Mesopotamian Zig-zag Function of Day Length from Indian Point of View

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I. INTRODUCTION

I would like to discuss the construction of ancient Mesopotamian zig-zag function of the day length from Indian point of view. I do not say that the ancient Mesopotamian astronomy and Indian astronomy had direct relationship. I think that they are independent, but it may be possible to understand Mesopotamian astronomy in better way from Indian point of view. What I want to say will become clear from the following discussion.

In the 20th century, some people tried to interpret ancient Indian "*Vedāṅga*" astronomy from Mesopotamian point of view. They conjectured that the ancient Indian astronomy was influenced by the Mesopotamian astronomy. However, I think that it was wrong. (See the following discussion.) Now, I would like to discuss from the opposite direction, and show that they are independent, and that the Mesopotamian astronomy can better be understood by the consideration of Indian case, although they must have been independent.

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II. Indian *Vedāṅga* astronomy

India has a long history of the development of astronomy. [For the general history of Indian astronomy, see Ôhashi (2009).] The early development in the Indus valley civilisation is not well known. The earliest development which we can study through existing decipherable literature is the astronomy in the Vedic period.

After Aryans migrated into India (ca. BC. 1600 or so), they created Vedic culture.

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Towards the end of the Later Vedic period in India, the *Vedāṅga*, which is a kind of supplemental learning for the *Veda* and consists of six divisions, was created in India, and one of its divisions is calendrical astronomy called *jyotiṣa*. Let us call the astronomy in this stage "*Vedāṅga* astronomy". Its fundamental text entitled *Jyotiṣa-vedāṅga* (or *Vedāṅga-jyotiṣa*) is extant [Sastry and Sarma (1984).]. I think that the *Vedāṅga* astronomy was created sometime between the 6th and 4th centuries BC.

A kind of linear zig-zag function was used for the length of daytime in the *Vedāṅga* astronomy. It can be expressed as follows:

$$T = \left(12 + \frac{2}{61} n\right) \tag{1}$$

Here, T is the length of daytime in terms of *muhūrta*s, and n is the number of days elapsed from or remaining until the winter solstice. One *muhūrta* is 1/30 of a day. According to this linear zig-zag function, the length of daytime changes by one *muh¯ urta* during one solar month.

According to this formula, the proportion of daytime and nighttime is 2:3 at the time of winter solstice. This proportion, at first sight, seems to have been observed at the latitude 35°N or so. (See Fig.1.) However, the main area of India when the *Vedāṅga* was produced was around Ganga and Yamuna rivers (around 25 ~ 30°N or so). So, some people conjectured that this function (1) was not Indian original function, but was imported from Mesopotamia which is located around the latitude 35°N or so. [See, for example, Pingree (1973).] This conjecture is based on the assumption that this function is the result of the interpolation from the data at the solstices.

Now, we should also consider that there is also a possibility that this function is the result of the extrapolation from the data around the equinoxes. Let us discuss about this second possibility.

A source belonging to the Later Vedic literature tells that Vedic people observed the sun (most probably the direction of sunrise) which moves constantly during its northward and southward courses, but considered that it is stationary around the solstices. The *Kaucītaki-brāhmaṇa* (XIX.3), one of the Later Vedic literature, reads:

"On the new moon of Māgha [name of a month, quoter] he rests, being about to turn northwards; ….. He goes north for six months; ….. Having gone north for six months he stands still, being about to turn southwards; ….. He goes south for six months; ….. Having gone south for six months he stands still, being about to turn north; .....” [Keith (1920), p.452.]

In this text, “he” must have referred to the sun. It is seen from this text that Vedic people noticed that the sun (probably the direction of sunrise) moves constantly except for a certain period around the solstices. The sun was considered that it “stands still” around the solstices. It means that the seasonal movement of the sun around the equinoxes was more important rather than its stationary position around the solstices at that time, Vedic period.

From this fact, we are obliged to think that Vedic people thought that a linear function should be based on the data excluding those around the solstices, and that the formula (1) of the *Jyotiṣa-vedāṅga* was not obtained by the interpolation from the observations at the solstices, but was obtained by the extrapolation from the observations of the length of daytime around the equinoxes. Practically, there are two possibilities. (A) If the formula was obtained from one *muhūrta*’s difference of the length of daytime during one solar month after the equinox, the most suitable latitude for this observation is around 27°N. (B) If it was obtained from two *muhūrta*s’ difference

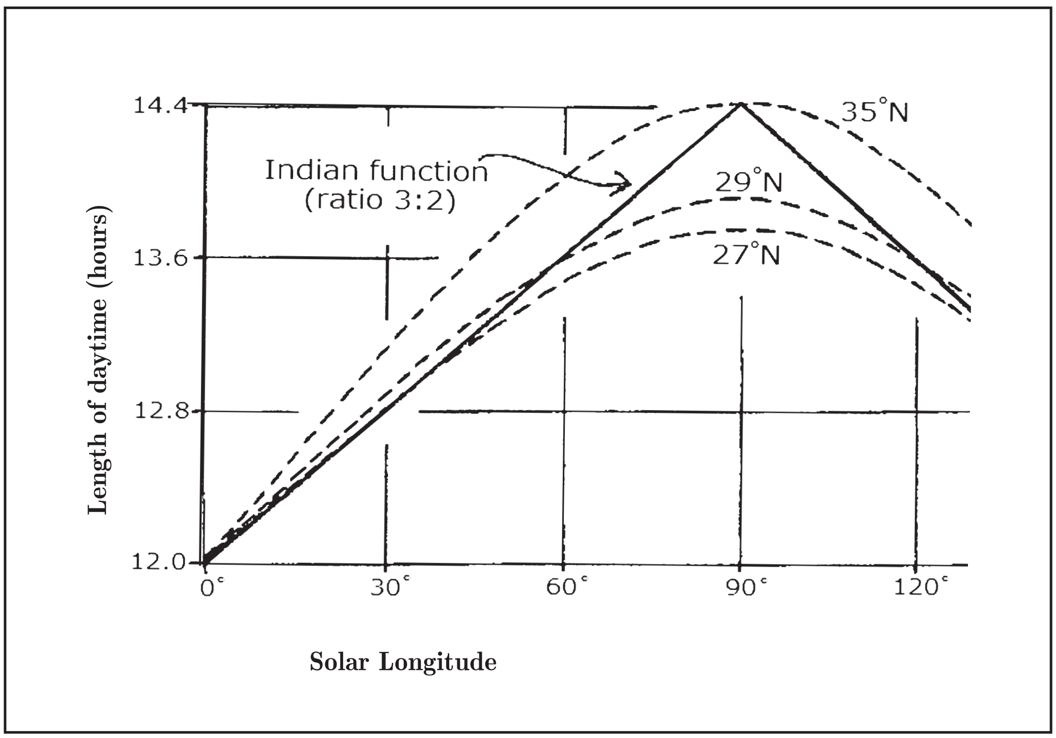

Fig.1. Indian function of the seasonal variation of the length of daytime

during two solar months, the most suitable latitude is around 29°N. I graphed the formula (1) together with the actual seasonal change of the length of daytime at the latitudes 35°N , 29°N , and 27°N in Fig.1.

It is clear from the Fig.1 that the function of the *Vedāṅga* astronomy is based on the actual observation in North India (around 27~29°N), and that it is mathematically based on the extrapolation and not the interpolation [See Ôhashi (1993) and (2011).].

Now, when we interpret ancient zig-zag functions, we should consider the possibility of the extrapolation as well as the interpolation. This point of view will be useful in order to interpret ancient Mesopotamian zig-zag function as we shall see in the following section.

III. ANCIENT MESOPOTAMIAN ASTRONOMY

III.1. Introduction

The history of Ancient Mesopotamia can roughly be divided as follows:

1. Sumerian and Old Akkadian periods,
2. Old Babylonian and Old Assyrian period (from ca. 20th century BC to ca. 16th century BC),
3. Middle Babylonian and Middle Assyrian period (from ca. 16th century BC to ca.1000 BC),
4. Neo-Babylonian and Neo-Assyrian period (from ca.1000 BC to the end of the 7th century BC),
5. Late Babylonian period (from the end of the 7th century BC to the 1st century BC):

   (5.1) Neo-Babylonian (Chaldean) dynasty (625 BC ~ 539 BC),

   (5.2) Achaemenid dynasty (559 BC ~ 330 BC),

   (5.3) Seleucid dynasty (312 BC ~ 64 BC).

III.2. The Mesopotamian linear zig-zag function with the ratio 2:1

Ancient Mesopotamian records were written by cuneiform characters on clay tablets. Already in the Old Babylonian tablet BM 17175+17284 (The abbreviation "BM" is used to denominate tablets in the British Museum.) [Hunger and Pingree (1989) pp.163 – 164 and Plate XIIIa. Also see Hunger and Pingree (1999) p.50, and Brown (2000) pp.128 – 129 and 249.], which might have been produced in the first half of the second millennium BC, the length of daytime and nighttime at equinoxes and solstices are given. The length is given in terms of "mina", which is used to measure the weight of water poured into a water clock. One "mina" corresponds to 4 hours, and one day corresponds to 6 "mina". It gives the following value.

Summer solstice: daytime = 4 "mina", nighttime = 2 "mina".

Autumnal equinox: daytime = 3 "mina", nighttime = 3 "mina".

Winter solstice: daytime = 2 "mina", nighttime = 4 "mina",

Vernal equinox: daytime = 3 "mina", nighttime = 3 "mina".

It is not clear whether the above value was meant for a kind of step function or for linear zig-zag function. However, we know that the linear zig-zag function with the ratio 2:1 did exist in Mesopotamia from the following source.

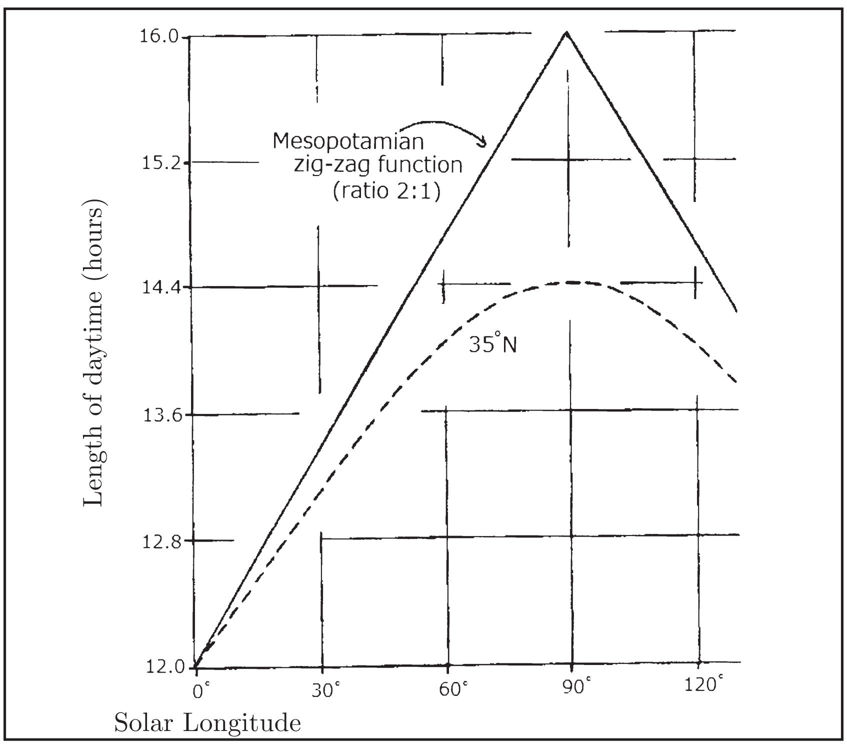

Fig.2. Mesopotamian linear zig-zag function with the ratio 2:1

A clear description of the linear zig-zag function of the length of daytime with the ratio 2:1 is given in *Enūma Anu Enlil* XIV, Table C [Al-Rawi and George (1992/1993). Also see Hunger and Pingree (1999) pp.44 – 50, Brown (2000) pp.128 – 129 and 254 – 256, and Rochberg (2004) pp.73 – 75.], which might have been composed in the second half of the second millennium BC. It tells that the length of daytime (or nighttime) changes from 2 "mina" to 4 "mina", and that the length changes linearly by one sixth "mina" per a quarter month. I graphed this function of the length of daytime in Fig.2 together with the actual seasonal change of the length of daytime at the latitude 35°N.

This Mesopotamian function looks different from the actual change at first sight, and was, wrongly, considered to be "very inaccurate", "incorrect for Mesopotamia" etc. by previous researchers. However, I suspect that the Mesopotamian linear function is the result of the extrapolation from the data around the equinoxes. We should note that the Mesopotamian linear function gives more or less acceptable value around the equinoxes. We already have seen that the extrapolation must have been used in ancient India. So, we have to consider the same possibility in ancient Mesopotamia also, that is to say we should consider the preference of the extrapolation in ancient astronomies from Indian point of view, although ancient Indian astronomy and Mesopotamian astronomy must have been independent. This point will become clearer when we discuss about the later development of polygonal function.

This linear zig-zag function with the ratio 2:1 is the earliest function of the length of daytime in ancient Mesopotamia, which is most probably based on the observations at the latitude 35°N, and is the beginning of the later development in Mesopotamia.

III.3. The Mesopotamian linear zigzag function with the ratio 3:2

The date of the beginning of the ancient Mesopotamian linear zig-zag function with the ratio 3:2 for the length of daytome is controversial. According to Pingree's interpretation [Hunger and Pingree (1989) pp.153 – 154. Also see Hunger and Pingree (1999) pp. 79 – 83.], the linear zigzag function with the ratio 3:2 is implied in the shadow table of Mul.Apin (II.ii.21 – 42), which is at least as old as the early first millennium BC according to Hunger and Pingree. However, Brown [Brown (2000) p.120.] objected to Pingree's interpretation, because the ratio 2:1 is otherwise used throughout the series of Mul.Apin.

According to Brown [Brown (2000) p.120 and 261.], the earliest attestation of the ratio 3:2 is in the late Neo-Assyrian period in BM 36731 [Neugebauer and Sachs (1967).], which probably describes the period 616 – 588 BC. This text implies the ratio 3:2 of the length of daytime and nighttime at the solstices, but it is not clear whether the linear zig-zag function was implied or not.

A table which is clearly based on the linear zig-zag function with the ratio 3:2 is given in the Late Babylonian tablet BM 29371 [Brown, Fermor and Walker (1999/2000).]. In this tablet, the value is given for every 5 days.

I graphed this function of the length of daytime in Fig.3 together with the actual seasonal change of the length of daytime at the latitude 35°N. The Mesopotamian function is evidently the result of the interpolation from the data at the solstices.

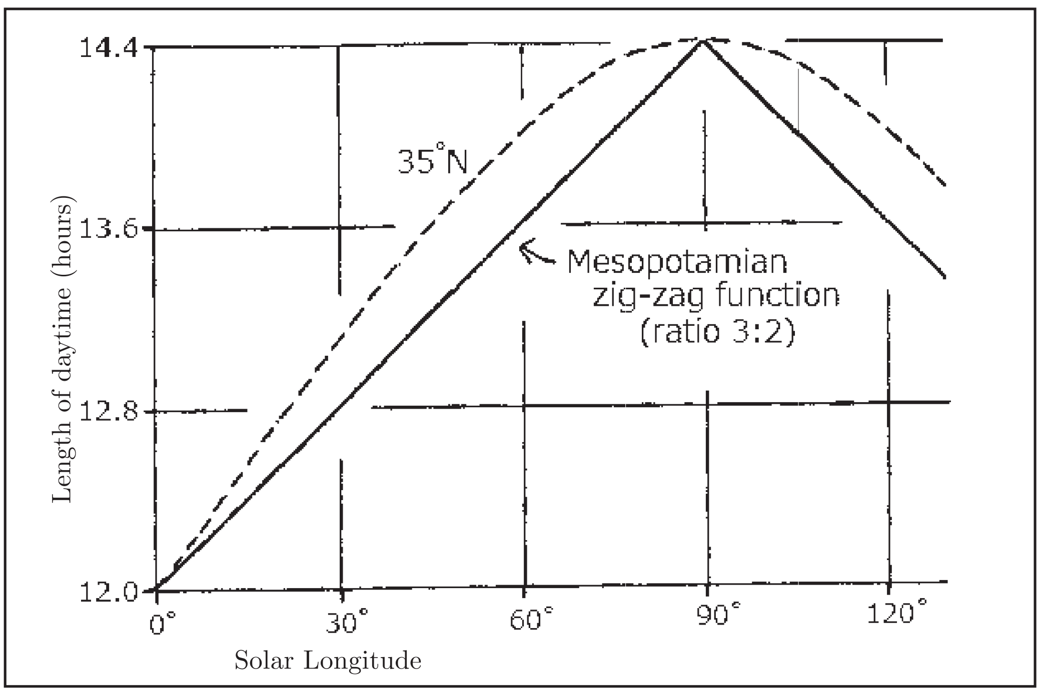

Fig.3. Mesopotamian linear zig-zag function with the ratio 3:2

This function is the same as the ancient Indian function of the *Vedāṅga* astronomy. According to the above sources, Mesopotamian ratio 3:2 already existed by the end of the Neo-Assyrian period (ca. 600 BC), and the linear zig-zag function of this ratio existed sometime in the Late Babylonian period (end of the 7th century – the 1st century BC). On the other hand, according to my study, the date of the creation of the *Vedāṅga* astronomy is sometime between the 6th and 4th centuries BC or so, but is not definite, and its date is controversial. So, it is difficult to say the definite chronological order of this linear zig-zag function among Mesopotamia and India, and is open to the future study. It may be safe to say at present that the same zig-zag function was produced in Mesopotamia and India in the similar period, most probably independently. We should note that the Mesopotamian function must have been the result of interpolation, while the Indian function must have been the result of extrapolation, although the function is the same.

III.4. The Mesopotamian polygonal function with the ratio 3:2

The mathematical astronomical texts in the Seleucid period (312 BC ~ 64 BC), which is the last stage of the Ancient Mesopotamian astronomy, was collected in the *Astronomical Cuneiform Texts* (hereafter ACT) of Otto Neugebauer [Neugebauer (1955).].

The polygonal function of the length of daytime with the ratio 3:2 is given in the "Section 2" of ACT 200 (= BM 32651) [Neugebauer (1955) Vol.I, p.187 and Vol.III, Plates 223 and 234, and also Ossendrijver (2012), pp.358 – 359.] and ACT 200b (= BM 33631) [Neugebauer (1955) Vol.I, p.214 and Vol.III, Plate 236, and also Ossendrijver (2012) pp.382 – 383.], which belong to the "Procedure texts" of "System A" [Also see Neugebauer (1955) Vol.I, p.47 and Neugebauer (1975), Part 1, pp. 369 – 371, and also Ossendrijver (2012) pp.130 – 132.]. The value is given for every 30° of the solar longitude starting from the vernal equinox as follows. For other longitudes, the value is obtained by the linear interpolation. Therefore, this is a kind of polygonal function. The length of daytime is given in terms of "large-hours", where one "large-hour" is 4 hours, and one day consists of 6 "large-hours". Fractions are sexagesimally expressed. For example, "3;20" means "3 + 20/60" large-hours, that is $(3+20/60)\times 4 = (13+1/3)$ hours. (Or, it is also possible to say that the time is expressed by degrees, where 360° is 24h, and "3,20" (rather than 3;20, where the mark ";" means the boundary of integer and fraction) means $3\times 60° + 20° = 200°$, that is (13+1/3)h.)

Solar longitude	Length of daytime
0°	3
30°	3;20
60°	3;32
90°	3;36
120°	3;32

150°	3;20
180°	3
210°	2;40
240°	2;28
270°	2;24
300°	2;28
330°	2;40

I graphed this polygonal function of the length of daytime in Fig.4, together with the linear zig-zag function with the ratio 2:1 and that with the ratio 3:2, and also the actual change at the latitude 35°N.

From Fig.4, it is seen clearly that the polygonal function from the solar latitude 0° to 30° is exactly the same as the linear zigzag function with the ratio 2:1. From this fact, I suppose that the astronomers in the Seleucid period considered that the linear zig-zag function with the ratio 2:1 is the result of the extrapolation from the data around the equinoxes as I suspected, and used this old function partially. So, it will be

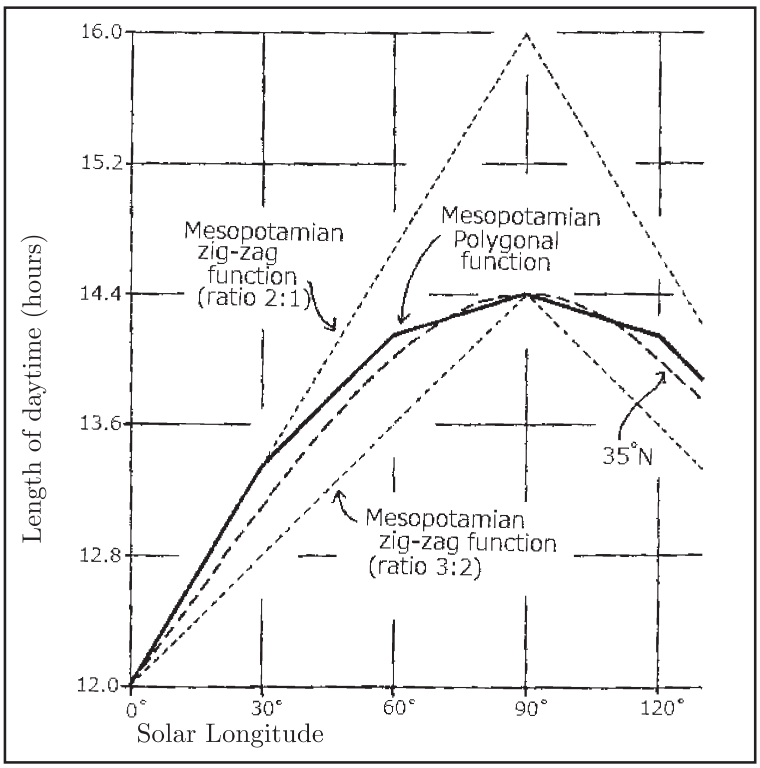

Fig.4. Mesopotamian polygonal function with the ratio 3:2

justified to say that this polygonal function is the result of the synthesis of extrapolation (linear function with the ratio 2:1) and interpolation (linear function with the ratio 3:2), both of which are the results of the observations at the latitude 35° N or so. Here, we can see the development of a theory using actual observations at a particular terrestrial latitude 35° N or so.

IV. CONCLUSION

The Indian *Vedāṅga* zig-zag function of the seasonal variation of the day length was based on the data observed at the latitude 27° ~ 29°N or so, and their preference of extrapolation can be understood from a description in the Vedic literature. What is important is that we should consider the preference of extrapolation in order to interpret ancient astronomy. Considering this point of view, we have seen that the Mesopotamian functions can be understood that they were based on the data at the latitude 35° N or so, and used the extrapolation, interpolation, and their combination. We can see that Mesopotamian people developed the function of the day length at their own latitude, and finally created the polygonal function which is a quite good approximation. I think that the Indian function and the Mesopotamian functions are independent, because both of which are understood that they were created at their own different latitudes. However, there is certain similarity of way of thinking between them. We should further study ancient astronomies impartially.

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