area bounded by the section of the whole cone* and a straight line, assuming lemmas not easily conceded, so that it was recognised by most people that the problem was not solved. But I am not aware that any one of my predecessors has attempted to square the segment bounded by a straight line and a section of a right-angled cone [a parabola], of which problem I have now discovered the solution. For it is here shown that every segment bounded by a straight line and a section of a right-angled cone [a parabola] is four-thirds of the triangle which has the same base and equal height with the segment, and for the demonstration

* There appears to be some corruption here: the expression in the text is τᾶς ὅλου τοῦ κώνου τομᾶς, and it is not easy to give a natural and intelligible meaning to it. The section of 'the whole cone' might perhaps mean a section cutting right through it, i.e. an ellipse, and the 'straight line' might be an axis or a diameter. But Heiberg objects to the suggestion to read τᾶς ὀξυγωνίου κώνου τομᾶς, in view of the addition of καὶ εὐθείας, on the ground that the former expression always signifies the whole of an ellipse, never a segment of it (*Quaestiones Archimedeae*, p. 149).

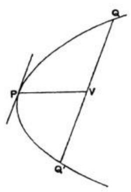
234 ARCHIMEDES

of this property the following lemma is assumed: that the excess by which the greater of (two) unequal areas exceeds the less can, by being added to itself, be made to exceed any given finite area. The earlier geometers have also used this lemma; for it is by the use of this same lemma that they have shown that circles are to one another in the duplicate ratio of their diameters, and that spheres are to one another in the triplicate ratio of their diameters, and further that every pyramid is one third part of the prism which has the same base with the pyramid and equal height; also, that every cone is one third part of the cylinder having the same base as the cone and equal height they proved by assuming a certain lemma similar to that aforesaid. And, in the result, each of the aforesaid theorems has been accepted* no less than those proved without the lemma. As therefore my work now published has satisfied the same test as the propositions referred to, I have written out the proof and send it to you, first as investigated by means of mechanics, and afterwards too as demonstrated by geometry. Prefixed are, also, the elementary propositions in conics which are of service in the proof (στοιχεῖα κωνικὰ χρεῖαν ἔχοντα ἐς τὰν ἀπόδειξιν). Farewell."

Proposition 1.

If from a point on a parabola a straight line be drawn which is either itself the axis or parallel to the axis, as PV, and if QQ′ be a chord parallel to the tangent to the parabola at P and meeting PV in V, then

$$QV = VQ'.$$



Conversely, *if $QV = VQ'$, the chord QQ' will be parallel to the tangent at P.*

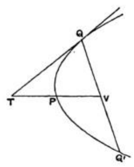
* The Greek of this passage is: συμβαίνει δὲ τῶν προειρημένων θεωρημάτων ἕκαστον μηδὲν ἧσσον τῶν ἄνευ τούτου τοῦ λήμματος ἀποδεδειγμένων πεπιστευκέναι. Here it would seem that πεπιστευκέναι must be wrong and that the passive should have been used.

QUADRATURE OF THE PARABOLA. 235

Proposition 2.

If in a parabola QQ′ be a chord parallel to the tangent at P, and if a straight line be drawn through P which is either itself the axis or parallel to the axis, and which meets QQ′ in V and the tangent at Q to the parabola in T, then

$$PV = PT.$$



Proposition 3.

If from a point on a parabola a straight line be drawn which is either itself the axis or parallel to the axis, as PV, and if from two other points Q, Q′ on the parabola straight lines be drawn parallel to the tangent at P and meeting PV in V, V′ respectively, then

$$PV : PV' = QV^2 : Q'V'^2.$$

"*And these propositions are proved in the elements of conics.**"

Proposition 4.

If Qq be the base of any segment of a parabola, and P the vertex of the segment, and if the diameter through any other point R meet Qq in O and QP (produced if necessary) in F, then

$$QV : VO = OF : FR.$$

Draw the ordinate RW to PV, meeting QP in K.

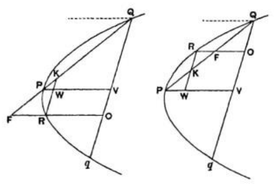
* i.e. in the treatises on conics by Euclid and Aristaeus.

236 ARCHIMEDES

Then $PV : PW = QV^2 : RW^2$;

whence, by parallels,

$$PQ : PK = PQ^2 : PF^2.$$



In other words, PQ, PF, PK are in continued proportion; therefore

$$PQ : PF = PF : PK$$
$$= PQ \pm PF : PF \pm PK$$
$$= QF : KF.$$

Hence, by parallels,

$$QV : VO = OF : FR.$$

[It is easily seen that this equation is equivalent to a change of axes of coordinates from the tangent and diameter to new axes consisting of the chord Qq (as axis of x, say) and the diameter through Q (as axis of y).

For, if $QV = a$, $PV = \frac{a^2}{p}$, where p is the parameter of the ordinates to PV.

Thus, if $QO = x$, and $RO = y$, the above result gives

$$\frac{a}{x - a} = \frac{OF}{OF - y},$$

whence $$\frac{a}{2a - x} = \frac{OF}{y} = \frac{x \cdot \frac{a}{p}}{y},$$

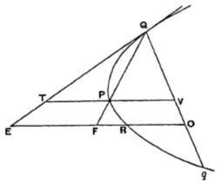
or $$py = x(2a - x).]$$

Proposition 5.

If Qq be the base of any segment of a parabola, P the vertex of the segment, and PV its diameter, and if the diameter of the parabola through any other point R meet Qq in O and the tangent at Q in E, then

$$QO : Oq = ER : RO.$$

Let the diameter through R meet QP in F.



Then, by Prop. 4,

$$QV : VO = OF : FR.$$

Since $QV = Vq$, it follows that

$$QV : qO = OF : OR \quad \text{..................(1).}$$

Also, if VP meet the tangent in T,

$$PT = PV, \text{ and therefore } EF = OF.$$

Accordingly, doubling the antecedents in (1), we have

$$Qq : qO = OE : OR,$$

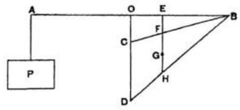
whence $$QO : Oq = ER : RO.$$

Propositions 6, 7*.

Suppose a lever AOB placed horizontally and supported at its middle point O. Let a triangle BCD in which the angle C is right or obtuse be suspended from B and O, so that C is attached to O and CD is in the same vertical line with O. Then, if P be such an area as, when suspended from A, will keep the system in equilibrium,

$$P = \tfrac{1}{3} \triangle BCD.$$

Take a point E on OB such that $BE = 2OE$, and draw EFH parallel to OCD meeting BC, BD in F, H respectively. Let G be the middle point of FH.



Then G is the centre of gravity of the triangle BCD.

Hence, if the angular points B, C be set free and the triangle be suspended by attaching F to E, the triangle will hang in the same position as before, because EFG is a vertical straight line. "For this is proved†."

Therefore, as before, there will be equilibrium.

Thus $$P : \triangle BCD = OE : AO$$
$$= 1 : 3,$$
or $$P = \tfrac{1}{3} \triangle BCD.$$

* In Prop. 6 Archimedes takes the separate case in which the angle BCD of the triangle is a right angle so that C coincides with O in the figure and F with E. He then proves, in Prop. 7, the same property for the triangle in which BCD is an obtuse angle, by treating the triangle as the difference between two right-angled triangles BOD, BOC and using the result of Prop. 6. I have combined the two propositions in one proof, for the sake of brevity. The same remark applies to the propositions following Props. 6, 7.

† Doubtless in the lost book περὶ ζυγῶν. Cf. the Introduction, Chapter II., *ad fin.*

(From Heath: *The Works of Archimedes*, Cambridge, 1897, pp.233 – 238.)

And also, some of the propositions in the "On the equilibrium of planes" of Archimedes should be considered,

Proposition 1.: Weights which balance at equal distances are equal.

Proposition 2.: Unequal weights at equal distances will not balance but will incline towards the greater weight.

Proposition 3.: Unequal weights will balance at unequal distances, the greater weight being at the lesser distance.

Proposition 4.: If two equal weights have not the same centre of gravity, the centre of gravity of both taken together is at the middle point of the line joining their centres of gravity.

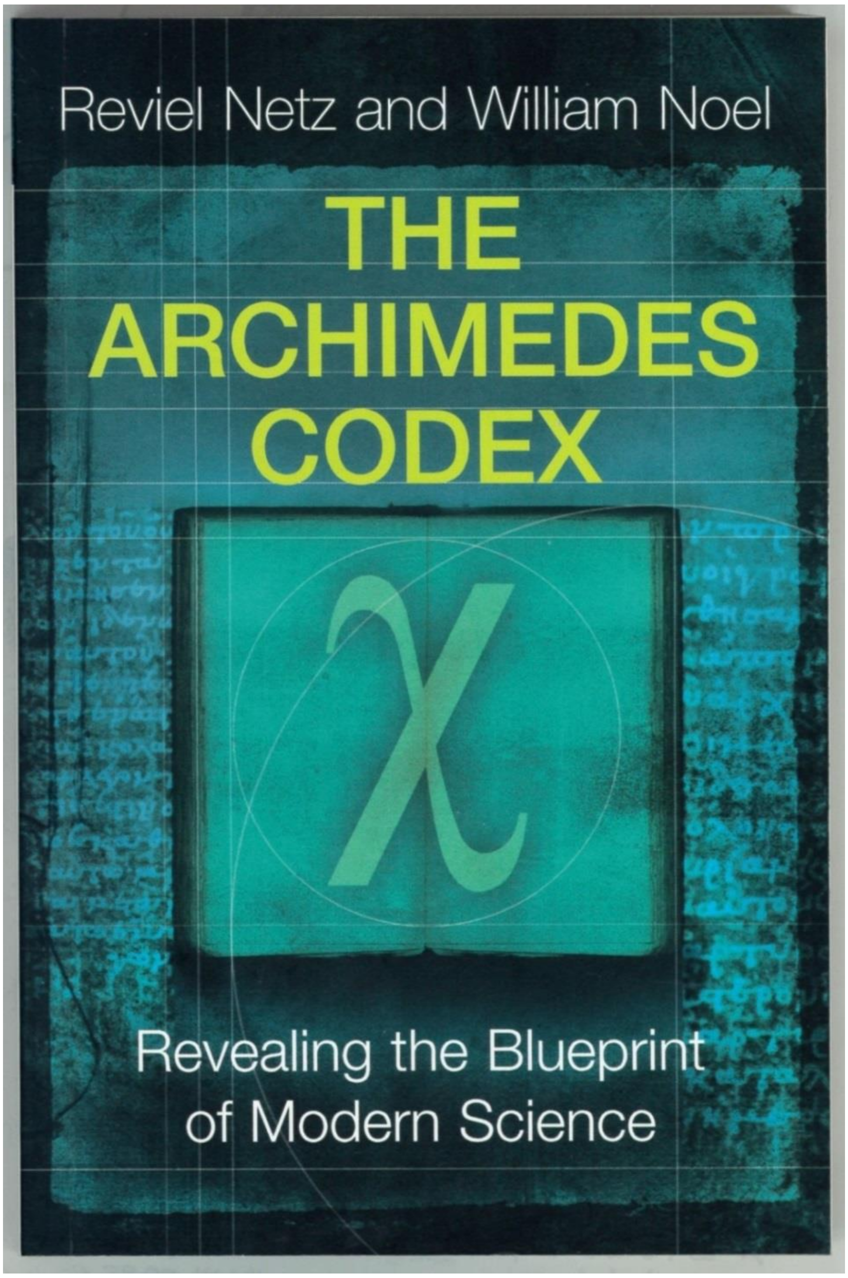
Proposition 5.: If three equal magnitudes have their centres of gravity on a straight line at equal distances, the centre of gravity of the system will coincide with that of the middle magnitude.

Propositions 6, 7.: Two magnitudes, whether commensurable [Prop. 6] orr incommensurable [Prop. 7], balance at distances reciprocally proportional to the magnitude.

(For more information, see Heath: *The Works of Archimedes*, Cambridge, 1897, and/or Japanese translation by Mita in Tamura (1972).)

The Archimedes codex

A codex of Archimedes was sold at an auction in 1998, and is being studied now. The following is an interesting document of this research.



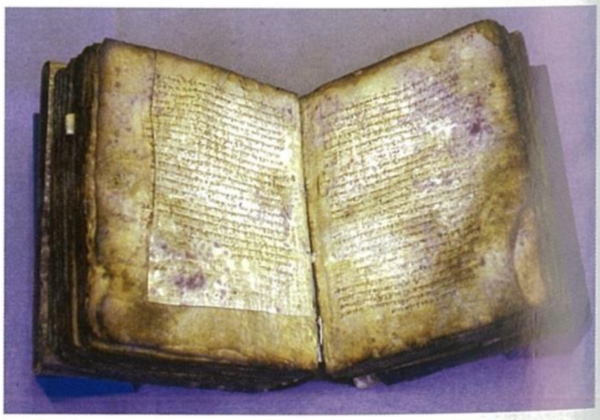
(Nets and Noel: *The Archimedes Codex*, A Phoenix Paperback, London, Orion Books Ltd., 2008. There is a Japanese translation of this book: リヴィエル・ネッツ/ウィリアム・ノエル（吉田監訳）『解読！ アルキメデス写本』、光文社、2008.)

(“Codex” ----- book-style manuscript (handwritten work).)

解読！アルキメデス写本

羊皮紙から甦った天才数学者

1999年1月19日、ウォルターズ美術館に到着したアルキメデスのパリンプセスト。



パリンプセストを開いたところ。右側のページには、アルキメデスの『方法』命題14の唯一のテキストが収められている。目に見えるのは祈禱書の文章だけだ。下部右側のへこみは、ページの材料となった山羊の脇腹の部分である。

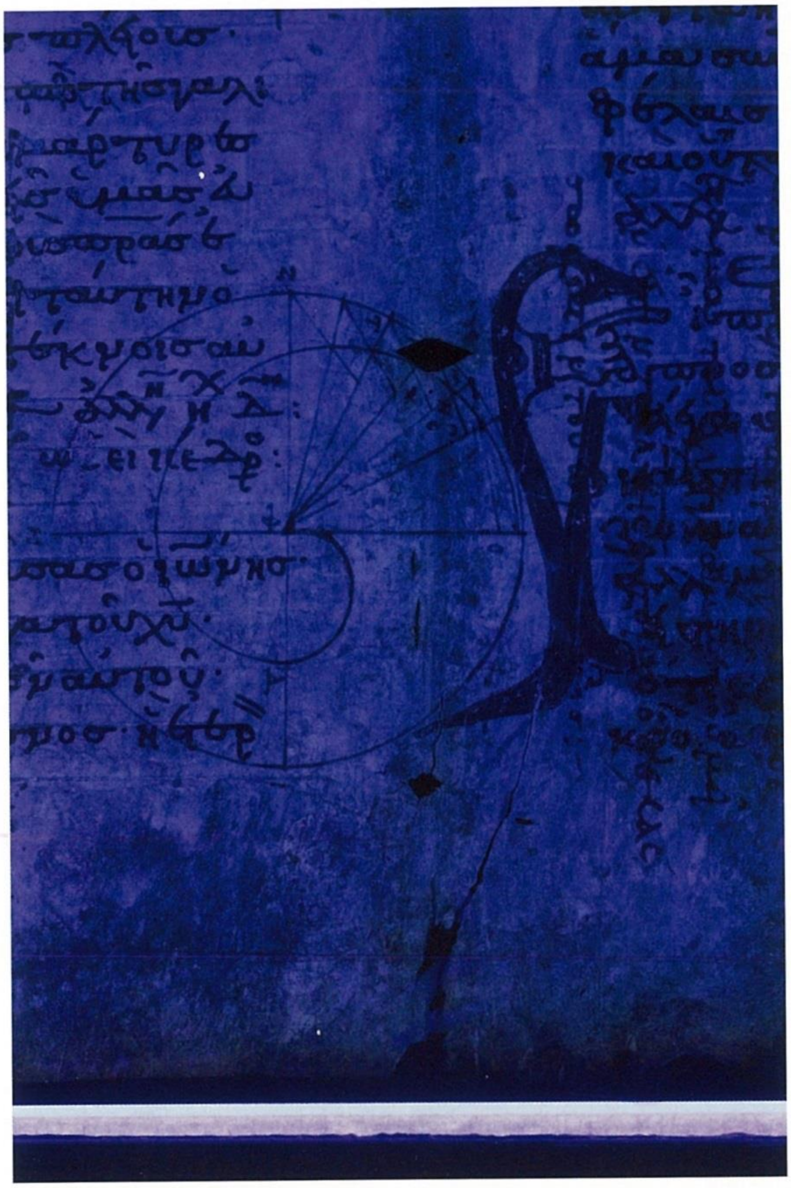
1

(As the above mentioned English paperback edition does not have coloured pictures, the pictures quoted here are quoted from its Japanese translation.)

The above figures are: Archimedes Codex, or Palimpsest containing the works of Archimedes.

("Palimpsest" ----- the parchment (writing material made from skins of animals) used to make it has been scraped more than once.)

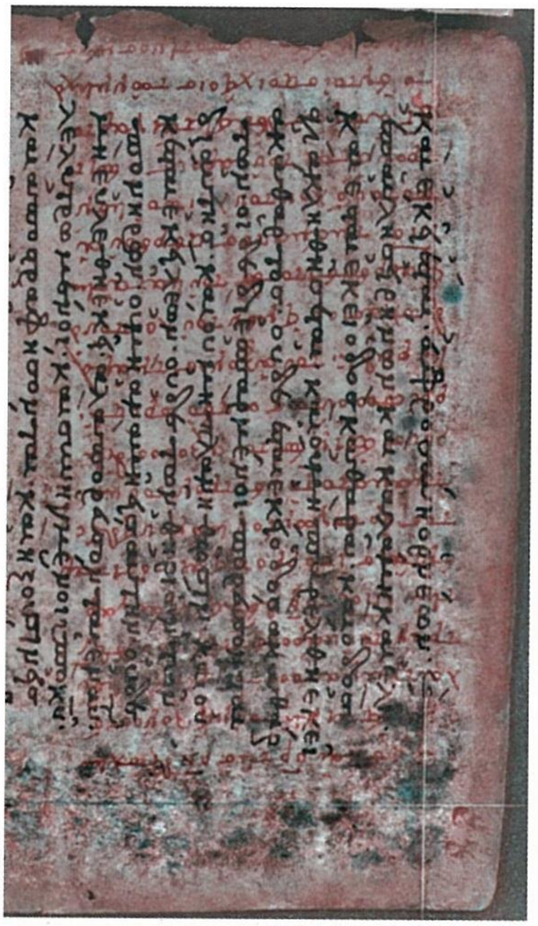
Only a prayer book can be seen by visible light. The works of Archimedes have been scraped, and then the prayer book was written.



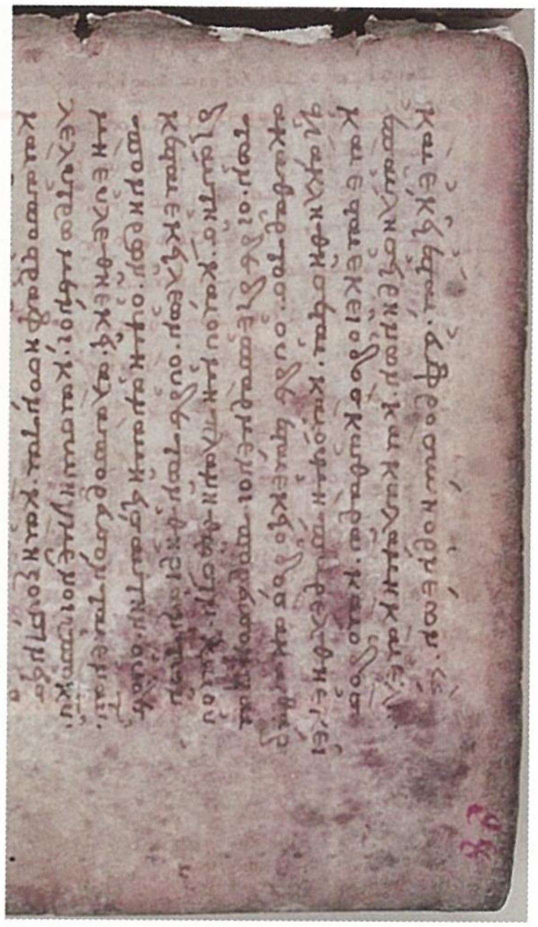
蛍光紫外線の鮮やかな光質で美しく変身したアルキメデスのパリンプセスト。アルキメデスの『螺線について』の命題21の図形が描かれている（第4章参照）。祈禱書の文字と装飾が重なっているため見づらい。祈禱書の冒頭の装飾文字のひとつ（図形の直線上の右上にある「手」）は、図形の直線を（衣服の）「袖」として見えるように配置されている。

The works of Archimedes can be seen by ultraviolet light.

In the above picture, a figure in Archimrdes’ “On Spirals” can be seen.



同じページに擬似カラー画像処理をしたもの。通常光で撮影した画像と紫外線で撮影した画像とを組み合わせ、キース・ノックスが考案した。



通常光で見たパリンプセストの細部。アルキメデスの文章はほとんど見えない。

(Right:)　Only the prayer book can be seen by visible light.

(Left:)　A combination of the images by visible light and ultraviolet light.　The work of Archimedes can be seen under the prayer book.

The above mentioned book "Nets and Noel: *The Archimedes Codex*, (『解読！ アルキメデス写本』)" is well written, and is highly recommended to read,

Eratosthenes of Cyrene

(around the latter half of the 3rd century BCE)

Eratosthenes' determination of the size of the earth

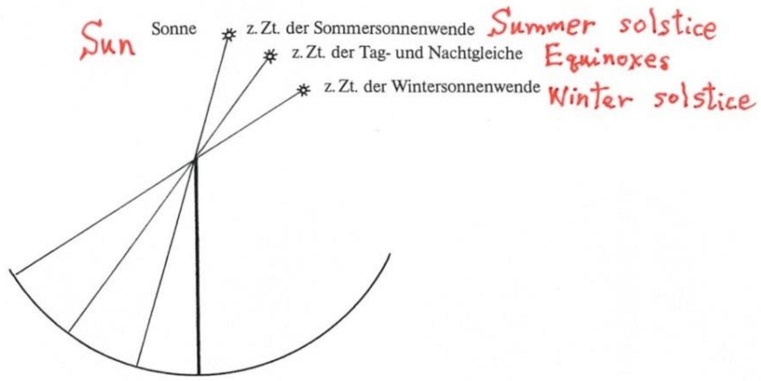


Abb. 2.44

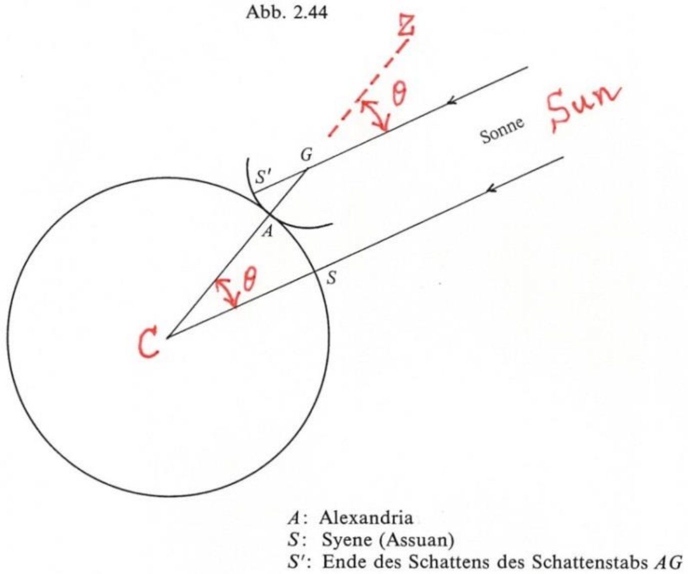


Abb. 2.45

(From Gericke: *Mathematik in Antike und Orient*, Berlin,1984, p.149 with my notes.)

Alexandria and Syene were assumed to be in the same meridian.
At the time of summer solstice, the Sun was at the zenith of Syene.
At the same time, the arc AS' was one-fiftieth part of its proper circle.
The distance AS = 5000 stades.

⇒ **The complete great circle of the earth = 250000 stades.**

Apollonius of Perga

(aroud the latter half of the 3rd century BCE)

Apollonius' study of conic section

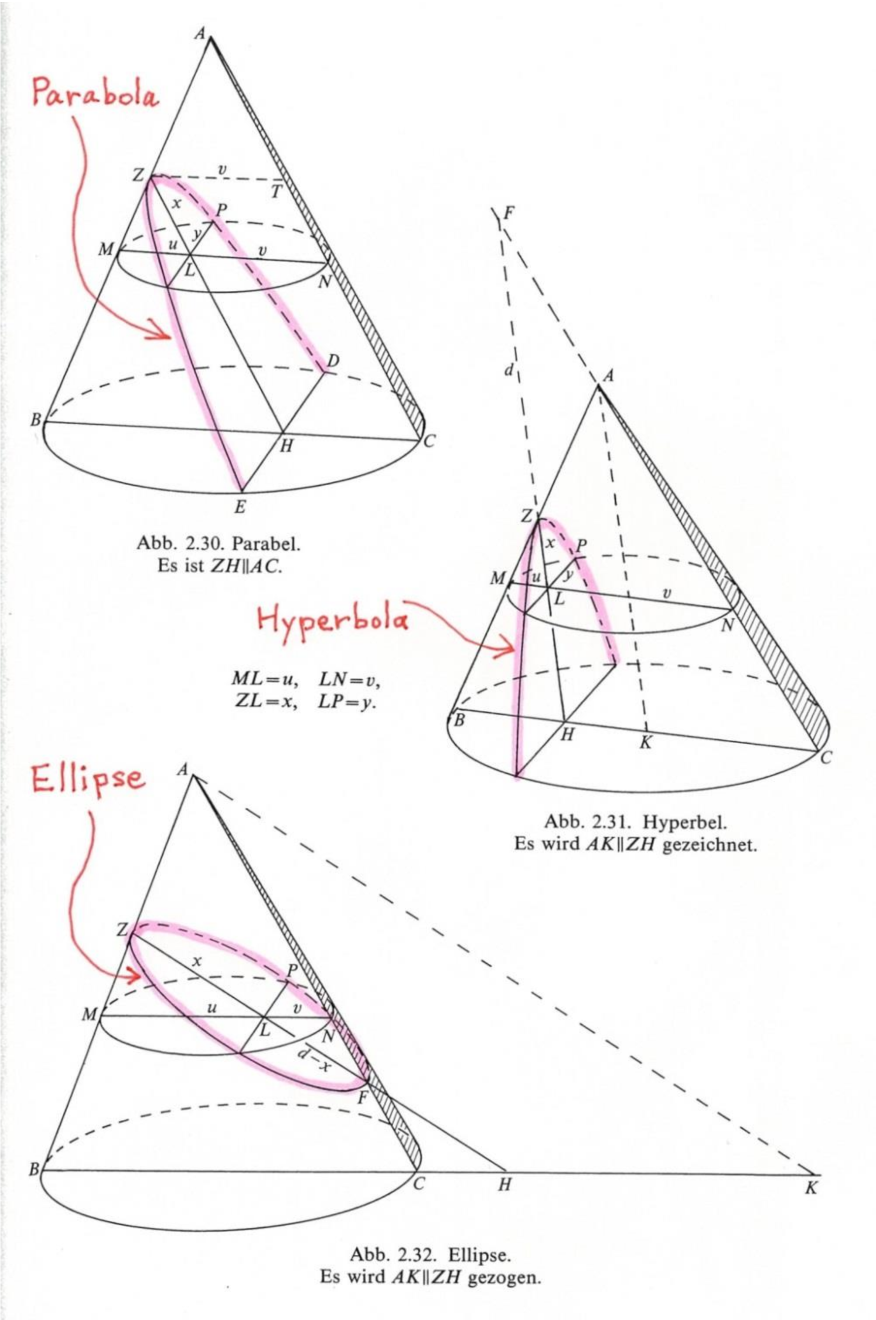


Abb. 2.30. Parabel.
Es ist $ZH \parallel AC$.

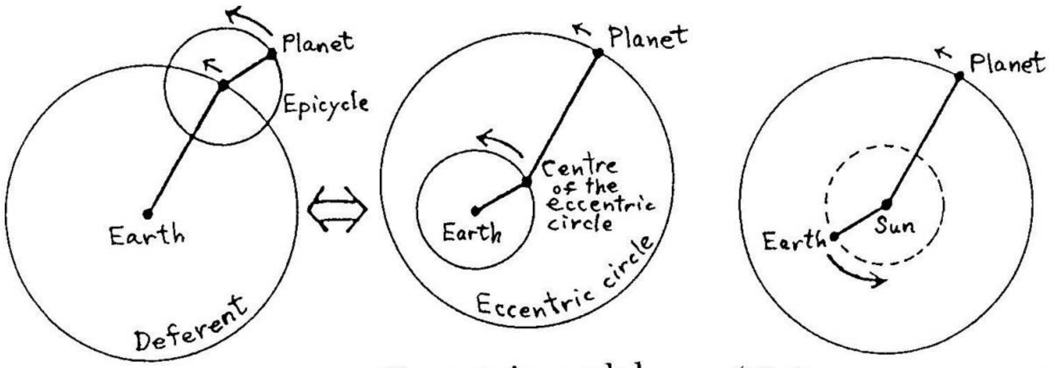
Abb. 2.31. Hyperbel.
Es wird $AK \parallel ZH$ gezeichnet.

Abb. 2.32. Ellipse.
Es wird $AK \parallel ZH$ gezogen.

(From Gericke: *Mathematik in Antike und Orient*, Berlin,1984, p.133 with my notes.)

For more detail, see Heath: *Apollonius of Perga*, Cambridge, Cambridge University Press, 1896.

Apollonius' planetary model



Epicyclic model　　Eccentric model　　(Heliocentric model)

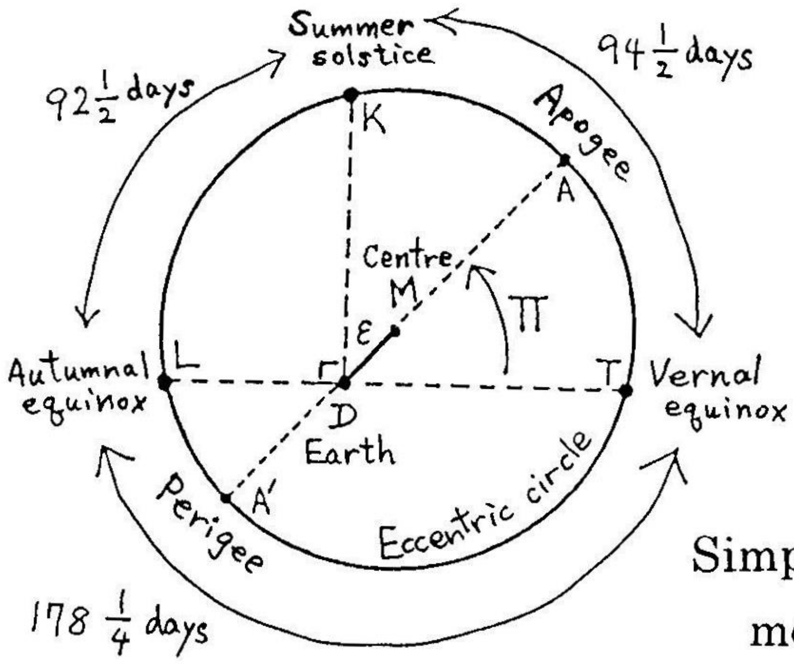
We do not know who invented the eccentric and epicyclic models, but the first astronomer who mathematically treated these models seems to be Apollonius (ca. end of the 3rd century BC). Ptolemy wrote in his *Almagest* (XII.1) that Apollonius explained the retrograde motion of planets by the epicyclic model as well as the eccentric model which is mathematically equivalent to the epicyclic model.

Hipparchus of Nicaea (2nd century BCE)

The precession of the equinoxes:

Hipparchus discovered the precession of the equinoxes. The equinoxes move westward along the ecliptic relative to the fixed stars.

The equation of centre of the sun and moon was explained by Hipparchus using the eccentric model as well as the epicyclic model which is mathematically equivalent to the eccentric model. Ptolemy explained the method of Hipparchus in his *Almagest* (III~IV). The model of Hipparchus is that the sun (or moon) revolves along an eccentric circle with a constant speed. Let this model be called “Simple eccentric model”.



Simple eccentric model of the solar orbit

According to the *Almagest* (III.4), the solar theory of Hipparchus is as follows. Hipparchus based on the observational data that the period from the vernal equinox to the summer solstice is 94½ days, and that the period from the summer solstice to the autumnal equinox is 92½ days, and determined the eccentric distance ε and the longitude of the apogee Π of the eccentric circle (radius = 1). If the length of a year is assumed to be 365¼ days, the period from the autumnal equinox to the vernal equinox becomes 178¼ days. From these data, ε and Π can be determined by plane geometry. In this model, the eccentric distance ε is a double of the modern eccentricity.

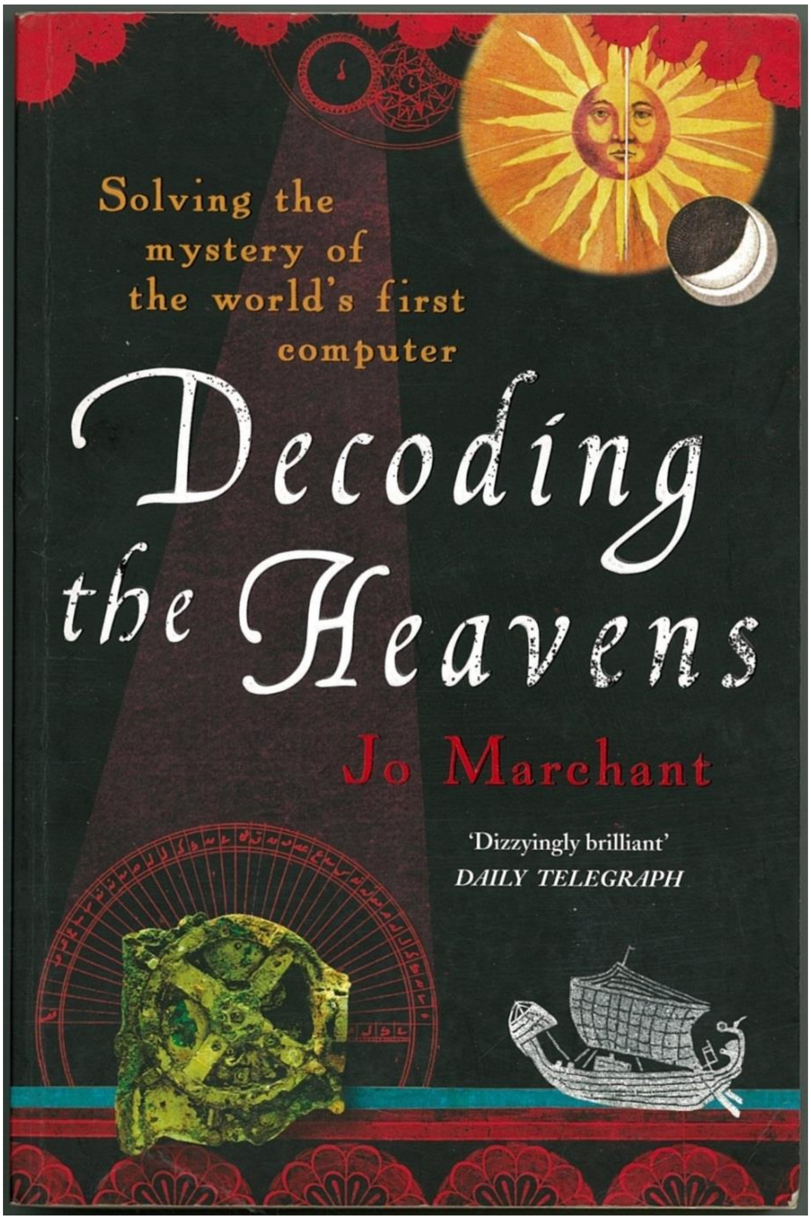
According to the *Almagest* (IV), Hipparchus also determined the orbit of the moon (with certain error) from the observational data of three lunar eclipses.

Antikythera mechanism

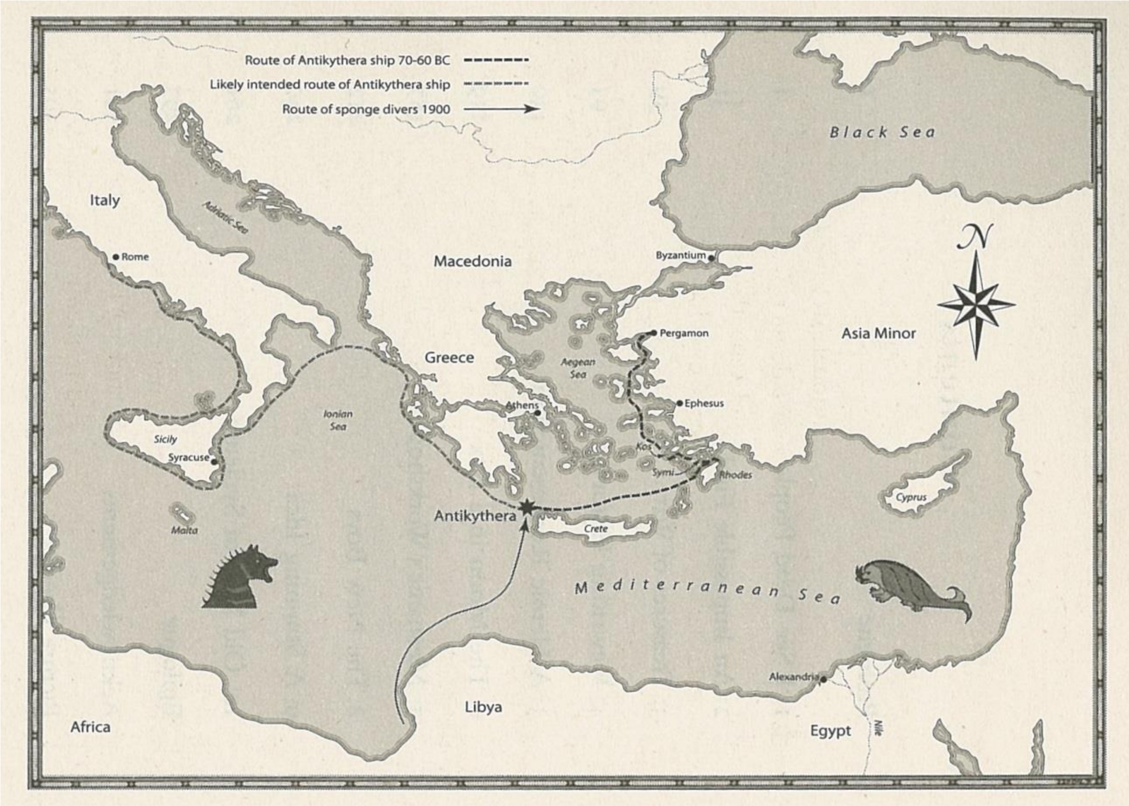
(1st century BCE)

Some fragments of the Antikythera mechanism were discovered from a shipwreck (1st century BCE) at Antikythera in the Mediterranean in 1901. This is a kind of astronomical machine.

32



(Marchant, Jo: *Decoding the Heavens*, first published in Great Britain in 2008 by William Heinemann; Paperback: London, Windmill Books, 2009. Japanese translation: ジョー・マーチャント（木村訳）『アンティキテラ 古代ギリシアのコンピュータ』、文藝春秋、2009)



A fragment of the Antikythera mechanism.

(The above mentioned English paperback edition has coloured pictures, but I quote pictures from Japanese translation just for my convenience.)

Reconstructions of the Antikythera mechanism.

(Right:) Derek de Solla Price.

(Left above:) Allan Bromley.

(Left below:) Michael Wright.

The above mentioned book "Marchant, Jo: *Decoding the Heavens*, (『アンティキテラ古代ギリシアのコンピュータ』)" is well written, and is recommended to read,

Philosophical schools in the age of Hellenism

Besides Platonism and Peripateticism (Aristoterianism), there were philosophical schools in the age of Hellenism, such as Epicureanism, Stoicism, Pyrrhonism (Skepticism) etc.

Epicureans developed atomism.　Stoics had their own natural philosophy.

V. Greco-Roman Period

Heron of Alexandria (around 60 CE)

Applied geometry and applied mechanics.

Measuring instruments and machines.

Ptolemy of Alexandria

(2nd century CE)

Ptolemy (Claudius Ptolemaeus) was a great astronomer, mathematician and astronomer. He established Ptolemaic geocentric planetary system.

Ptolemy's works:

Almagest **(a treatise of astronomy)**

The firsts English translation is Taliaferro (1952). A new scholarly translation is Toomer (1998). For its Japanese translation, see Yabuuti (1949-58).

Geography

See Berggren and Jones (2000), and/or Nakatsukasa (1986) (Japanese translation of the text).

Harmonics **(on musical theory)**

See Barker: *Scientific Method in Ptolemy's 'Harmonics'*, Cambridge, (2000), and/or Yamamoto (2008) (Japanese translation).

Tetrabiblos **(on astrology)**

(See *Ptolemy Tetrabiblos*, (Greek and English), Loeb Classical Library, 1940.)

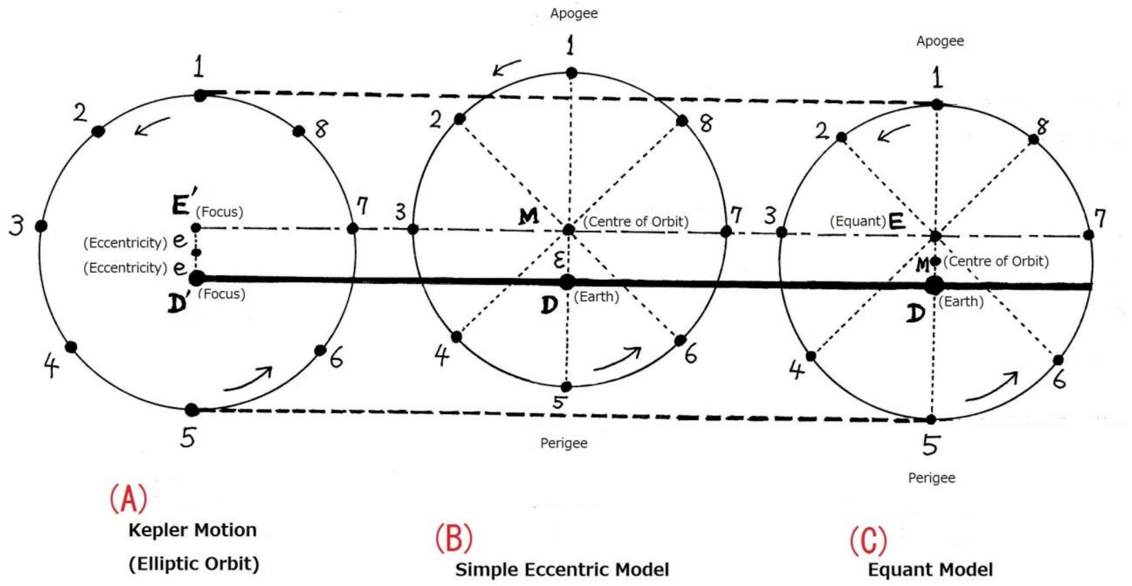
----- And some other works.

Ptolemaic geocentric planetary system

Further development of the theories of Apollonius, Hipparchus etc.
The most authentic theory of Western astronomy until Copernican theory.

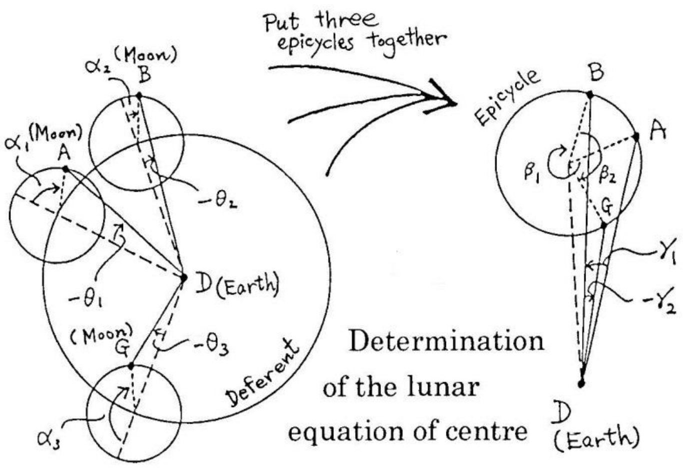
Equant model:

Ptolemy created the "equant" model for the deferent of the planetary orbits. In this model, the distance of the deferent (eccentric orbit) is almost the same as the actual elliptic orbit, and the apparent diameter of its epicycle is harmonious with the actually observed elongation of the planets from the supposed centre of the epicycle. In order to make the revolution of the centre of the epicycle correspond to the actual inequality, which corresponds to the equation of center of the Keplerian motion, the centre of the epicycle revolves at a constant angular velocity around the "equant" (E in Fig.(C)). It is seen that this "equant" model gives a quite good result. Solar orbit of Ptolemywas the same as the simple eccentric model of Hipparchus.

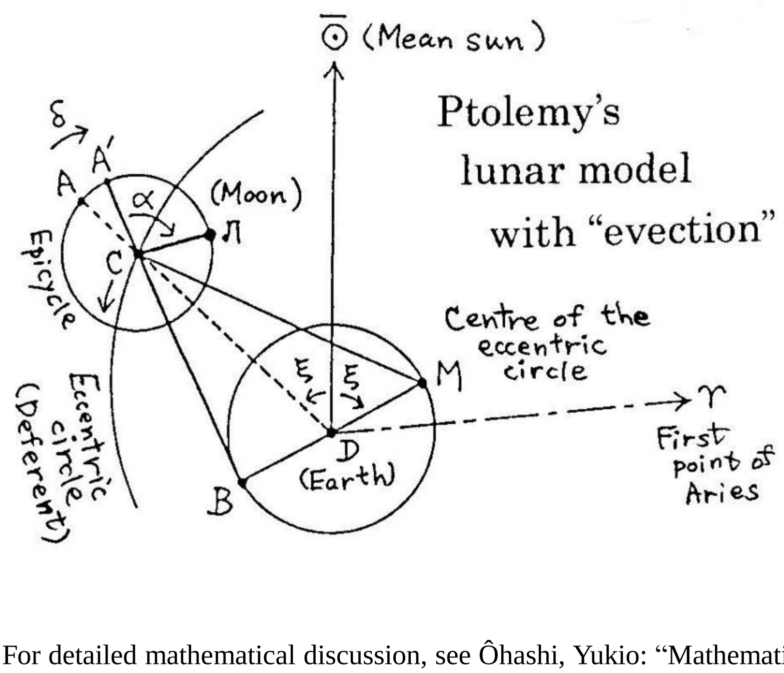


Ptolemy also developed the theory of lunar motion.

Firstly, the "equation of centre" is explained by an epicyclic model, obtained from three observations of lunar eclipses, as follows. This method is originated from Hipparchus' method.



Ptolemy further considered the "evection", and made a model of lunar motion as follows. This is a good model for lunar longitude, but quite inaccurate regarding the distance to the moon.



For detailed mathematical discussion, see Ôhashi, Yukio: "Mathematical structure of the eccentric and epicyclic models in ancient Greece and India", in Yadav and Singh (eds.): *History of the Mathematical Sciences II*, Cambridge, Cambridge Scientific Publishers, 2011, pp.83 – 102.

A map based on the Geography of Ptolemy.

PLATE 6. Map of the world in Ptolemy's second projection (Ulm edition of 1482)

(From Berggren, J. Lennart and Alexander Jones: *Ptolemy's Geography, An Annotated Translation of the Theoretical Chapters*, Princeton, Princeton University Press, 2000. For the maps based on Ptolemy's Geography, also see 『プトレマイオス世界図』、岩波書店、1978.)

The geocentric system of Ptolemy was the most precise mathematical astronomy even after the system of Copernicus. Copernicus used the data of Ptolemy, and the accuracy of the theory of Copernicus was almost the same as the accuracy of the theory of Ptolemy. More precise theories were created by the assistants of Tycho Brahe ----- Longomontanus and Kepler!

Diophantus of Alexandria (around 250 CE ?)

Diophantus discussed algebraic problems in his *Arithmetica*.

(For detail, see Heath: *Diophantus of Alexandria*, Cambridge, 1885.)

Pappus of Alexandria (fl. 320 CE)

Pappus composed a handbook of Greek geometry.

Theon of Alexandria and his daughter Hypatia (d. 418 CE)

Hypatia was a mathematician, astronomer, Neoplatonist. She was assassinated by mob of Christians in 418 CE. This is the end of the tradition of Greek mathematics and astronomy in Alexandria.

(For more detail, see Dzielska, Maria (Translated by F. Lyra): *Hypatia of Alexandria*, Cambridge, Mass., Harvard University Press, 1995.)

Roman science and technology

Julian Calendar

----- Julius Caesar (100 – 44 BCE) established in 45 BCE.

1 year = 365¼ days.

Egyptian solar calendar was utilized.

One intercalary day is inserted in every 4 years, (regularly inserted since 8 CE (at the time of Emperor Augustus)).

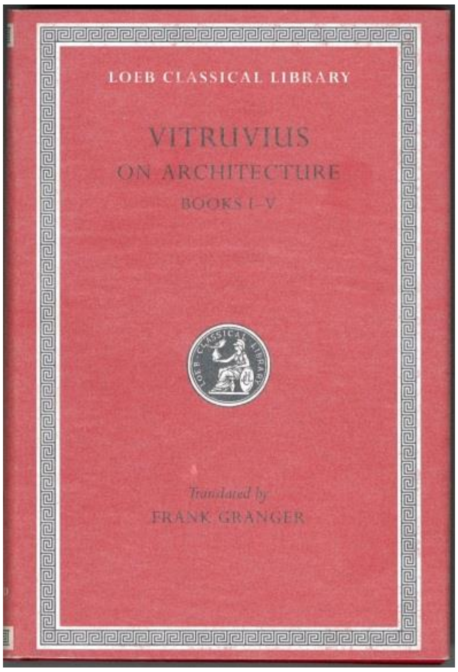
(For the history of Western calendar, see Duncan: *Calendar*, New York, 1998.)

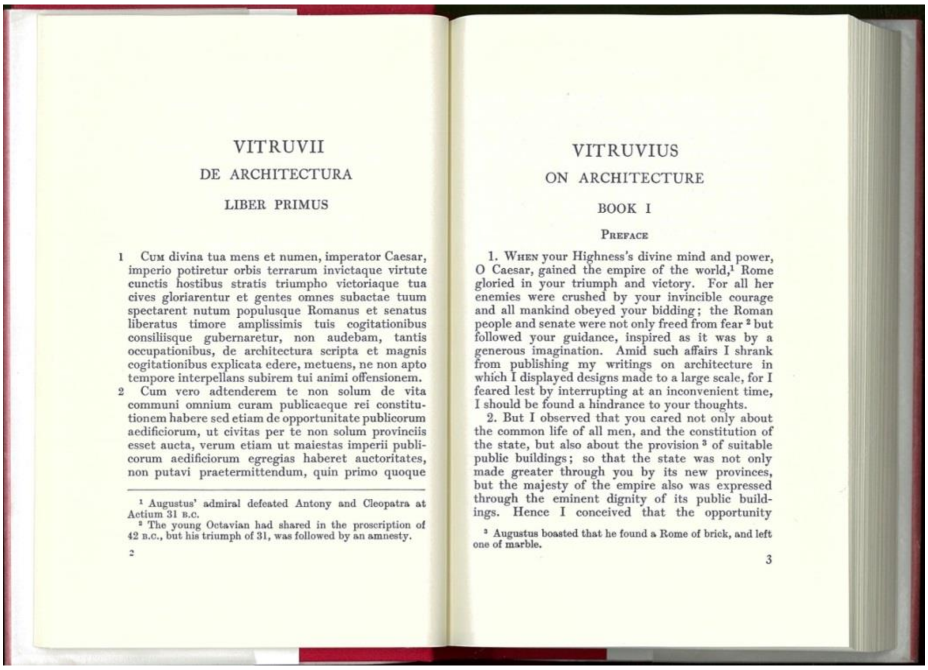
Roman technology

Vitruvius (1st century BCE) ----- *On Architecture*.

The earliest treatise on architecture and related technology.
Mathematics and astronomy were also utilized in architecture.

(For its English translation, see Vitruvius: *The Ten Books on Architecture*, (tr. by Morgan), (Latin text and English translation), Cambridge, 1914, or *Vitruvius on Architecture*, 2 vols, (Loeb Classical Library), (Latin text and English translation), Cambridge, Mass., Harvard University Press, and London, William Heineman Ltd, 1931-1934 (See below.). For its Japanese translation, see Morita (1979).)





VITRUVII

DE ARCHITECTURA

LIBER PRIMUS

1 Cum divina tua mens et numen, imperator Caesar, imperio potiretur orbis terrarum invictaque virtute cunctis hostibus stratis triumpho victoriaque tua cives gloriarentur et gentes omnes subactae tuum spectarent nutum populusque Romanus et senatus liberatus timore amplissimis tuis cogitationibus consiliisque gubernaretur, non audebam, tantis occupationibus, de architectura scripta et magnis cogitationibus explicata edere, metuens, ne non apto tempore interpellans subirem tui animi offensionem.

2 Cum vero adtenderem te non solum de vita communi omnium curam publicaeque rei constitutionem habere sed etiam de opportunitate publicorum aedificiorum, ut civitas per te non solum provinciis esset aucta, verum etiam ut maiestas imperii publicorum aedificiorum egregias haberet auctoritates, non putavi praetermittendum, quin primo quoque

[1] Augustus' admiral defeated Antony and Cleopatra at Actium 31 B.C.

[2] The young Octavian had shared in the proscription of 42 B.C., but his triumph of 31, was followed by an amnesty.

2

VITRUVIUS

ON ARCHITECTURE

BOOK I

PREFACE

1. When your Highness's divine mind and power, O Caesar, gained the empire of the world,[1] Rome gloried in your triumph and victory. For all her enemies were crushed by your invincible courage and all mankind obeyed your bidding; the Roman people and senate were not only freed from fear [2] but followed your guidance, inspired as it was by a generous imagination. Amid such affairs I shrank from publishing my writings on architecture in which I displayed designs made to a large scale, for I feared lest by interrupting at an inconvenient time, I should be found a hindrance to your thoughts.

2. But I observed that you cared not only about the common life of all men, and the constitution of the state, but also about the provision [3] of suitable public buildings; so that the state was not only made greater through you by its new provinces, but the majesty of the empire also was expressed through the eminent dignity of its public buildings. Hence I conceived that the opportunity

[3] Augustus boasted that he found a Rome of brick, and left one of marble.

3

(The beginning of *Vitruvius' On Architecture* (Loeb Classical Library), Vol.1.

And also, water supply was highly developed in Roman Empire.

(See Frontinus, (ed. and tr. by Herschel), *The Two books on the Water Supply of the City of Rome*, Boston, Dana Estes and Company, 1899, and/or *Frontinus*, (Loeb Classical Library), 1925, and/or Imai (1987) (in Japanese).)

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