6. Dielectric Waveguide

$$\varepsilon_r = n^2$$

6.1 Dielectric slab waveguide

EM waves are classified into two groups, TE and TM mode.

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon E_x$$

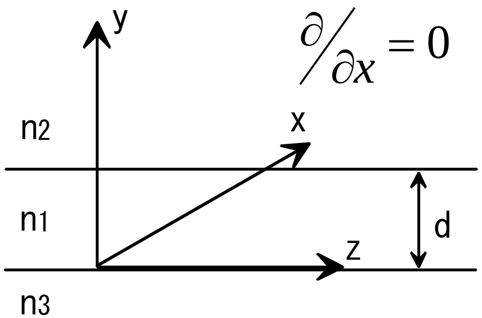
$$-j\beta H_x = j\omega\varepsilon E_y$$

$$-\frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x$$

$$-j\beta E_x = -j\omega\mu_0 H_y$$

$$-\frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z$$



asymmetric slab waveguide

$$\frac{\partial^2 E_x}{\partial y^2} + (\omega^2\varepsilon\mu_0 - \beta^2)E_x = 0$$

Dielectric slab waveguide : TE mode

$$E_x = \begin{cases} (A\cos ud + B\sin ud)e^{-w_2(y-d)} & : \quad y \geqq d \\ A\cos uy + B\sin uy & : \quad d \geqq y \geqq 0 \\ Ae^{w_3 y} & : \quad 0 \geqq y \end{cases}$$

$$\mathrm{w}_2 = \sqrt{\beta^2 - n_2^2\omega^2\varepsilon_0\mu_0} = \sqrt{\beta^2 - n_2^2 k_0^2}$$

$$\mathrm{u} = \sqrt{n_1^2\omega^2\varepsilon_0\mu_0 - \beta^2} = \sqrt{n_1^2 k_0^2 - \beta^2}$$

$$\mathrm{w}_3 = \sqrt{\beta^2 - n_3^2\omega^2\varepsilon_0\mu_0} = \sqrt{\beta^2 - n_3^2 k_0^2}$$

$$k_0 = \omega\sqrt{\varepsilon_0\mu_0} = \frac{2\pi}{\lambda}$$

$$H_z = \frac{1}{j\omega\mu_0}\frac{\partial E_x}{\partial y} = \frac{1}{j\omega\mu_0} \times \begin{cases} -w_2(A\cos ud + B\sin ud)e^{-w_2(y-d)} & : \quad y \geqq d \\ u\left(-A\sin uy + B\cos uy\right) & : \quad d \geqq y \geqq 0 \\ w_3 A e^{w_3 y} & : \quad 0 \geqq y \end{cases}$$

boundary condition : E_x and H_z must be continuous at y=0, d .

$$\tan ud = \frac{u(w_2 + w_3)}{u^2 - w_2 w_3}$$

characteristic equation

Dielectric slab waveguide : TM mode

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon E_x$$

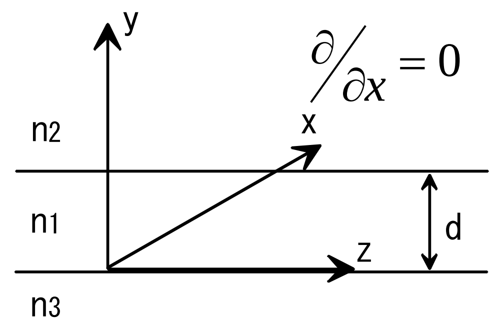
$$- j\beta H_x = j\omega\varepsilon E_y$$

$$-\frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x$$

$$- j\beta E_x = -j\omega\mu_0 H_y$$

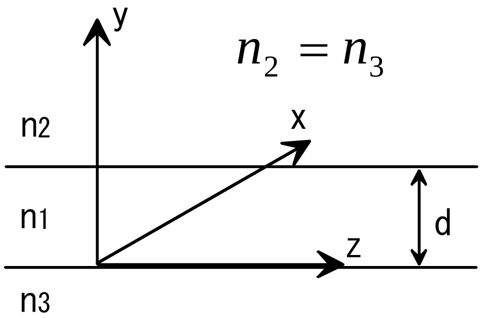
$$-\frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z$$



$$\tan ud = \frac{\frac{u}{n_1^2}\left(\frac{w_2}{n_2^2} + \frac{w_3}{n_3^2}\right)}{\left(\frac{u}{n_1^2}\right)^2 - \frac{w_2}{n_2^2}\frac{w_3}{n_3^2}}$$

symmetric slab waveguide : TE mode

symmetric slab waveguide.



$$n_2 = n_3$$

$$\tan 2u\frac{d}{2} = \frac{2uw}{u^2 - w^2} = \frac{2\frac{w}{u}}{1-\left(\frac{w}{u}\right)^2} \qquad \text{(TE mode)}$$

$$w = w_2 = w_3$$

$$\tan^2\frac{ud}{2} + \left(\frac{u}{w} - \frac{w}{u}\right)\tan\frac{ud}{2} - 1 = 0$$

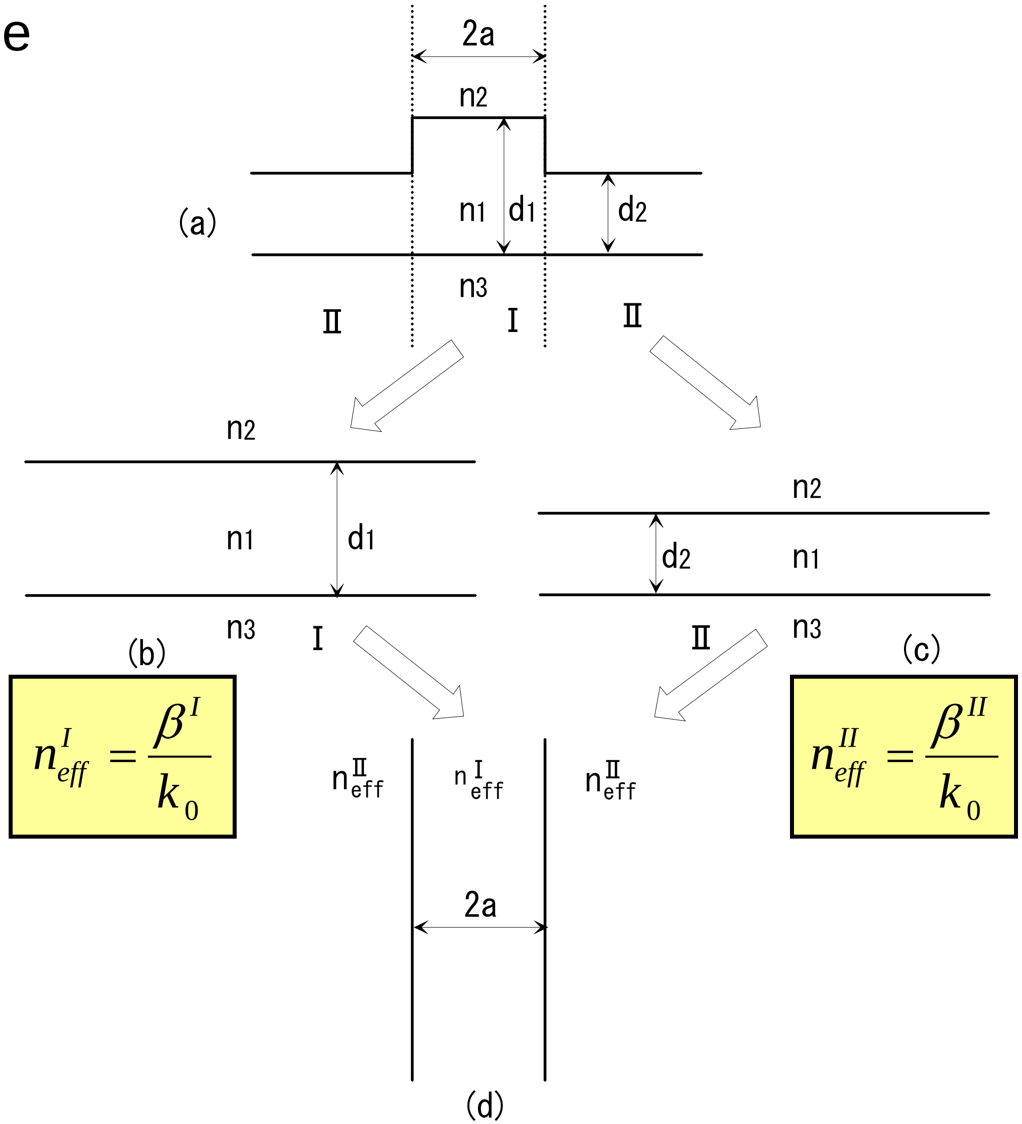
$$\left(\tan\frac{ud}{2} - \frac{w}{u}\right)\left(\tan\frac{ud}{2} + \frac{u}{w}\right) = 0$$

$$\tan\frac{ud}{2} = \frac{w}{u} \qquad \text{(even mode)}$$

$$\tan\frac{ud}{2} = -\frac{u}{w} \qquad \text{(odd mode)}$$

6.2 Effective Index Method (EIM)

Approximate method to determine the propagation constant of 3D- waveguide

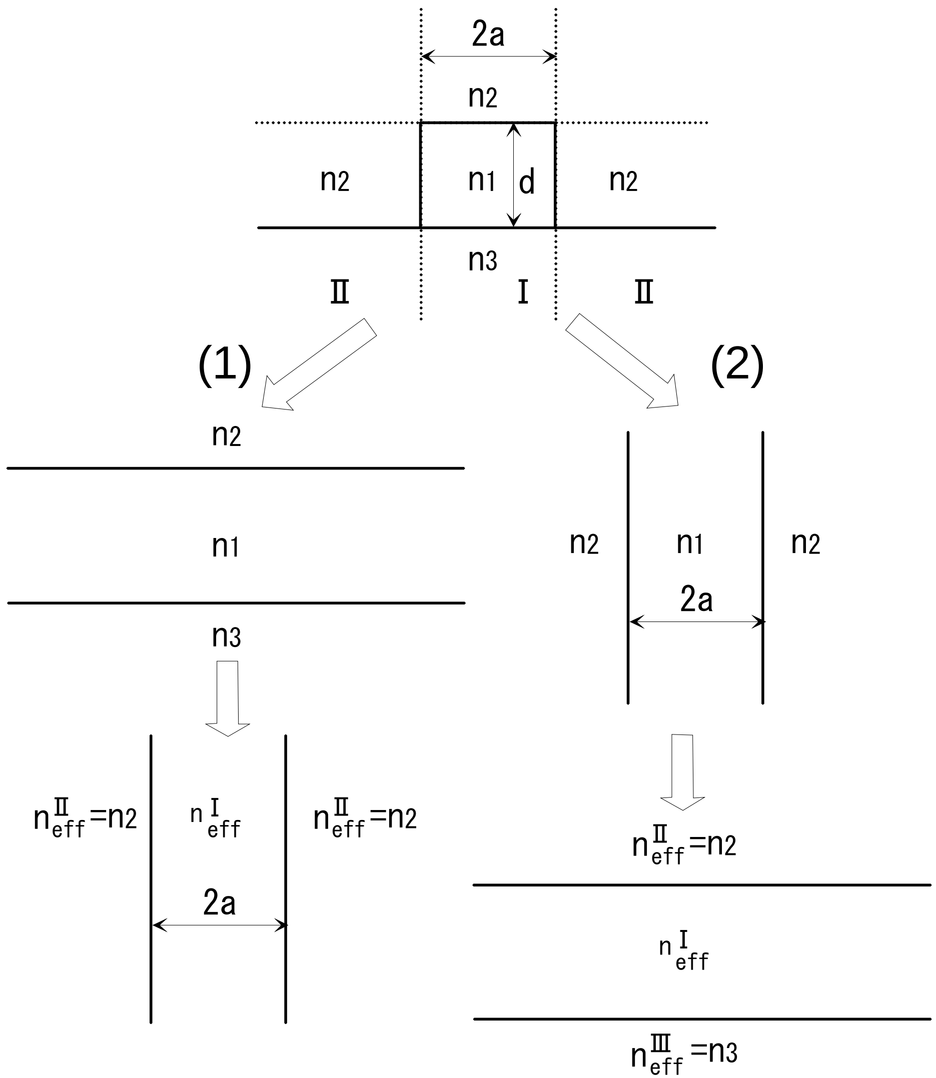


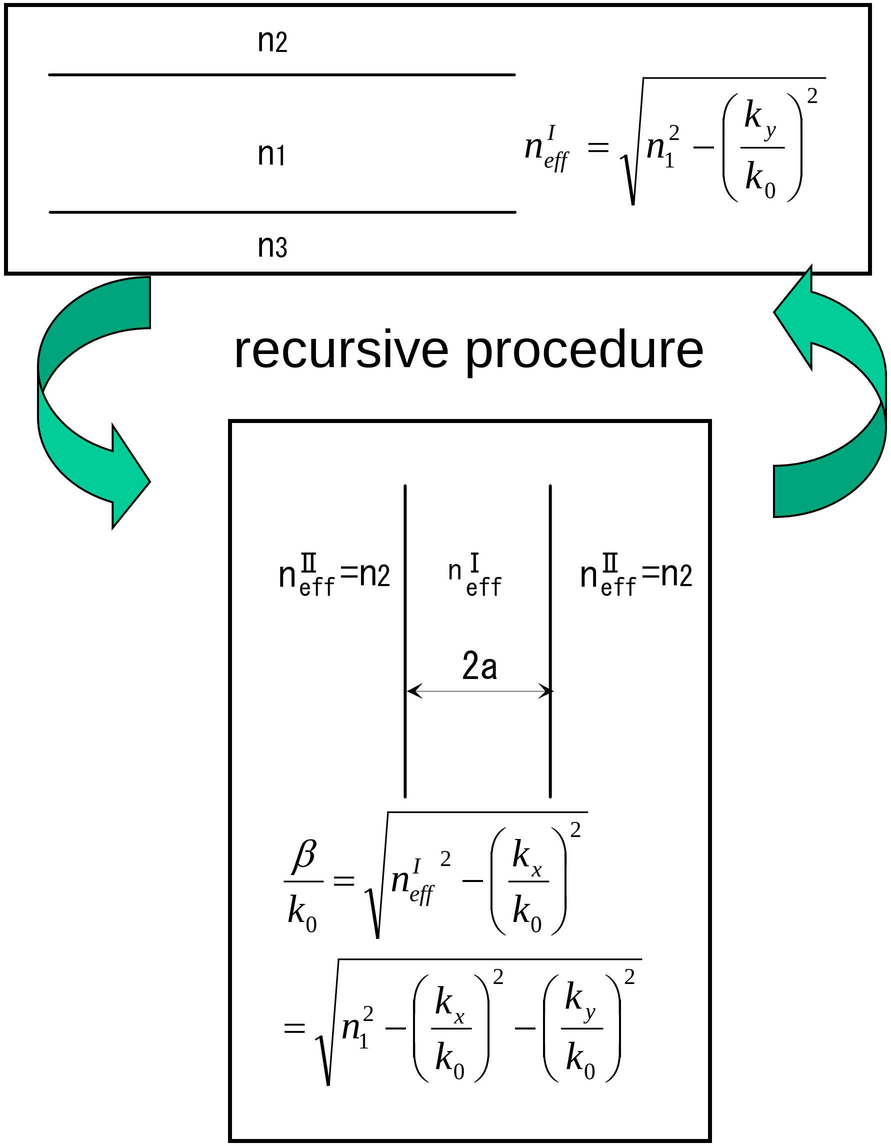
$$n_{eff}^{I} = \frac{\beta^{I}}{k_0}$$

$$n_{eff}^{II} = \frac{\beta^{II}}{k_0}$$

6.3 Generalized Effective Index Method (GEIM)

issue to be considered :





$$n_{eff}^{I} = \sqrt{n_1^2 - \left(\frac{k_y}{k_0}\right)^2}$$

$$\frac{\beta}{k_0} = \sqrt{n_{eff}^{I\,2} - \left(\frac{k_x}{k_0}\right)^2} = \sqrt{n_1^2 - \left(\frac{k_x}{k_0}\right)^2 - \left(\frac{k_y}{k_0}\right)^2}$$

6.4 Perturbation Feedback Method

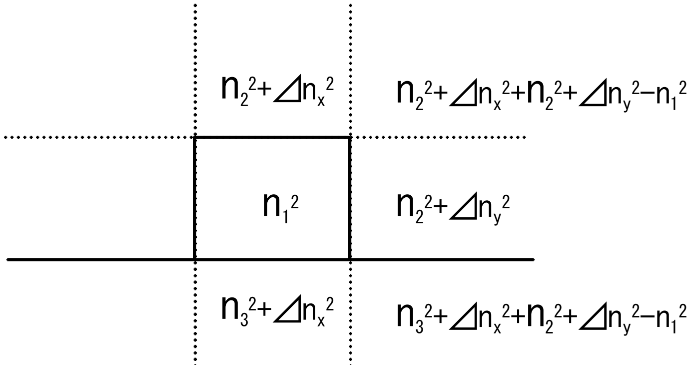
better approximation than EIM and GEIM

(i)Calculate index perturbation.

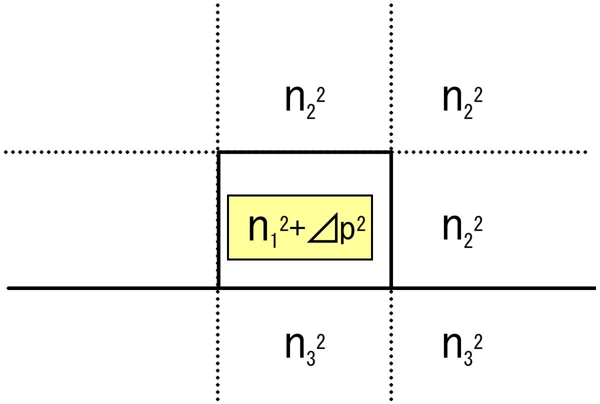
$$\Delta n_x^2 = \left(\frac{k_x}{k_0}\right)^2, \qquad \Delta n_y^2 = \left(\frac{k_y}{k_0}\right)^2$$

(ii)Calculate perturbation weighted with a field distribution.

$$\Delta_p^2 = \frac{k_0^2 \int_S \left| f(x) g(y) \right|^2 \delta n^2 dS}{\int_S \left| f(x) g(y) \right|^2 dS}$$



(iii)Modify the waveguide structure. Then, repeat procedure until perturbation converges.



6.5 Marcatili's Method

E_{pq}^{x} mode $\quad (H_x = 0)$

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon E_x$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z$$

(1) $H_x = 0$:

$$E_z = \frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

(2), (3)

$$E_y = \frac{1}{\beta\omega\varepsilon}\frac{\partial^2 H_y}{\partial x \partial y}$$

(4)

$$H_z = \frac{1}{j\beta}\frac{\partial H_y}{\partial y}$$

(5)

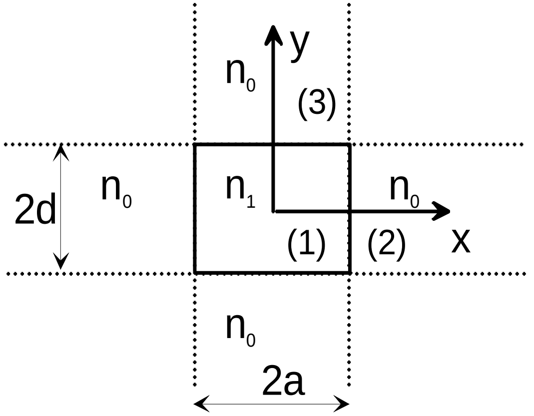
$$E_x = \frac{\omega\mu_0}{\beta} H_y + \frac{1}{\beta\omega\varepsilon}\frac{\partial^2 H_y}{\partial x^2}$$

(6)

wave equation

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (\omega^2\varepsilon\mu_0 - \beta^2) H_y = 0$$

Marcatili's Method



ignore shaded region

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (\omega^2 \varepsilon \mu_0 - \beta^2) H_y = 0$$

$$H_y = \begin{cases} A\cos(k_x x - \phi)\cos(k_y y - \varphi) & : in\ (1) \\ A\cos(k_x a - \phi)\exp(-\gamma_x (x-a))\cos(k_y y - \varphi) & : in\ (2) \\ A\cos(k_x x - \phi)\cos(k_y d - \varphi)\exp(-\gamma_y (y-d)) & : in\ (3) \end{cases}$$

$$-k_x^2 - k_y^2 + (n_1^2 k_0^2 - \beta^2) = 0$$

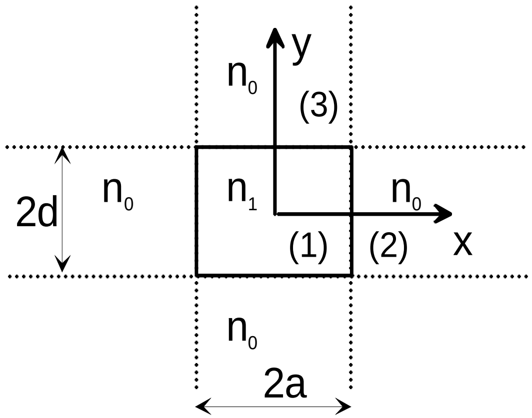
$$\gamma_x^2 - k_y^2 + (n_0^2 k_0^2 - \beta^2) = 0$$

$$-k_x^2 + \gamma_y^2 + (n_0^2 k_0^2 - \beta^2) = 0$$

$\phi = (p-1)\pi/2$

$\varphi = (q-1)\pi/2$

Marcatili's Method



ignore shaded region

interface between (1) and (2)

$$E_z : \left.\frac{1}{n_1^2}\frac{\partial H_y}{\partial x}\right|_{x=a} = \left.\frac{1}{n_0^2}\frac{\partial H_y}{\partial x}\right|_{x=a}$$

$$\tan(k_x a - \phi) = \frac{n_1^2 \gamma_x}{n_0^2 k_x}$$

$$\therefore k_x a - \phi = \tan^{-1}\left(\frac{n_1^2 \gamma_x}{n_0^2 k_x}\right)$$

interface between (1) and (3)

$$\tan(k_y d - \varphi) = \frac{\gamma_y}{k_y}$$

$$\therefore k_y d - \varphi = \tan^{-1}\left(\frac{\gamma_y}{k_y}\right)$$

$$\gamma_x^2 = -k_x^2 + (n_1^2 - n_0^2)k_0^2$$

$$\gamma_y^2 = -k_y^2 + (n_1^2 - n_0^2)k_0^2$$

$$\Rightarrow \beta^2 = n_1^2 k_0^2 - (k_x^2 + k_y^2)$$

Marcatili's Method

E^y_{pq} mode $(H_y = 0)$

$$\begin{cases} E_x = -\dfrac{1}{\beta\omega^2\varepsilon}\dfrac{\partial^2 H_x}{\partial x \partial y} \\ E_y = -\dfrac{\omega\mu_0}{\beta}H_x - \dfrac{1}{\beta\omega\varepsilon}\dfrac{\partial^2 H_x}{\partial y^2} \\ E_z = -\dfrac{1}{j\omega\varepsilon}\dfrac{\partial H_x}{\partial y} \\ H_y = 0 \\ H_z = \dfrac{1}{j\beta}\dfrac{\partial H_x}{\partial x} \end{cases}$$

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + (\omega^2 \varepsilon \mu_0 - \beta^2) H_x = 0$$