Lecture note 1: (2019)

(Introduction;　Mathematics and astronomy in ancient Egypt and Mesopotamia.)

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Introduction:

(A) Traditional sciences:

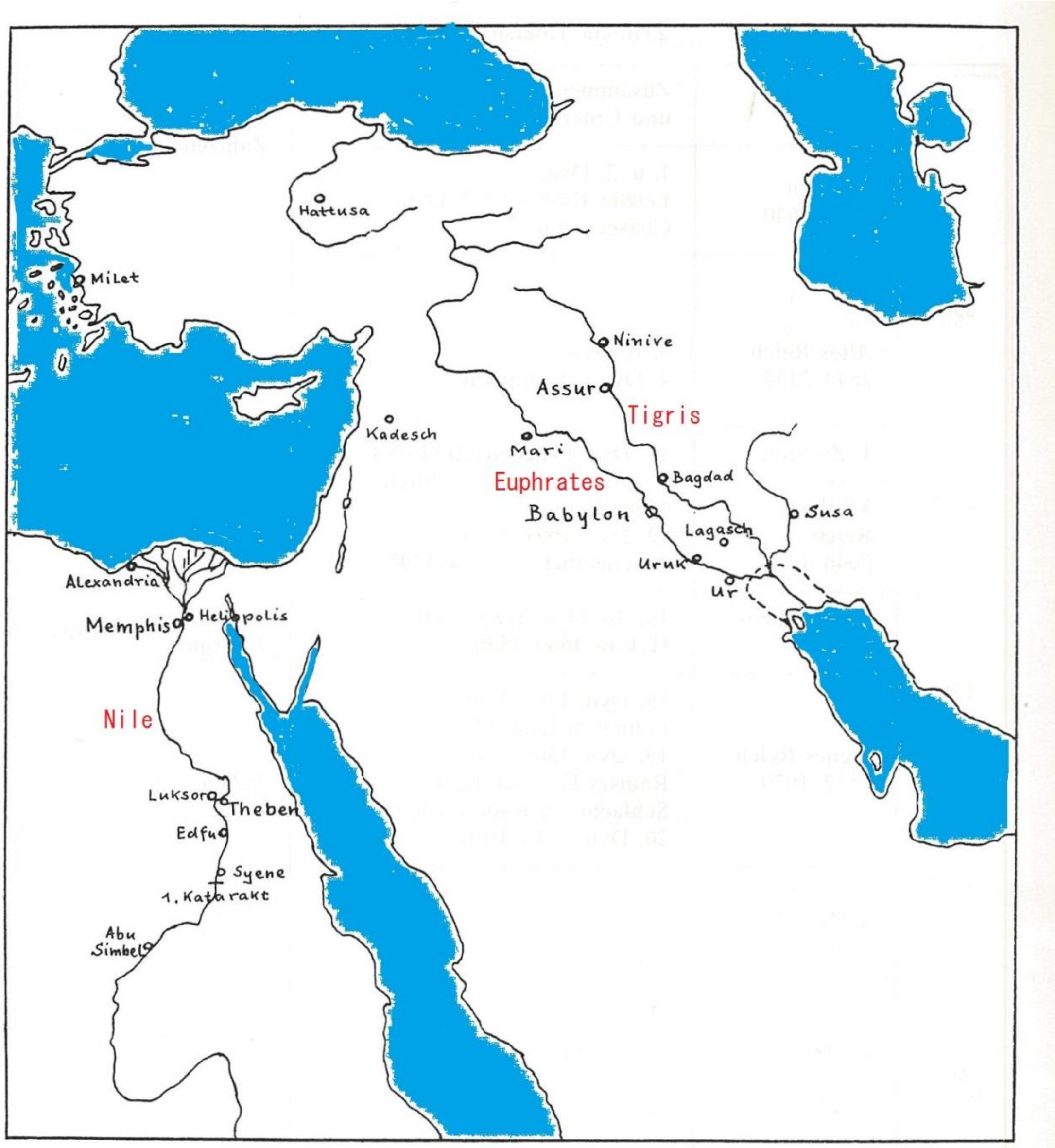
(A.1) Ancient Mediterranean Sciences
⇒ Sciences in the Islamic World
⇒ Sciences in Medieval Europe.
(A.2) South Asian Sciences (India etc.).
(A.3) East Asian Sciences (China, Korea, Japan, etc.)
And other traditional sciences.

(B) Modern Science.

Plan of the lecture:

1. Introduction;
   Mathematics and astronomy in ancient Egypt and Mesopotamia.
2. Mathematics and astronomy in ancient Greece and Rome.
3. Mathematics and astronomy in traditional India.
4. Mathematics and astronomy in traditional China.
5. Mathematics and astronomy in traditional Korea and Japan.
6. Mathematics and astronomy in traditional Tibet and Southeast Asia.
7. Mathematics and astronomy in the Islamic World and Medieval Europe.
8. Development of modern mathematics and astronomy;
   Conclusion.

A map of Egypt and Mesopotamia:



Ägypten und Mesopotamien

(From Gericke: *Mathematik in Antike und Orient*, 1984, p.276, with my additions.)

I. Mathematics and astronomy in ancient Egypt

THE EGYPTIANS

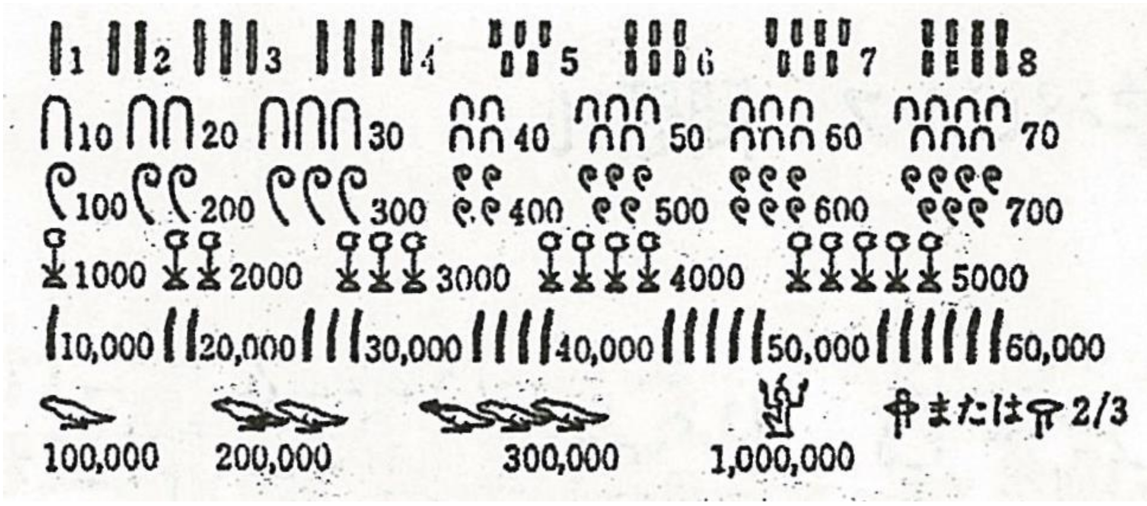
Chronological Summary

General History	History of Civilization	History of Science
3000 Menes The Old Kingdom	Hieroglyphics Pyramids	Number symbols to 100,000
2000–1800 The Middle Kingdom	Literature The goldsmith's art	Rhind papyrus and Moscow papyrus Star calendars on sarcophagi
1700 The Hyksos domination		Ahmes copies the Rhind papyrus
1600–1100 The New Kingdom	New theology (Echnaton) Architecture Sculpture	Very primitive astronomy (Senmuth's tomb)
300 B.C. – 300 A.D. Hellenism	Alexandria as center of Greek art and science Rise of astrology	Highest development of Greek science Egyptian arithmetic and astronomy remain very primitive

(From van der Waerden: *Science Awakening I*, Groningen, 1961, p.15.)

(I.1) Egyptian numerals

Hieroglyph (Standard writing system):



(From Hirata (1974), p.51.)

Hieratic (Usual writing system written on papyrus):

(From Takasaki (1977), p.229.)

Egyptian number system:

Decimal system.

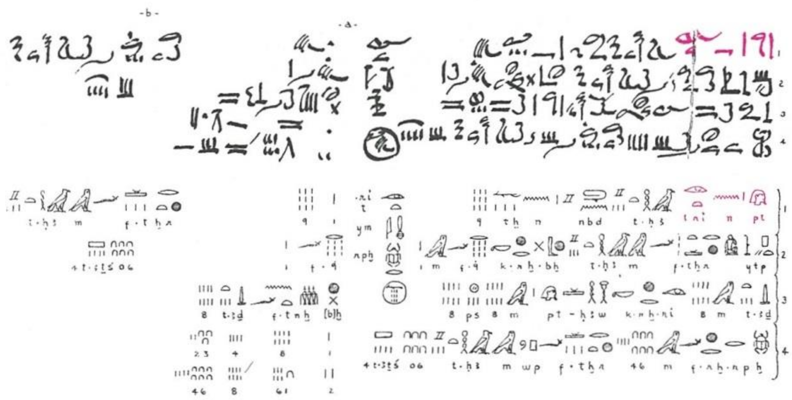
Unit fractions and 2/3.

(I.2) Rhind papyrus

(A well known Egyptian mathematical papyrus composed in the middle kingdom period.) An example:

Plate 72

PROBLEM 50



Photographs xx-xxi, *Register* 2 *B. M. Facsimile, Plate* xiv

Fig. IV.2jj Hieratic text and Hieroglyphic transcription of the Rhind Mathematical Papyrus from Chace et al., *The Rhind Mathematical Papyrus*, Vol. 2 (Oberlin, Ohio, 1929), Plates 71-72.

[Problem 50]

Example of producing [the area of] a round field of diameter of 9 khet. What is the reckoning (*lit.*, *rḫt*, knowledge) of its area (*3ḥt*)?

Take away 1/9 of it (the diameter), namely 1; the remainder is 8. Multiply 8 times 8; it makes 64. [Therefore,] the amount of it in area is 64 setjat.

The procedure is as follows:

1	9
1/9	1;

this taken away leaves 8.

1	8
2	16
4	32
\8	64.

The amount of it in area is 64 setjat.[67]

(From Clagett: *Ancient Egyptian Science*, Vol.3, Philadelphia, 1999, pp.162-163.)

(For its Egyptian text with Japanese translation, see Yoshinari (1985).)

$$\text{The area of a circle} = (\text{diameter} \times 8/9)^2$$
$$= (\text{radius} \times 2 \times 8/9)^2$$
$$= (\text{radius})^2 \times 3.16 \cdots$$

(I.3) Egyptian solar calendar

1 civil year = 365 days.

1461 civil years = 1460 Sirius years.

1 Sirius year: From a heliacal rising of Sirius to the next heliacal rising of Sirius.

"Heliacal rising": The first visibility of a star just before sunrise above the eastern horizon.

1 Sirius year = 365.25 days.

$(365 \times 1461 \div 1460 = 365.25)$

The Egyptian Sirius year is the origin of the Julian Calendar. Is it too long or too short? Consider:

1 Tropical year = 365.24219 days

1 Sidereal year = 365.25636 days

(I.4) Egyptian star map

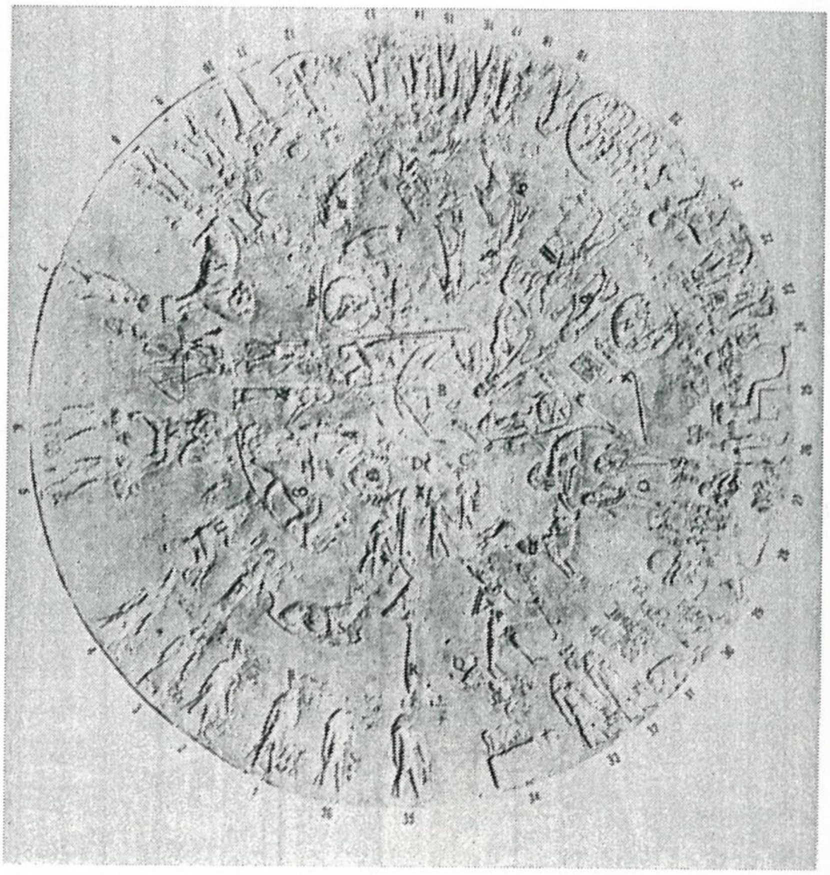


Fig. III.76a The round zodiac of Dendera ("Dendera B"), dated to late Ptolemaic times. The plate given here is from Neugebauer and Parker, *Egyptian Astronomical Texts*, Vol. 3 (Plates), Plate 35, and includes their additions of numbers, letters and symbols to identify decans, constellations, and zodiacal signs.

(From Clagett: *Ancient Egyptian Science*, Vol.2, Philadelphia)

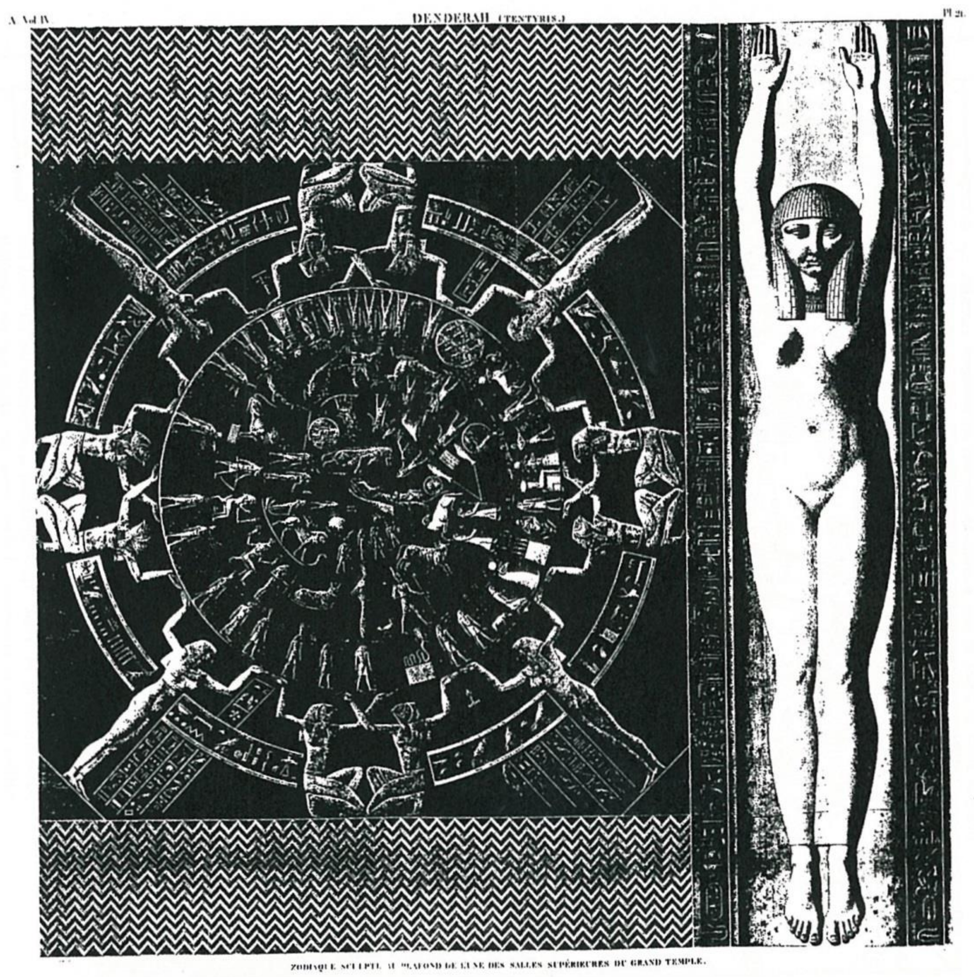


Fig. III.76b The zodiac of Fig. III.76a, as given in the *Description de l'Egypte*, Vol. 4, Plate 21. And see the drawing on the back of that plate.

(From Clagett: *Ancient Egyptian Science*, Vol.2, Philadelphia)

II. Mathematics and astronomy in ancient Mesopotamia

BABYLONIAN MATHEMATICS

Chronological Summary.

General History	History of Civilization	History of Science
3000 Sumerian city states	Cuneiform writing High level of culture	Sexagesimal system
2800—1800 Semitification		Tables for dividing and multiplying
First Babylonian dynasty 1700 Hammurabi	Cultural flowering Legislatio, administration of justice	Phenomenal flowering of algebra and geometry. Observations of Venus.
1500—1250 Babylon under the rule of the Cassites	Series of astrological omens: "Enuma Anu Enlil"	Primitive astronomical calculations. Observations of heliacal rising of fixed stars.
747, Nabonassar, king of Babylon	Beginning of the astronomical "Era Nabonassar"	Dated observations of eclipses in Babylon.
729 The Assyrian Tiglatpilesar II ascends the throne of Babylon as Pulu.		
722 Sargon II 700 Sanherib	Assyrian royal palaces Court astrologers	Astronomical compendia: I·NAM·GIS·HAR̆ and mul APIN of Babylonian origin, copied in Assyria about 700
650 Assurbanipal	Library of Assurbanipal	
612 Destruction of Niniveh. End of the Assyrian empire New Babylonian empire of the Chaldeans 580 Nebukadnezar II	New period of flowering of arts and sciences	Observation of moon and planets.
540 Cyrus, founder of the Persian empire 500 Darius	Babylonian religion not affected Calendar periods	Increased accuracy of observations of the Zodiac. Periods of planets
333 Alexander the Great 311 Beginning of the era of the Seleucids 247 Beginning of the Arsacid era	Hellenism Birth-horoscopes	Flowering of astronomy. Lunar and planetary tables. Revival of algebra. Extensive calculation tables.

(From van der Waerden: *Science Awakening I*, Groningen, 1961, p.62.)

(II.1) Cuneiform script

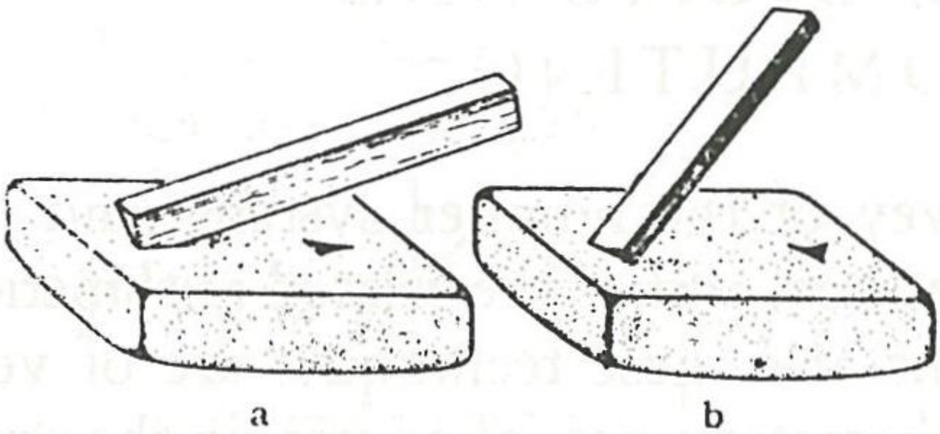
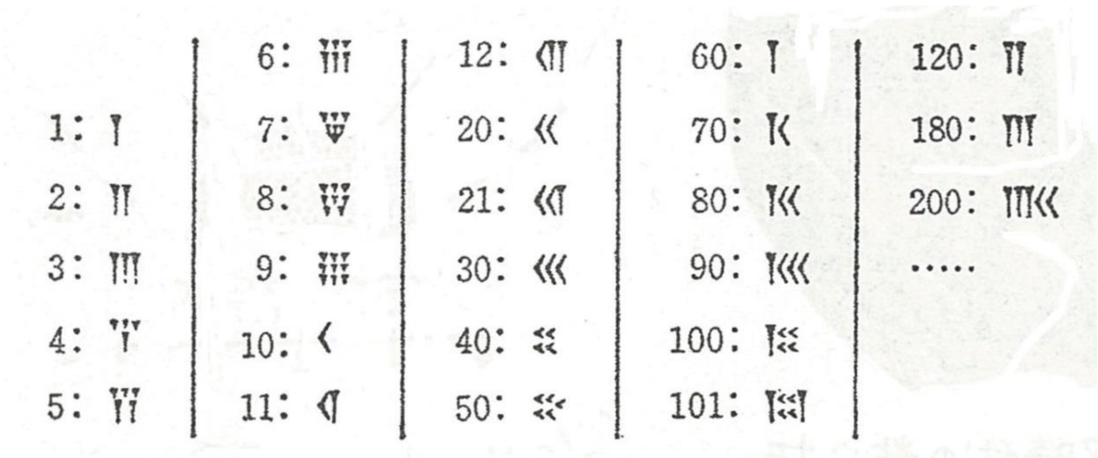


Fig. 8. Stylus for cuneiform script
From O. Neugebauer, Vorgriechische Mathematik

(From van der Waerden, *Science Awakening*, p.38.)

(II.2) Cuneiform numerals



(From Hirata (1974), p.113)

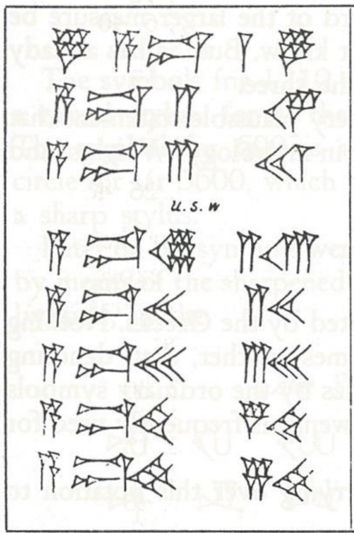
Sexagesimal place value system.

(II.3) Multiplication table

Sumerian technique of computation.

The most ancient Sumerian texts, from which the Sumerian number system was deduced, dating from the time of Shulgi (about 2000), were tables of inverses ($1/x$) and multiplication tables. The latter tables appear singly or in combination. A separate table contains the multiples of a single number. Two examples:

Table of 7 and of 16,40.



7 a-rá 1	7	a-rá 1	16, 40
a-rá 2	14	a-rá 2	33, 20
a-rá 3	21	a-rá 3	50
...		...	
a-rá 19	2, 13	a-rá 19	5, 16, 40
a-rá 20	2, 20	a-rá 20	5, 33, 20
a-rá 30	3, 30	a-rá 30	8, 20
a-rá 40	4, 40	a-rá 40	11, 6, 40
a-rá 50	5, 50	a-rá 50	13, 53, 20

Obviously, a-rá means "times".

(From van der Waerden: *Science Awakening I*, Groningen, 1961, p.42.)

(II.4) Diagonal of a square (√2)

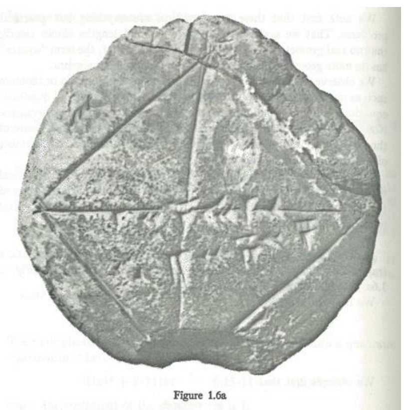


Figure 1.6a

(A tablet of Old Babylonian Period)

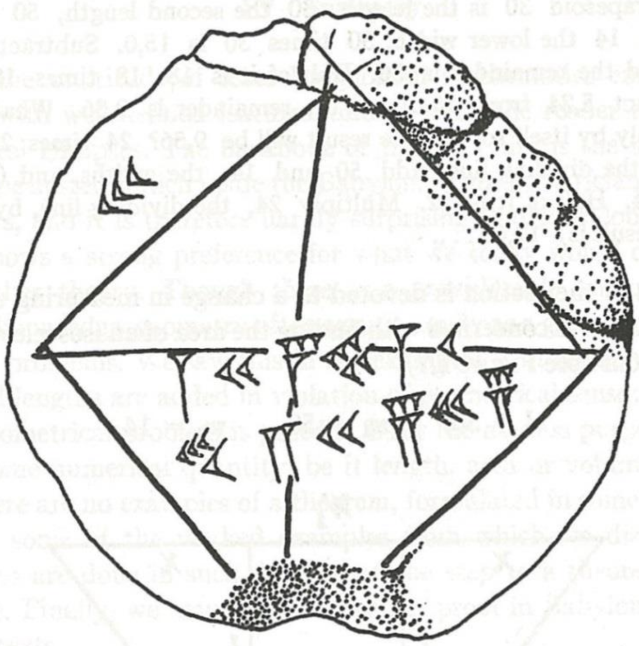


Figure 1.6b

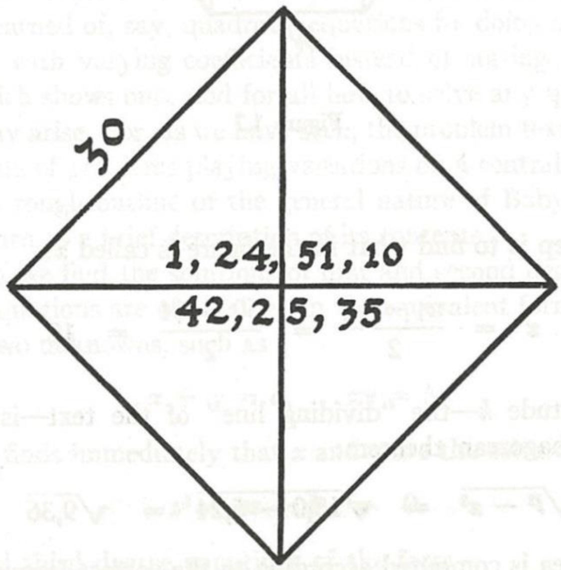


Figure 1.6c

(From Aaboe: *Episodes from the early history of mathematics*, pp.26-27. Originally published in Neugebauer and Sachs, *Mathematical Cuneiform Texts*, 1945.)

1; 24, 51, 10 in sexagesimal system:

In decimal system:

$1 + 24/60 + 51/60^2 + 10/60^3$

$= 1 + 0.4 + 0.01416666\cdots + 0.00046296\cdots$

$= 1.4146296\cdots$

Is this enough exact?

Convert the exact value in decimal system $1.41421356\cdots$ into sexagesimal system ⇒

$$
\begin{aligned}
1.41421356\cdots = 1 & \\
& + 24/60 \\
& + 51/60^2 \\
& + 10/60^3 \\
& + 7/60^4 \\
& \text{-----------}
\end{aligned}
$$

Perfectly exact!

(II.5) Mesopotamian astronomy:

Luni-solar calendar.

One year is a solar year (about 365.25 days).

One month is a synodic month (about 29.5306 days).

($29.5306 \times 12 \approx 354.367$ days.)

Sometimes, an intercalary month is inserted. From the early 5th century BCE, 7 intercalary months were inserted in 19 years.

Zodiacal signs (since ca.400 BCE or so).

The ecliptic is divided into 12 signs (30° each).

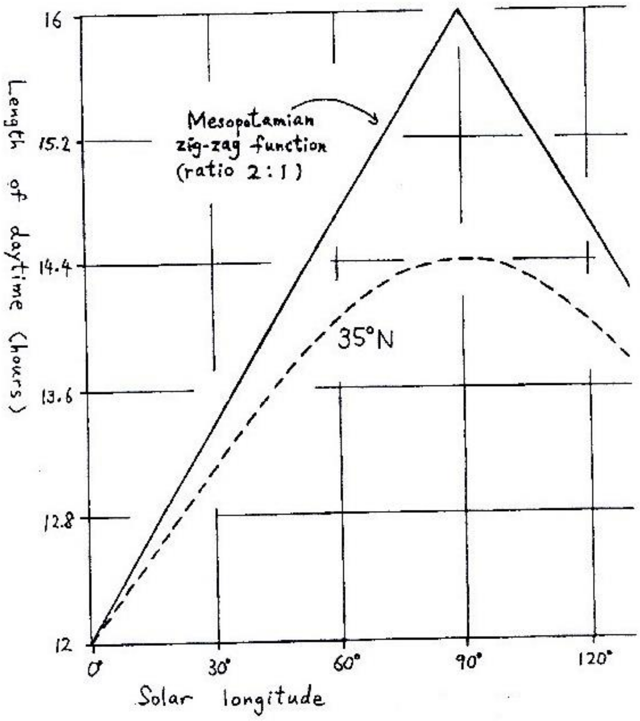
(The signs of the zodiac: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius, and Pisces)

They were used to express longitude of heavenly bodies in ancient and medieval western world.

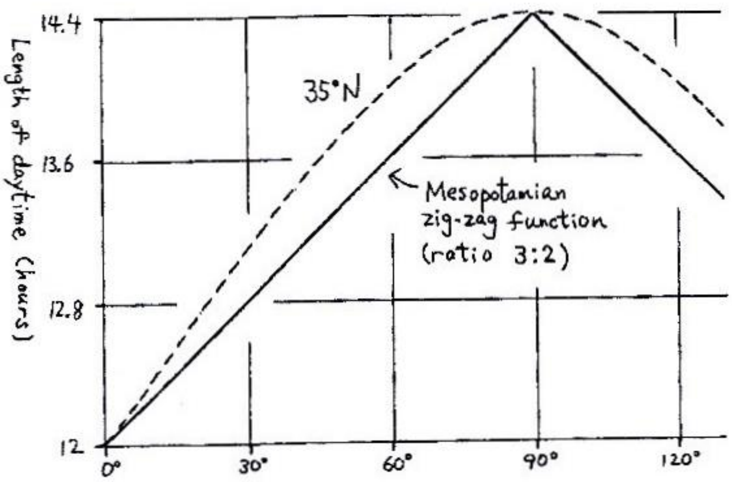
Linear zig-zag function.

An example --- Length of daytime:

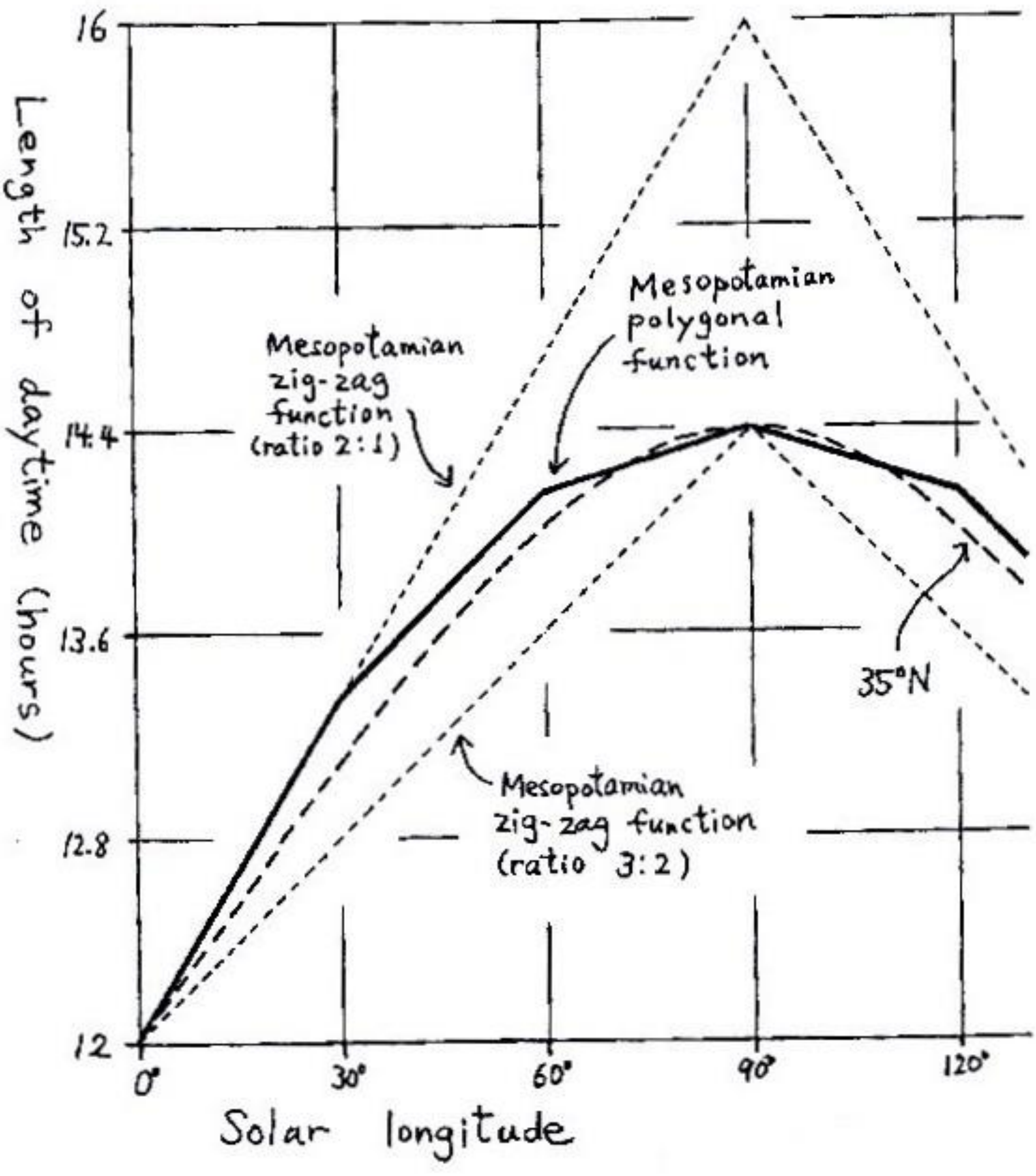
Old Babylonian period:



Neo-Assyrian period:



Seleucid period:



(The above three figures are from: Ôhashi, Yukio: "On *Vedāṅga* astronomy: The Earliest Systematic Indian Astronomy", in Nakamura, Orchiston, Sôma and Strom (eds.): *Mapping the Oriental Sky. Proceedings of the Seventh International Conference on Oriental Astronomy*, Tokyo, National Astronomical Observatory of Japan, 2011, pp.164 – 170.)

(Also see Ôhashi, Yukio: Mesopotamian Zig-zag Function of Day Length from Indian Point of View, *Gaṇita Bhāratī*, **34**, 2012, 53 – 63.)

(II.6) A Babylonian map:

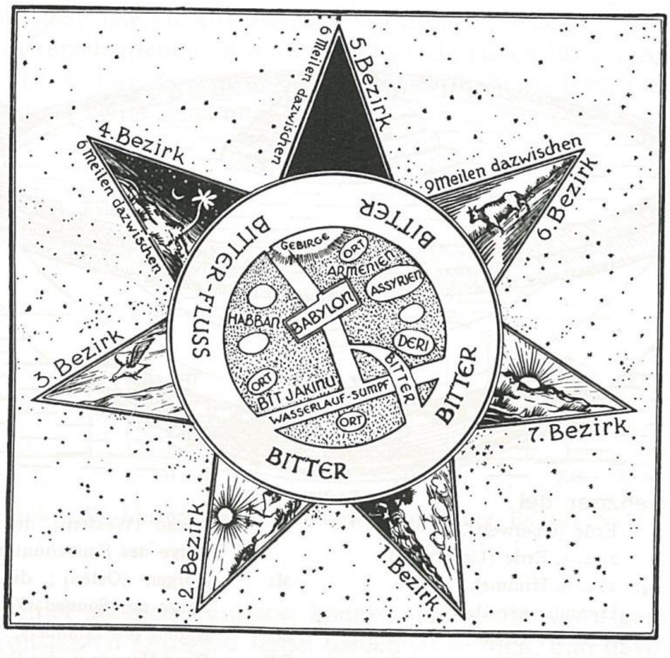


Abb. 1.35. Babylonische Weltkarte (Tontafel aus dem 7. Jh. v. Chr. – BM 92687)
[Jeremias, Abb. 89, 90]

(From Gericke: *Mathematik in Antike und Orient*, Berlin,1984, p.46.)

References:

General references of the history of mathematics and astronomy:

Katz, Victor J.: *A History of Mathematics*, 3rd Edition, Essex, Pearson, 2017, (Japanese translation of its 2nd edition: 『カッツ 数学の歴史』, 共立出版, 2005.).

North, John: *Cosmos: An Illustrated History of Astronomy and Cosmology*, Chicago, The University of Chicago Press, 2008.

References of the history of ancient (mainly Mediterranean) mathematics and astronomy:

Neugebauer, O.: *The Exact Sciencces in Antiquity*, Princeton, Princeton University Press, 1952, Second edition: Providence, Brown University Press, (Japanese translation: ノイゲバウアー（矢野・斎藤訳）『古代の精密科学』, 恒星社厚生閣, 1984).

Neugebauer, O.: *A History of Ancient Mathematical Astronomy*, 3 pts., Berlin, Springer-Verlag, 1975.

Neugebauer, O.: *Vorgriechische Mathematik*, Berlin, Springer-Verlag, 1969 (Egyptian and Mesopotamian mathematics).

van der Waerden, B. L.: *Science Awakening I*, (English translation by Arnold Dresden), Groningen, Wolters Noordhoff Publishing, 1954, 1961, (Japanese translation: ヴァン・デル・ウァルデン（村田・佐藤訳）『数学の黎明』, みすず書房, 1984).

van der Waerden, B. L.: *Science Awakening II*, The birth of astronomy, Leyden, Noordhoff International Publishing, 1974.

Aaboe, Asgar: *Episodes from the early history of mathematics*, Washington, The Mathematical Association of America, 1964, (Japanese translation: アーボー（中村幸四郎訳）『古代の数学』, 河出書房新社, 1971.).

Aaboe, Asgar: *Episodes from the early history of astronomy*, New York, Springer, 2001.

Gericke, Helmuth: *Mathematik in Antike und Orient*, Berlin, Springer-Verlag,1984.

Hirata: 平田寛、『科学の起原』、岩波書店、1974.

References of the history of ancient Egyptian mathematics and astronomy:

Clagett, Marshall: *Ancient Egyptian Science*, Volume I, Knowledge and order, Philadelphia, American Philosophical Society, 1989.

Clagett, Marshall: *Ancient Egyptian Science*, Volume II, Calendars, Clocks, and Astronomy, Philadelphia, American Philosophical Society, 1995.

Clagett, Marshall: *Ancient Egyptian Science*, Volume III, Ancient Egyptian Mathematics, Philadelphia, American Philosophical Society, 1999.

Takasaki: 高崎昇、『古代エジプトの数学』、（初版：1937）、総合科学出版、1977.

Yoshinari: 吉成薫訳、A. B. Chace:『リンド数学パピルス』、全２冊、朝倉書店、1985.

Miura: 三浦伸夫、『古代エジプトの数学問題集を解いてみる』、NHK 出版、2012.

References of the history of ancient Mesopotamian mathematics and astronomy:

Neugebauer, Otto: *Mathematische Keilschrift-Texte*, Berlin, Springer-Verlag, 1935.

Neugebauer and Sachs, *Mathematical Cuneiform Texts*, New Heaven, American Oriental Society, 1945.

Neugebauer, O.: *Astronomical Cuneiform Texts*, 3 vols, London, Lund Humphries, 1955, reprinted: New York, Springer, 1983.

Brown, David: *Mesopotamian Planetary Astronomy-Astrology* (Cuneiform Monographs 18), Groningen, STYX, 2000.

Hunger, Hermann and David Pingree: *Astral Sciences in Mesopotamia* (Handbuch der Orientalistik 44), Leidenn, Brill, 1999.

Rochberg, Francesca: *The Heavenly Writing*, Cambridge, Cambridge University Press, 2004.

Muroi: 室井和男『バビロニアの数学』、東京大学出版会、2000.

Muroi: 室井和男『シュメール人の数学』、共立出版、2017.

Additional reading:

Ôhashi, Yukio: Mesopotamian Zig-zag Function of Day Length from Indian Point of View, *Gaṇita Bhāratī*, **34**, 2012, 53 – 63.