

14. Nonreciprocal circuits

14.1 Loss-less characteristics of circuits

- isolators

- circulators

14.2 Microwave response of magnetic material

14.3 Isolator

14.4 Edge-guided mode

14.5 Circulator

14.6 Application of circulators

- Bi-directional transmission

- Applicable to a phase modulator

- Add Drop Multiplexer

Loss-less or lossy ?

Check whether S-matrix can be unitary or not.

(1) Isolator

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \mathbf{S}\mathbf{a} \quad \Rightarrow \quad \mathbf{S}\mathbf{S}^+ = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \neq \mathbf{I}$$

Isolator is never a loss-less device.

(2) Circulator

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \Rightarrow \quad \mathbf{S}\mathbf{S}^+ = \mathbf{I}$$

Circulator can be a loss-less device.

Microwave response of magnetic material

Apply a magnetic field along the z-direction, then

$$[\mu] = \mu_0 \begin{bmatrix} \mu_r & -j\kappa & 0 \\ j\kappa & \mu_r & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} \quad \text{Poldar's tensor}$$

$$\mu_{rz} \approx 1$$

$$\omega_0 = -\gamma H_0 \quad (\gamma = -2.8 \text{ MHz/Oe})$$

$$\omega_s = -\gamma M_s \quad (M_s: \text{saturation magnetization})$$

ω : operating frequency

$$\mu_r = 1 + \frac{\omega_0 \omega_s}{\omega_0^2 - \omega^2}$$

$$\kappa = \frac{-\omega_s \omega}{\omega_0^2 - \omega^2}$$

Electromagnetic field in magnetic material

H and B of RF field : $\mathbf{h}e^{j\omega t}$ $\mathbf{b}e^{j\omega t}$

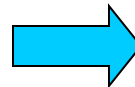
right-handed circularly polarized wave:

$$\mathbf{h}_+ = \mathbf{i}h_x + \mathbf{j}h_y$$

$$h_y = -jh_x$$

$$h_x = \text{Re}[e^{j\omega t}] = \cos \omega t$$

$$h_y = \text{Re}[-je^{j\omega t}] = \cos(\omega t - \frac{\pi}{2})$$



$$\mathbf{b}_+ = \mu_0 \begin{bmatrix} \mu_r & -j\kappa & 0 \\ j\kappa & \mu_r & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} \begin{bmatrix} h_x \\ -jh_x \\ 0 \end{bmatrix}$$

$$= \mu_0 \begin{bmatrix} (\mu_r - \kappa)h_x \\ -j(\mu_r - \kappa)h_x \\ 0 \end{bmatrix} = \mu_0(\mu_r - \kappa) \begin{bmatrix} h_x \\ -jh_x \\ 0 \end{bmatrix} = \boxed{\mu_0(\mu_r - \kappa)} \mathbf{h}_+$$

left-handed circularly polarized wave:

$$\mathbf{h}_- = \mathbf{i}h_x + \mathbf{j}h_y$$

$$h_y = +jh_x$$

$$h_x = \text{Re}[e^{j\omega t}] = \cos \omega t$$

$$h_y = \text{Re}[+je^{j\omega t}] = \cos(\omega t + \frac{\pi}{2})$$



$$\mathbf{b}_- = \mu_0 \begin{bmatrix} \mu_r & -j\kappa & 0 \\ j\kappa & \mu_r & 0 \\ 0 & 0 & \mu_{rz} \end{bmatrix} \begin{bmatrix} h_x \\ +jh_x \\ 0 \end{bmatrix}$$

$$= \mu_0 \begin{bmatrix} (\mu_r + \kappa)h_x \\ j(\mu_r + \kappa)h_x \\ 0 \end{bmatrix} = \mu_0(\mu_r + \kappa) \begin{bmatrix} h_x \\ jh_x \\ 0 \end{bmatrix} = \boxed{\mu_0(\mu_r + \kappa)} \mathbf{h}_-$$

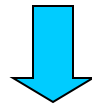
Faraday rotation

linearly polarized wave: $e_x = E_0 \cos \omega t$, $e_y = e_z = 0$

$$\mathbf{ie}_x = \mathbf{i}E_0 \cos \omega t$$

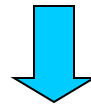
$$= \frac{E_0}{2} \left\{ \mathbf{i} \cos \omega t + \mathbf{j} \cos\left(\omega t - \frac{\pi}{2}\right) \right\} + \frac{E_0}{2} \left\{ \mathbf{i} \cos \omega t + \mathbf{j} \cos\left(\omega t + \frac{\pi}{2}\right) \right\}$$

r-circular polarized



$$\beta_+ = \omega \sqrt{\epsilon \mu_0} (\mu_r - \kappa)$$

l-circular polarized



$$\beta_- = \omega \sqrt{\epsilon \mu_0} (\mu_r + \kappa)$$

$$\frac{E_0}{2} \{ \mathbf{i} \cos(\omega t - \beta_+ z) + \mathbf{j} \cos(\omega t - \frac{\pi}{2} - \beta_+ z) \}$$

$$\frac{E_0}{2} \{ \mathbf{i} \cos(\omega t - \beta_- z) - \mathbf{j} \sin(\omega t - \beta_- z) \}$$

$$= \frac{E_0}{2} \{ \mathbf{i} \cos(\omega t - \beta_+ z) + \mathbf{j} \sin(\omega t - \beta_+ z) \}$$

$$e_x = \frac{E_0}{2} \{ \cos(\omega t - \beta_+ z) + \cos(\omega t - \beta_- z) \} = E_0 \cos\left(\frac{\beta_- - \beta_+}{2} z\right) \cos\left(\omega t - \frac{\beta_- + \beta_+}{2} z\right)$$

$$e_y = \frac{E_0}{2} \{ \sin(\omega t - \beta_+ z) - \sin(\omega t - \beta_- z) \} = E_0 \sin\left(\frac{\beta_- - \beta_+}{2} z\right) \cos\left(\omega t - \frac{\beta_- + \beta_+}{2} z\right)$$

Polarization plane rotates by an angle

$$\theta_F = \frac{\beta_- - \beta_+}{2} z = \frac{\omega \sqrt{\epsilon}}{2} (\sqrt{\mu_-} - \sqrt{\mu_+})$$

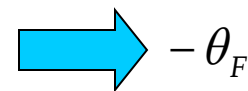
in the reverse direction:

$$H_0 \rightarrow -H_0$$

$$M_s \rightarrow -M_s$$

$$\mu_+ = \mu_0 (\mu_r + \kappa)$$

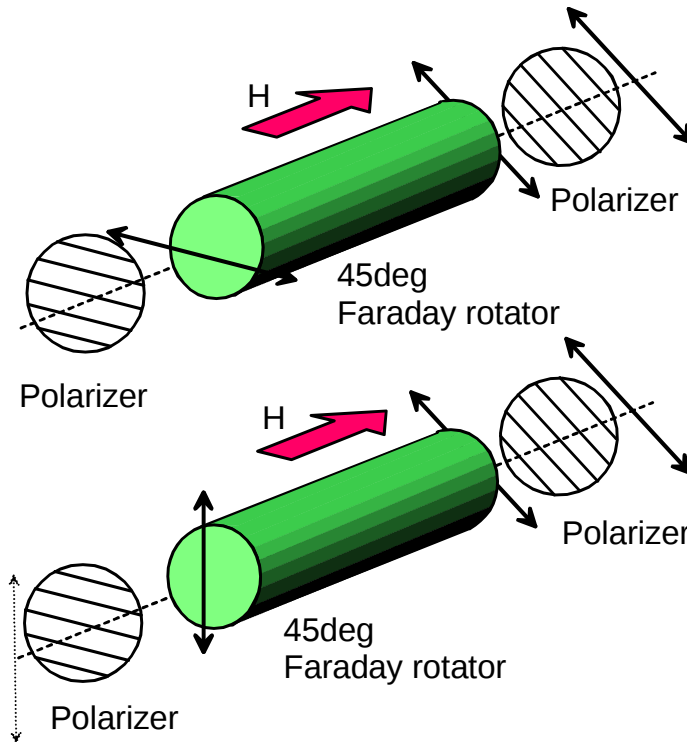
$$\mu_- = \mu_0 (\mu_r - \kappa)$$



$$-\theta_F$$

Isolator

polarizer + Faraday rotator + polarizer

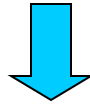


edge guided mode

Edge-guided mode

$$\nabla \times \mathbf{E} = -j\omega\mu_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu_r & -j\kappa \\ 0 & j\kappa & \mu_r \end{pmatrix} \mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$

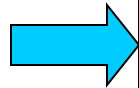


$$\begin{cases} \frac{\partial E_x}{\partial z} = -j\omega\mu_0(\mu_r H_y - j\kappa H_z) & -\frac{\partial E_x}{\partial y} = -j\omega\mu_0(j\kappa H_y + \mu_r H_z) \\ \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x \end{cases}$$

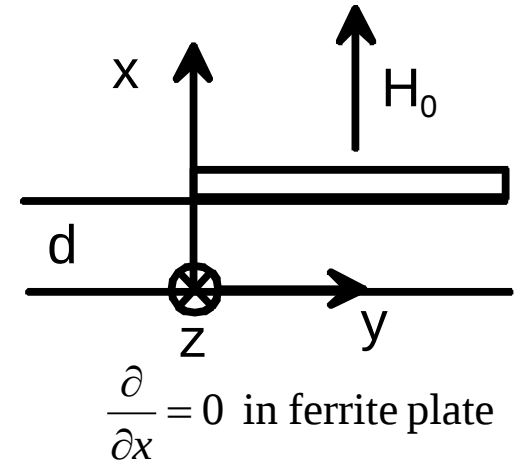
$$H_y = \frac{\mu_r}{\omega\mu_0(\mu_r^2 - \kappa^2)} \left(j \frac{\partial E_x}{\partial z} + \frac{\kappa}{\mu_r} \frac{\partial E_x}{\partial y} \right)$$

$$H_z = \frac{\mu_r}{\omega\mu_0(\mu_r^2 - \kappa^2)} \left(\frac{\kappa}{\mu_r} \frac{\partial E_x}{\partial z} - j \frac{\partial E_x}{\partial y} \right)$$

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu_0 \mu_{\text{eff}} \epsilon E_x = 0 \quad \frac{(\mu_r^2 - \kappa^2)}{\mu_r} = \mu_{\text{eff}}$$



$$\frac{\partial^2 E_x}{\partial y^2} = (\beta^2 - \omega^2 \mu_0 \mu_{\text{eff}} \epsilon) E_x \quad \left(\frac{\partial}{\partial z} = -j\beta \right)$$



Edge-guided mode

$$\frac{\partial^2 E_x}{\partial y^2} = (\beta^2 - \omega^2 \mu_0 \mu_{\text{eff}} \varepsilon) E_x \quad \left(\frac{\partial}{\partial z} = -j\beta \right)$$

$$E_x = (Ae^{-\alpha y} + Be^{\alpha y}) e^{-j\beta z} \quad \alpha = \sqrt{\beta^2 - \omega^2 \mu_0 \mu_{\text{eff}} \varepsilon}$$

$$E_x = 0 \text{ at } y = \infty$$

$$\therefore E_x = Ae^{-\alpha y} e^{-j\beta z}$$

$$H_y = \frac{1}{\omega \mu_0 \mu_{\text{eff}}} \left(\beta - \frac{\kappa}{\mu_r} \alpha \right) E_x = \frac{1}{\omega \mu_0 \mu_{\text{eff}}} \left(\beta - \frac{\kappa}{\mu_r} \alpha \right) Ae^{-\alpha y} e^{-j\beta z}$$

$$H_z = \frac{j}{\omega \mu_0 \mu_{\text{eff}}} \left(\alpha - \frac{\kappa}{\mu_r} \beta \right) Ae^{-\alpha y} e^{-j\beta z}$$

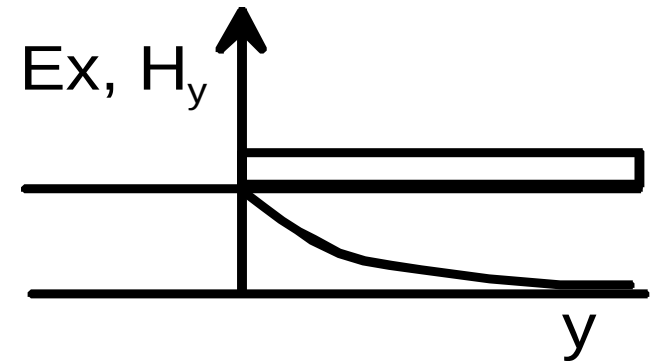
$$H_z = 0 \text{ on } y = 0$$

$$\alpha = \frac{\kappa}{\mu_r} \beta \quad \alpha = \sqrt{\beta^2 - \omega^2 \mu_0 \mu_{\text{eff}} \varepsilon}$$

$$\beta = \omega \sqrt{\mu_0 \mu_r \varepsilon}$$

$$\alpha = \frac{\kappa}{\mu_r} \omega \sqrt{\mu_0 \mu_r \varepsilon}$$

$$H_y = \sqrt{\frac{\varepsilon}{\mu_0 \mu_r}} Ae^{-\alpha y} e^{-j\beta z}$$

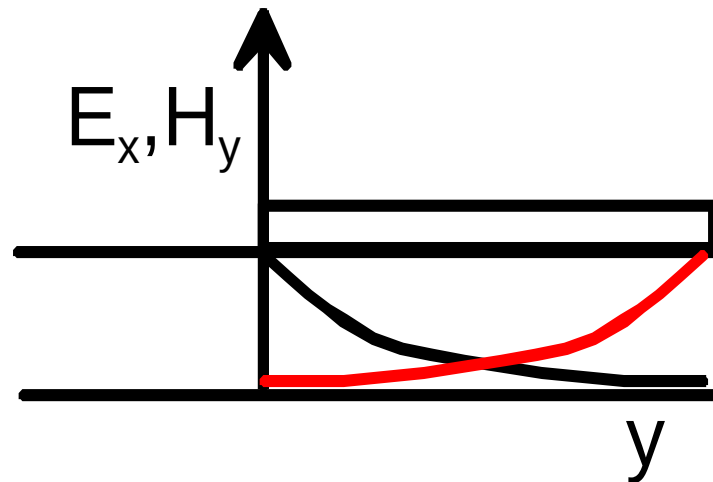


Edge-guided mode

Reversing the propagation direction ($\beta \rightarrow -\beta$)

$$\alpha = \frac{\kappa}{\mu_r} \beta \quad \longrightarrow \quad \alpha_{\text{back}} = \frac{\kappa}{\mu_r} (-\beta) = -\alpha_{\text{forward}}$$

$$H_y = \sqrt{\frac{\epsilon}{\mu_0 \mu_r}} A e^{\alpha y} e^{j\beta z}$$



Junction type circulator

eigen excitation of 3 port rotationally symmetric circuit

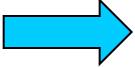
$$\mathbf{u}_0 = \frac{1}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \mathbf{u}_+ = \frac{1}{3} \begin{pmatrix} 1 \\ e^{-j2\pi/3} \\ e^{j2\pi/3} \end{pmatrix} \quad \mathbf{u}_- = \frac{1}{3} \begin{pmatrix} 1 \\ e^{j2\pi/3} \\ e^{-j2\pi/3} \end{pmatrix}$$

$$\begin{bmatrix} S_{11} & S_{31} & S_{21} \\ S_{21} & S_{11} & S_{31} \\ S_{31} & S_{21} & S_{11} \end{bmatrix} \mathbf{U}_i = S_i \mathbf{U}_i \quad \Rightarrow \quad \begin{aligned} S_{11} &= \frac{1}{3}(S_0 + S_+ + S_-) \\ S_{21} &= \frac{1}{3}(S_0 + e^{-j\frac{2}{3}\pi} S_+ + e^{j\frac{2}{3}\pi} S_-) \\ S_{31} &= \frac{1}{3}(S_0 + e^{j\frac{2}{3}\pi} S_+ + e^{-j\frac{2}{3}\pi} S_-) \end{aligned}$$

Junction type circulator -- design

basic configuration: $\frac{2\pi}{\lambda} R \sqrt{\epsilon_r \mu_{rz}} \approx 1.84$

U_0 excitation: μ_e  Disk center is replaced with open circuit.
 $S_0 = e^{j\phi_0}$ f_0 : disk center - port electrical length

U_+ excitation: μ_+  $\mu_+ L_0 \parallel C$

$$S_+ = \frac{\frac{1}{j\omega C + \frac{1}{j\omega\mu_+ L_0}} - Z_c}{\frac{1}{j\omega C + \frac{1}{j\omega\mu_+ L_0}} + Z_c} = \frac{-Z_c + j \frac{\omega\mu_+ L_0}{1 - \omega^2 \mu_+ L_0 C}}{Z_c + j \frac{\omega\mu_+ L_0}{1 - \omega^2 \mu_+ L_0 C}} = e^{j\phi_+}$$

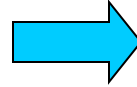
U_- excitation: μ_-  $\mu_- L_0 \parallel C$

$$S_- = \frac{-Z_c + j \frac{\omega\mu_- L_0}{1 - \omega^2 \mu_- L_0 C}}{Z_c + j \frac{\omega\mu_- L_0}{1 - \omega^2 \mu_- L_0 C}} = e^{j\phi_-}$$

Junction type circulator -- design

$M_s, H_0 \rightarrow \mu_e, \mu_+, \mu_-$

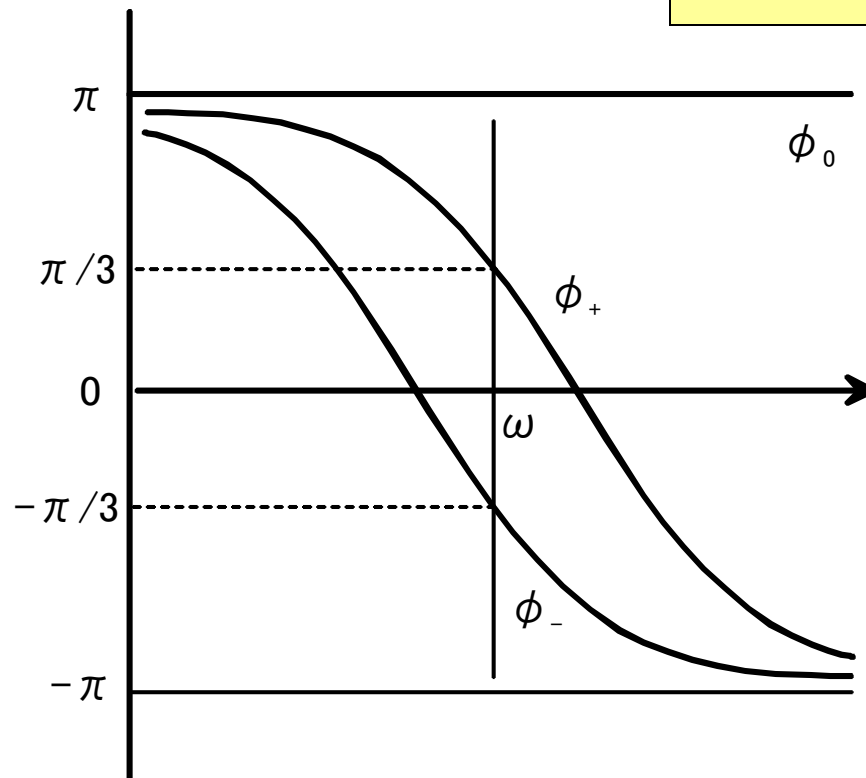
disk and line structure $\rightarrow L_0, C$



$$\phi_0 = \pi$$

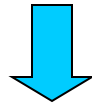
$$\phi_+ = \frac{\pi}{3}$$

$$\phi_- = -\frac{\pi}{3}$$



Junction type circulator -- design

$$S_0 = e^{j\pi}, S_+ = e^{j\frac{\pi}{3}}, S_- = e^{-j\frac{\pi}{3}}$$



$$S_{11} = \frac{1}{3}(S_0 + S_+ + S_-) = 0$$

$$S_{21} = \frac{1}{3}(S_0 + e^{-j\frac{2}{3}\pi} S_+ + e^{j\frac{2}{3}\pi} S_-) = 0$$

$$S_{31} = \frac{1}{3}(S_0 + e^{j\frac{2}{3}\pi} S_+ + e^{-j\frac{2}{3}\pi} S_-) = e^{j\pi}$$

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

Circulator !

Application of circulator

- (1) Bi-directional transmission
- (2) Isolator is constructed in combination with an anti-reflection terminator.
- (3) Applicable to a phase modulator
- (4) Add Drop Multiplexer (ADM)

