

6. Dielectric Waveguide

$$\epsilon_r = n^2$$

6.1 Dielectric slab waveguide

EM waves are classified into two groups, TE and TM mode.

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega \epsilon E_x$$

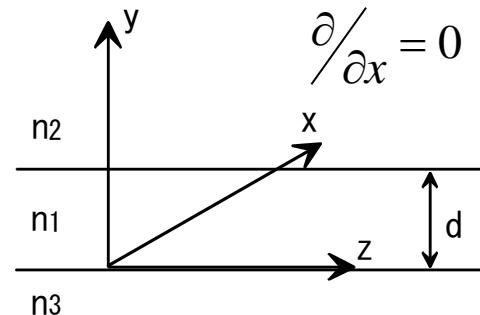
$$-j\beta H_x = j\omega \epsilon E_y$$

$$-\frac{\partial H_x}{\partial y} = j\omega \epsilon E_z$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega \mu_0 H_x$$

$$-j\beta E_x = -j\omega \mu_0 H_y$$

$$-\frac{\partial E_x}{\partial y} = -j\omega \mu_0 H_z$$



asymmetric slab waveguide

$$\frac{\partial^2 E_x}{\partial y^2} + (\omega^2 \epsilon \mu_0 - \beta^2) E_x = 0$$

Dielectric slab waveguide : TE mode

$$E_x = \begin{cases} (A \cos ud + B \sin ud) e^{-w_2(y-d)} & : y \geq d \\ A \cos uy + B \sin uy & : d \geq y \geq 0 \\ Ae^{w_3 y} & : 0 \geq y \end{cases}$$

$$w_2 = \sqrt{\beta^2 - n_2^2 \omega^2 \epsilon_0 \mu_0} = \sqrt{\beta^2 - n_2^2 k_0^2}$$

$$u = \sqrt{n_1^2 \omega^2 \epsilon_0 \mu_0 - \beta^2} = \sqrt{n_1^2 k_0^2 - \beta^2}$$

$$w_3 = \sqrt{\beta^2 - n_3^2 \omega^2 \epsilon_0 \mu_0} = \sqrt{\beta^2 - n_3^2 k_0^2}$$

$$k_0 = \omega \sqrt{\epsilon_0 \mu_0} = \frac{2\pi}{\lambda}$$

$$H_z = \frac{1}{j\omega\mu_0} \frac{\partial E_x}{\partial y} = \frac{1}{j\omega\mu_0} \times \begin{cases} -w_2(A \cos ud + B \sin ud) e^{-w_2(y-d)} & : y \geq d \\ u(-A \sin uy + B \cos uy) & : d \geq y \geq 0 \\ w_3 A e^{w_3 y} & : 0 \geq y \end{cases}$$

boundary condition : E_x and H_z must be continuous at $y=0$, d .

$$\tan ud = \frac{u(w_2 + w_3)}{u^2 - w_2 w_3}$$

characteristic equation

Dielectric slab waveguide : TM mode

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega \epsilon E_x$$

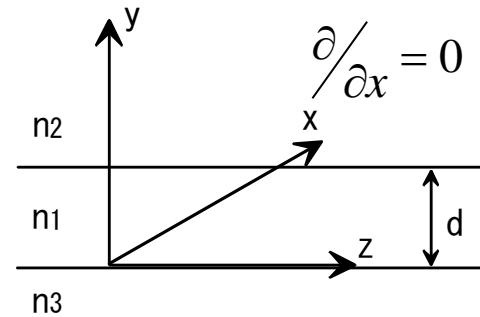
$$-j\beta H_x = j\omega \epsilon E_y$$

$$-\frac{\partial H_x}{\partial y} = j\omega \epsilon E_z$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega \mu_0 H_x$$

$$-j\beta E_x = -j\omega \mu_0 H_y$$

$$-\frac{\partial E_x}{\partial y} = -j\omega \mu_0 H_z$$



$$\tan ud = \frac{\frac{u}{n_1^2} \left(\frac{w_2}{n_2^2} + \frac{w_3}{n_3^2} \right)}{\left(\frac{u}{n_1^2} \right)^2 - \frac{w_2}{n_2^2} \frac{w_3}{n_3^2}}$$

symmetric slab waveguide : TE mode

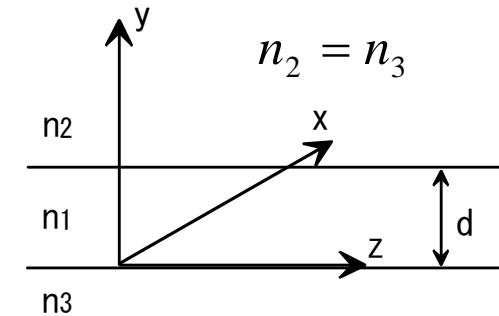
symmetric slab waveguide.

$$\tan 2u \frac{d}{2} = \frac{2uw}{u^2 - w^2} = \frac{2\frac{w}{u}}{1 - \left(\frac{w}{u}\right)^2} \quad (\text{TE mode})$$

$$w = w_2 = w_3$$

$$\tan^2 \frac{ud}{2} + \left(\frac{u}{w} - \frac{w}{u} \right) \tan \frac{ud}{2} - 1 = 0$$

$$\left(\tan \frac{ud}{2} - \frac{w}{u} \right) \left(\tan \frac{ud}{2} + \frac{u}{w} \right) = 0$$

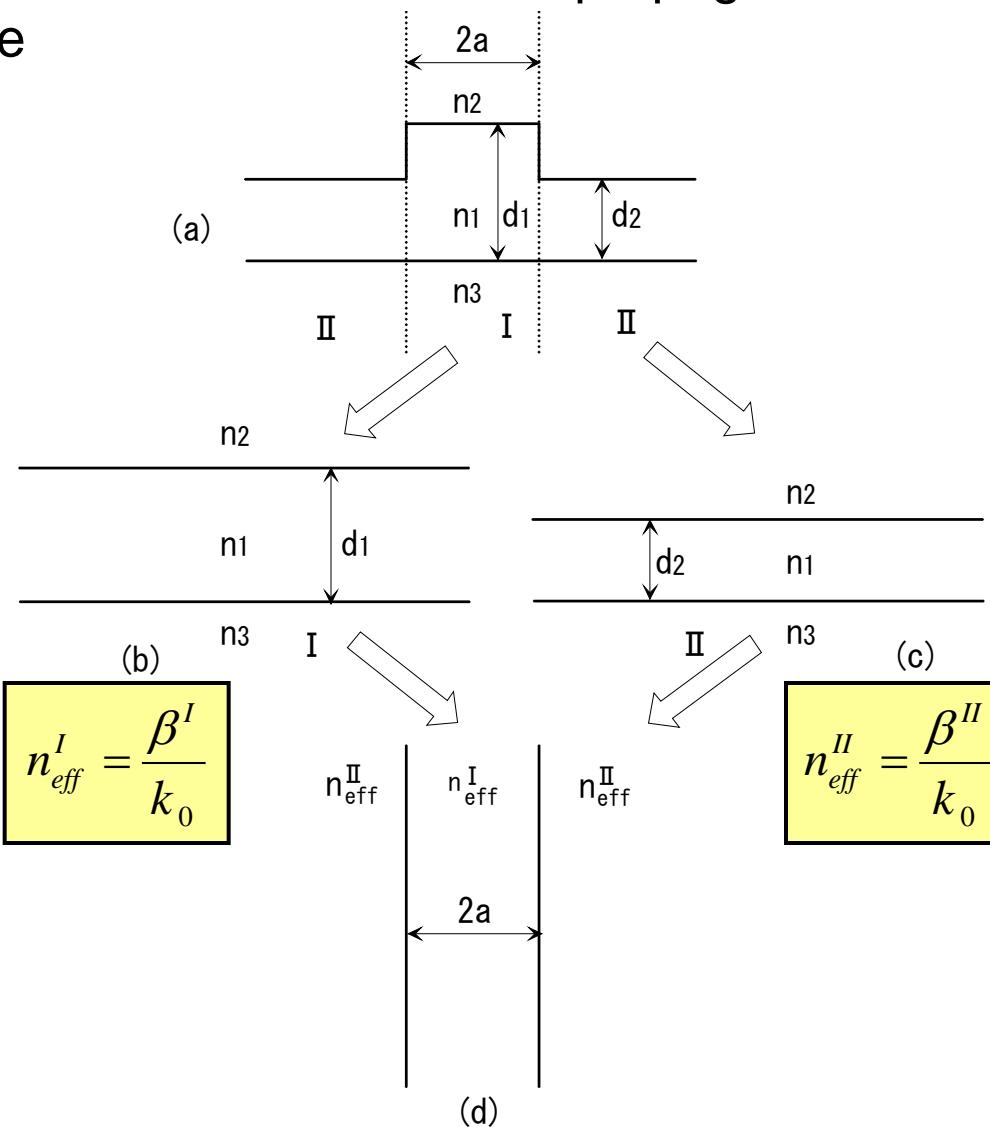


$$\tan \frac{ud}{2} = \frac{w}{u} \quad (\text{even mode})$$

$$\tan \frac{ud}{2} = -\frac{u}{w} \quad (\text{odd mode})$$

6.2 Effective Index Method (EIM)

Approximate method to determine the propagation constant of 3D- waveguide



6.3 Generalized Effective Index Method (GEIM)

issue to be considered :

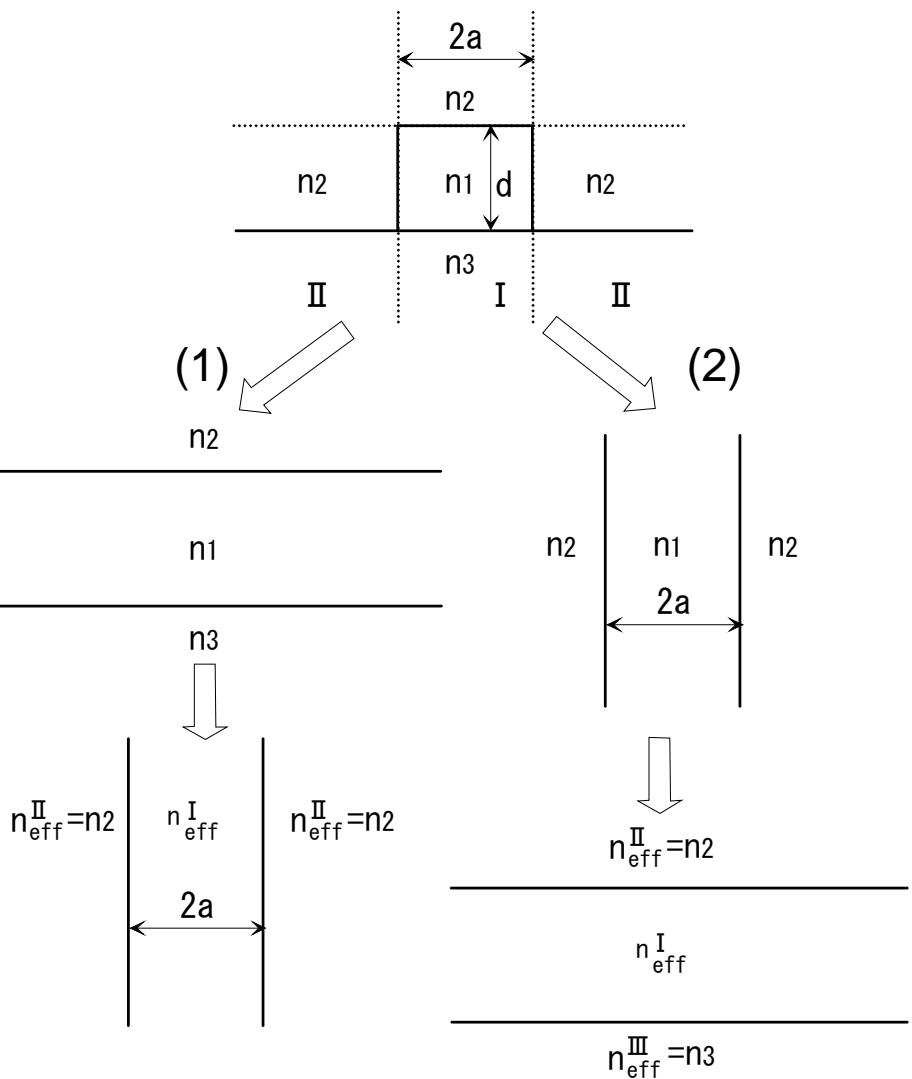


Diagram illustrating the recursive procedure for the GEIM method. The diagram shows a stack of three layers (n₂, n₁, n₃) with a central air gap of width d. The top layer has thickness 2a. The diagram is divided into two regions, I and II, by vertical dashed lines. Region I is the central air gap, and Region II is the outer regions. Two arrows labeled (1) and (2) point downwards from the top layer to the bottom layer, indicating the recursive steps. The bottom layer is shown with its own air gap of width 2a and effective index $n_{eff}^{III} = n_3$. The effective indices for the top and middle layers are labeled as $n_{eff}^{II} = n_2$.

$$n_{eff}^I = \sqrt{n_1^2 - \left(\frac{k_y}{k_0}\right)^2}$$

recursive procedure

Diagram illustrating the recursive procedure for the GEIM method. The diagram shows a stack of three layers (n₂, n₁, n₃) with a central air gap of width d. The top layer has thickness 2a. The diagram is divided into two regions, I and II, by vertical dashed lines. Region I is the central air gap, and Region II is the outer regions. Two arrows labeled (1) and (2) point downwards from the top layer to the bottom layer, indicating the recursive steps. The bottom layer is shown with its own air gap of width 2a and effective index $n_{eff}^{III} = n_3$. The effective indices for the top and middle layers are labeled as $n_{eff}^{II} = n_2$.

$$\frac{\beta}{k_0} = \sqrt{n_{eff}^{I\ 2} - \left(\frac{k_x}{k_0}\right)^2}$$

$$= \sqrt{n_1^2 - \left(\frac{k_x}{k_0}\right)^2 - \left(\frac{k_y}{k_0}\right)^2}$$

6.4 Perturbation Feedback Method

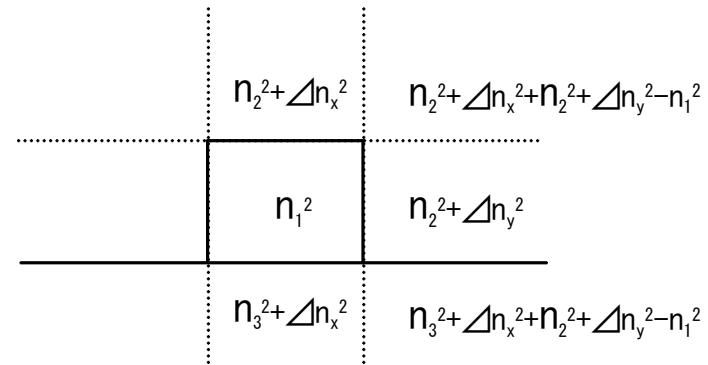
better approximation than EIM and GEIM

(i) Calculate index perturbation.

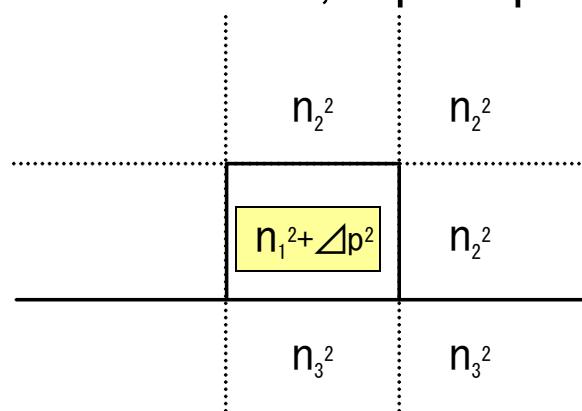
$$\Delta n_x^2 = \left(\frac{k_x}{k_0} \right)^2, \quad \Delta n_y^2 = \left(\frac{k_y}{k_0} \right)^2$$

(ii) Calculate perturbation weighted with a field distribution.

$$\Delta_p^2 = \frac{k_0^2 \int |f(x)g(y)|^2 \delta n^2 dS}{\int |f(x)g(y)|^2 dS}$$



(iii) Modify the waveguide structure. Then, repeat procedure until perturbation converges.



6.5 Marcatili's Method

E_{pq}^x mode ($H_x = 0$)

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x \quad \boxed{- j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad H_z = \frac{1}{j\beta} \frac{\partial H_y}{\partial y}} \quad (6)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad \boxed{\begin{matrix} H_x = 0 \\ (1) \end{matrix}} \quad E_z = \frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial x} \quad (4)$$

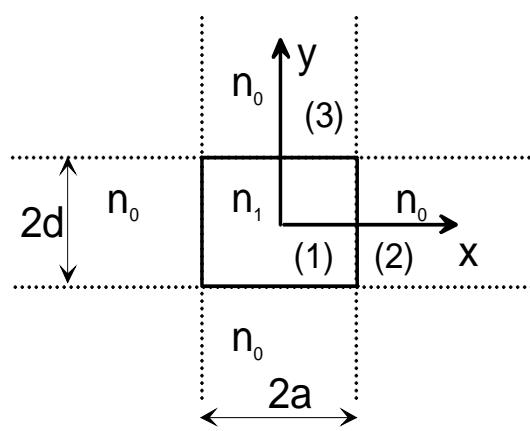
$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x \quad \boxed{(2)} \quad E_y = \frac{1}{\beta\omega\epsilon} \frac{\partial^2 H_y}{\partial x \partial y} \quad (3)$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y \quad \boxed{(5)} \quad E_x = \frac{\omega\mu_0}{\beta} H_y + \frac{1}{\beta\omega\epsilon} \frac{\partial^2 H_y}{\partial x^2} \quad (6)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z$$

wave equation $\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (\omega^2 \epsilon \mu_0 - \beta^2) H_y = 0$

Marcatili's Method



ignore shaded region

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (\omega^2 \epsilon \mu_0 - \beta^2) H_y = 0$$

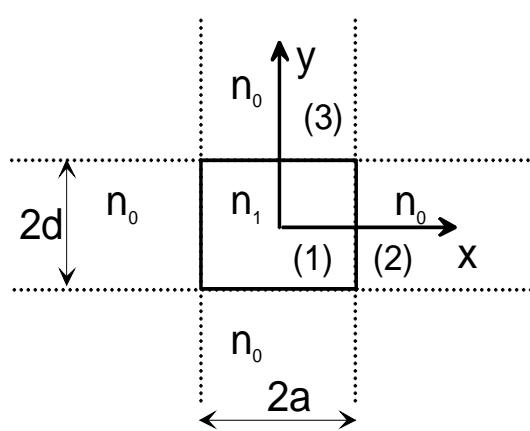
$$H_y = \begin{cases} A \cos(k_x x - \phi) \cos(k_y y - \varphi) & : \text{in (1)} \\ A \cos(k_x a - \phi) \exp(-\gamma_x (x - a)) \cos(k_y y - \varphi) & : \text{in (2)} \\ A \cos(k_x x - \phi) \cos(k_y d - \varphi) \exp(-\gamma_y (y - d)) & : \text{in (3)} \end{cases}$$

$$-k_x^2 - k_y^2 + (n_1^2 k_0^2 - \beta^2) = 0 \quad \phi = (p-1)\pi/2$$

$$\gamma_x^2 - k_y^2 + (n_0^2 k_0^2 - \beta^2) = 0 \quad \varphi = (q-1)\pi/2$$

$$-k_x^2 + \gamma_y^2 + (n_0^2 k_0^2 - \beta^2) = 0$$

Marcatili's Method



ignore shaded region

interface between (1) and (2)

$$E_z : \frac{1}{n_1^2} \frac{\partial H_y}{\partial x} \Big|_{x=a} = \frac{1}{n_0^2} \frac{\partial H_y}{\partial x} \Big|_{x=a}$$

$$\tan(k_x a - \phi) = \frac{n_1^2 \gamma_x}{n_0^2 k_x}$$

$$\therefore k_x a - \phi = \tan^{-1} \left(\frac{n_1^2 \gamma_x}{n_0^2 k_x} \right)$$

interface between (1) and (3)

$$\tan(k_y d - \varphi) = \frac{\gamma_y}{k_y}$$

$$\therefore k_y d - \varphi = \tan^{-1} \left(\frac{\gamma_y}{k_y} \right)$$

$$\gamma_x^2 = -k_x^2 + (n_1^2 - n_0^2)k_0^2$$

$$\gamma_y^2 = -k_y^2 + (n_1^2 - n_0^2)k_0^2$$



$$\beta^2 = n_1^2 k_0^2 - (k_x^2 + k_y^2)$$

Marcatili's Method

E_{pq}^y mode ($H_y = 0$)

$$\begin{cases} E_x = -\frac{1}{\beta\omega^2\varepsilon} \frac{\partial^2 H_x}{\partial x \partial y} \\ E_y = -\frac{\omega\mu_0}{\beta} H_x - \frac{1}{\beta\omega\varepsilon} \frac{\partial^2 H_x}{\partial y^2} \\ E_z = -\frac{1}{j\omega\varepsilon} \frac{\partial H_x}{\partial y} \\ H_y = 0 \\ H_z = \frac{1}{j\beta} \frac{\partial H_x}{\partial x} \end{cases}$$

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + (\omega^2\varepsilon\mu_0 - \beta^2)H_x = 0$$