ICT.H409

Optics in Information Processing II

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Geometrical optics, ray-tracing, lens aberration

- Focal point and focal length of a lens
- Image formation, real image and virtual image
- Thick lens, principal plane
- Ray-tracing
- Paraxial approximation
- Lens aberration
  - Five Seidel aberrations
  - Chromatic aberration

Lens

- Thin lens: lens thickness can be ignored.
- Positive lens / Negative lens
- Gaussian Optics
- Primary focal point
- Secondary focal point

Focal points

- Primary focal point ***F*** :
  Any ray coming from ***F***, or proceeding toward F travels parallel to the optical axis after refraction.
  - - - [1]
- Secondary focal point ***F'*** :
  Any incident ray traveling parallel to the optical axis will proceed toward ***F'*** or appear to come from ***F'*** after refraction.
  - - - [2]

Focal plane

A plane perpendicular to the optical axis and passing through the focal point.

Parallel incident rays making an angle θ with the optical axis are converged at a point Q' on the focal plane.
- - - [3]

Focal length

- Primary focal length: f
- Secondary focal length: f'

If refractive index is the same in both sides of the lens,

$$f = f'$$

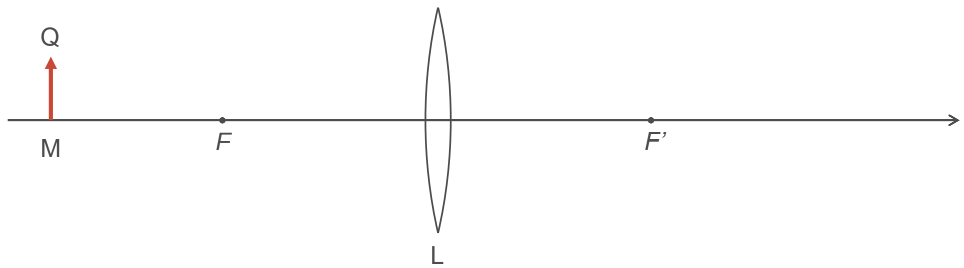
Chief ray: the ray passing through the center of the lens, i.e., $incident\ angle = \ exit\ angle = \theta$.
- - - [4]

Image formation

Real image:
Virtual image:

Quiz 2.1

The primary and secondary focuses of the lens L are ***F*** and ***F'***, respectively. MQ is an object. Draw 3 rays that emerge from the top of the object, Q, based on the characteristics of a lens [1], [2], and [4], respectively.



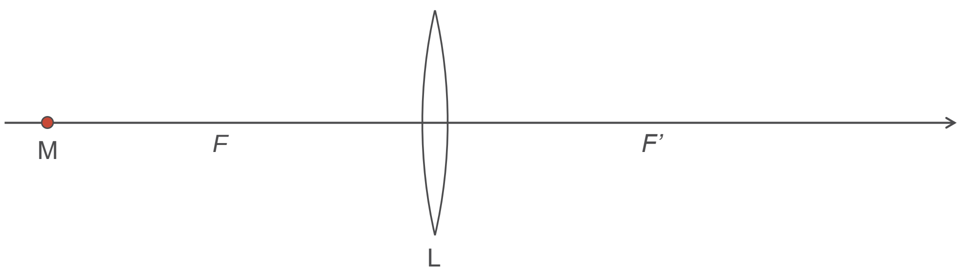
"Real image"

Quiz 2.2
Derive the relationship between f, f', s, and s', from the result of Quiz 2.1.

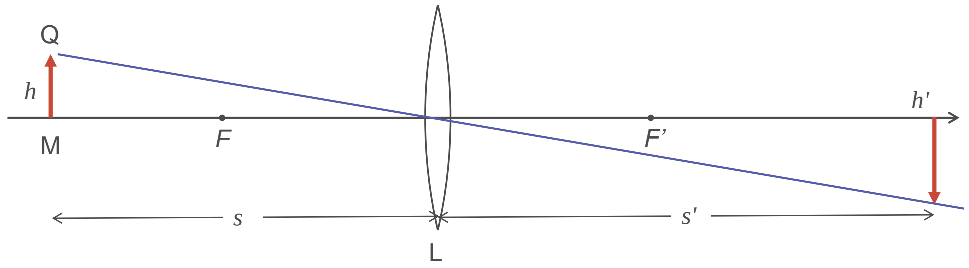
Lens formula (for thin lens) $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$

Quiz 2.3

The primary and secondary focuses of the lens L are ***F*** and ***F'***, respectively. Now let us consider an object M on the optical axis. Draw 3 rays that emerge from the object M, based on the characteristics of a lens [1] and [3], respectively.



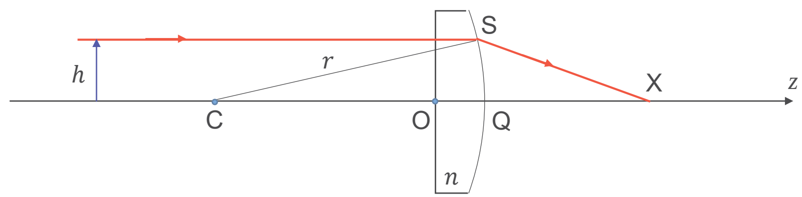
Magnification



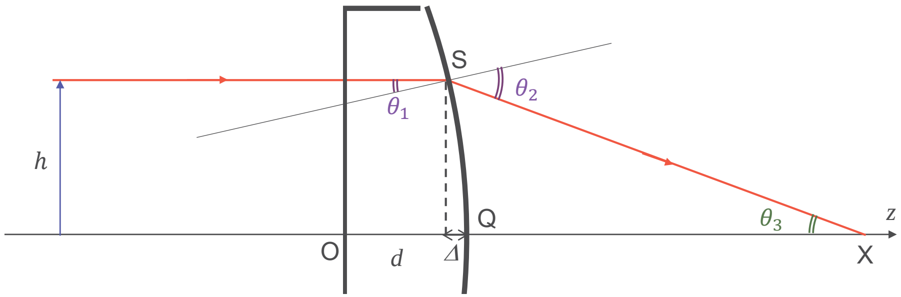
Magnification $M = \frac{h'}{h} = -\frac{s'}{s}$

Exercise 2

Consider a lens made of glass with refractive index n=1.5 is placed in the air (refractive index =1.0), as shown in the figure. The first surface is plane and the second surface is spherical convex shape whose radius of curvature is r=100 [mm]. z-axis is called optical axis. The first surface perpendicularly intersects the optical axis at O, and second surface intersects the optical axis at Q. The thickness of the lens OQ = d = 20 [mm]. A light beam parallel to the optical axis is incident to the lens, where the distance (height) of the beam from the optical axis is h. The refracted beam intersects the optical axis at X. Derive the distance between the lens and the intersection X, QX for $h = 1, 10, 20$ [mm], respectively. Discuss what is a problem here.



Hint for Exercise 2



(1)Derive Δ.

$$\Delta = r - r\cos\theta_1$$

$$r\sin\theta_1 = h$$

$$\cos\theta_1 = \sqrt{1-\sin^2\theta_1} = \sqrt{1-\left(\frac{h}{r}\right)^2}$$

$$\Delta = r - \sqrt{r^2-h^2}$$

(2) What is the relation between θ_1 and θ_2.

$$n\sin\theta_1 = 1\cdot\sin\theta_2$$

(3) What is the relation between θ_1, θ_2 and θ_3.

$$\theta_3 = \theta_2 - \theta_1$$

(4) Derive Δ + QX.

$$\tan\theta_3 = h/(\Delta + QX)$$

$$QX = \frac{h}{\tan\theta_3} - \Delta$$

$$QX = \frac{h}{\tan(\theta_2-\theta_1)} - r\,(1-\cos\theta_1)$$

Exercise 2, calculated by Excel

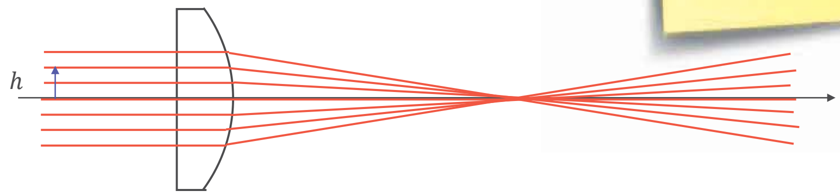
r	100
n	1.5
d	20

h		40.00	30.00	20.00	10.00	1.00
sin θ1	h/r	0.4	0.3	0.2	0.1	0.01
θ1		0.4115	0.3047	0.2014	0.1002	0.0100
sin θ2	n sin θ1	0.6000	0.4500	0.3000	0.1500	0.0150
θ2		0.6435	0.4668	0.3047	0.1506	0.0150
θ3	θ2- θ1	0.2320	0.1621	0.1033	0.0504	0.0050
Δ	r-sqrt(r^2-h^2)	8.3485	4.6061	2.0204	0.5013	0.0050
QX	h/tan(θ3) - Δ	160.97	178.87	190.84	197.74	199.98

Lens maker's formula

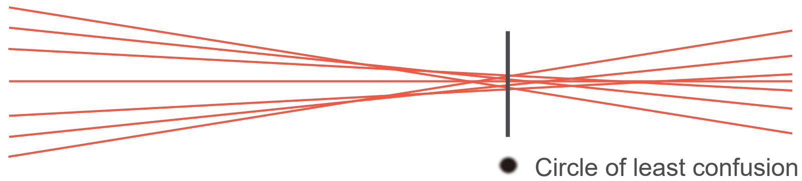
$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$n = 1.5$
$r_1 = \infty$
$r_2 = r = 100$
$\rightarrow$

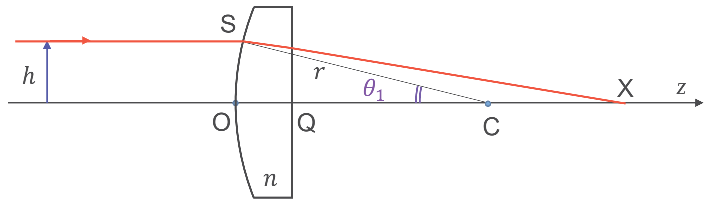


Magnified

The rays do not converge to a single point when the spherical aberration exists.



Flip the lens:
If rays are incident from opposite side…

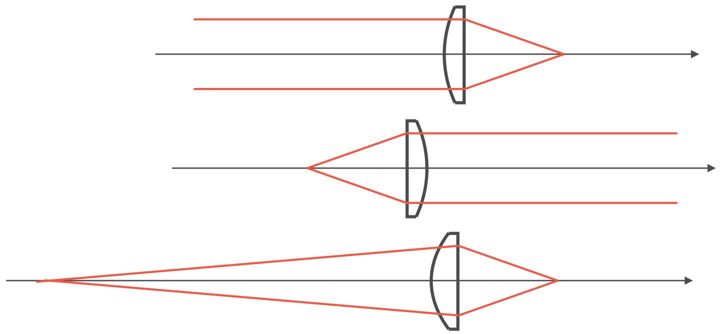


The radius of curvature of the first surface: r
C: Center of curvature
The radius of curvature of the second surface: infinity
(The second surface is planar.)

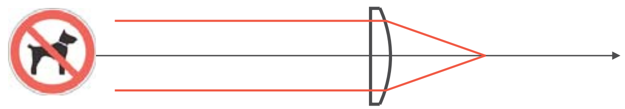
Rays are reflected twice.

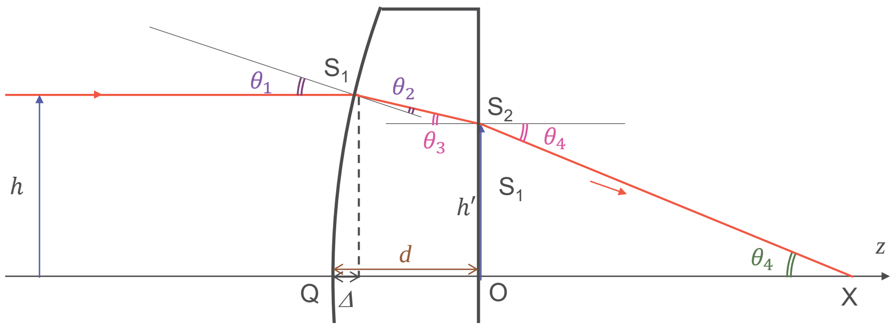
Plano-convex lens

Use this side!



Not like this!





$\varDelta = r - r\cos\theta_1$

$\sin\theta_1 = n \cdot \sin\theta_2$

$\theta_3 = \theta_1 - \theta_2$

$n \cdot \sin\theta_3 = \sin\theta_4$

$h' = h - (d - \Delta)\tan\theta_4$

$\tan\theta_4 = \dfrac{h'}{QX}$

$QX = \dfrac{h'}{\tan\theta_4} + d$

h		40.00	30.00	20.00	10.00	1.00
sin θ1	h/r	0.4	0.3	0.2	0.1	0.01
θ1		0.4115	0.3047	0.2014	0.1002	0.0100
sin θ2	(1/n) sin θ1	0.2667	0.2000	0.1333	0.0667	0.0067
θ2		0.2699	0.2014	0.1337	0.0667	0.0067
θ3	θ2- θ1	0.1416	0.1033	0.0676	0.0335	0.0033
Δ	r-sqrt(r^2-h^2)	8.3485	4.6061	2.0204	0.5013	0.0050
sin θ4	n sin θ3	0.2117	0.1547	0.1014	0.0502	0.0050
tan θ4		0.2166	0.1566	0.1019	0.0502	0.0050
h'	h-(d-Δ)tan θ4	37.4766	27.5891	18.1681	9.0206	0.9000
QX	h' / tan(θ4) + d	193.04	196.16	198.32	199.58	200.00

Paraxial Approximation

If θ is small enough, $\sin\theta \cong \theta,\ \cos\theta \cong 1$

Maclaurin series expansion

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Table 9A VALUES OF sin θ AND ITS FIRST THREE EXPANSION TERMS

	sin θ	θ	$\frac{\theta^3}{3!}$	$\frac{\theta^5}{5!}$
10°	0.1736482	0.1745329	0.0008861	0.0000135
20°	0.3420201	0.3490658	0.0070888	0.0000432
30°	0.5000000	0.5235988	0.0239246	0.0003280
40°	0.6427876	0.6981316	0.0567088	0.0013829

If $z \gg x, z \gg y$,

$$\sqrt{x^2 + y^2 + z^2} = z\sqrt{1 + \left(\frac{x}{z}\right)^2 + \left(\frac{y}{z}\right)^2} \cong z\left\{1 + \frac{1}{2}\,\frac{x^2 + z^2}{z}\right\}$$

Spherical surface is approximated as quadratic surface.

Summary of lens aberrations

- Seidel aberrations
  - Spherical aberration
  - Coma
  - Astigmatism
  - Curvature of field
  - Distortion
- Chromatic aberration

- ザイデルの5収差
  - 球面収差
  - コマ収差
  - 非点収差
  - 像面湾曲
  - 歪曲収差
- 色収差

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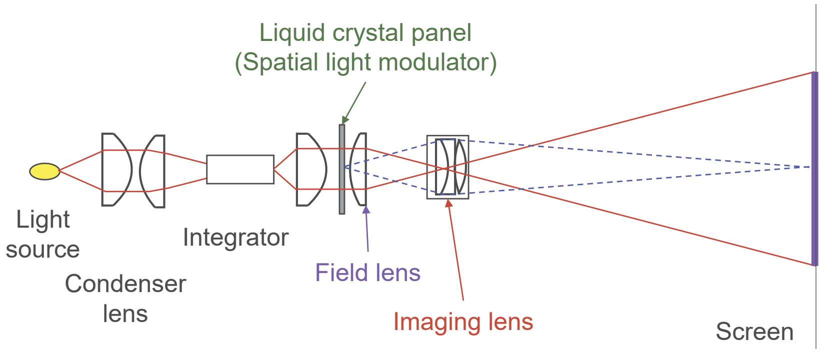
☑ You may think that the design of optical system is difficult, but the combination of high-quality camera lens and other optical devices as well as the basic knowledge of optics greatly help you to design the optical system for your experiment.

Basic optical system for projector



Red: Light flux propagation
Blue: Image formation
The quality of imaging lens is critical to the projected image quality.