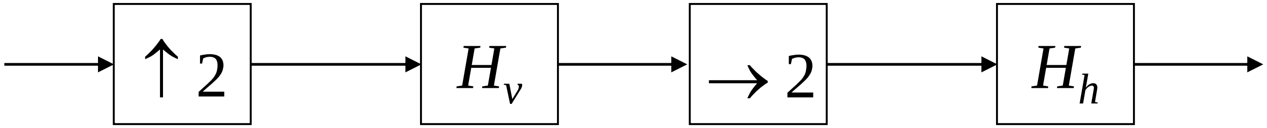
MR-SP Applications: Image Resizing

image enlarging



vertical direction

horizontal direction

Symmetric Extension

Linear convolution for finite-duration signal

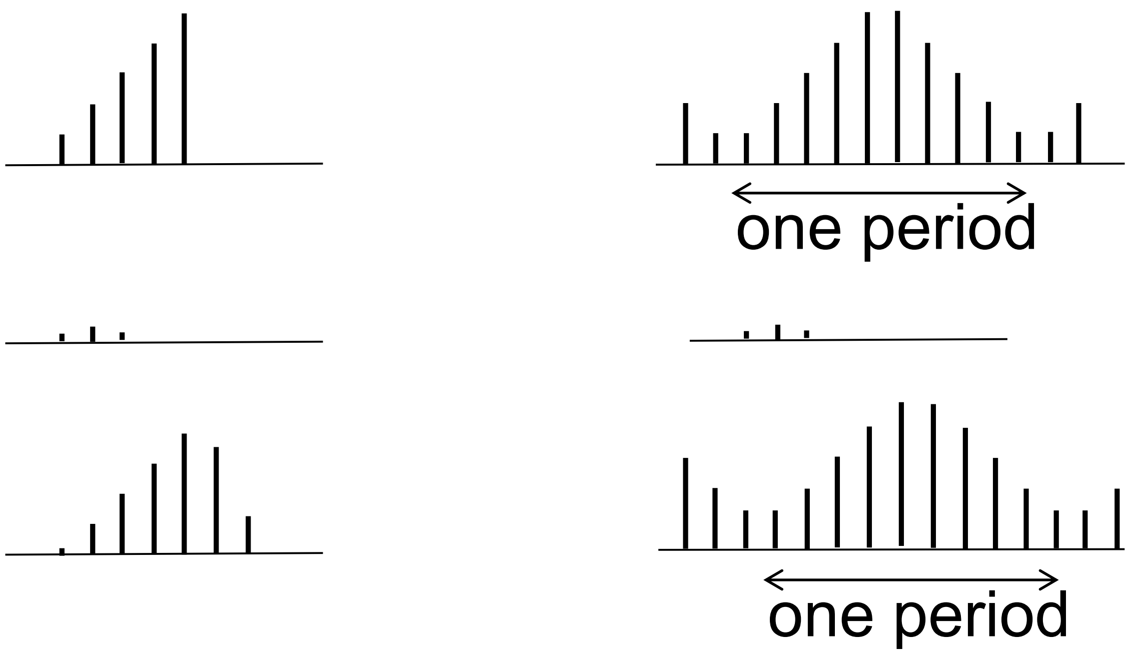
e.g. convolution of data {1 2 3 4 5} with IR {1/4 1/2 1/4}

Finite signal | Extended periodic signal

$x(n)$

$h(n)$

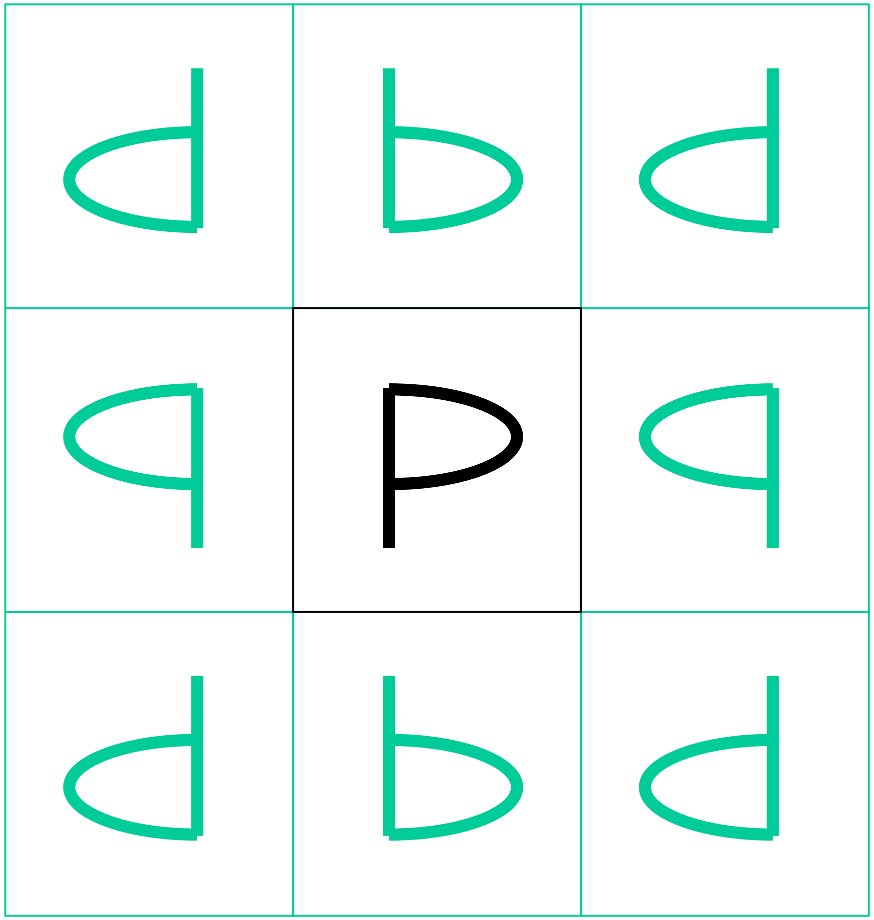
$y(n) = x(n) * h(n)$



circular convolution

Symmetric Extension

2-dimensional case for images

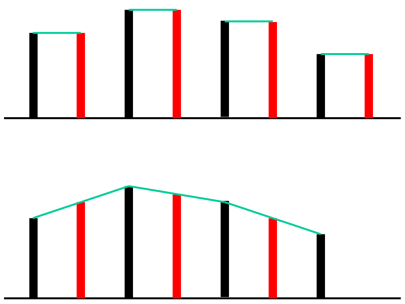


Interpolation Filter

Simple filters like

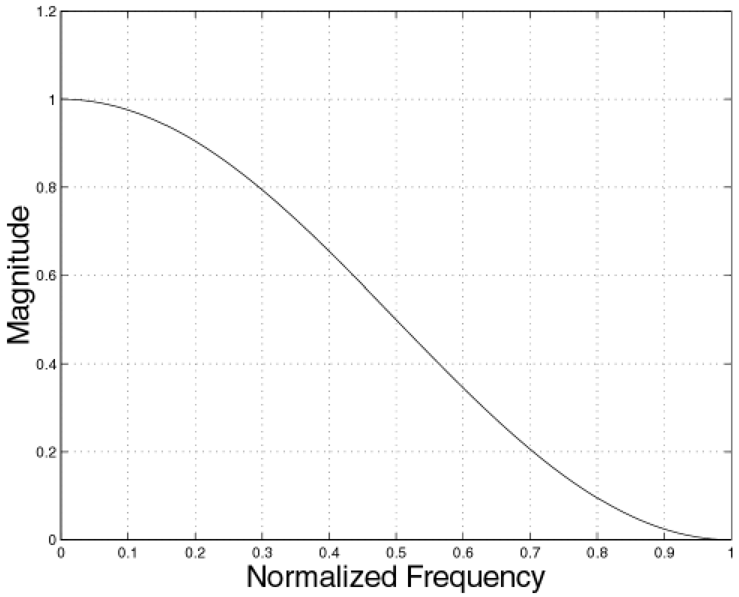
- zero-order hold
- linear interpolation

are often used



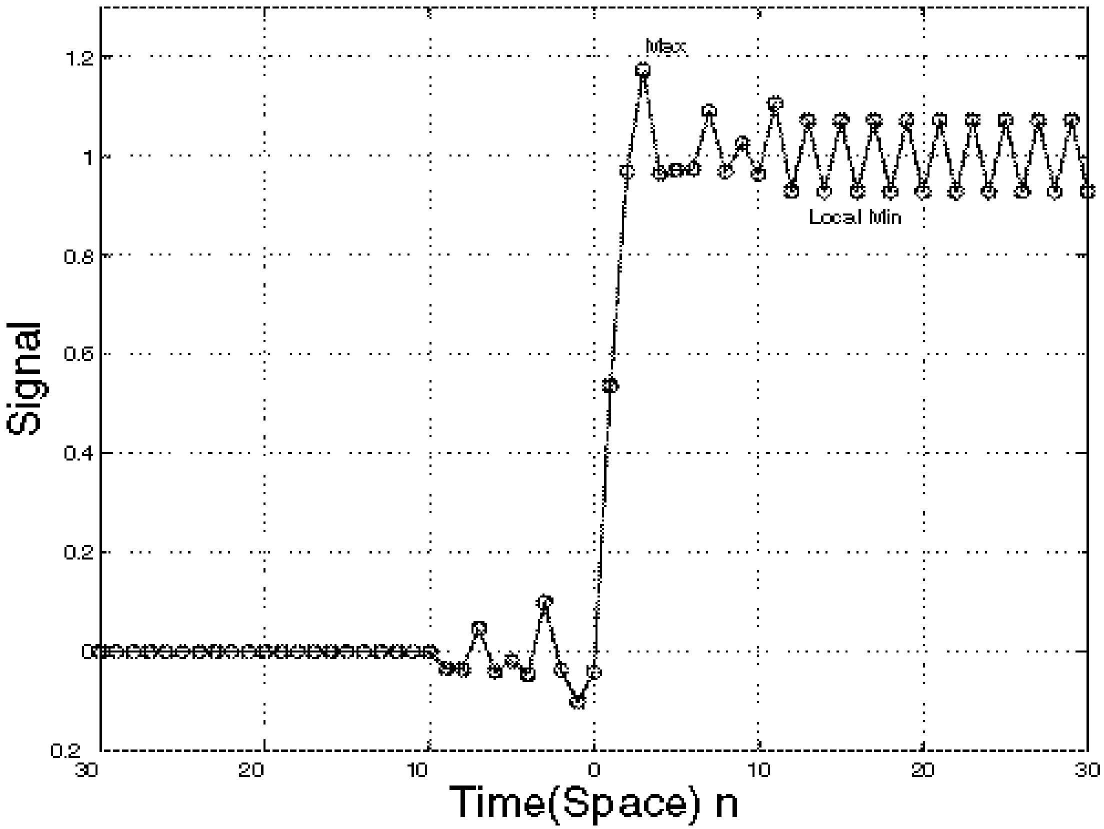
amplitude response of such a filter is

poor



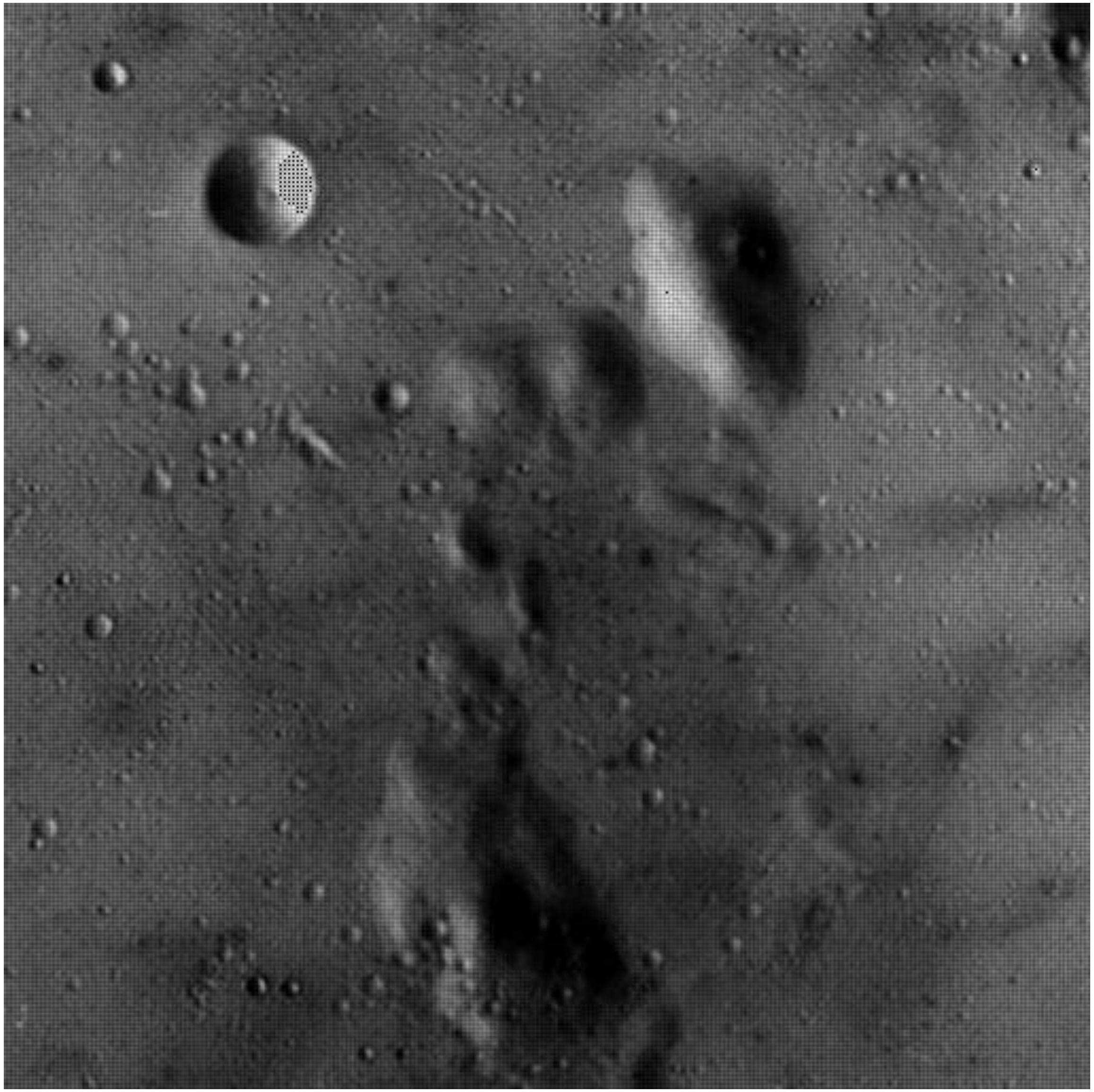
Checkerboard Effect

Oscillation of step response due to time-varying nature



can be avoided by a zero at $z = -1$ $(\omega = \pi)$

Example of Checkerboard Effect



Picture Quality

Objective error such as mean square error

Human visual system is sensitive to

- Blur attenuation of high frequency component
- Jaggy remaining alias component
- Ringing narrow transition bandwidth

Blurred Image

by a filter with passband edge of 0.3π

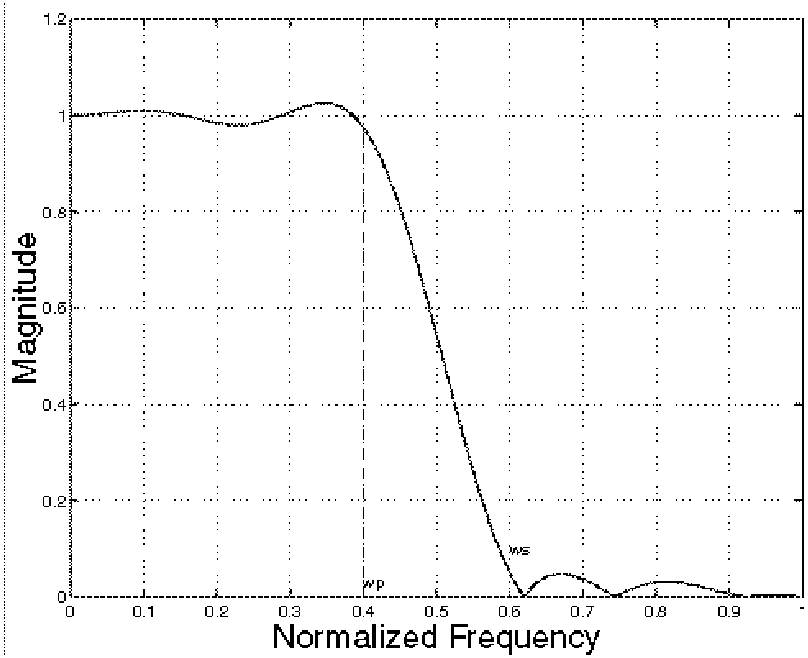
Jaggy Image

by a filter with passband edge of 0.6π

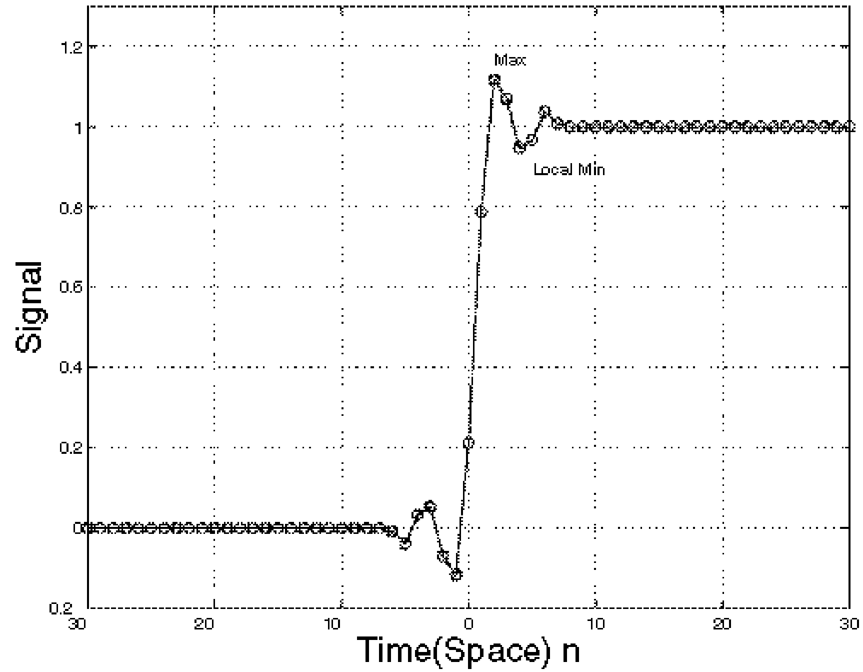
Image with Ringing

by a very sharp cutoff filter

Best Filter



Amplitude response



Step response

Jun Murakawa and Akinori Nishihara: "Design and Visual Evaluation of Digital Filters for Enlargement and Reduction of Images", IEEE Asia Pacific Conference on Circuits and Systems, pp.128-131, Dec. 2000 (Best Paper Award)

Half Band Filters

$$H(z) = E_0(z^2) + z^{-1}E_1(z^2)$$

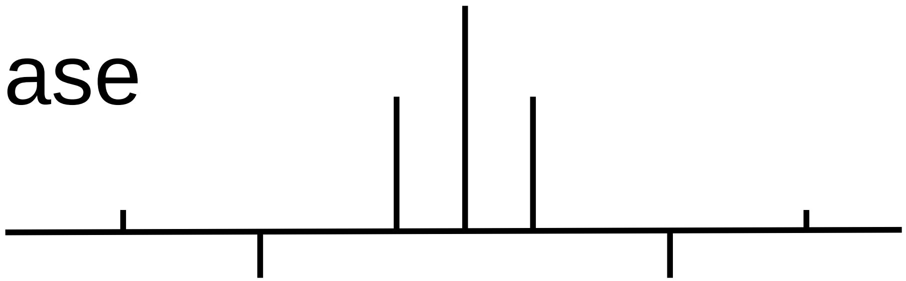
either of $E_k(z)$ is cz^{-n_k}

$$h(2n+k) = \begin{cases} c & n = n_k \\ 0 & otherwise \end{cases}$$

$$\begin{cases} k=0 & H(z)+H(-z) = 2cz^{-2n_k} \\ k=1 & H(z)-H(-z) = 2cz^{-2n_k-1} \end{cases} \Rightarrow 2cz^{-(2n_k+k)}$$

Impulse Response of HB Filters

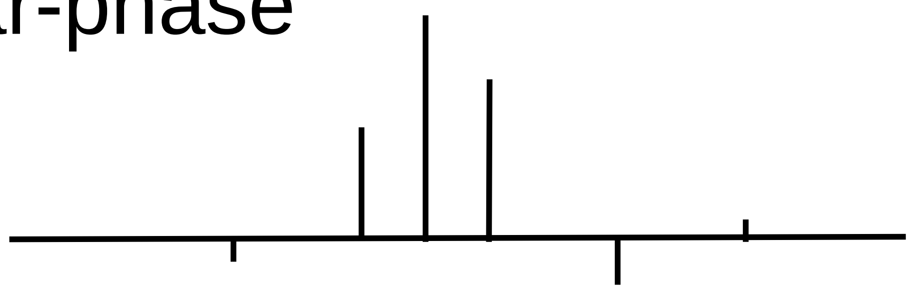
linear-phase



$N = 4K + 3$

11-tap

nonlinear-phase



$N = 2K + 1$

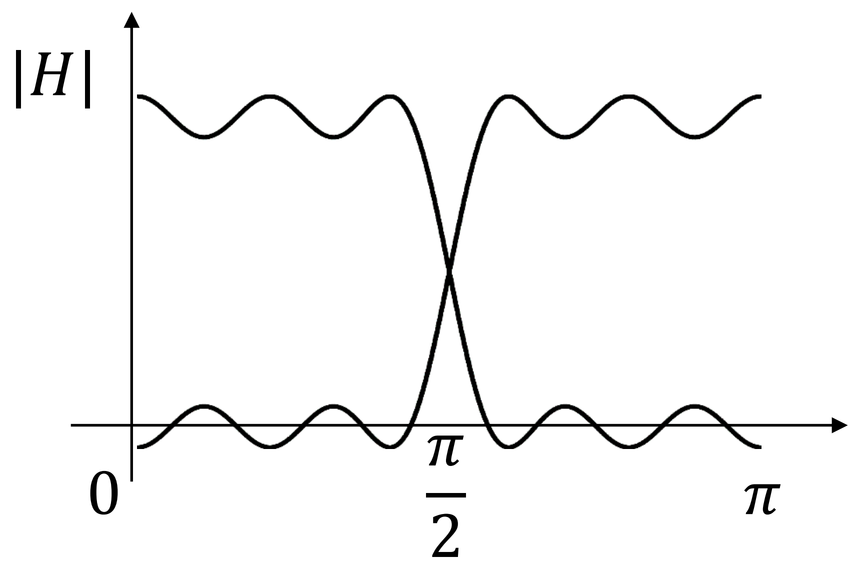
9-tap

Frequency Response of HB Filters

If $H(z)$ has real coefficients,

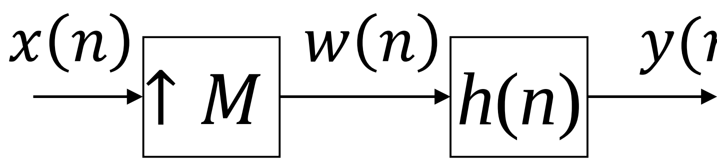
$$H\left(-e^{j\omega}\right) = H\left(e^{j(\pi-\omega)}\right)$$

$$H\left(e^{j\omega}\right) \pm H\left(e^{j(\pi-\omega)}\right) = 2ce^{j(2n_k+k)\omega}$$



M-th Band Filters
Nyquist(M) Filters

M-fold interpolation filter



$$Y(z) = X(z^M)H(z)$$

Polyphase decomposition

Suppose $E_0(z)$ is a constant c, *i.e.*

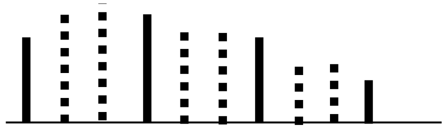
$$H(z) = c + z^{-1}E_1(z^M) + \cdots + z^{-(M-1)}E_{M-1}(z^M)$$

M-th Band Filters (cont'd)

$$Y(z) = cX(z^M) + \sum_{k=1}^{M-1} z^{-k} E_k(z^M) X(z^M)$$

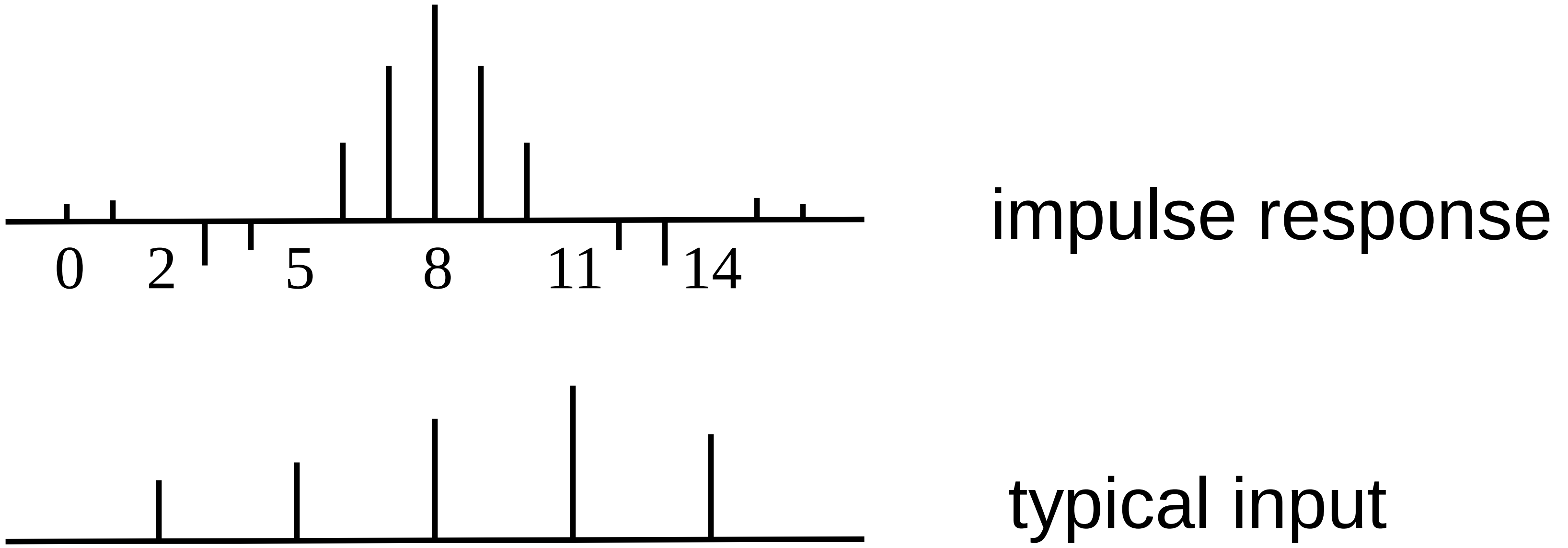
$$y(Mn) = cx(n)$$

Even though the interpolation filter inserts new samples, the existing samples are never altered.



Impulse Response of *M*-th Band Filters

$$h(Mn+k)=\begin{cases} c & n=n_k \\ 0 & otherwise \end{cases}$$



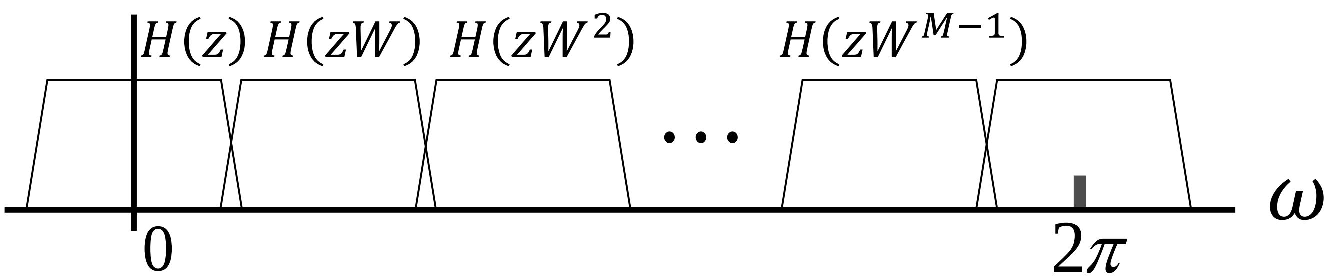
M-th Band Filters (cont'd)

If $H(z)$ is an M-th band filter

$$\sum_{k=0}^{M-1} H\left(zW^{k}\right) = Mc = 1 \quad (\text{assuming } c = \frac{1}{M})$$

$$W = e^{-j2\pi/M}$$

All shifted versions of H add up to a constant.



Exercise 5

1. Derive the transfer function of the zero-order holding filter, and calculate its frequency response.
2. Give comments on the following type of extension;

   PPP

   PPP

   PPP
3. Show that the checkerboard effect occurs when an interpolation filter with L=2 does not have enough attenuation at $\omega = \pi$.
4. Show that the impulse response length of FIR half-band filters is odd.
5. Design an 11-tap half-band FIR filter as you want. You may use a program at http://www.nh.cradle.titech.ac.jp/old/maxflat/ with N=10, K=5 and an even integer for d. Show that $\left|H\left(e^{j\omega}\right) - H\left(e^{j(\pi-\omega)}\right)\right| = 1/2$.
6. Show that for an M-th band filter $H(z)$ satisfies

   $$\sum_{k=0}^{M-1} H\left(zW^k\right) = Mc$$
7. Read the following paper;

   Jun Murakawa and Akinori Nishihara: "Design and Visual Evaluation of Digital Filters for Enlargement and Reduction of Images", IEEE Asia Pacific Conference on Circuits and Systems, pp.128-131, Dec. 2000