Electron Transport

Electricity and heat are carried by free electrons in metals.

(1) Phase velocity and group velocity

Wave is represented in general

$$U = U_0 e^{i(kx-\omega t)}$$

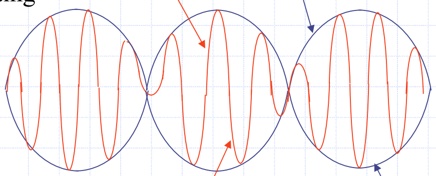
Convolution of two slightly different waves with $\omega \pm \Delta\omega$ and $k \pm \Delta k$ is,

$$U = U_0[e^{i[(k+\Delta k)x-(\omega+\Delta\omega)t]} + e^{i[(k-\Delta k)x-(\omega-\Delta\omega)t]}]$$

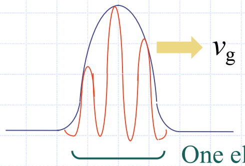
$$= U_0 e^{i(kx-\omega t)}[e^{i(\Delta k\cdot x-\Delta\omega\cdot t)} + e^{-i(\Delta k\cdot x-\Delta\omega\cdot t)}]$$

$$= 2U_0 e^{i(kx-\omega t)}\cos(x\Delta k - t\Delta\omega)$$

Beating



A lump of wave (wave packet) moves by v_g.



Velocity of the original wave (Phase velocity) $v = \dfrac{\Delta x}{\Delta t} = \Big[\quad\Big]$

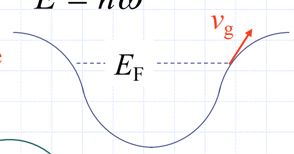
Velocity of the envelop (Group velocity) $v_g = \Big[\quad\Big]$

(2) From energy band $E(k)$

$$E = \hbar\omega$$

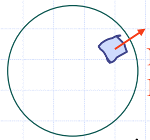
$$v_g = \frac{\partial\omega}{\partial k} = \frac{1}{\hbar}\frac{\partial E(k)}{\partial k}$$

Tangent of the energy band



For 3D

$$\vec{v}_g = \frac{1}{\hbar}\left(\frac{\partial E(k)}{\partial k_x}, \frac{\partial E(k)}{\partial k_y}, \frac{\partial E(k)}{\partial k_z}\right)$$



Normal to the Fermi surface

Differentiated by t

$$\frac{\partial v_g}{\partial t} = \frac{1}{\hbar}\frac{\partial^2 E(k)}{\partial k^2}\frac{\partial k}{\partial t}$$

External field ε during δt gives rise to work $\delta E = -e\,\varepsilon\, v_g\,\delta t$ (force · moved distance = impulse)

But $\delta E = \left(\dfrac{\partial E}{\partial k}\right)\delta k = \hbar v_g \delta k$ leads to $-e\,\varepsilon\, v_g\,\delta t = \hbar\, v_g\,\delta k$ giving $\hbar\dfrac{\partial k}{\partial t} = \Big[\quad\Big]$

Thus

$$\frac{\partial v_g}{\partial t} = \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k^2}(-e\varepsilon)$$

$ma=F$ ⇒ 加速度 · $1/m^*$ · 力

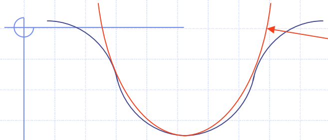
Alternately, differentiate $p=\hbar k$

$$\frac{\partial p}{\partial t} = \hbar\frac{\partial k}{\partial t} = F = -e\varepsilon$$

よって

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\partial^2 E(k)}{\partial k^2}$$

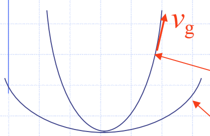
有効質量 effective mass
= curvature of the energy band



$$E(k) = \frac{\hbar^2 k^2}{2m^*}$$

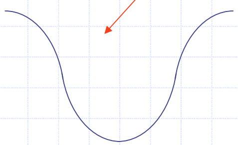
General energy band is approximated by free electron and the coefficient gives m^*

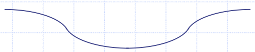
Acceleration of the wave packet under a certain force (electric field)

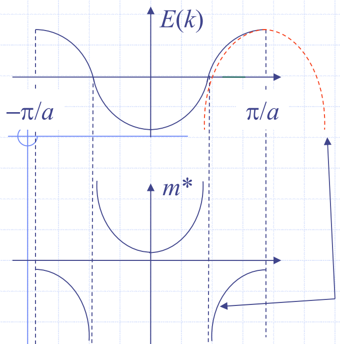


m^* small → light electron → v_g [ ] → mobile
←bandwidth [ ]

m^* large → heavy electron → v_g [ ] → inmobile
←bandwidth [ ]







Effective mass of 1D tight-binding band

$$E(k) = \alpha + 2\beta\cos ka$$

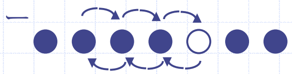
$$\frac{\partial^2 E}{\partial k^2} = \Big[\quad\Big]$$

$$m^* = -\frac{\hbar^2}{2\beta a^2\cos ka}$$

$m^*<0$ for upper half of the band: hole

The wave packet is accelerated in the opposite direction as like a positively charged particle.

Nearly occupied band



$$E(k) = \frac{h^2k^2}{2m^*} = -\frac{h^2k^2}{2|m^*|} < 0$$

The upper, the more stable like a balloon.

Hole: $m^*<0$, $E<0$, a particle with positive charge

(3) Electric conduction

Equation of motion under electric field ε (classical mechanics!)

$$m\frac{dv}{dt}+\frac{m}{\tau}v=-e\varepsilon$$

加速度　摩擦力　電場による力

1) Remove the force, so ε =0 leads to

$$\frac{dv}{dt}=[\quad] \Rightarrow v=[\quad]$$

τ ：relaxation time

Electron is scattered to v=0 within time τ .

2) Steady state at $\varepsilon\neq0$ leads to $\frac{dv}{dt}=0$

$v=[\quad]$ so the current density is $j=-nev=[\quad]$ and

conductivity is $\sigma=\frac{j}{\varepsilon}=[\quad]$

or using mobility $\mu=\frac{e\tau}{m}$ to afford $\sigma=[\quad]$

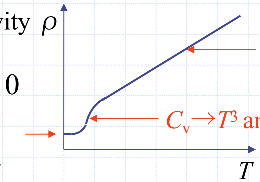
Resistivity $\rho=\frac{1}{\sigma}\propto\frac{1}{\tau}$ Only τ is temperature dependent

In reality $\frac{1}{\tau}=\frac{1}{\tau_{ph}}+\frac{1}{\tau_{i}}$

Phonon scattering / Impurity scattering (independent of temperature)

$\frac{1}{\tau_{ph}}=C_v\cdot T \Rightarrow$ at high T above room T $\frac{1}{\tau_{ph}}\propto T$

Lattice specific heat：const. at high T (Dulong-Petit's rule)



at low T $\frac{1}{\tau_{ph}}\to0$

thus $\rho\propto\frac{1}{\tau_i}$ = const.

Constant resistivity from impurity scattering (残留抵抗 residual resistance)

Resistivity of metals decreases at low temperatures,
and becomes constant at very low temperatures.

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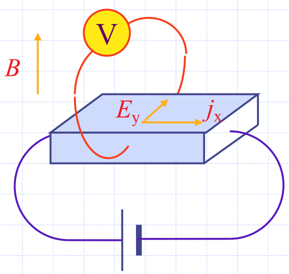
(4) Hall effect

x//: apply current j_x,
z//: apply magnetic field B
y//: measure the generated voltage E_y.



Electrons in electric and magnetic fields feel

$$\bar{F}=e(\bar{v}\times\bar{B}+\bar{E})$$

Lorentz force　force of electric field

No current //y leads to

$$F_y=ev_xB-eE_y=0 \Rightarrow E_y=[\quad]$$

Put this in j_x=-nev_x and $E_y=[\quad]j_xB$

R_H：Hall coefficient cf. hole

Hall coefficient R_H>0 hole
R_H<0 electron

$R_H\to$ carrier concentration n gives $\to$ [Hall] $\mu=\frac{e\tau}{m}$

$\sigma=ne\mu$

---

(5) Thermal conductivity (of metals is due to free electrons)

Thermal conductivity carried by free electrons
in analogy with thermal conductivity carried by phonons (lattice vibration)

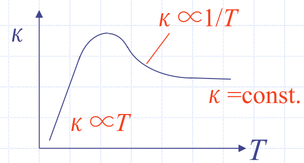
$$\kappa=\frac{1}{3}C_v v_F l_F$$

1/3 for x, y, z directions; electron heat capacity; velocity (at E_F); mean free path (at E_F)

$l_F=v_F\tau$

$$\kappa=\frac{1}{3}\frac{\pi^2}{3}D(E_F)k_B^2T\cdot v_F\cdot v_F\tau \qquad D(E_F)=\frac{3N}{2E_F}=\frac{3N}{mv_F^2} \qquad E_F=\frac{1}{2}mv_F^2$$

$$=\frac{\pi^2Nk_B^2T\tau}{3m}$$



$$\frac{\kappa}{\sigma}=\frac{\frac{\pi^2Nk_B^2T\tau}{3m}}{\frac{Ne^2\tau}{m}}=[\quad]T$$

$\frac{\kappa}{\sigma T}=L$ Lorenz number＝const.
(Wiedeman-Franz law)

High T $\sigma\propto1/T$ κ= [ ]
Intermediate $\sigma\propto1/T^2$ $\kappa\propto$ [ ]
Low T σ =const. $\kappa\propto$ [ ]

Thermal conductivity of various materials

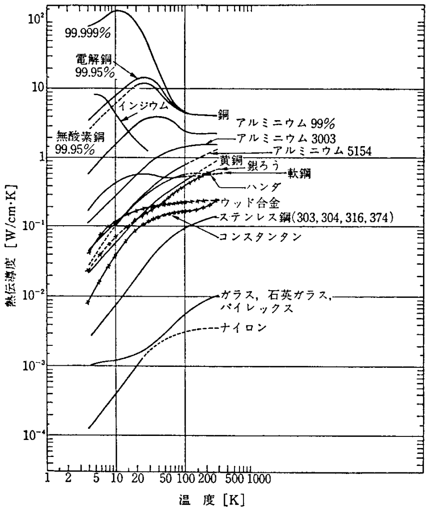


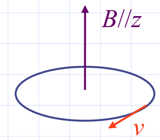
図 5.2 熱伝導率
(BOC 社, "Cryogenic Data Chart" より)

(6) Cyclotron oscillation

Electron moves circularly under magnetic field.

Equation of motion under electric and magnetic fields

$$\frac{d\bar{p}}{dt} = m\frac{d\bar{v}}{dt} = e\bar{v} \times \bar{B}$$ Lorentz force



For magnetic field $B // z$

$$\frac{dv_x}{dt} = -\frac{eB}{m}v_y = -\omega_c v_y \qquad \frac{dv_y}{dt} = \omega_c v_x \qquad \omega_c = \frac{eB}{m}$$

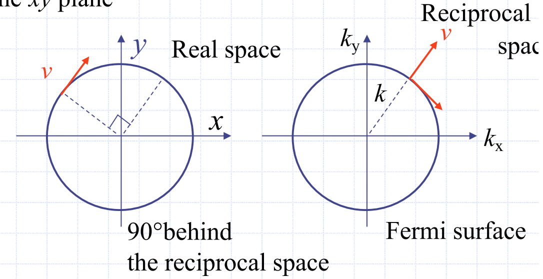
Cyclotron frequency

Electron moves circularly in the *xy* plane

$$\begin{cases} v_x = v_0 \cos \omega_c t \\ v_y = v_0 \sin \omega_c t \end{cases}$$

Thus

$$\begin{cases} x = -\dfrac{v_0}{\omega_c} \sin \omega_c t \\ y = \dfrac{v_0}{\omega_c} \cos \omega_c t \end{cases}$$



where

$$|r| = \frac{v_0}{\omega_c} = \frac{m}{eB}v_0 = \frac{|p|}{eB}$$

and

$$\frac{dE(k)}{dt} = \frac{dE}{d\bar{p}}\frac{d\bar{p}}{dt} = v \cdot (ev \times B) = 0$$

Exterior product of *v* and *B* is $\perp v$. Its inner product is 0.

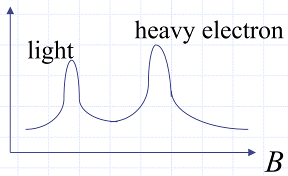
Electron moves on a constant energy surface (= Fermi surface).

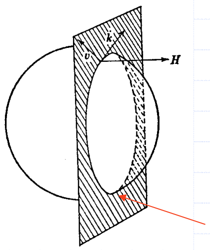
Absorbs the microwave with $\omega_c = \dfrac{eB}{m}$

Cyclotron resonance

$\hbar\omega_c$=const.

Experimental estimation of m^*





Cross section of $H \perp$ surface and the Fermi surface

(7) Quantum Oscillation

$$\omega_c = \frac{eB}{m}$$

Circular motion is quantized just like a hydrogen atom.

(circle)=(wavelength) × (integer) Bohr's quantization condition

$$2\pi r = n\lambda = n\frac{h}{p} \Rightarrow r \cdot p = n\hbar$$

Using $|r| = \dfrac{|p|}{eB}$ for circular motion $\dfrac{p}{eB} \cdot p = n\hbar$

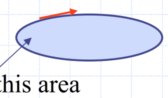
$$\| \quad k^2 = \frac{eB}{\hbar}n$$

Kinetic energy

$$E = \frac{\hbar^2}{2m}(k_x^2 + k_y^2 + k_z^2)$$

$$= \frac{\hbar^2}{2m} 2\frac{eB}{\hbar} n + \frac{\hbar^2}{2m}k_z^2$$

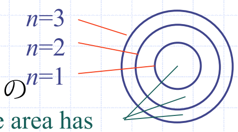
$$= \left[\qquad \right] n + \frac{\hbar^2}{2m}k_z^2$$



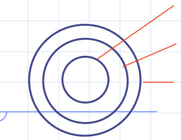
$S_k = \pi(k_x^2 + k_y^2)$ using the area of this circle

$$S_k = \frac{2\pi eB}{\hbar}n$$

S_k is quantized as integer times of $\dfrac{2\pi eB}{\hbar}$



The area has the same interval.



n=1 circle= λ E=$\hbar\omega_c$

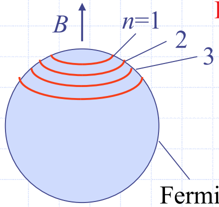
n=2 circle=2 λ − E=2$\hbar\omega_c$

n=3 circle=3 λ − E=3$\hbar\omega_c$

$$\hbar\omega_c = \frac{\hbar e B}{m}$$

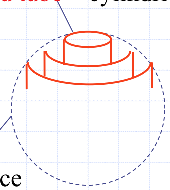
Orbit is restricted to these circles (also circle in the real space).

In three dimension



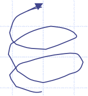
Landau tube：cylindrical slices of the Fermi surface

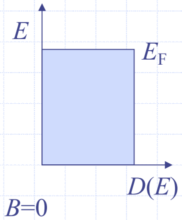
Fermi surface

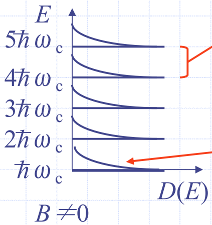


z direction motion $\frac{\hbar^2}{2m}k_z^2$ remains →

Spiral in the real space



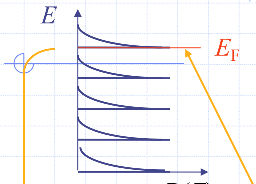




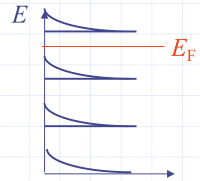
Interval of the Landau levels is 0 at B = 0, and increases with increasing B.

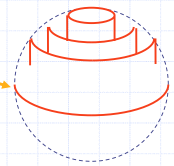
1D energy band for z has density of dates, $D(E)\propto E^{-1/2}$ with a peak at E =0.

Changing B



large B





Occupied upper most Laudau level

= Most outer orbit

= Equator of the Fermi surface

= (In general) extremal cross section

When this crosses E_F by changing B, $D(E_F)$ makes peaks.

The periodicity is

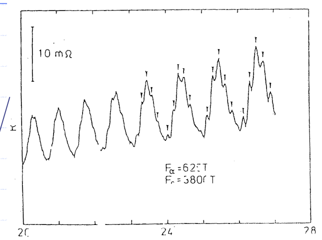
$$S_k = \frac{2\pi e}{\hbar}B_n n = \frac{2\pi e}{\hbar}B_{n+1}(n+1)$$

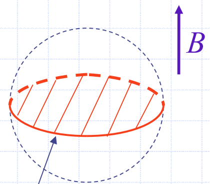
Erasing n from this to give

$$\frac{1}{B_{n+1}} - \frac{1}{B_n} = \Delta(\frac{1}{B}) = \left(\qquad\right)$$

Many properties of metals oscillate when B is scanned at low T.

The periodicity is $\Delta(\frac{1}{B}) = \frac{2\pi e}{\hbar S_k}$





This area

Magnetic field

Electrical Resistivity — Shubnikov-de Haas oscillation

Magnetic Susceptibility — de Haas-van Alphen oscillation

Specific Heat

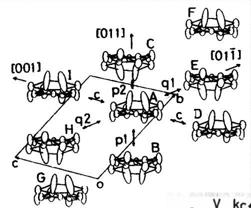
Fermiology to investigate Fermi surface

D. Shoenberg, *Magnetic Oscillations in Metals*, Cambridge (1984).

J. Wosnitza, *Fermi Surfaces of Low-Dimensional Organic Metals and Superconductors*, Springer (1996).

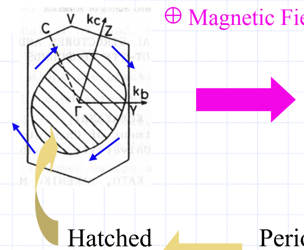
M. V. Kartsovnik, *Chem. Rev.* **104**, 5737 (2004).

Shubnikov- de Haas Oscillation in β -(BEDT-TTF)$_2$I$_3$



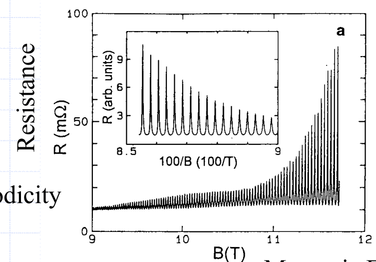
Oscillation of electrons rotating around the Fermi surface is observed at low T (< 1 K) and under strong magnetic field (> 10 T).

1 electron per unit cell



⊕ Magnetic Field

100 mK



Hatched Area

Periodicity

Magnetic Field

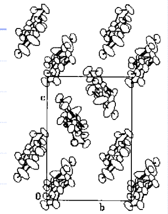
Fermi surface is 50% of the 1st Brillouin zone.

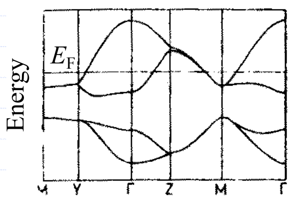
$$\Delta(\frac{1}{B}) = \frac{2\pi e}{\hbar S_k}$$

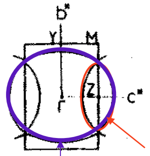
Kang, et al. *Phys. Rev. Lett.* **62**, 2559 (1989).

Shubnikov- de Haas Oscillation in κ-(BEDT-TTF)$_2$Cu(NCS)$_2$

4 molecules in a unit cell → 2 electrons in a cell → S_{FS}/S_{BZ}=100%

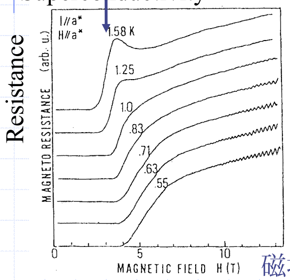




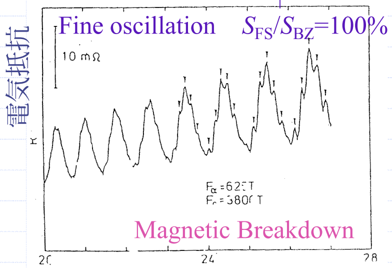


S_{FS}/S_{BZ} =18%

Superconductivity



S_{FS}/S_{BZ} =18%



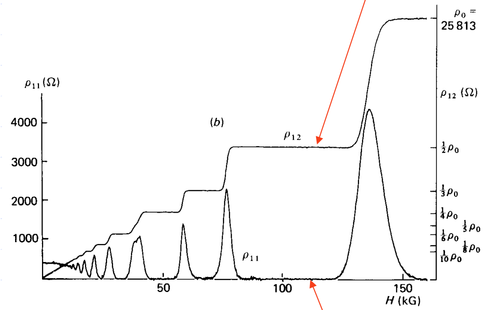
Oshima et al. *Phys. Rev*. B **38**, 938 (1988).

Quantum Hall Effect

Amplitude of Shubnikov-de Haas oscillation grows to reach R→0.
At large B, near the quantum limit （n → 1）.

GaAs MOS FET 0.35 K

Hall effect = const.
抵抗標準

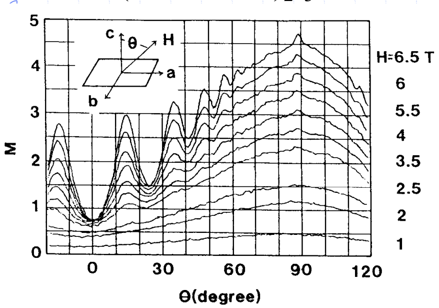


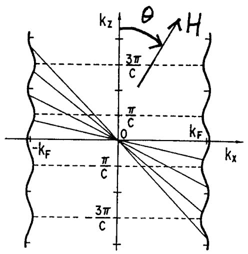
抵抗=0

Angular Dependent Magnetoresistance Oscillation (ADMRO)

θ-(BEDT-TTF)$_2$I$_3$

Measure resistance by titling the magnetic field.





Kajita oscillation (Yamaji oscillation)

$$\tan\theta = \frac{n\frac{\pi}{c}}{k_F}$$

$$ck_F\tan\theta = n\pi$$

Kajita, *Solid State Commun*. **70**, 1189 (1989).
Kartsovnik, *JETP Lett*. **48**, 541 (1988).
Yamaji, *J. Phys. Soc. Jpn*. **58**, 1520 (1989).

Angular Dependent Magnetoresistance Oscillation (ADMRO)

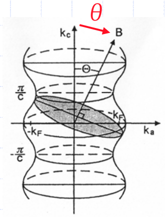
β-(BEDT-TTF)$_2$I$_3$

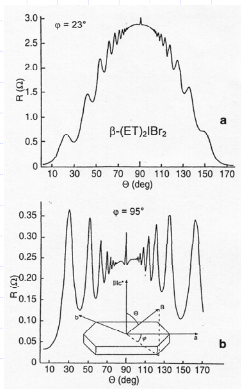
Measure resistance by titling the magnetic field.

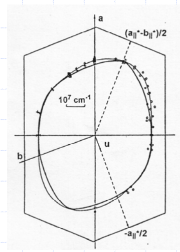
Interval of peaks

positions of k_F

θ





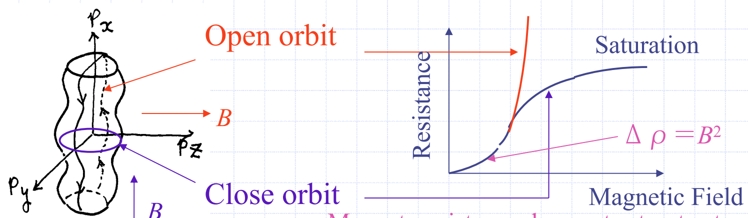


$$\tan\theta = \frac{n\frac{\pi}{c}}{k_F}$$

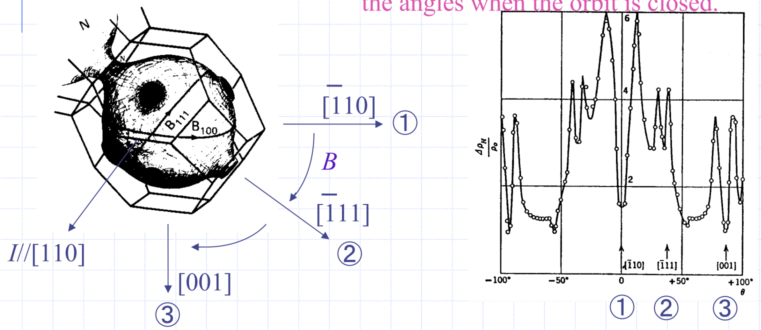
$$ck_F\tan\theta = n\pi$$

θ

Conventional ADMRO due to the Fermi surface of Cu



Magnetoresistance does not saturate at the angles when the orbit is closed.

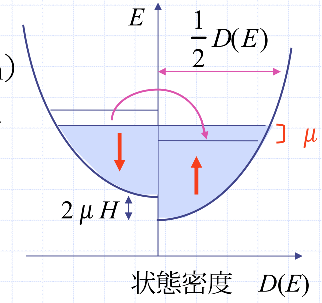


Magnetic Susceptibility of Metal Electrons (Pauli paramagnetism)

↓ is more stable than ↑ under magnetic field.

$$E_{\uparrow} = \frac{\hbar^2 k^2}{2m} - \mu H$$

$$E_{\downarrow} = \frac{\hbar^2 k^2}{2m} + \mu H$$



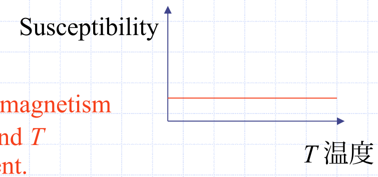
Magnetization is

$$M = \mu(N_{\uparrow} - N_{\downarrow}) = \mu\frac{1}{2}D(E)\times 2\mu H$$

Susceptibility

$$\chi = \frac{M}{H} = \mu^2 D(E_{\mathrm{F}})$$

Pauli paramagnetism is $\chi > 0$ and T independent.

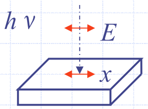


電子比熱係数 $\gamma = \frac{\pi^2}{3}D(E_{\mathrm{F}})k_{\mathrm{B}}^2$ leads to $\frac{\chi}{\gamma} = \left(\qquad\right)$ = const.

Wilson比

Ratio of χ and γ does not change depending on m^*.

Optical Properties of Metals (Plasmon)



When light is irradiated, how the ac electric field enters.

$D(\omega) = \varepsilon_0 E(\omega) + P$　　　$D(\omega) = \varepsilon\,\varepsilon_0 E(\omega)$

These lead to

$$\varepsilon(\omega) = \frac{D(\omega)}{\varepsilon_0 E(\omega)} = 1 + \frac{P(\omega)}{\varepsilon_0 E(\omega)}$$

Eq. of motion vibrated by the electric field E: $m\frac{d^2x}{dt^2} = -eE$

$E, x \propto e^{i\omega t}$ leads to　$-m\omega^2 x = -eE$　→　$x = \left(\qquad\right)$

$P = -nex = \left(\qquad\right)$

Plasma frequency

Put this in the above eq. to　$\varepsilon(\omega) = \left(\qquad\right) = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}$

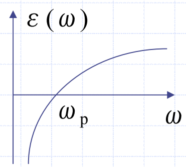
Maxwell eq.　$\mu_0\frac{\partial^2 D}{\partial t^2} = \nabla^2 E$　with $E \propto e^{i(Kr-\omega t)}$ to give

$\mu_0\varepsilon_0\varepsilon\,\omega^2 = K^2$　→　$\omega^2 - \omega_{\mathrm{p}}^2 = c^2K^2$

$1/c^2$

$\omega^2 - \omega_{\mathrm{p}}^2 = c^2K^2$

$$\varepsilon(\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}$$



$\omega < \omega_{\mathrm{p}}$　→　K imaginary

Diminish as $E \propto e^{-Kr}$ in metals

Light cannot enter, and is reflected: metallic luster.

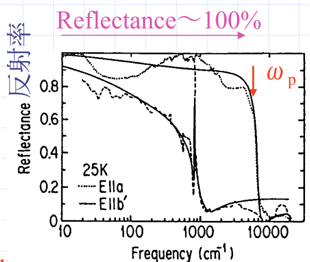
ω_{p}: near ultraviolet in ordinary metals　near infrared in organics

$(TMTSF)_2PF_6$

$\omega > \omega_{\mathrm{p}}$　→　K real

$E \propto e^{-iKr}$　Metals are transparent to UV.

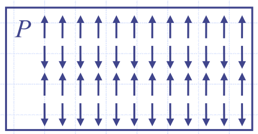
$\omega = \omega_{\mathrm{p}}$　→　$D = 0$　→　$E = -\frac{1}{\varepsilon_0}P = \frac{nex}{\varepsilon_0}$



$$m\frac{d^2x}{dt^2} = -\frac{ne^2}{\varepsilon_0}x \quad\rightarrow\quad \frac{d^2x}{dt^2} + \omega_{\mathrm{p}}^2 x = 0$$

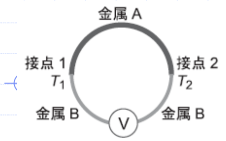
Characteristic oscillation of electron ω_{p}

: plasma oscillation



longitudinal rarefaction wave

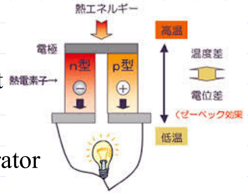
Thermoelectric power: temperature defference → voltage
cf. Thermocouple



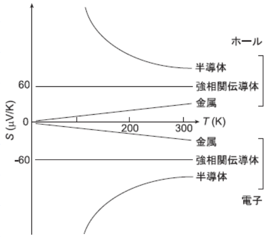
$$\Delta V = S(T_2 - T_1)$$

Themopower or Seebeck coefficient

1 μV/K ~ 1 mV/K



Waste heat → Several 100 K → Generator



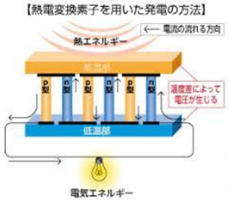
$S > 0$ hole

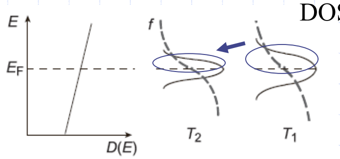
$S < 0$ Electron

Charge polarity determination instead of Hall effect

$S \propto T$ metal

$S \propto 1/T$ semiconductor





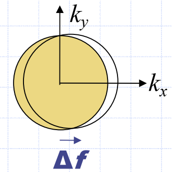
DOS gradient → Different electron numbers above E_F → voltage

---

Transpor equation

Electron# = 2×(Fermi surface volume)/$(2\pi/L)^3$

$$n = \frac{N}{V} = \frac{2}{8\pi^3}\int dSdk = \frac{1}{4\pi^3\hbar}\int \frac{dS}{v_k}dE$$



Electric field $\boldsymbol{\varepsilon}$ → displace the Fermi distribution by Δf

$$\frac{df}{dt} = \frac{\partial f}{\partial \mathbf{p}}\frac{\partial \mathbf{p}}{\partial t} = \frac{\partial f}{\partial E}\frac{\partial E}{\partial \mathbf{p}}\frac{\partial \mathbf{p}}{\partial t} = \frac{\partial f}{\partial E}\mathbf{v}e\boldsymbol{\varepsilon}$$

Relaxation time $\frac{df}{dt} = -\frac{\Delta f}{\tau}$

$$\Delta f = \left(-\frac{\partial f_0}{\partial E}\right)\mathbf{v}e\boldsymbol{\varepsilon}\tau$$

→ delta function

The current is

$$\mathbf{J} = e\iiint \mathbf{v}\Delta f dk = e\int \mathbf{v}\Delta f \frac{1}{4\pi^3\hbar}\frac{dS}{v_k}dE = \frac{e^2\tau\boldsymbol{\varepsilon}}{4\pi^3\hbar}\int \mathbf{v}^2\left(-\frac{\partial f_0}{\partial E}\right)\frac{dS}{v_k}dE$$

$J = e^2K_0\varepsilon$ or $\sigma = e^2K_0$ but $K_0 = \frac{\tau}{4\pi^3\hbar}\int \mathbf{v}^2\left(-\frac{\partial f_0}{\partial E}\right)\frac{dS}{v_k}dE = \frac{\tau}{4\pi^3\hbar}\int \mathbf{v}^2\frac{dS}{v_k}$

Band → integrate v^2 → conductivity (but τ is unknown)

Transpor by T gradient is obtained by replacing $e\varepsilon \to \frac{E(k)-\mu}{T}(-\nabla T)$

$$J = \frac{e}{T}K_1(-\nabla T) \qquad K_1 = \frac{e^2\tau}{4\pi^3\hbar}\int \mathbf{v}^2(E((k)-\mu)\left(-\frac{\partial f_0}{\partial E}\right)\frac{dS}{v_k}dE$$

Both ε and ΔT

$$J = e^2K_0\varepsilon + \frac{e}{T}K_1(-\nabla T) \qquad ①$$

Heat flow U from the next order of $(E(k)-\mu)/e$

$$U = eK_1\varepsilon + \frac{1}{T}K_2(-\nabla T) \qquad ② \qquad K_n = \frac{e^2\tau}{4\pi^3\hbar}\int \mathbf{v}^2(E((k)-\mu)^n\left(-\frac{\partial f_0}{\partial E}\right)\frac{dS}{v_k}dE$$

Integration of any Φ around the Fermi surface is represented by

$$\int \Phi(E)\left(-\frac{\partial f}{\partial E}\right)dE = \Phi(E_F) + \frac{\pi^2}{6}(k_BT)^2\left[\frac{\partial^2\Phi(E)}{\partial E^2}\right]_{E=E_F} + \cdots$$

$\Phi(E) = (E_F - \mu)K_0(E)$ is inserted $\qquad \Phi(E) = (E_F - \mu)^2K_0(E)$ is inserted

$$K_1 = \frac{\pi^2}{3}(k_BT)^2\left[\frac{\partial K_0(E)}{\partial E}\right]_{E=E_F} \qquad \sigma = e^2K_0 \qquad K_2 = \frac{\pi^2}{3}(k_BT)^2K_0(E_F)$$

Putting $J = 0$ in ① $\quad \varepsilon = \frac{1}{eT}\frac{K_1}{K_0}\nabla T$

$$S = \frac{\pi^2k_B^2T}{3e}\left[\frac{\partial \ln\sigma(E)}{\partial E}\right]_{E=E_F} \qquad \sigma = \frac{ne^2\tau}{m}$$

$$N = \frac{V}{3\pi^2}\times\left(\frac{2mE_F}{\hbar^2}\right)^{\frac{3}{2}} \qquad S = \frac{\pi^2k_B^2T}{3e}\left[\frac{\partial \ln N(E)}{\partial E}\right]_{E=E_F}$$

Thermopewer of metal $\propto T$ $\qquad S = \frac{\pi^2k_B^2}{6eE_F}T$

For a one-dimensional band:

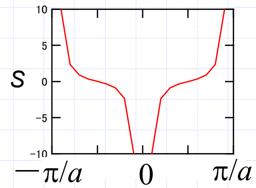
$$D = \frac{\partial N}{\partial E} = \frac{L}{\pi}\frac{\partial k}{\partial E} \qquad \frac{1}{D}\frac{\partial D}{\partial E} = \frac{1}{D}\frac{\partial}{\partial E}\left(\frac{\partial N}{\partial E}\right) = \frac{1}{\frac{L}{\pi}\frac{\partial k}{\partial E}}\frac{\partial k}{\partial E}\frac{\partial}{\partial k}\left(\frac{L}{\pi}\frac{\partial k}{\partial E}\right) = \frac{\partial}{\partial k}\left(\frac{1}{\frac{\partial E}{\partial k}}\right) = -\frac{\frac{\partial^2E}{\partial k^2}}{\left(\frac{\partial E}{\partial k}\right)^2}$$

$$S = -\frac{\pi^2k_B^2T}{3e}\left[\left(\frac{\partial^2E}{\partial k^2}\right)\Big/\left(\frac{\partial E}{\partial k}\right)^2\right]_{E=E_F}$$

$$E = \alpha + 2\beta\cos ka \qquad \frac{\partial^2E}{\partial k^2} = -2\beta a^2\cos ka$$

Themopower ∝ band curvature

$$S = \frac{\pi^2k_B^2T}{6e\beta}\frac{\cos k_Fa}{1-\cos^2k_Fa} = \frac{\pi^2k_B^2T}{6e\beta}\frac{\cos(\pi\rho/2)}{1-\cos^2(\pi\rho/2)}$$



Using $\sigma \propto \exp(E_a/T)$ for semiconductors

$$S = \frac{1}{e}\frac{E_a}{T}$$

Thermopower of semiconfuctor ∝ $1/T$