5 August 2013

Advanced Signal Processing – Final Exam

Prof. A. Nishihara

1. Consider a digital filter shown in Figure 1.

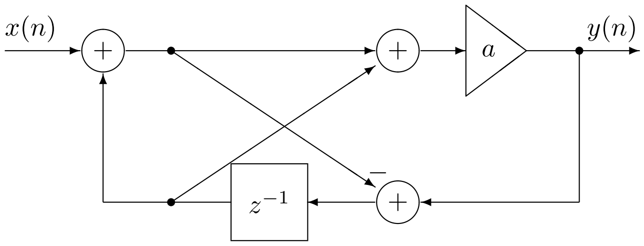

Figure 1: A filter

   (a) Derive its transfer function.
   (b) What is the range of the parameter a for the filter to be stable?
   (c) What are the amplitude responses at $\omega = 0$ and at $\omega = \pi$?
   (d) What is the filter type (lowpass, highpass, $\cdots$)?
   (e) Sketch the rough frequency–amplitude response.
   (f) What happens to the transfer function when the direction of subtraction is reversed.
   (g) What are the critical path and iteration bound? Use T_M for multiplication time and T_A for additon/subtraction time.

2. Let $x(n)$ be periodic with period N (N is the smallest interger such that $x(n) = x(n+N)$). Let $y(n)$ be the M-fold decimated version ($y(n) = x(Mn)$). Show that $y(n)$ is periodic, i.e. there exists $L < \infty$ such that $y(n) = y(n+L)$ for all n. Assuming no further knowledge about the input, what is the smallest value of L in terms of M and N?

3. Consider a sinusoid in white noise

$$x(n) = \sqrt{2}\sin(n\omega) + e(n).$$

   (a) Show that the autocorrelation function is $r(\ell) = \cos(\ell\omega) + \sigma_e^2\delta(\ell)$.
   (b) Show that the eigenvalues of the 3×3 autocorrelation matrix

$$R = \begin{pmatrix} r(0) & r(1) & r(2) \\ r(1) & r(0) & r(1) \\ r(2) & r(1) & r(0) \end{pmatrix}$$

   are

$$\begin{aligned} \lambda_1 &= \sigma_e^2, \\ \lambda_2 &= \sigma_e^2 + 1 - \cos 2\omega, \\ \lambda_3 &= \sigma_e^2 + 2 + \cos 2\omega. \end{aligned}$$

4. Consider the DFG shown in Figure 2, where the number at each node denotes its execution time.

   (a) What is the critical path length?
   (b) What is the iteration bound?
   (c) Manually retime this DFG to minimize the sampling period.

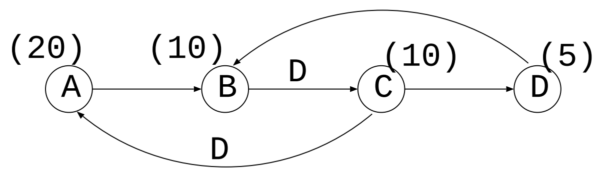

Figure 2: A data flow graph

30 July 2012

Advanced Signal Processing – Final Exam

Prof. A. Nishihara

1. Consider a digital filter described by

$$H(z) = \frac{1}{16}(3 + 8z^{-1} + 6z^{-2} - z^{-4}).$$

   (a) What is the DC gain of this filter?
   (b) What is the filter type (low-pass, high-pass, or $\cdots$)?
   (c) What is the relation between $H(e^{j\omega})$ and $H(e^{j(\pi-\omega)})$
   (d) What is the particular feature of this filter (What is the class of this filter?)
   (e) Show a structure to realize this transfer function, and show its critical path length (You don't need to minimize the critical path length).

2. Consider a signal $x(n)$ whose spectrum $X(e^{j\omega})$ is given by Figure 1. $y(n)$ is made by $y(n) = x(2n)$.

   (a) Sketch the spectrum of $y(n)$.
   (b) Show how you can recover $x(n)$ from $y(n)$.

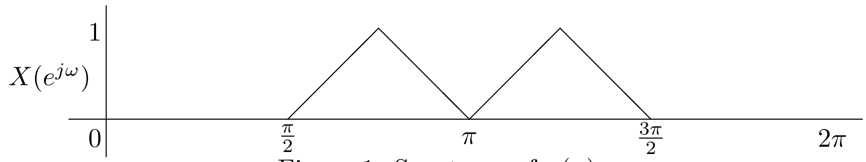

Figure 1: Spectrum of $x(n)$

3. Find the first three terms of the autocorrelation function of the auto-regressive signal

$$x(n) = 0.8x(n-1) - 0.2x(n-2) + e(n),$$

   where $e(n)$ is a unit-power zero-mean white noise.

4. Write the transfer functions of the three filters shown in Figure 2. State the relationship among the three structures.

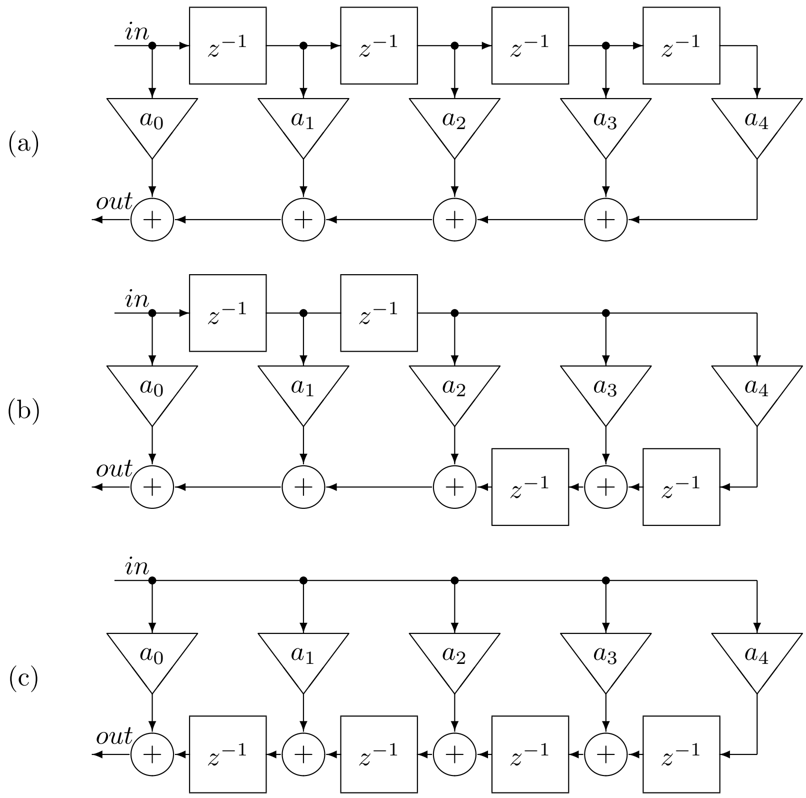

Figure 2: Three Filters

11 July 2011

Advanced Signal Processing – Final Exam

Prof. A. Nishihara

1. The first row of the 8-point DFT is written as

$$X(0) = x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7).$$

   (a) Consider the above as a filter, and write its transfer function.
   (b) Calculate its frequency response.
   (c) What is the filter type (lowpass, highpass, $\cdots$)?
   (d) Write the next row of the 8-point DFT.
   (e) Write its transfer function.
   (f) Calculate its frequency response.
   (g) How does the frequency response related to that of the first row?

2. For the filter bank shown in Figure 1,

   (a) Derive the transfer functions $H_0(z)$ and $H_1(z)$ of the analysis filter bank.
   (b) Derive the transfer functions $F_0(z)$ and $F_1(z)$ of the synthesis filter bank.
   (c) Show that the filter bank has perfect reconstruction property.
   (d) When $E_0(z) = E_1(z) = 1 + 0.6z^{-1}$, show transfer functions $H_0(z), H_1(z), F_0(z)$ and $F_1(z)$. Confirm IIR transfer functions are stable. Calculate the overall transfer function $T(z) = \frac{\hat{X}(z)}{X(z)}$.
   (e) What is the main disadvantage of this filter bank?

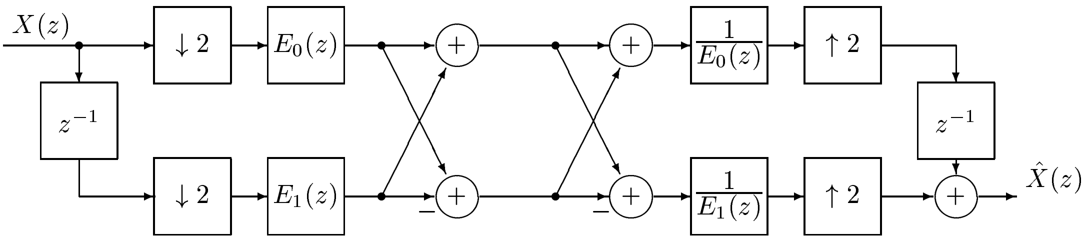

Figure 1: A 2-channel filter bank

3. Consider an FIR fiter

$$y(n) = a_0x(n) + a_2x(n-2) + a_3x(n-3).$$

   Assume that the time required for 1 multiply operation is T_M and the time required for 1 add operation is T_A.

   (a) Draw a direct-form realization structure of the above filter.
   (b) What is the critical path length?
   (c) Pipeline this filter such that the clock period is $T_M + T_A$.
   (d) Draw the broadcast filter architecture for the above filter. What are the advantages and disadvantages of the broadcast architecture compared to the direct-form?

26 July 2010

Advanced Signal Processing – Final Exam

Prof. A. Nishihara

1. For the filter shown in Figure 1,
   - (a) Derive the transfer function $H(z) = Y(z)/X(z)$.
   - (b) What is the filter type (lowpass, highpass, $\cdots$)?
   - (c) Show that the transfer function is FIR.
   - (d) Derive the transfer function $F(z) = W(z)/X(z)$.
   - (e) When the input signal is a unit step function $u(n)$, derive $w(n)$ and $y(n)$.
   - (f) What kind of potential problem do you find in the above $w(n)$?
   - (g) How can you solve that problem by modifying the structure?

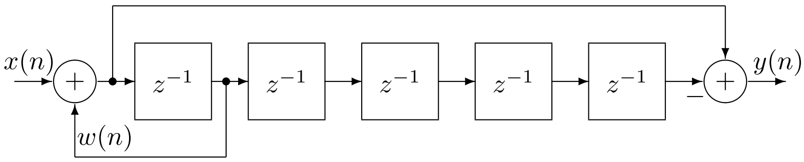

Figure 1: A filter

2. Consider the structure shown in Figure 2, where $H(z)$ is given by

$$H(z) = \frac{(1+z^{-1})^2}{(1-0.5jz^{-1})(1+0.5jz^{-1})}.$$

   - (a) Show a structure of $H(z)$ using only real coefficients.
   - (b) What is the filter type (lowpass, highpass, $\cdots$)?
   - (c) Show the critical path. Suppose one multiplication time is $200ns$ and one addition time is $50ns$. What is the maximum input sampling frequency?
   - (d) Decompose $H(z)$ using two polyphase components and show the structure.
   - (e) Use the Noble Identity and show thus modified structure.
   - (f) What is the maximum input sampling frequency in this case?

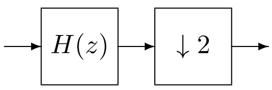

Figure 2: A system

3. A measurement gives the signal autocorrelation values $r(0) = 1.0, r(1) = 0.6$, and $r(2) = 0.2$. Suppose the signal is generated by a second-order linear prediction filter excited by a white noise. Calculate the two coefficients of the second-order linear predictor and the white noise power.

27 July 2009

Advanced Signal Processing – Final Exam

Prof. A. Nishihara

1. For the filter shown in Fig.1,
   (a) Derive the transfer function $H(z) = Y(z)/X(z)$.
   (b) What is the filter type (lowpass, highpass, $\cdots$)?
   (c) When $w(n)$ is made as $y(n) - x(n)$, what is the transfer function $W(z)/X(z)$?
   (d) Derive the iteration bound using T_M, one multiplication time, and T_A, one addition time.

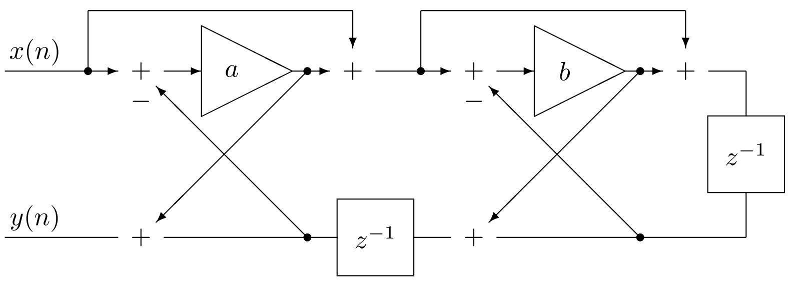

Figure 1: A filter

2. Suppose $H_0(z)$ is a half-band lowpass filter and $H_1(z)$ is its complementary highpass filter. Consider the structure shown in Fig. 2.
   (a) Derive the overall transfer function.
   (b) Draw rough plot of the overall amplitude response. Show the values corresponding to a_k in the plot.
   (c) What kind of applications would you find for this circuit?
   (d) Draw a diagram when the number of channels is further increased.

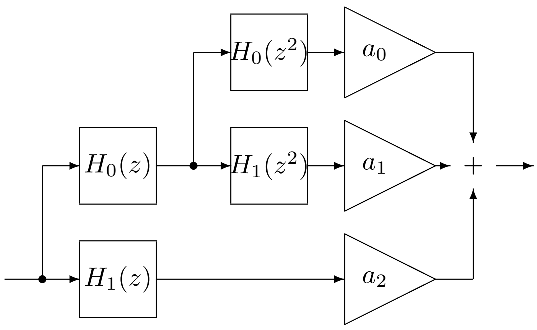

Figure 2: A circuit

3. LMS adaptive filters have coefficient update formula given by

$$H(n+1) = H(n) + \mu e(n) X(n),$$

where $H(n)$ is the N-tap FIR filter coefficient vector, $e(n)$ is the error, $X(n)$ is the N most recent input signal vector, at time n. The stepsize parameter μ is a scalar constant, i.e., uniform coefficient update gain. There is a variable stepsize algorithm to vary μ according to the situation. It is further possible to replace this scaler $\mu(n)$ by a matrix $M(n)$ given by

$$M(n) = \begin{bmatrix} \mu_0(n) & 0 & 0 & \cdots & 0 \\ 0 & \mu_1(n) & 0 & \cdots & 0 \\ 0 & 0 & \mu_2(n) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \mu_{N-1}(n) \end{bmatrix}.$$

   (a) What will be an advantage of this replacement?
   (b) What kind of strategy can you consider for the choice of $\mu_k(n)$ to improve the performance of this adaptive filter?