

Dielectrics in a capacitor

Capacitors:

$$Q = CV \quad q = \epsilon E \quad E = V/d$$

$$C = \epsilon S/d \quad Q = qS$$

$$\epsilon_0 = 0.088 \text{ pF/cm}$$

Insert dielectrics with keeping q

Actual field Original electric field

$$E = E_0 - 4\pi P \quad \epsilon_0 E = D - P \quad q' = q - \sigma$$

$$\rightarrow D = E + 4\pi P \text{ cgs} \quad D = \epsilon_0 E + P \text{ MKS}$$

$$D = \epsilon E \text{ dielectric const.} \quad D = \epsilon_0 \epsilon_r E \rightarrow$$

E decreases $1/\epsilon$ of D e.g. Si ($\epsilon = 11.9$) $\rightarrow 1/11.9$ water $\rightarrow 1/80$

Insert dielectrics with keeping $V \rightarrow C$ is $\propto \epsilon$

In particular, $\sigma = q$ perfect polarization $\rightarrow E = 0$ and $\epsilon = \infty$

$\rightarrow$ Metal ($E = 0$ in a metal)

Always $P > 0 \rightarrow \epsilon > 1$

1

Displacement current when charging a capacitor

$$I = \frac{dQ}{dt} = \frac{dCV}{dt} = i\omega CV \quad \text{using } V = Ve^{i\omega t}$$

Conductance of the dielectric, $G = 1/R$, is added,

$$I = GV + i\omega CV = (G + i\omega C)V$$

In the Maxwell equation

$$\nabla \times H = \frac{1}{c} \frac{\partial D}{\partial t} + \frac{4\pi}{c} j \quad \text{cgs (for MKS: } 4\pi \rightarrow 1, c \rightarrow 1)$$

Insert $D = \epsilon E$ and $E = Ee^{i\omega t}$

$$\nabla \times H = \frac{1}{c} \epsilon \frac{\partial E}{\partial t} + \frac{4\pi}{c} \sigma E = \frac{1}{c} \left(\epsilon + \frac{4\pi i \sigma}{\omega} \right) \frac{\partial E}{\partial t}$$

Inside () is replaced by $\epsilon^* = \epsilon' + i\epsilon''$

$$\epsilon^* = \epsilon + \frac{4\pi i \sigma}{\omega}$$

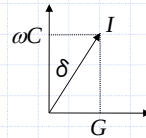
Imaginary part of the complex dielectric constant is conductivity.

2

$$I = GV + i\omega CV = (G + i\omega C)V$$

$$\text{where } \tan \delta = \frac{G}{\omega C} = \frac{\epsilon''}{\epsilon'}$$

$\tan \delta = \epsilon'' = 0$ No dielectric loss (no conductance)



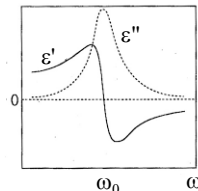
To obtain ω dependence of ϵ , in the eq. of motion of an electron

$$\frac{d^2 x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = \frac{e}{m} E$$

$P = Nex = \epsilon_0 (\epsilon_r^* - 1)E$, and $E = Ee^{i\omega t}$ are inserted.

$$\epsilon_r^* = 1 + \frac{Ne^2 / \epsilon_0 m}{\omega_0^2 - \omega^2 + i2\omega\gamma}$$

ω_0 : characteristic frequency



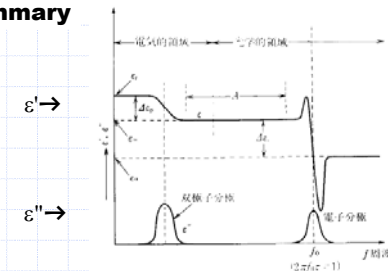
Polarization P due to the dipole relaxes according to $\frac{dP}{dt} = \frac{P_b}{\tau} - \frac{P}{\tau}$

Inserting $P = P_b e^{i\omega t}$ affords

$$\epsilon_r^* = \epsilon_{\infty} + \frac{\epsilon_{r0} - \epsilon_{\infty}}{1 + i\omega\tau}$$

3

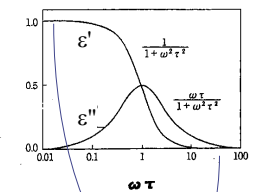
In summary



Deleting ω and τ from the above equation ϵ^*

$$\left(\epsilon' - \frac{\epsilon_0 + \epsilon_{\infty}}{2} \right)^2 + \epsilon''^2 = \left(\frac{\epsilon_0 - \epsilon_{\infty}}{2} \right)^2$$

Cole Cole plot



High freq. limit Low freq. limit

4

Maxwell equation $\nabla \times E = \frac{\mu}{c} \frac{\partial H}{\partial t}$ affords

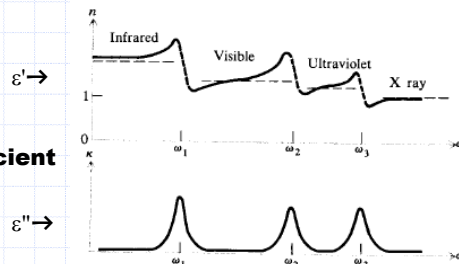
$$\nabla^2 E = \frac{\epsilon \mu}{c^2} \frac{\partial^2 E}{\partial t^2} + \frac{4\pi \sigma \mu}{c^2} \frac{\partial E}{\partial t}$$

Inserting $E = E_0 e^{i(qr - \omega t)}$ gives

$$q^2 = \mu \frac{\omega^2}{c^2} \left(\epsilon + i \frac{4\pi \sigma}{\omega} \right)$$

Light velocity $v = \omega/q$, refractive index $n = c/v$ in a medium, and complex refractive index $n^* = n + ik$ give $|n^*|^2 = \epsilon^*$

Refractive index



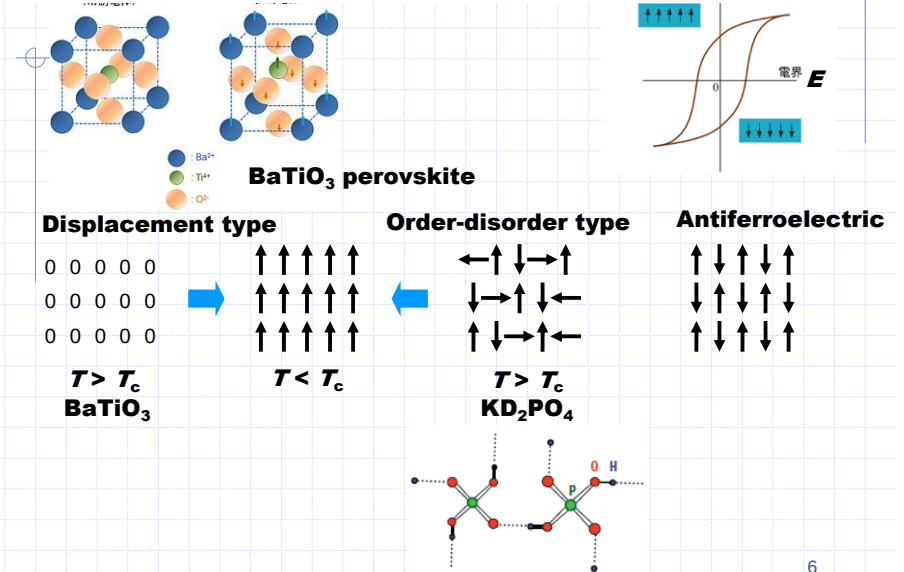
Absorption coefficient

$$\alpha = \frac{2\omega}{c} k$$

Fowler, Introduction to Modern Optics

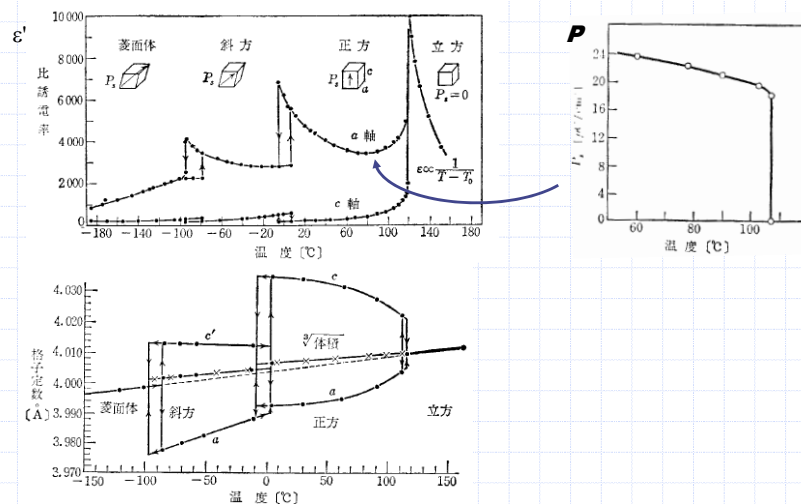
5

Ferroelectrics: spontaneous polarization in a direction P ($E = 0$ leads to $P \neq 0$)



6

BaTiO₃



7

Free energy F is expanded by P (because $P \sim 0$ at $T \sim T_c$)

$$F = aP^2 + bP^4 + \dots$$

P at F minimum is obtained by differentiating F by P .

$$\frac{\partial F}{\partial P} = 2aP + 4bP^3 = 0$$

Assuming $a = a_0(T - T_c)$,

$$T > T_c \quad P = 0$$

$$T < T_c \quad P = \sqrt{\frac{-a}{2b}} = \sqrt{\frac{a_0(T_c - T)}{2b}}$$

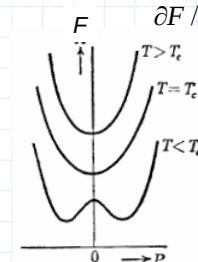
Energy in a field E is

$$F = aP^2 + bP^4 + \epsilon_0 P E$$

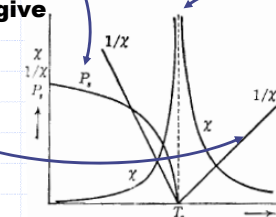
differentiated by P to give

$$\epsilon' = \frac{P}{E} = \frac{\epsilon_0}{2a_0(T - T_c)}$$

Curie law



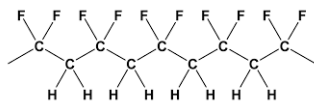
2nd order transition



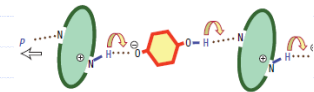
8

Organic ferroelectrics

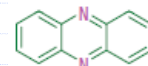
ポリフッ化ビニリデン PVDF (Polyvinylidenedifluoride)



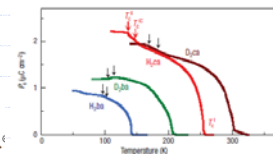
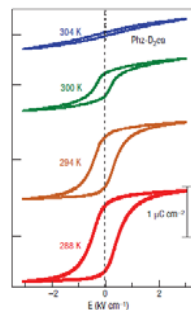
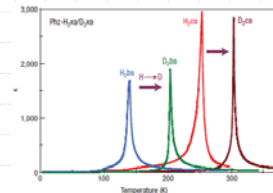
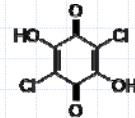
Polyvinylidene fluoride (PVDF)



Phenazine (Phz)



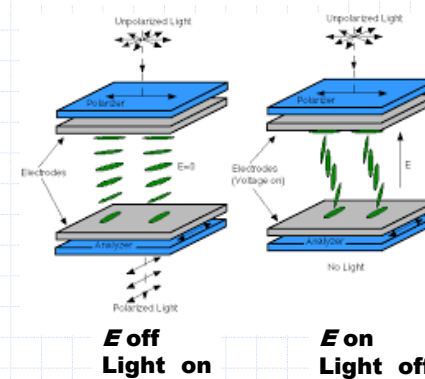
Chloranilic acid



Nature Mater. 7, 357 (2008).

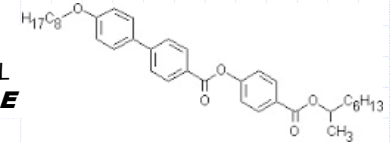
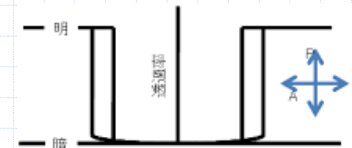
9

Liquid crystal display Twisted Nematic



In a rod-like molecule
Largely different ϵ in $\parallel$ and $\perp$
 $\rightarrow$ Parallel alignment in a field E

Antiferroelectric liquid crystal



MHPOBC

10

Optical Properties of Metals (Plasmon)

Eq. of motion vibrated by an electric field E

$$m \frac{dp}{dt} = -eE - \frac{p}{m\tau} \quad p = p(\omega)e^{i\omega t}$$

These lead to $-i\omega p(\omega) = -eE(\omega) - \frac{p(\omega)}{m\tau}$

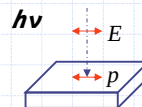
$$\text{then } j(\omega) = -enp(\omega)/m = \frac{ne^2\tau}{m} \frac{E}{1-i\omega\tau}$$

$$\text{Using } \sigma_0 = \frac{ne^2\tau}{m} \rightarrow \sigma(\omega) = \frac{\sigma_0}{1-i\omega\tau}$$

$$\text{or } \text{Re}\sigma = \frac{\sigma_0}{1+\omega^2\tau^2} \quad \text{Im}\sigma = \frac{\sigma_0\omega\tau}{1+\omega^2\tau^2}$$

Dielectric constant of a metal

$$\epsilon' = \epsilon - \frac{4\pi\sigma_0\tau}{1+\omega^2\tau^2} \quad \epsilon'' = \frac{4\pi\sigma_0}{\omega(1+\omega^2\tau^2)} \quad \epsilon \rightarrow \epsilon + \frac{4\pi\sigma}{\omega}$$



11

At low frequency: $\omega\tau \ll 1 \rightarrow \text{real} = 0 \quad \epsilon \sim i \frac{4\pi\sigma}{\omega}$

Refractive index $n^* = \sqrt{\epsilon} = n + ik$ leads to $n \sim k = \sqrt{\frac{\text{Im}\epsilon}{2}} = \sqrt{\frac{2\pi\sigma_0}{\omega}}$

Reflectivity $R = \frac{|1-n^*|^2}{|1+n^*|^2} = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}$ is $R \sim 1$

Light cannot enter, and is reflected: metallic luster.

At high frequency: $\omega\tau \gg 1 \rightarrow$ real part: **Plasma frequency**

$$\epsilon' = \epsilon - \frac{4\pi\sigma_0}{\omega^2\tau} = \epsilon - \frac{4\pi ne^2}{\omega^2 m} = \epsilon \left(1 - \frac{\omega_p^2}{\omega^2} \right) \quad \omega_p^2 = \frac{4\pi ne^2}{m}$$

$\omega < \omega_p \quad \epsilon < 0 \quad n = 0 \rightarrow R = 1$ Total reflection

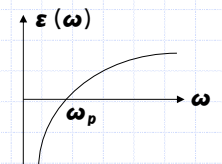
Light cannot enter, and is reflected: metallic luster.

ω_p near ultraviolet in ordinary metals

$\omega > \omega_p \quad \epsilon > 0 \quad k = 0$

$\rightarrow$ Absorption $\alpha = \frac{2\omega}{c} k \rightarrow 0$

$\rightarrow$ Metals are transparent for UV light.



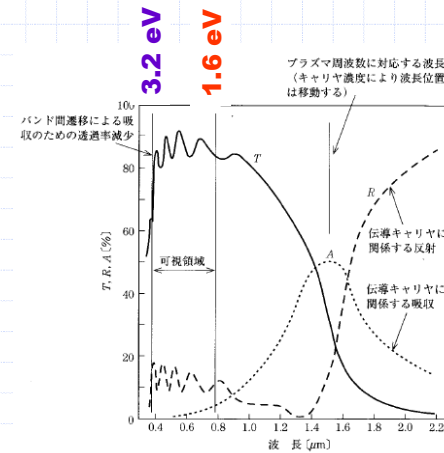
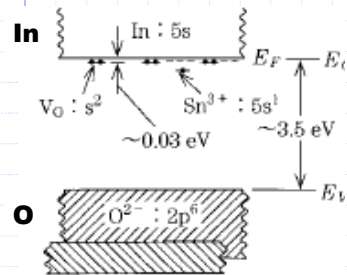
12

Transparent conductors Doped wide-gap semiconductors



Liquid crystal display
LED display
Solar cell

ITO In_2O_3 In^{3+} (+ Sn^{4+})



13

Energy units conversion

	10 ⁻⁴	10 ⁻³	10 ⁻²	10 ⁻¹	1 eV	10	100
Energy							
Wave length $E = hc/\lambda$	10	1 mm	100	10	1 mm	100 nm	
Wave number	1 cm ⁻¹	10 ¹	10 ²	10 ³	10 ⁴	10 ⁵	
Temperature $E = k_B T$	1 K	10 ¹	10 ²	10 ³	10 ⁴	10 ⁵	

room temperature $E = (1240 \text{ nm/eV}) / \lambda$

1 cm⁻¹ ~ 1 K ~ 1 T

Magnetic field $E = k_B T$ $E = \mu_B H$

1 eV = 1.24 μm $eV = hc/\lambda$
 = 8065 cm⁻¹
 = 11605 K $E = k_B T$
 = 0.96x10⁵ J/mol or C/mol eN_A Faraday const. = 1 mol charge

1 eV ~ 1 μm ~ 10⁴ cm⁻¹
 ~ 10⁴ K ~ 100 kJ/mol

14

Various transparent conductors

In_2O_3 several %Sn doped → ITO (Indium tin oxide)

In (3+) → Sn (4+) Electron doped cf. Si (4+) → As (5+)

Oxygen deficient → In⁰ from charge neutrality
→ Electron doped

Electron doped in most oxides

Films from magnetron sputtering

$2 \times 10^{-4} \Omega\text{cm}$, $E_g \sim 3.3 \text{ eV}$

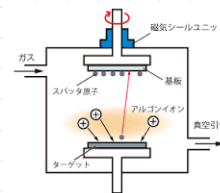
SnO_2 Rutile, oxygen deficiency

ZnO Wurtzite, oxygen deficiency

$\text{Ti}_{1-x}\text{Nb}_x\text{O}_2$ Rutile

SrTiO_3 Perovskite, oxygen deficiency

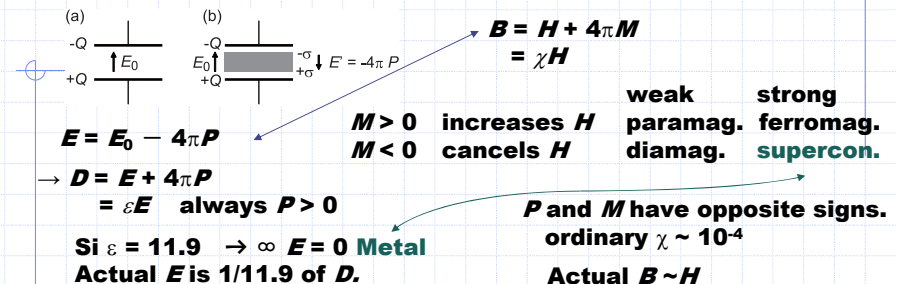
$12\text{CaO} \cdot 7\text{Al}_2\text{O}_3$ Partly reduced, electron doped
(Electrolyde)



15

Dielectrics in a capacitor

Magnets



(a) 誘電体

(b) 磁性体

外場 D ↑
 実際の電場 E ↑
 分極 P ↑

外場 H ↑
 実際の磁場 B ↑
 磁化 M ↑

$$D = E + 4\pi P$$

$$D = \epsilon_0 E + P$$

$$B = H + 4\pi M$$

$$B = \mu_0 H + M$$

cgs
MKS

16

Curie Paramagnetism

Magnetic moment from unpaired electrons

$$\mu = \gamma \hbar \mathbf{S} = -g \mu_B \mathbf{S}$$

Zeemann splitting due to the magnetic field $\mathbf{H}$

$$E = -g \mu_B \mathbf{H} \mathbf{S} = -\mu H$$

Split to two for $S=1/2$. In thermal equilibrium:

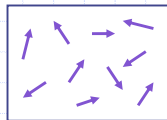
$$\frac{N_{\uparrow}}{N} = \frac{e^{\mu H / k_B T}}{e^{\mu H / k_B T} + e^{-\mu H / k_B T}} \quad \frac{N_{\downarrow}}{N} = \frac{e^{-\mu H / k_B T}}{e^{\mu H / k_B T} + e^{-\mu H / k_B T}}$$

lead to

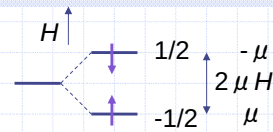
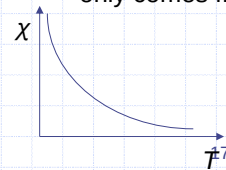
$$M = \mu_B (N_{\uparrow} - N_{\downarrow}) = N \mu \tanh \frac{\mu H}{k_B T} \approx N \mu \left(\frac{\mu H}{k_B T} \right) \leftarrow \frac{\mu H}{k_B T} \ll 1 \quad \text{so}$$

$$\chi = \frac{M}{H} = \frac{N \mu^2}{k_B T} = \frac{C}{T} \quad \text{Curie constant} \quad \text{Except for } S=1/2 \quad C = \frac{NS(S+1)g^2 \mu_B^2}{3k_B}$$

χ is inversely proportional to T . only comes from S .



Random spins
Aligned under magnetic field.
More aligned at low T .



Magnetic Order

Ferromagnetism: all parallel
The whole material is a magnet.

$J > 0$
 S_i / S_j is stable

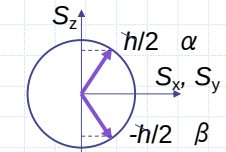
Antiferromagnetism:
alternately opposite directions

$J < 0$
Antiparallel S_i and S_j

Spin Hamiltonian

$$\hat{H} = -\sum_{i,j} 2J_{ij} \bar{S}_i \bar{S}_j - g \mu_B H \sum_i \bar{S}_i$$

Interaction Zeemann splitting



S_i is a vector like (S_x, S_y, S_z) : (Heisenberg model).

When spin is always directed in one direction (z) due to the large magnetic anisotropy coming from the crystal field, we only consider S_z .

(Ising model)

When $S_i = 1/2, S_j = 1/2$, (Interaction) $= -J/2$

When $S_i = 1/2, S_j = -1/2$, (Interaction) $= J/2$

Energy difference J

Note: (Interaction) is defined as $\sum_{i,j} J_{ij} \bar{S}_i \bar{S}_j$ in old literatures.
 J is twice larger. 18

Molecular Field (Mean Field) Approximation

Focusing on S_i , and use average sum for $\sum_j$



$$\hat{H} = \sum_i S_i \left(-\sum_j 2J_{ij} S_j - g \mu_B H \right) = -g \mu_B (H_{\text{eff}} + H) \sum_i S_i$$

$$H_{\text{eff}} = \frac{1}{g \mu_B} \sum_j 2J_{ij} \langle S_j \rangle$$

Interaction with S_j is replaced by a field (effective or internal field) (有効 or 内部磁場) generated on S_i .

S_j changes every moment, but S_j is approximated by the average $\langle S_j \rangle$
(分子場近似 or 平均場近似 Molecular Field or Mean Field Approximation)

Statistical distribution similar to the Curie paramagnetism gives

$$M = \frac{N \mu^2 H}{k_B T} \xrightarrow{H \rightarrow H_{\text{eff}} + H} M = \chi_0 (H_{\text{eff}} + H)$$

分子磁場係数

where $M = N g \mu_B \langle S \rangle$ leads to $\sum_j 2J_{ij} \langle S_j \rangle$

$$H_{\text{eff}} = \frac{\sum_j 2J_{ij} \langle S_j \rangle}{N (g \mu_B)^2} M = a M$$

Put this in the above eq. to give

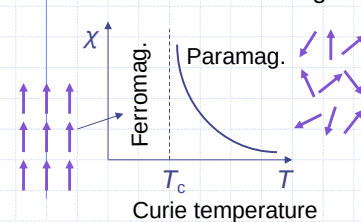
$$M = \chi_0 (a M + H) \quad M \left(\frac{1 - \chi_0 a}{\chi_0} \right) = \chi_0 H \rightarrow \chi = \frac{M}{H} = \frac{\chi_0}{1 - \chi_0 a} = \frac{C}{T - \theta}$$

Curie-Weiss則

$$\theta = \frac{1}{k_B} \sum_j 2J_{ij} = \frac{2zJ}{k_B} \quad \leftarrow \text{Coordination number } z \text{ for nearest neighbor } J\text{'s}$$

Weiss temperature

$J > 0 \rightarrow \theta > 0$ Ferromagnetism

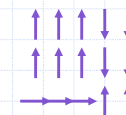
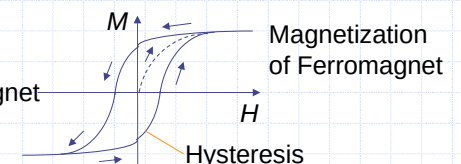


$T \rightarrow T_c: \chi \rightarrow \infty$

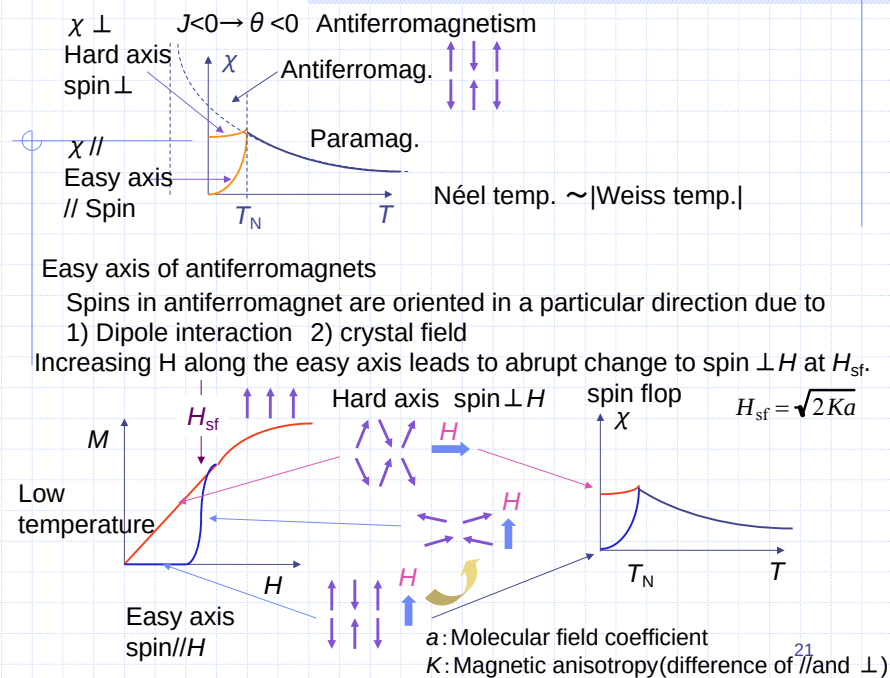
$T < T_c: M \neq 0$ for $H=0$

Spontaneous Magnetism $\rightarrow$ Magnet

T_c Fe 1043 K Ni 627 K

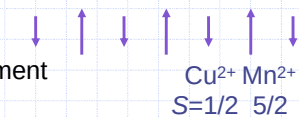


Usual ferromagnet has randomly oriented magnetic domains (磁区), but magnetic field aligns the spin orientation to make a bulk magnet.

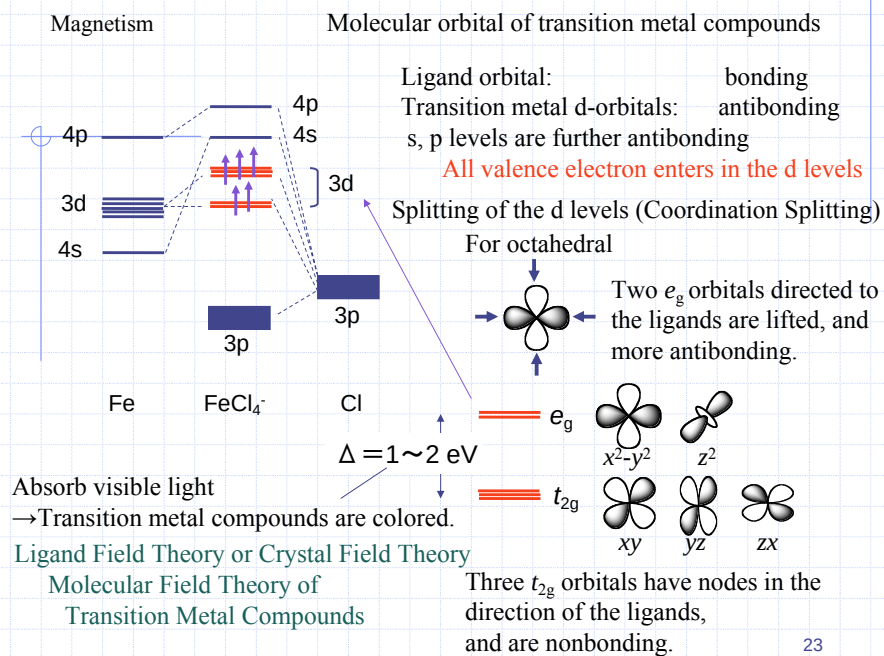


Ferrimagnetism

Alternating spins with different S
(e.g. different metals) lead to remaining moment
even for antiparallel order for $J < 0$.
Ferrite Fe_3O_4 has Fe^{3+} and Fe^{2+} .
Most molecular magnets.



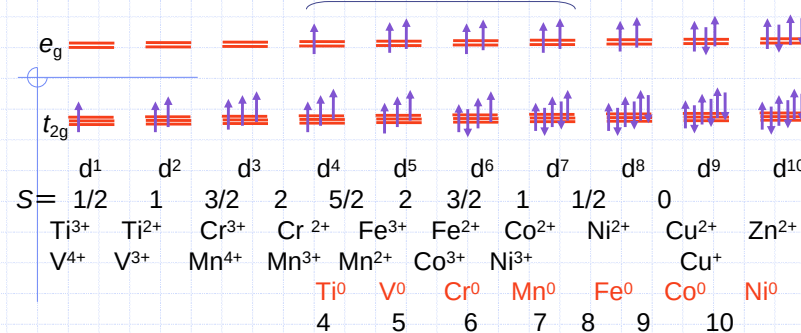
22



23

For octahedral

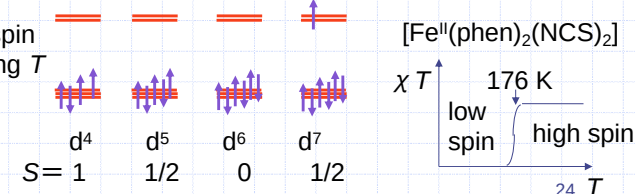
high spin



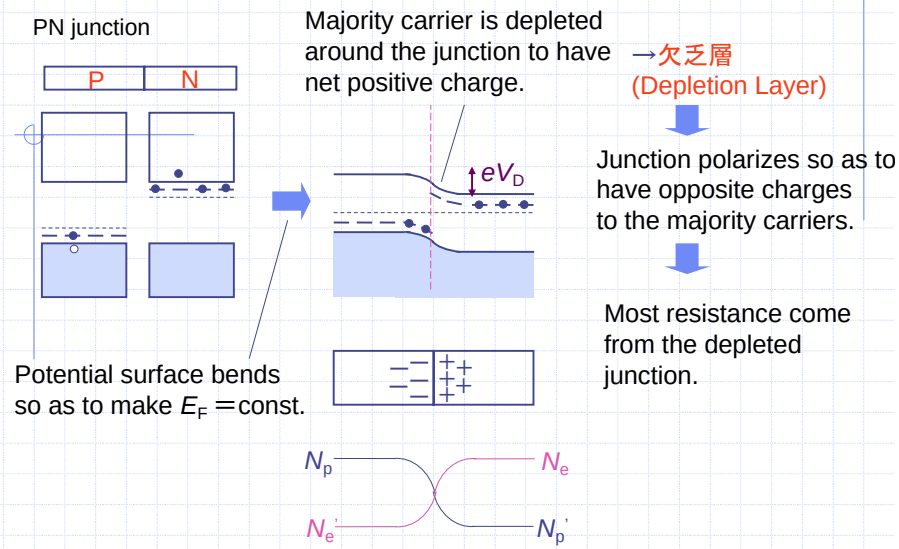
For $d = 4 \sim 7$, when crystal field $>$ Hund rule, low spin state achieved

Transition from high spin
to low spin by changing T

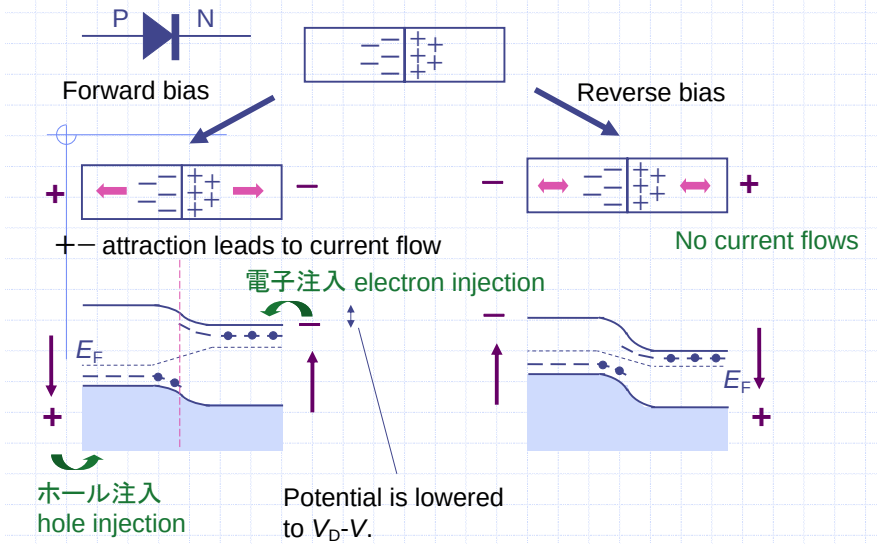
Spin crossover
Low spin d^6 is
nonmagnetic



24

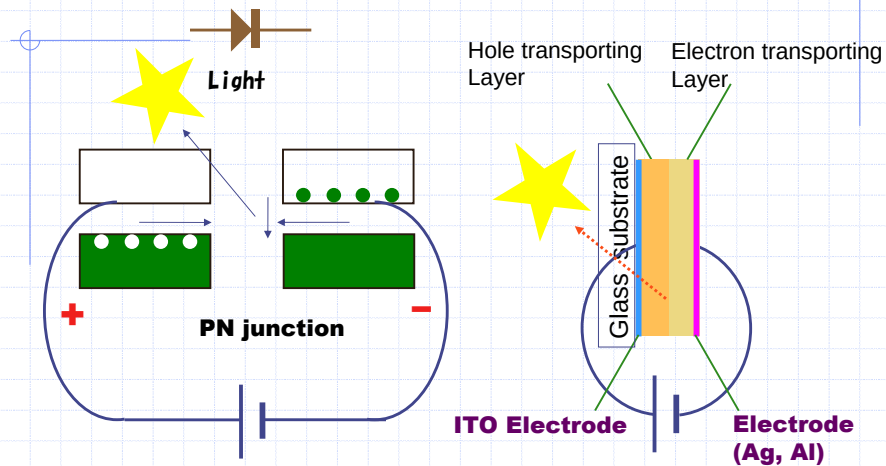


25



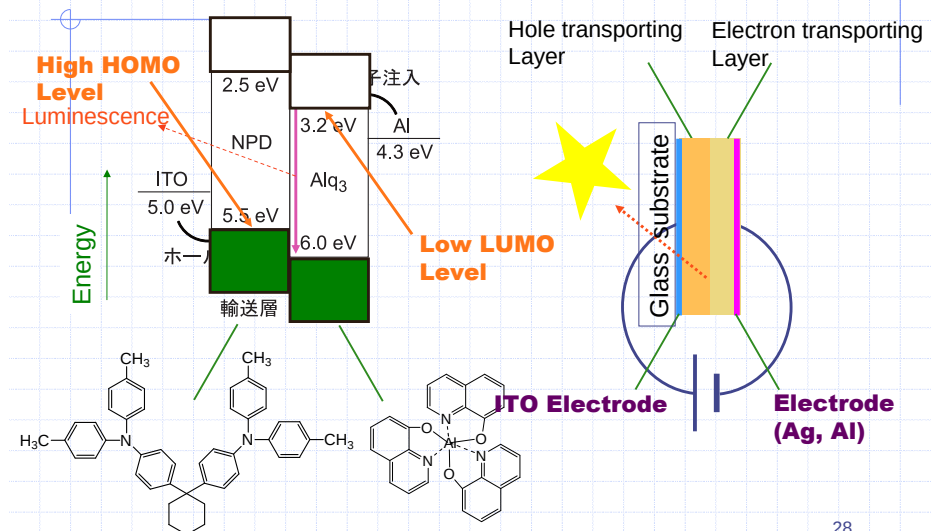
26

有機発光ダイオード Light Emitting Diode (LED) 有機EL (Electroluminescence)



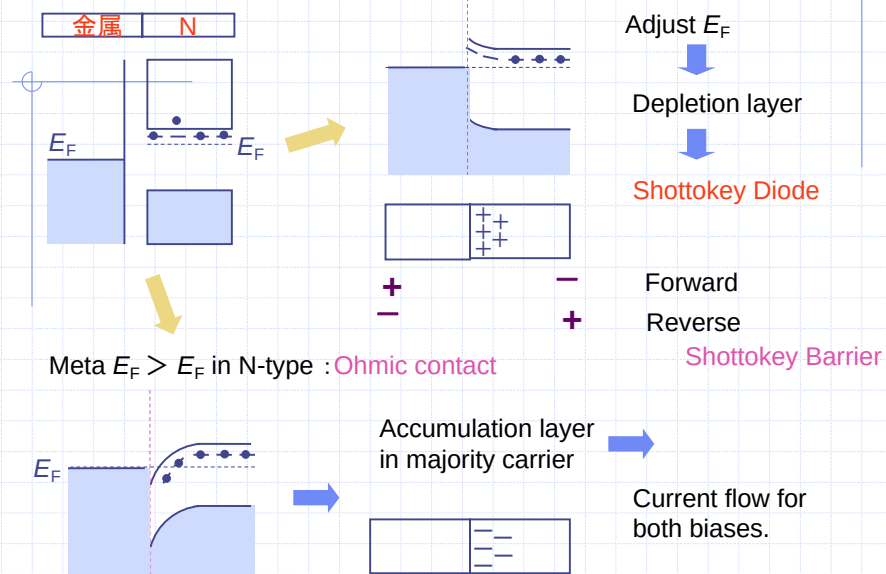
27

有機発光ダイオード Light Emitting Diode (LED) 有機EL (Electroluminescence)



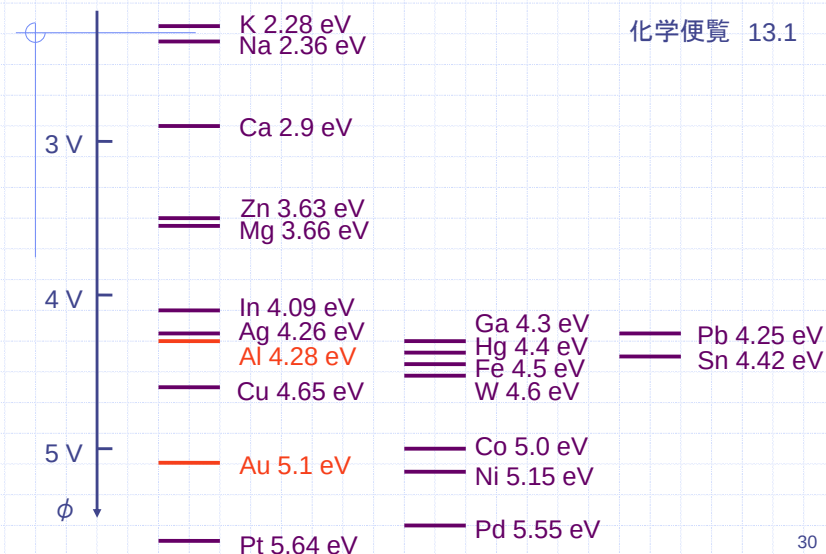
28

Metal-semiconductor junction



29

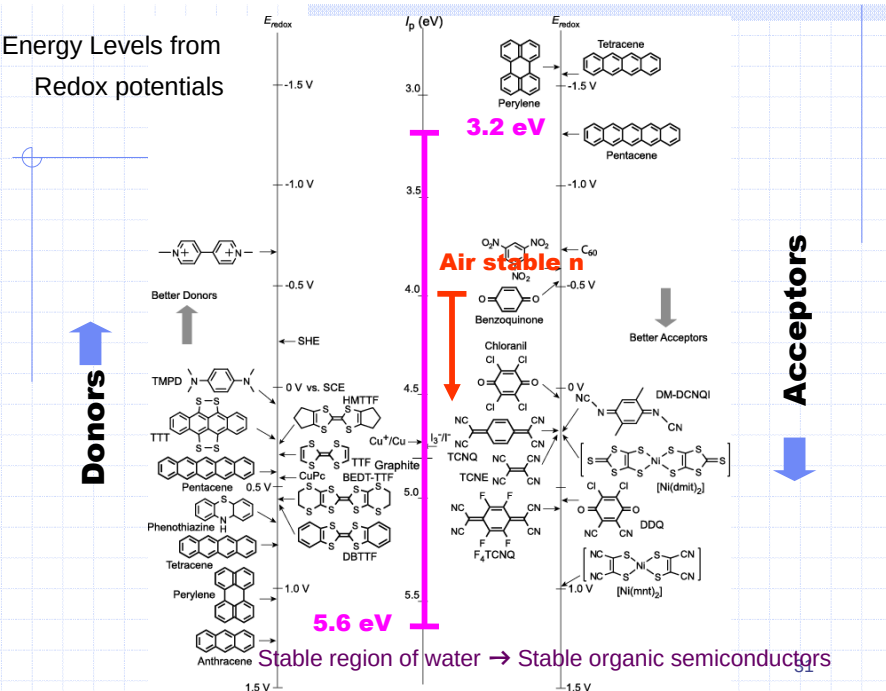
Work function of Metals (Position of E_F) cf. Ionization potential for semiconductors



化学便覧 13.1

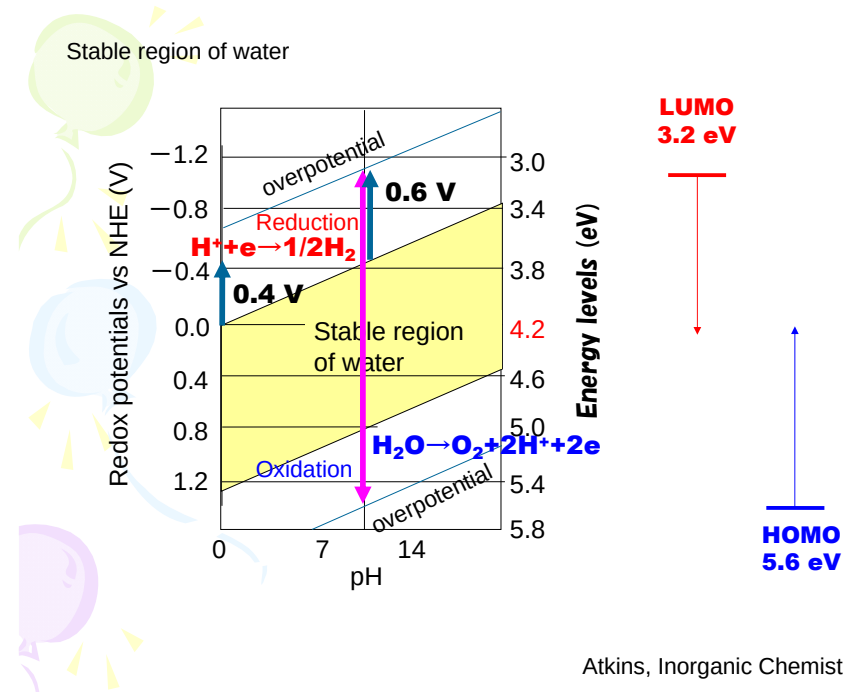
30

Energy Levels from Redox potentials



31

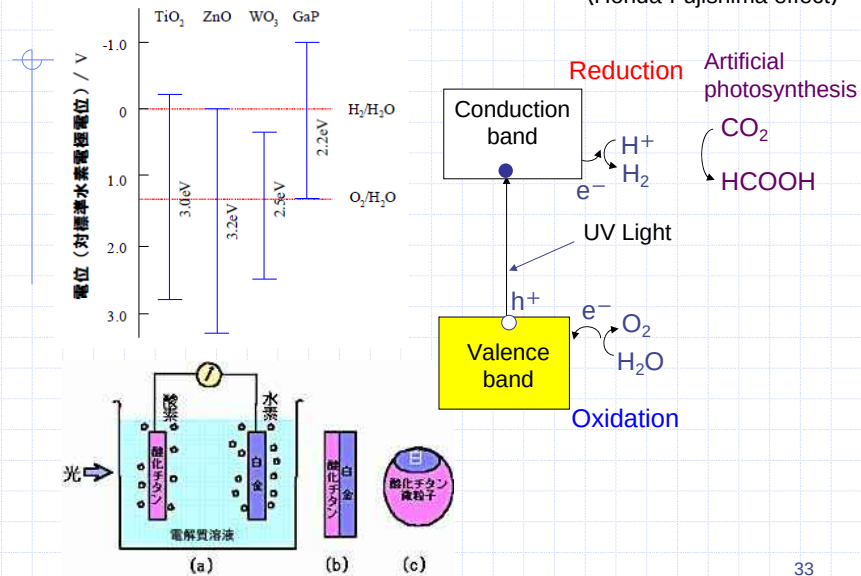
Stable region of water



Atkins, Inorganic Chemistry 32

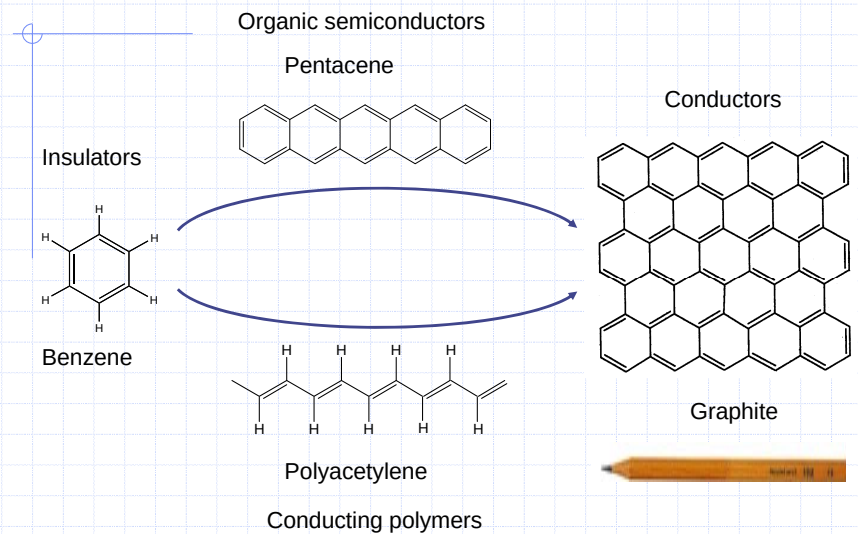
Photocatalysis: TiO_2 irradiation decomposes water to H_2 and O_2

(Honda-Fujishima effect)



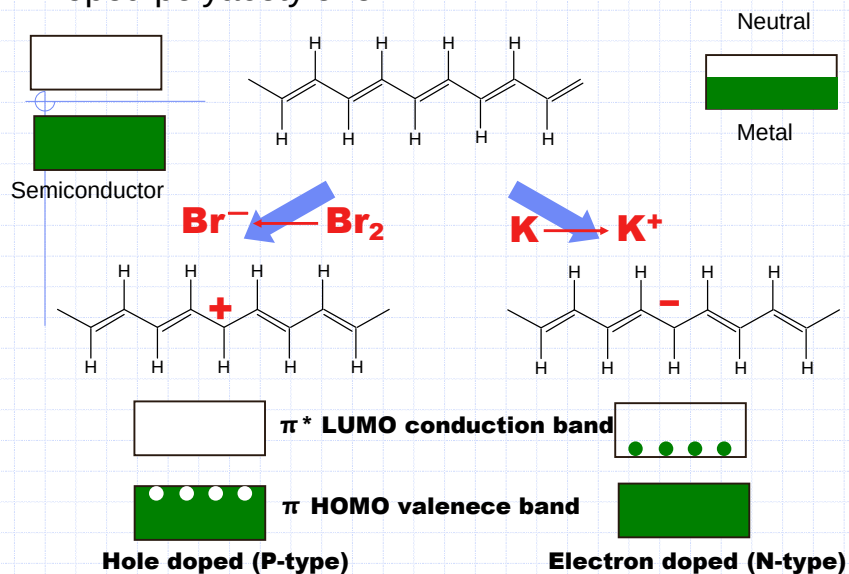
33

How organic materials conduct electricity?



34

Doped polyacetylene



35

Discovery of conducting polymers

The photograph shows the three scientists who discovered conducting polymers: H. Shirakawa, A. J. Heeger, and A. G. MacDiarmid. They were awarded the Nobel Prize in Chemistry in 1974.

H. Shirakawa
A. J. Heeger
A. G. MacDiarmid
1974~1977

Iodine doped polyacetylene

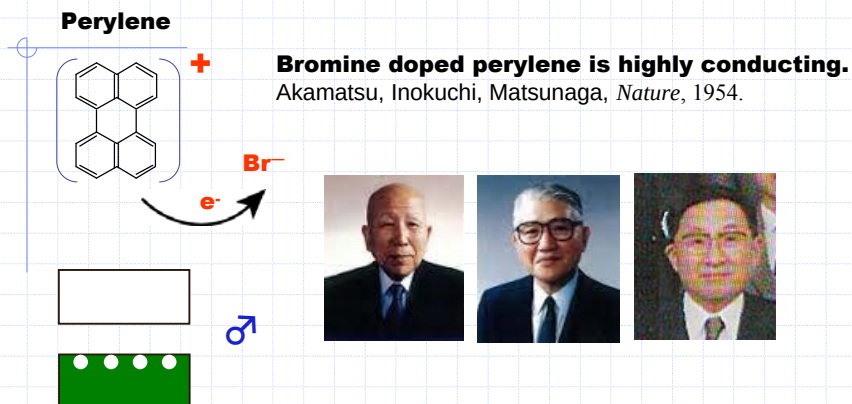
Polyacetylene film

Conductivity vs. time graph

Chemical structure of iodine doped polyacetylene

36

First conducting organics : charge-transfer complex

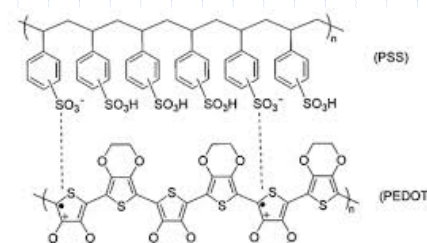


37

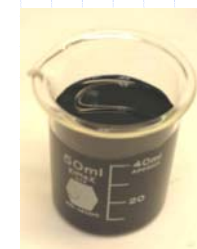
Polyacetylene → Polythiophene

PEDOT:PSS

poly(ethylenedioxythiophene) : polystyrene sulphonate)



Conductivity $\sigma \sim 300 \text{ S/cm}$
 $S = \Omega^{-1}$ Siemens

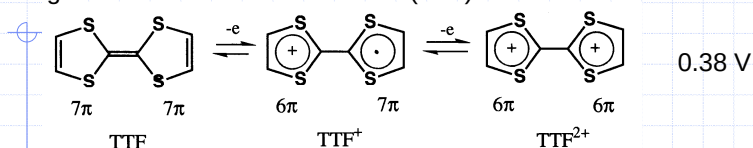


Water solution

38

Good donors and acceptors

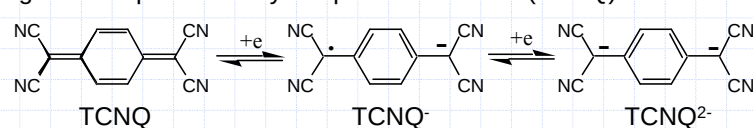
A good donor: Tetrathiafulvalene (TTF)



This 7π system easily gives up one electron to form a 6π system.
($C \rightarrow 1\pi$, $S \rightarrow 2\pi$)

Electron donating groups such as $-\text{NH}_2$, $-\text{OCH}_3$ strengthen donor ability.

A good acceptor Tetracyanoquinodimethane (TCNQ)

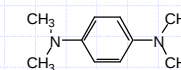
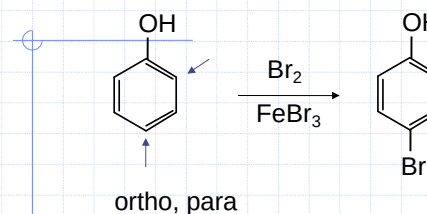


Reduction restores from the quinoid structure to an aromatic 6π system.
– emerges on the foot of two electron withdrawing groups (CN).

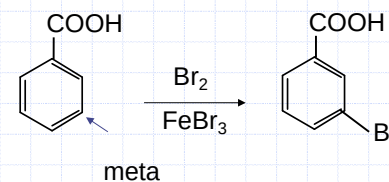
Electron withdrawing groups such as $-\text{CN}$, $-\text{NO}_2$ strengthen acceptor ability.

39

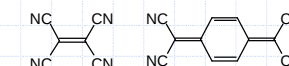
Electron donating : $-\text{OH}$, $-\text{OCH}_3$, $-\text{NH}_2$ → Electron Donors



Electron withdrawing : $-\text{NO}_2$, $-\text{CN}$, halogene, $-\text{COOH}$, $-\text{COOCH}_3$, $-\text{CHO}$

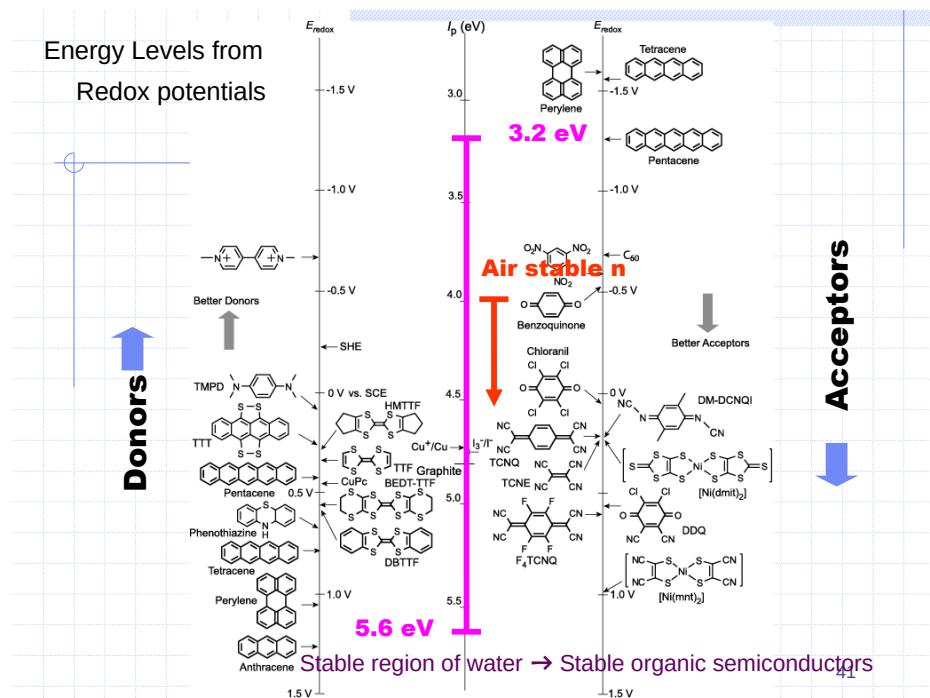


→ Electron Acceptors

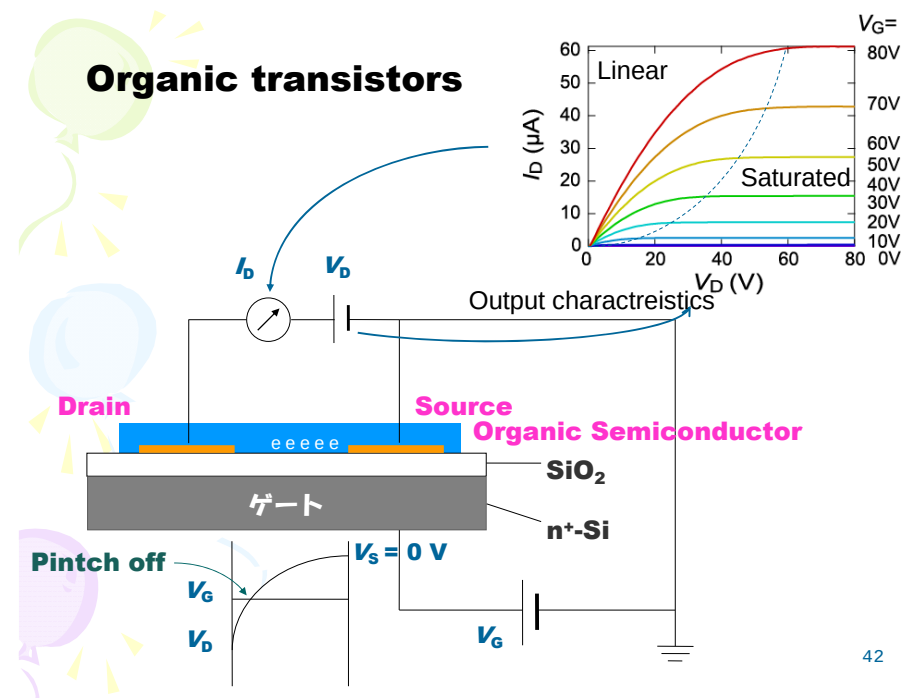


40

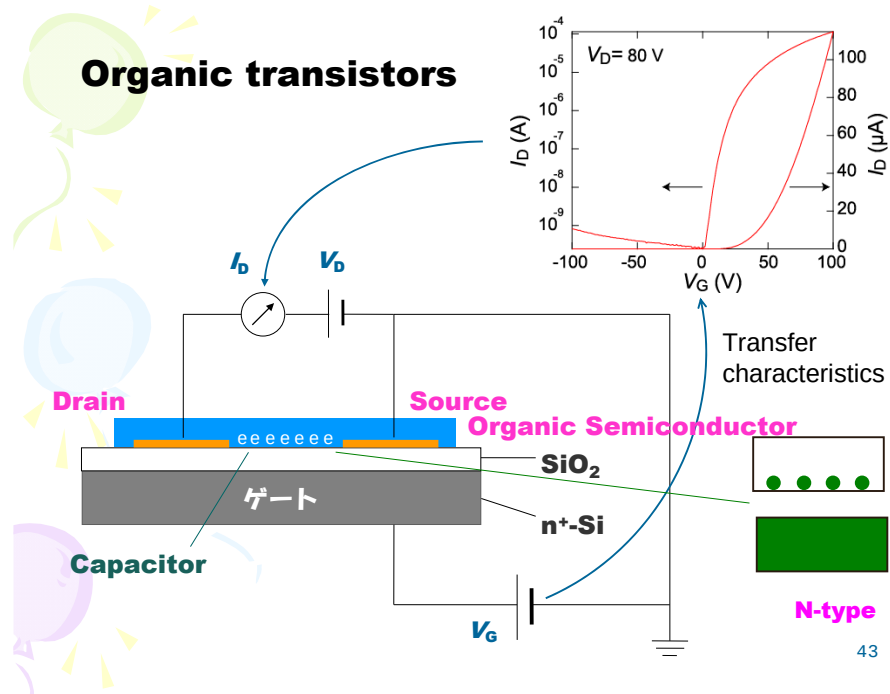
Energy Levels from Redox potentials



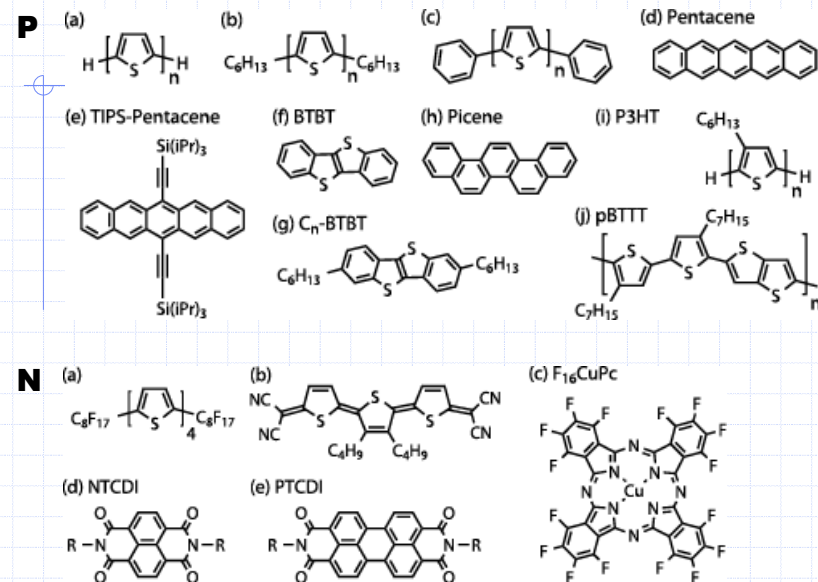
Organic transistors



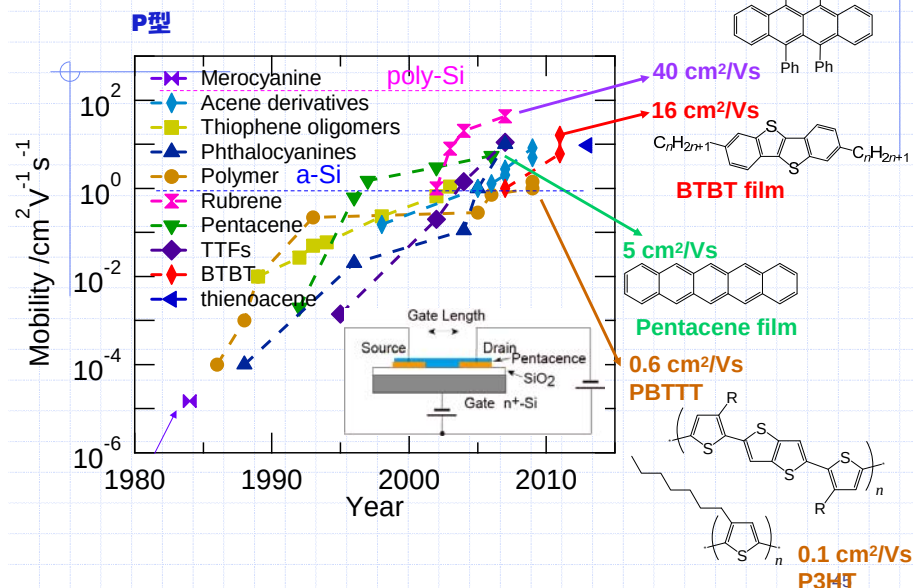
Organic transistors



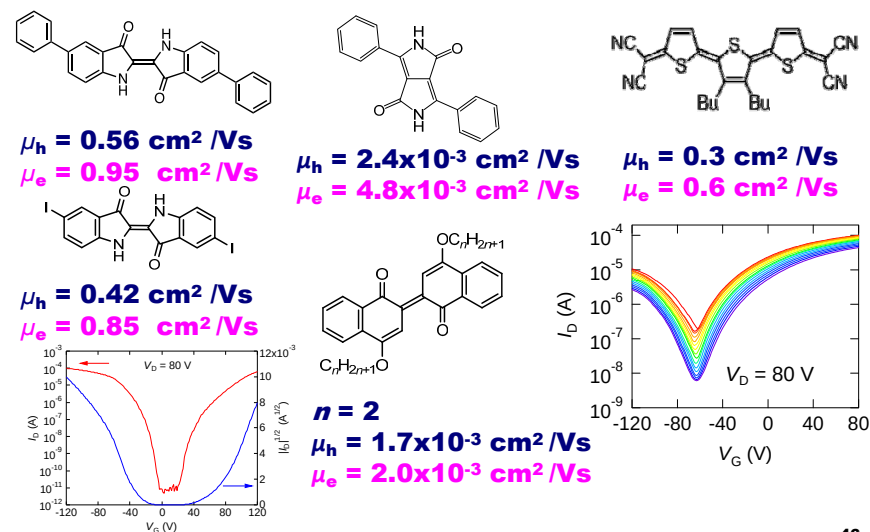
Organic transistor materials



History of mobility

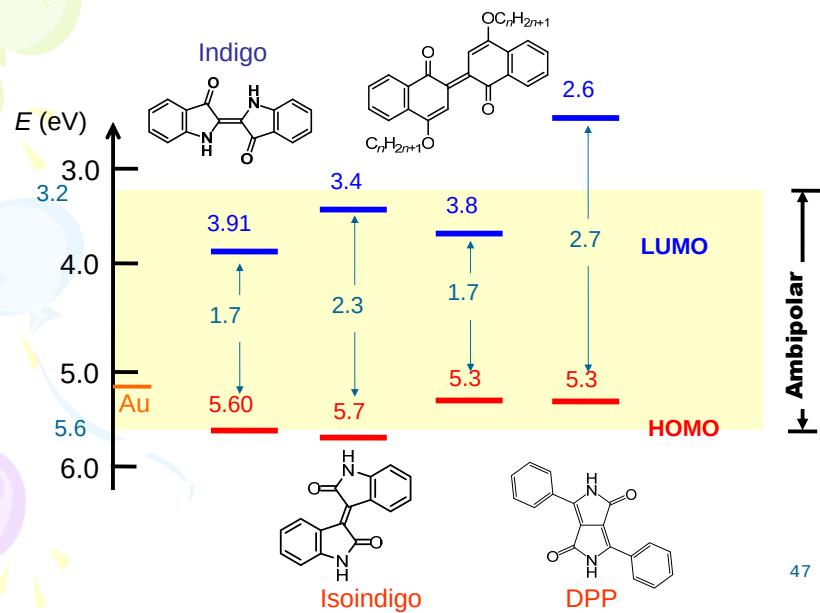


Ambipolar transistors (Both P and N)



46

HOMO/LUMO Levels



47

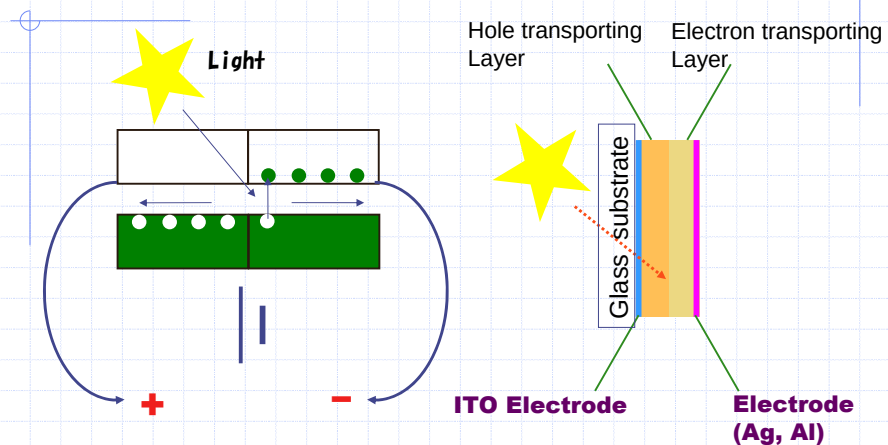
$$\text{Conductivity } \sigma = \frac{ne^2\tau}{m} = ne\mu \quad \text{Mobility}$$

Typical volume of organic molecule : 400 Å³
→ 1 electron / molecule → $n = 1.25 \times 10^{21} \text{ cm}^{-3}$

Mobility 1 cm²/Vs → $\sigma = 200 \text{ S/cm}$

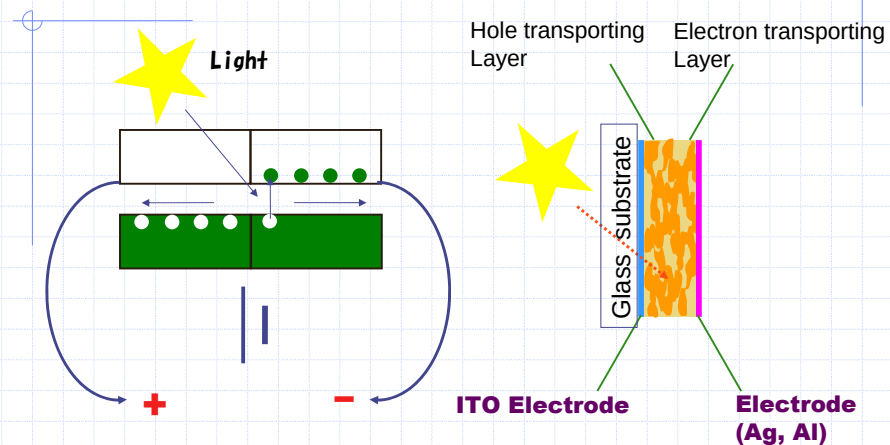
48

Organic solar cells are opposite of organic LED
Electricity ← Light



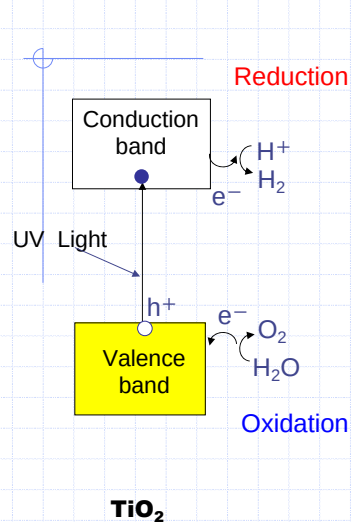
49

Organic solar cells are opposite of organic LED
Electricity ← Light

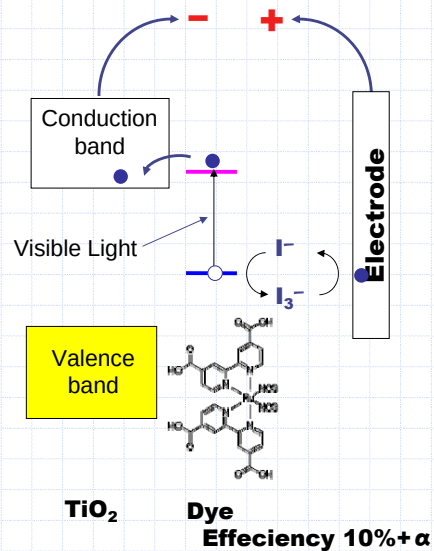


Scramble → Bulk heterojunction
Efficiency ~10%

Photocatalyst

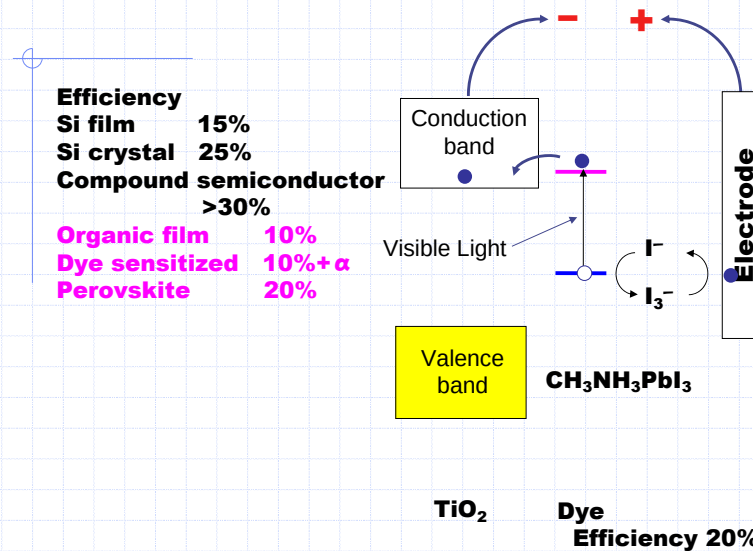


Dye sensitized solar cell



51

Perovskite dye sensitized solar cell



52