ICT.H409

Optics in Information Processing V

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Analysis of optical imaging system

- Resolution limit of optical imaging system
- NA: numerical aperture
- Depth of focus / Depth of field

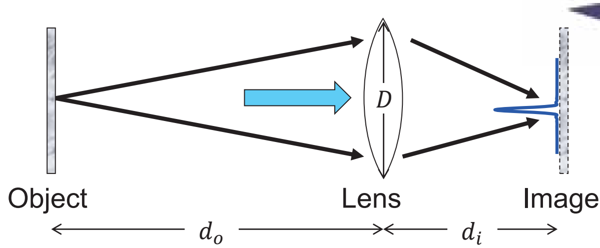
Computational optical imaging

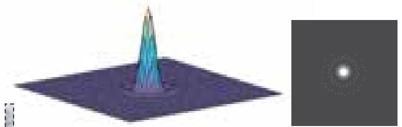
- Extended depth of field
- Computational super-resolution
- Wavefront coding
- Compressive sensing

Analysis of optical imaging system

Resolution limit of optical imaging system
NA: numerical aperture
Depth of focus / Depth of field

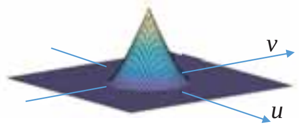
Resolution limit of optical imaging system PSF and OTF





PSF: | FT of pupil function |²
(Jinc function)

FT



OTF: Frequency response
Autocorrelation of pupil function

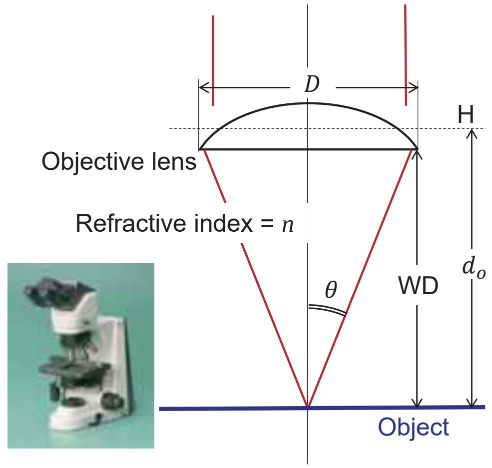
Resolution limit (Rayleigh limit) in circular aperture

Image plane $L_i = 1.22\frac{\lambda d_i}{D}$

Object plane $L_o = 1.22\frac{\lambda d_o}{D}$

Numerical Aperture (NA)

$$NA = n \sin\theta$$



$$\sin\theta \cong \tan\theta = \frac{D}{2d_o}$$

Rayleigh limit

$$L_o = 1.22\frac{\lambda d_0}{D} = 1.22 \cdot \frac{\lambda}{2\,NA}$$

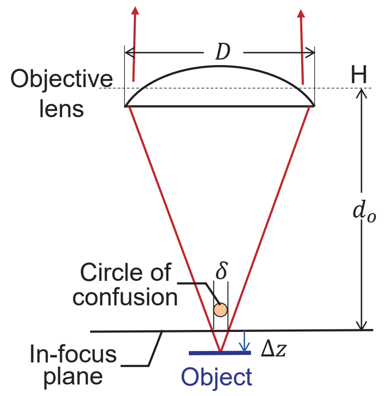
$$= 0.61 \cdot \frac{\lambda}{NA}$$

Resolution limit

WD: Working distance
f : focal length

http://www.nikon-instruments.jp/jpn/page/products/50i55i.aspx

Consider a microscopic imaging system in which the NA of the objective lens is 0.5. The refractive index of the medium is 1.0, and the wavelength of the light is assumed to be $\lambda = 500$nm. What is the resolution limit in the object plane based on Rayleigh criterion?

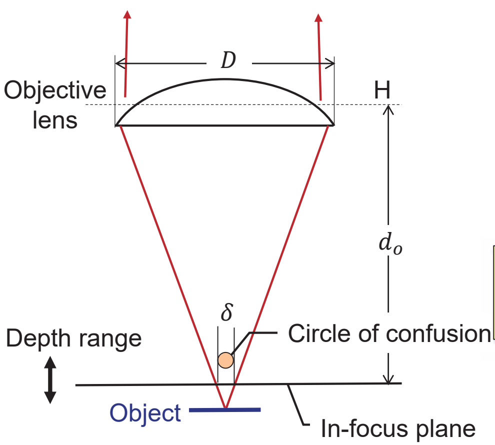


Consider a microscopic imaging system as shown in the left figure, where the diameter of the objective lens is D and the distance from the lens and the in-focus plane is d_o. For the object located Δz farther than the in-focus plane, derive the diameter of the circle of confusion (δ).

When the NA of the objective lens is 0.5 and $\Delta z = 5\ \mu m$, calculate δ approximately.

Assume that $\Delta z \ll d_o$.

Depth of field



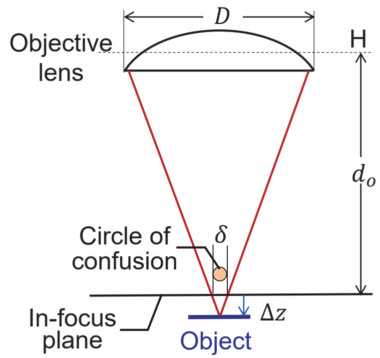
If the object is out of focus, the image is blurred. For point object, the blurred image becomes circle, which is called "circle of confusion."

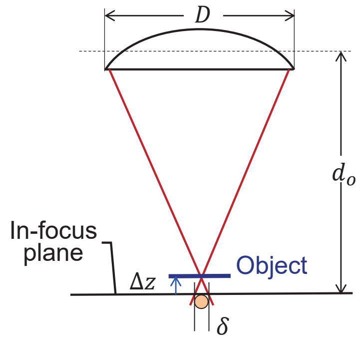
δ : Blur size
(Diameter of the circle of confusion)

If δ is smaller than the diffraction limit (δ_d) or the resolution of image sensor (δ_s) (or the resolution limit determined by other factors), the blur is imperceptible. Such δ is the diameter of "permissible circle of confusion" δ_p.

The depth range with $\delta < \delta_p$ is called "depth of field (DOF)."

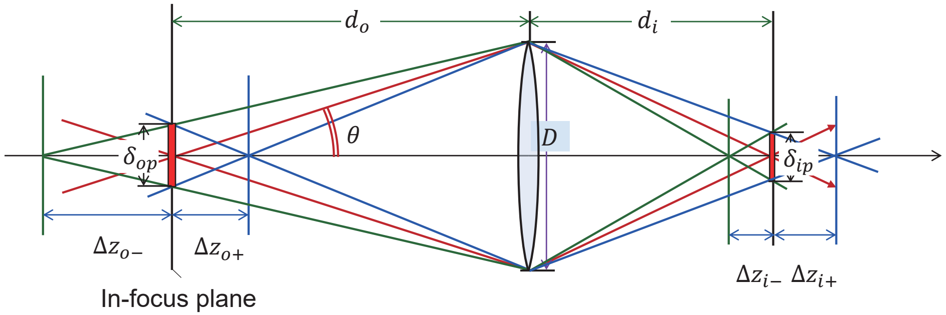
http://www.medlab.com.au/cytology/





For the object nearer than the in-focus plane,

$$\delta = \frac{\Delta z}{d_o - \Delta z} \cdot D \quad \cong \Delta z \cdot 2\,NA$$



$$\delta_{ip} = |M|\, \delta_{op} = \frac{d_i}{d_o}\delta_{op}$$

Diameter of permissible circle of confusion
-- Resolution of image sensor
-- Diffraction limit

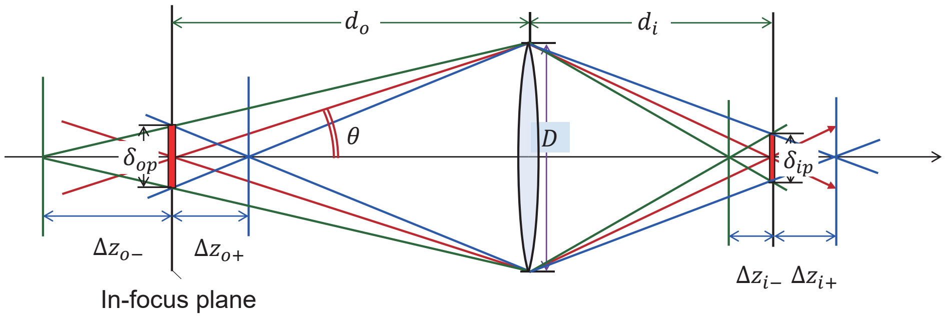
$|M| = M$ hereafter for simplicity.

$$\delta_{ip} = \frac{\Delta z_{i-}}{d_i - \Delta z_{i-}} \cdot D$$

$$\Delta z_{i-} = \frac{d_i \delta_{ip}}{D + \delta_{ip}}$$

$$\delta_{ip} = \frac{\Delta z_{i+}}{d_i + \Delta z_{i+}} \cdot D$$

$$\Delta z_{i+} = \frac{d_i \delta_{ip}}{D - \delta_{ip}}$$

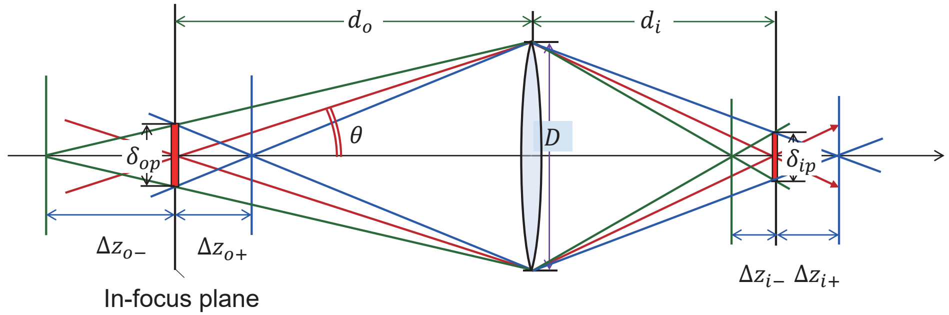


On the object plane

$$\Delta z_{o-} = \frac{d_o \delta_{op}}{D - \delta_{op}} \qquad \Delta z_{o+} = \frac{d_o \delta_{op}}{D + \delta_{op}}$$

$$\Delta z_{o-} + \Delta z_{o+} = \frac{d_o \delta_{op}}{D - \delta_{op}} + \frac{d_o \delta_{op}}{D + \delta_{op}} = \frac{2\, d_o D \delta_{op}}{D^2 - \delta_{op}^2} = \frac{2 d_o}{D} \frac{\delta_{op}}{1 - (\delta_{op}/D)^2}$$

$$= \frac{\delta_{ip}}{NA} \cdot \frac{1}{1 - 2\, NA\, (\delta_{op}/d_o)^2} \cong \frac{\delta_{op}}{NA}$$ Depth of field (DOF)



On the image plane

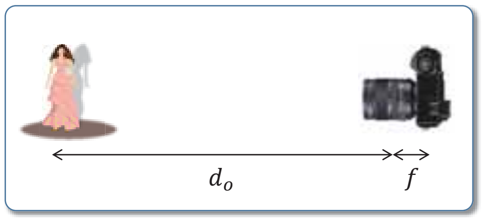
$$\Delta z_{i-} + \Delta z_{i+} = \frac{d_i \delta_{ip}}{D + \delta_{ip}} + \frac{d_i \delta_{ip}}{D - \delta_{ip}} = \frac{2\, d_i D \delta_{ip}}{D^2 - \delta_{ip}^2} = \frac{2 d_i}{D} \frac{\delta_{ip}}{1 - (\delta_{ip}/D)^2}$$

$(\delta_{ip} \ll D)$

$$\cong \frac{2 d_i}{D} \delta_{ip} = \frac{\delta_{ip}}{NA_i} = M \frac{\delta_{ip}}{NA}$$ Depth of focus

where $NA_i = \frac{D}{2 d_i} = \frac{1}{M} NA$ ➡ Depth of focus $= M^2$ Depth of field

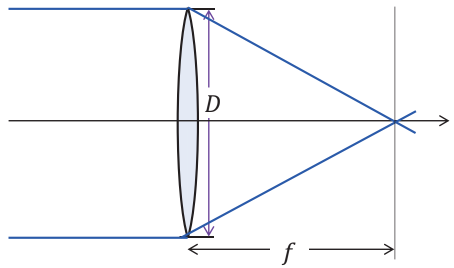
F-number (F#) $F = \frac{f}{D}$



If $d_o \gg f,\ d_i \cong f$

$$NA_i = \frac{D}{2 d_i} = \frac{1}{2F}$$

$$\text{Depth of field} \cong \frac{1}{M^2} \cdot \frac{\delta_{ip}}{NA_i} = \frac{2\, F \delta_{ip}}{M^2} = \frac{2\, F \delta_{op}}{M}$$



Brightness of the image (irradiance or illuminance) B is proportional to the area of lens aperture, and inversely proportional to M^2. Then

$$B \propto \frac{1}{F^2}$$

Summary of F-number, NA, brightness, resolution and DOF

F-number	Large	Small
NA	Small	Large
Image Brightness	High	Low
Resolution	Low	High
Depth of field Depth of focus	Long	Short

HD disk format war (2000-2008)

Blu-ray Disc Capacity ≈ DVD x 5 ≈ 25GB / single layer, single side
Beam spot size ≈ DVD x 1/5
Error margin for tilt becomes small (inversely proportional to NA^3) --> Thin cover layer (0.1mm)

HD DVD Capacity ≈ DVD x 3 = 15GB
Beam spot size ≈ DVD x 1/5
Same disk structure as DVD --> Cost, manufacturing equipment are almost equivalent to DVD
Error margin for tilt ≈ DVD x 1/2

Rayleigh criterion
$L = 0.61\ \lambda / NA$

Spot size

http://techon.nikkeibp.co.jp/NE/word/050512.html

BD
NA: 0.6/0.85 = 0.70
λ : 405/650 = 0.62
0.70 * 0.62 = 0.43
0.43 * 0.43 = 0.19 ~~ About 1/5

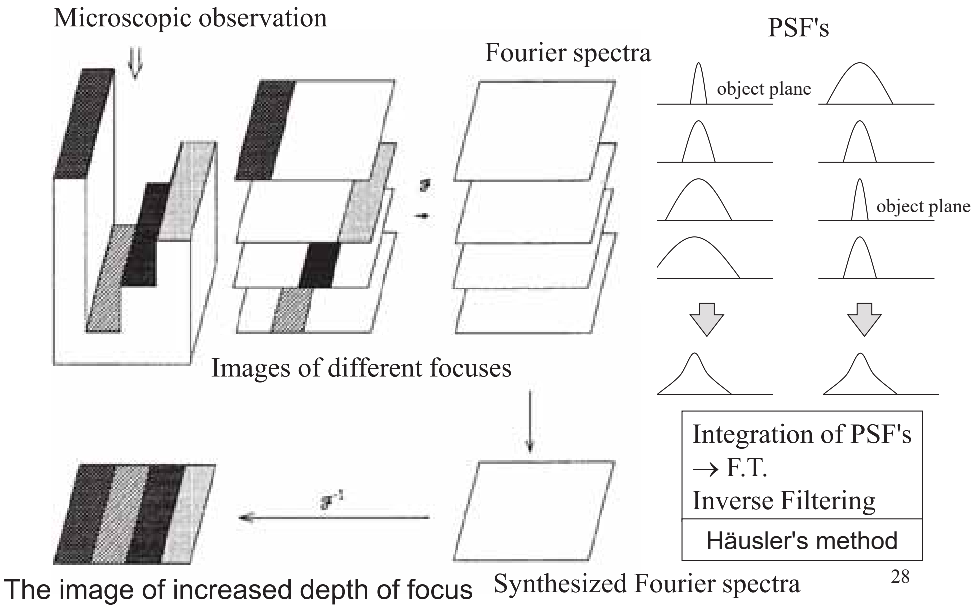
HD-DVD
NA: 0.6/0.65 = 0.92
λ : 405/650 = 0.62
0.92 * 0.62 = 0.57
0.57 * 0.57 = 0.32 ~~ About 1/3

Consider a microscopic imaging system in which the NA of the objective lens is 0.5. The refractive index of the medium is 1.0. The image is captured by an image sensor with $1\mu m$ resolution on the object plane. Derive the depth of field in this case.

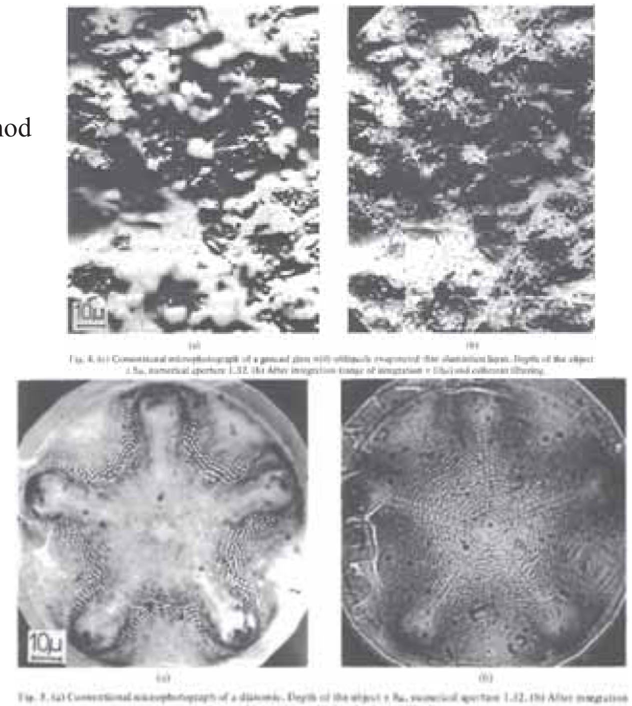
Computational optical imaging

- Extended depth of field
- Computational super-resolution
- Wavefront coding
- Compressive sensing

Increasing the depth of field in microscopic observations



Hausler's method

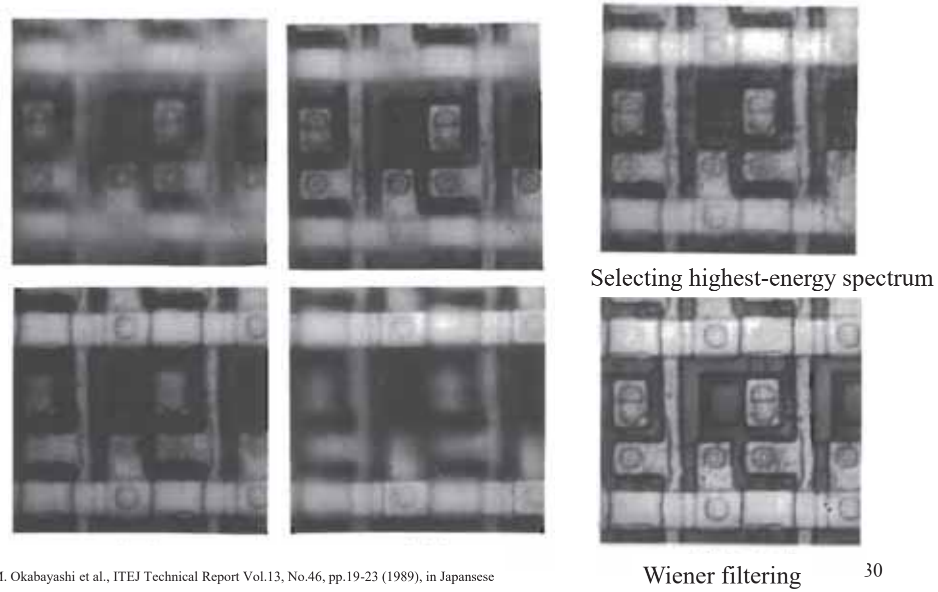


G. Häusler, "A method to increase the depth of focus by two step image processing," Optics Communications, Vol.6, Issue 1 , 1972, Pages 38-42

Wiener filtering for increasing depth of focus

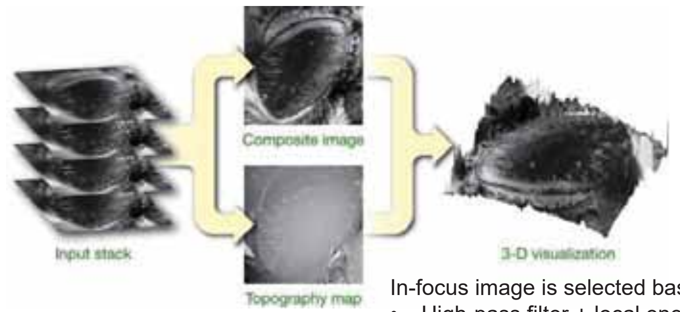
$$e = E\{\left| \sum_{l=0}^{L} F_l(u,v) - \sum_{k=1}^{K} M_k(u,v) G_k(u,v) \right|^2\}$$

l:depth of object, k: focus position



M. Okabayashi et al., ITEJ Technical Report Vol.13, No.46, pp.19-23 (1989), in Japansese

Extended depth of field (EDOF) and 3D visualization



In-focus image is selected based on

- High-pass filter + local energy
- Maximum absolute wavelet coefficients
- …

B. Forster, D. Van De Ville, J. Berent, D. Sage, M. Unser, "Complex Wavelets for Extended Depth-of-Field: A New Method for the Fusion of Multichannel Microscopy Images," Microscopy Research and Technique, vol. 65, no. 1-2, pp. 33-42, 2004

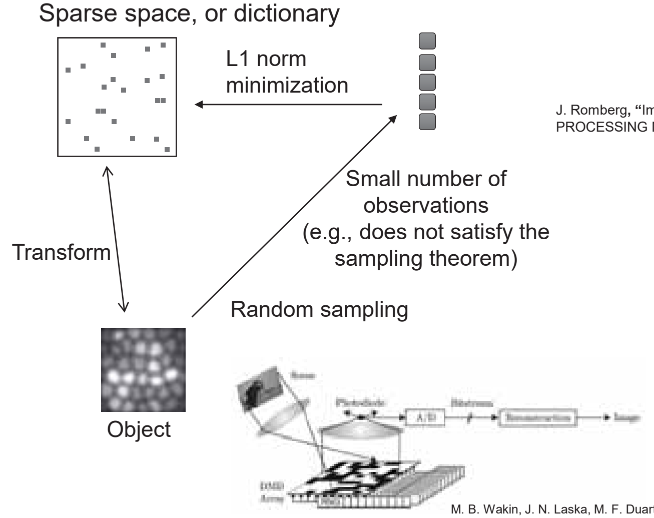
http://bigwww.epfl.ch/demo/edf/

Wavefront coding

- When NA of an optical system is large,
  - More light contributes the image formation
  - 👍 Brighter image (Higher S/N), Higher resolution
  - 👎 Shallow depth of field
    - → Image blur due in defocused objects
    - → Shift-variant blur
  - Difficult to be restored by digital image restoration
- Extending depth of field by “Wavefront coding”
  - Apply wavefront modulation at the lens aperture, such that the PSF(impulse response) becomes almost same shape in deep depth of field. → Shift-invariant blur.
  - Obtain a sharp image by digital image filtering

Compressive sensing

- Sparse signal



J. Romberg, “Imaging via Compressive Sampling,” IEEE SIGNAL PROCESSING MAGAZINE [14] MARCH 2008

M. B. Wakin, J. N. Laska, M. F. Duarte, D. Baron, S. Sarvotham, D. Takhar, K. F. Kelly, and R. G. Baraniuk, “An Architecture for Compressive Imaging,” Proc. ICIP 2006, 1273-1276

Superresolution

- A technology that enables higher resolution determined by the diffraction limit or the sampling theorem
- Optical superresolution
  - An optical system is modified such that higher frequency component contributes the image formation.
- Multiple image frames (Video)
  - Images with shifted sampling points are synthesized to obtain a higher resolution image.
- Learning-based superresolution
  - Lost high-frequency component is estimated by learning
- Constraint on the object (prior knowledge)
  - E.g., object support, point objects, nonnegative…

Summary

- Image formation by a lens system
  - Coherent case / Incoherent case
- PSF: Impulse response
- OTF: Frequency response
- Resolution limit
  - Rayleigh criterion
- The relationship between the resolution of optical imaging system, NA, F-number, and DOF.
- Computational imaging approaches for enhancing the resolution