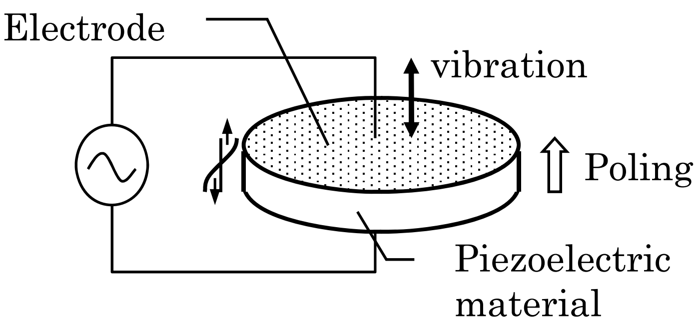
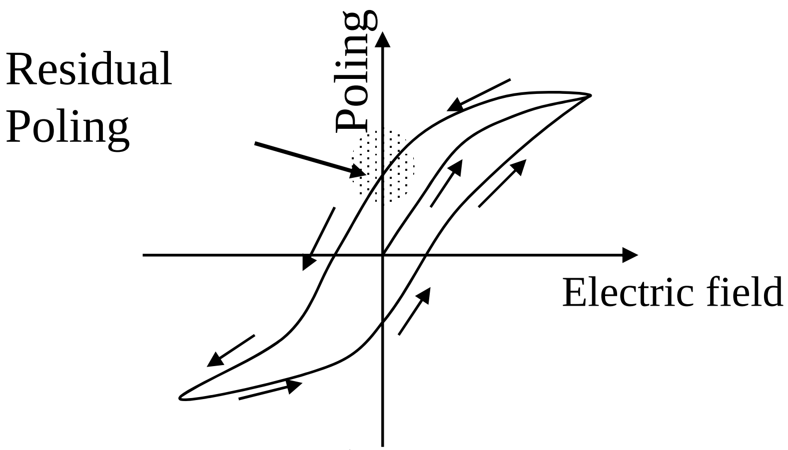
圧電振動子の基礎 Basics for piezoelectricity





•両面固定、電界 E を印加 Fixed and electric field E: 応力stress, $T = e\,E$

•両端短絡、ひずみ S を発生 Short circuited, strain S: 電束密度 Flux density, $D = e\,S$

•両端開放、応力 T を印加 Open circuited, stress T: 電界 electric field, $E = g\,T$

圧電方程式

Piezoelectric equations

$$\begin{cases} [T] = [c^E][S] - [e_t][E] \\ [D] = [e][S] + [\varepsilon^S][E] \end{cases}$$

$$\begin{cases} [S] = [s^E][T] + [d_t][E] \\ [D] = [d][T] + [\varepsilon^T][E] \end{cases}$$

弾性定数 Elastic const. c, s

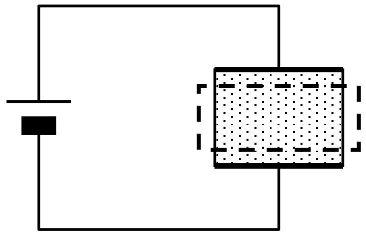
圧電定数 Piezoelectric const. d, e

誘電率 Permittivity ε

電気機械結合係数(静的) k^2
Electro-mechanical coupling factor (static)

$$k^2 = \frac{\text{Mechanically stored energy}}{\text{Electrically stored energy}}$$

Electro-mechanical coupling factor as the material constant.



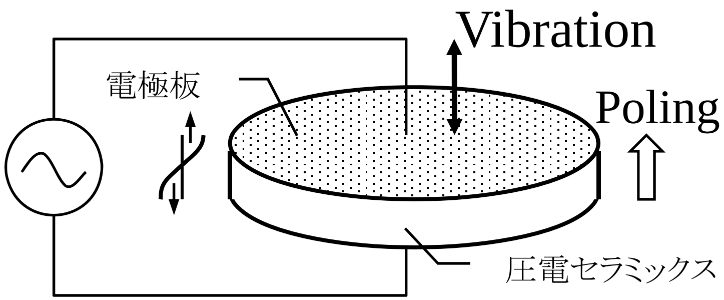
Mechanical $\frac{1}{2} S \cdot T = \frac{1}{2} S \cdot \frac{S}{c} = \frac{1}{2} (d \cdot E)^2 / c$

Electrical $\frac{1}{2} \varepsilon \cdot E^2$

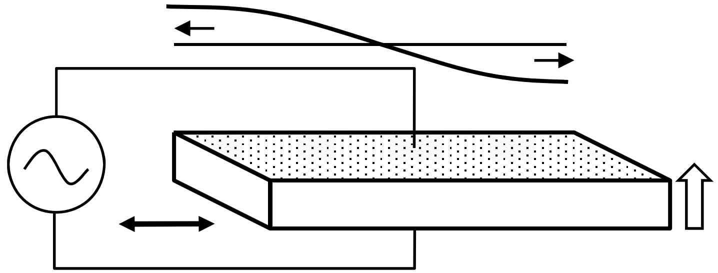
Per unit volume

Coupling factor $k^2 = \frac{d^2}{\varepsilon s}$

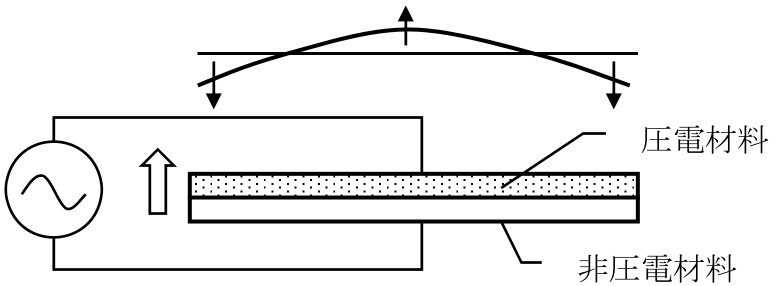
基本的な圧電振動子
Typical piezoelectric transducers



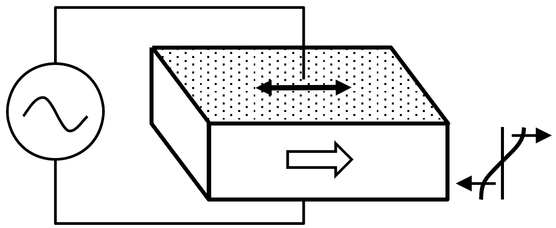
Thickness mode with longitudinal effect



Axial vibration with transverse effect



Bending mode with a mono-morph



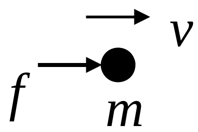
Shear mode

Electrical equivalent circuit (1)

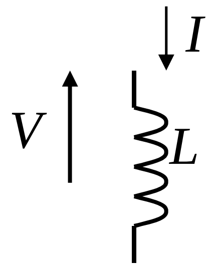
Analogy between the mechanical and electrical systems

Force→Voltage, Velocity→Current

1) Mass=Inductor

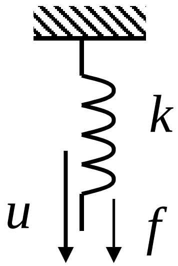


$$f = m\frac{dv}{dt}$$



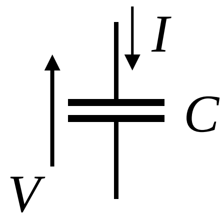
$$V = L\frac{dI}{dt}$$

2)Spring=Capacitor



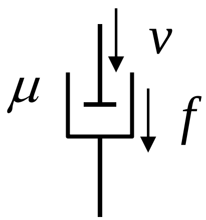
$$f = \frac{1}{c_m}u = \frac{1}{c_m}\int v\,dt$$

$$c_m = 1/k$$

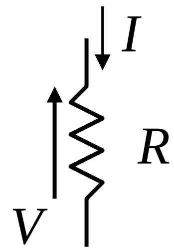


$$V = \frac{1}{c}\int I\,dt$$

3)Damper=Resistor



$$f = \mu v$$

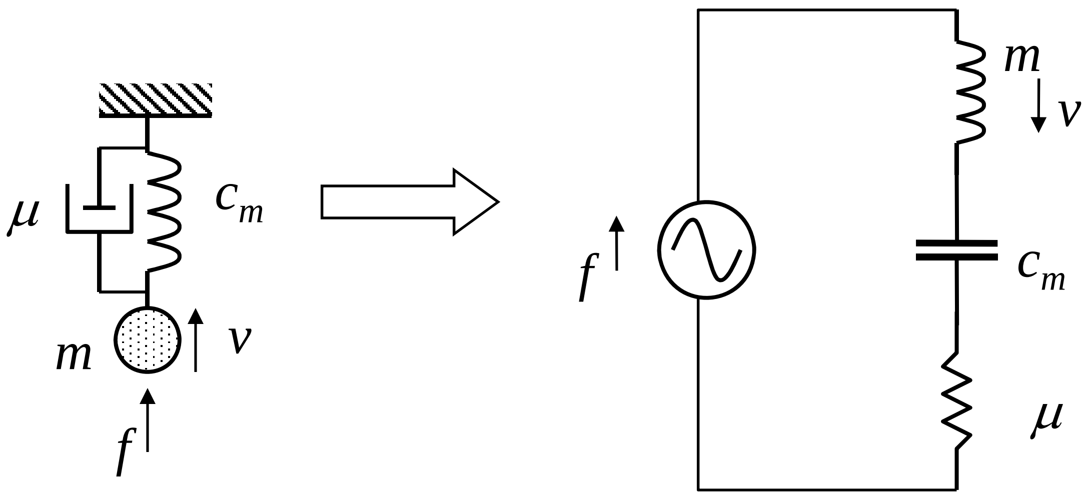


$$V = RI$$

Electrical equivalent circuit (2)

Harmonic oscillator with a mass and a spring

⇒ LCR resonance



Mechanical

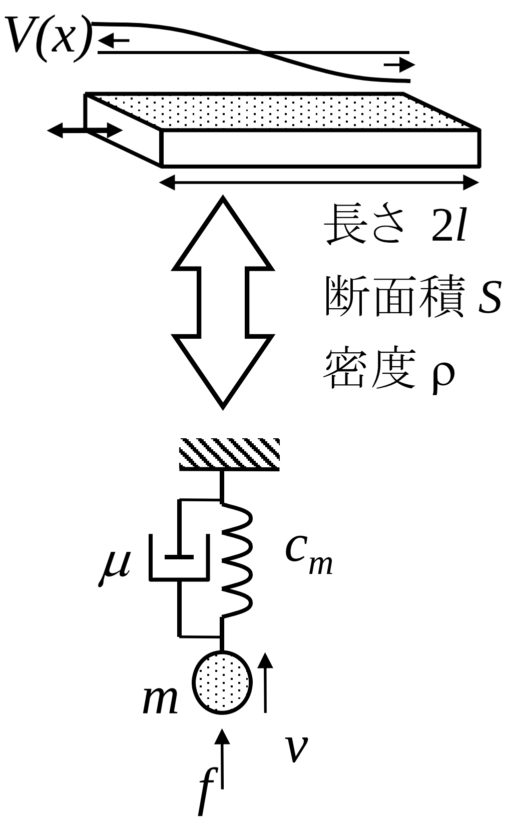
Electrical

Variable common among the circuit components?

→Velocity v

Electrical equivalent circuit (3)

Transducer=distributed circuit ⇨ Lumped circuit



> Determine *m* so that the kinetic energies in the actual transducer and the equivalent circuit be the same.

Total mass $M = 2L\rho S$

Resonance $\omega = \dfrac{\pi}{2L}\sqrt{\dfrac{E}{\rho}}$

Kinetic energy by the longitudinal vibration in the plate

$$U_D = \int_{-L}^{L} \frac{1}{2}\rho S v^2(x)dx$$

Kinetic energy in the lamped model

$$U_L = \frac{1}{2}mv^2(0)$$

> Equivalent mass
>
> $m = M / 2$

Electrical equivalent circuit (4)

弾性エネルギについても同様にして c_m を決定

c_m can be found through the same procedure, considering the elastic energy

$$\frac{1}{c_m} = \frac{\pi^2}{4}\frac{ES}{L}$$

等価スティッフネス

Equivalent stiffness

＊共振周波数から求めても良い。

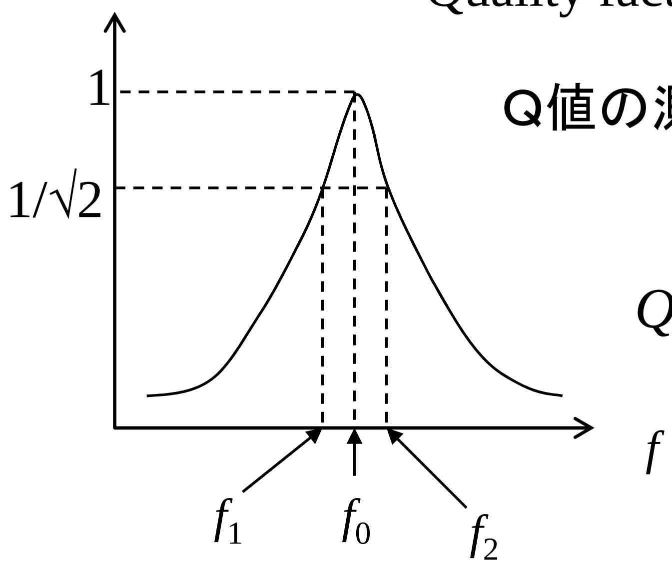
Alternatively, it can be found from the resonance frequency and the mass.

損失項 Loss　μ

機械的Q

Quality factor

$$Q_m = \frac{\omega m}{\mu}$$



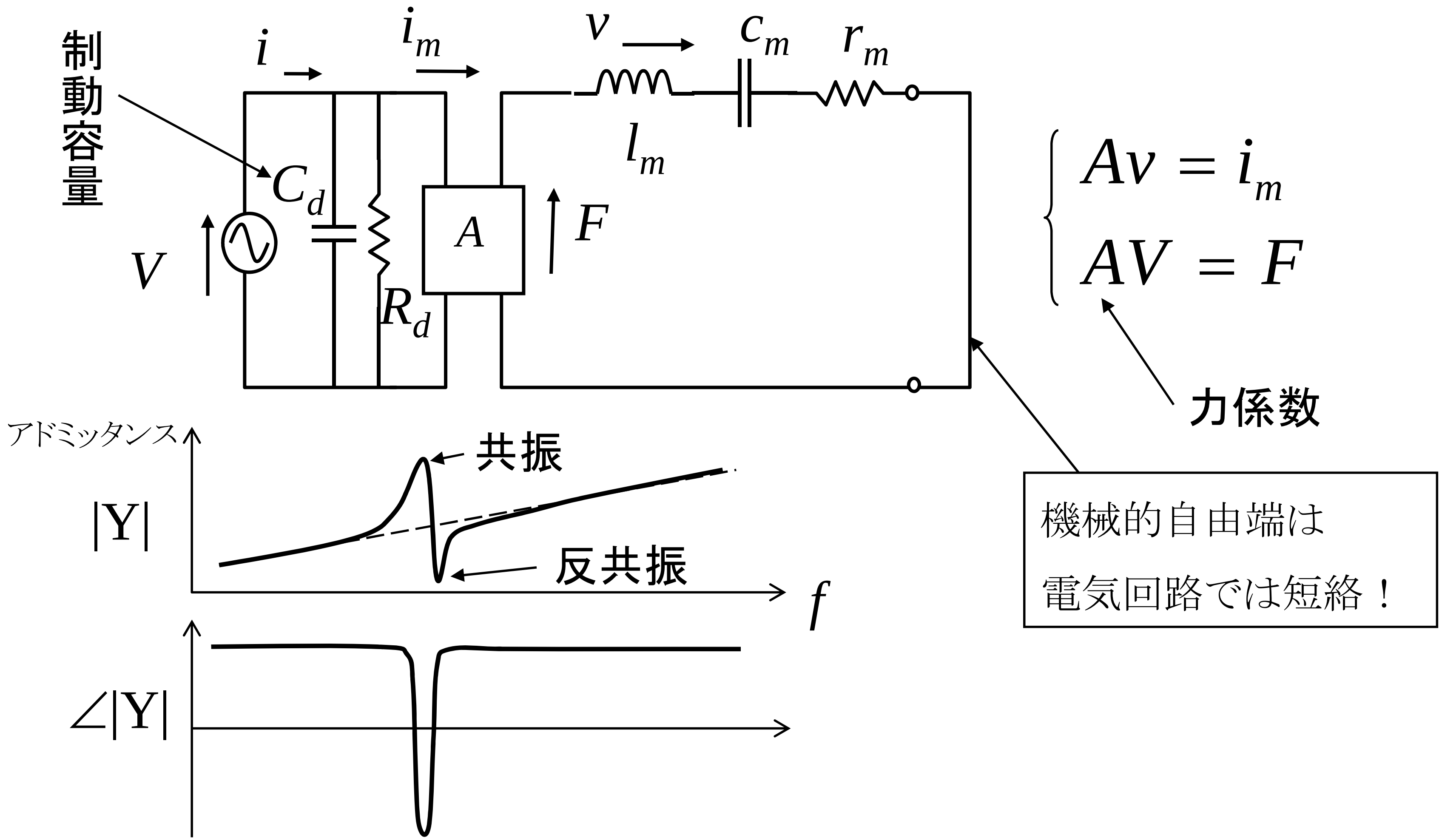
Q値の測定 measurement of Q

$$Q_m = \frac{f_0}{f_2 - f_1} \qquad (Q_m > 10)$$

圧電振動子の等価回路(圧電性の導入)

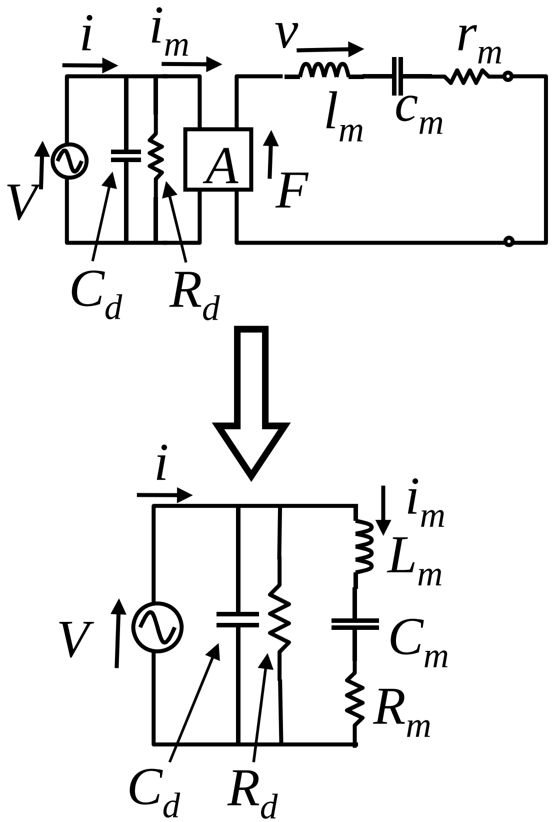
強誘電体＝キャパシタ(静電エネルギ)
機械変形＝ばね(弾性エネルギ)

変形量∝流れ込んだ電荷量　　∴速度∝電流，力∝電圧

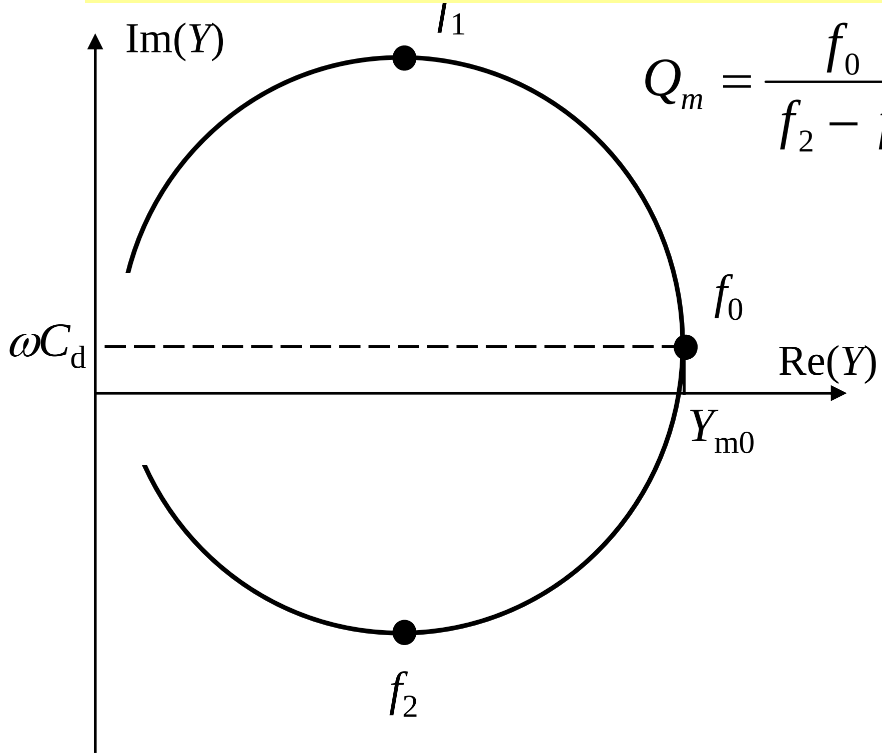


圧電振動子の電気端子から見た特性

Electrical characteristics



アドミッタンスループ Admittance loop



$$Q_m = \frac{f_0}{f_2 - f_1}$$

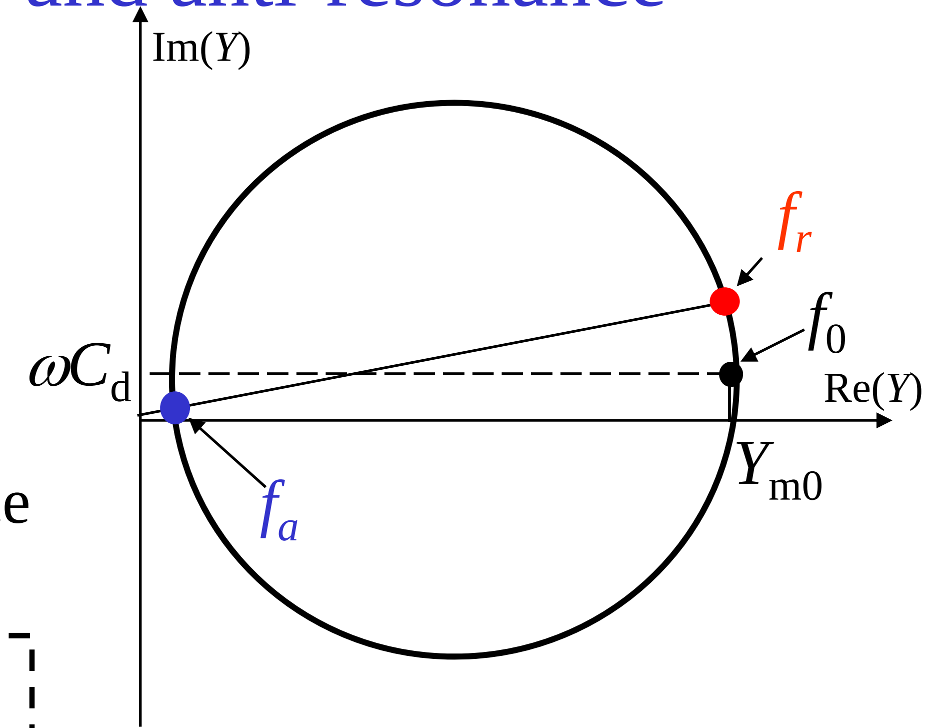
$$\begin{cases} R_m = r_m / A^2 \\ L_m = l_m / A^2 \\ C_m = A^2 c_m \end{cases}$$

$$R_m = 1 / Y_{m0}$$

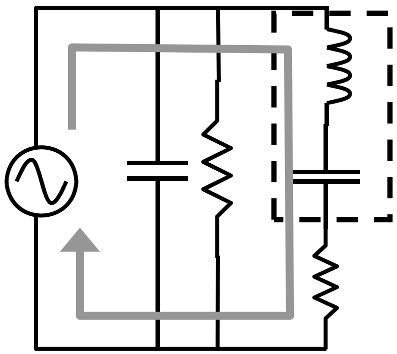
$$L_m = \frac{Q_m R_m}{2\pi f_0}, \quad C_m = \frac{1}{2\pi f_0 Q_m R_m}$$

共振と反共振 Resonance and anti-resonance

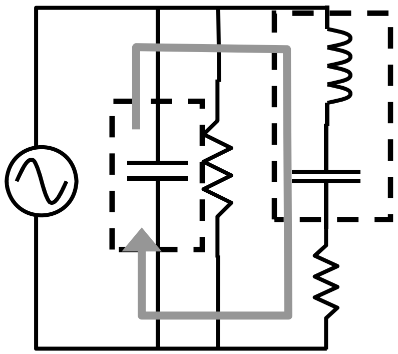
- 共振　＝インピーダンス小
- 反共振＝インピーダンス大



Resonance
共振状態



Anti-resonance
反共振状態

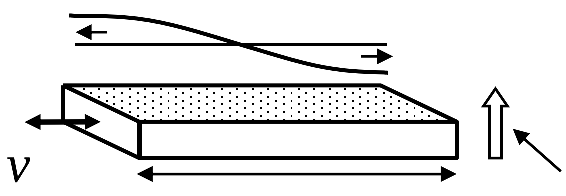


f_r < f_a

- 共振f_r　　Y_mの大きさ最大
- 反共振f_a　　Y_mの大きさ最小
- 機械共振f_0　　Y_mの実部最大

力係数の計算・測定 Calculation of force factor

横効果振動子の場合



力係数force factor $A = i / v$

$$\begin{cases} i = j\omega\int_S D dS, \quad D = d_{31}T \\ T(x) = E\dfrac{\partial u}{\partial x} = \dfrac{\pi EV}{j\omega \cdot 2l}\sin\dfrac{\pi}{2}\dfrac{x}{l} \end{cases}$$

$$A = 2wEd_{31} = \frac{2wd_{31}}{s_{11}}$$

測定 Measurement of force factor

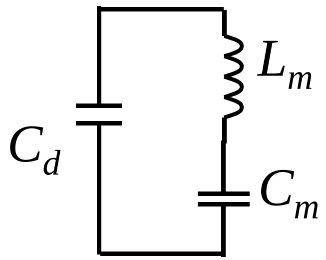
1）電流と振動速度の実測値から（動電流を抽出する必要がある）

From the ratio of the current to the vibration velocity

2）付加質量法（微小質量を付加した場合の共振周波数変化から算出）

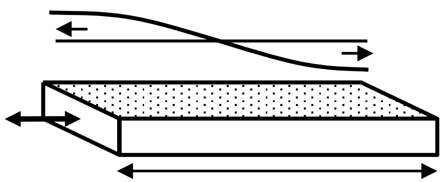
Form the frequency shift when a small additional mass is attached.

振動子としての電気機械結合係数
Electro-mechanical coupling factor as a transducer



電気エネルギElectrical＝C_dにたまる

機械エネルギMechanical＝ C_mにたまる



長さ $2l$

幅 w

厚さ t

$$A = \frac{2wd_{31}}{s_{11}}$$

$$\frac{1}{c_m} = \frac{\pi^2}{4}\frac{wt}{s_{11}l}$$

$$C_d = \frac{\varepsilon_{33} w \cdot 2l}{t}$$

$$k_v^{\ 2} = C_m \big/ C_d = A^2 c_m \big/ C_d = \frac{8}{\pi^2} k_{31}^{\ 2}$$

$$k_v^{\ 2} = \frac{f_a^{\ 2} - f_r^{\ 2}}{f_a^{\ 2}} \approx \frac{2(f_a - f_r)}{f_a}$$

共振・反共振から測定

It can be calculated from the resonance frequencies

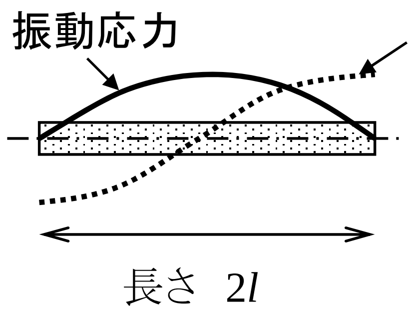
共振・反共振周波数が離れているほど電気機械結合係数が大きい

Larger separation between the resonance frequencies, higher coupling factor

最大振動速度の検討 Maximum vibration velocity

振動応力による破壊 ： 圧電セラミックスの疲労強度 T_{max}=40 MPa程度

Fatigue limit of the piezoelectric ceramics



振動速度 $v(x) = V \sin \frac{\pi}{2}\frac{x}{l}$ （端部の振動速度V）

λ/2共振子

振動応力

Stress

$$T(x) = E\frac{\partial u}{\partial x} = \frac{\pi E V}{j\omega \cdot 2l}\sin\frac{\pi}{2}\frac{x}{l}$$

$$V_{max} = \frac{2\omega l}{\pi E}T_{max} = \frac{T_{max}}{\sqrt{\rho E}}$$

最大応力を特性インピーダンスで除したもの

Question:

Find the maximum velocity of PZT ceramics, where the effect of temperature is ignored

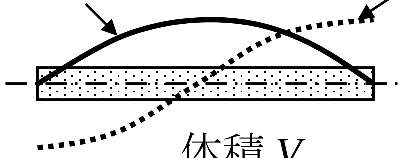
$$\rho = 7900\ \mathrm{kg/m^3}, \quad E = 66\ \mathrm{GPa}$$

$$V_{max} = \square\ \mathrm{m/s}_{(0-p)}$$

圧電振動子のパワー密度の検討

圧電セラミックスの疲労強度 T_{max}=40MPa程度

$$v_{max} = \frac{2\omega l}{\pi E} T_{max} = \frac{T_{max}}{\sqrt{\rho E}}$$



$$\rho = 7900\ \mathrm{kg/m^3}, \quad E = 66\ \mathrm{GPa}$$

$$v_{max} = 1.7\ \ \mathrm{m/s}_{(0-p)}$$

等価質量 $m=\rho V/2$ の振動子で考える

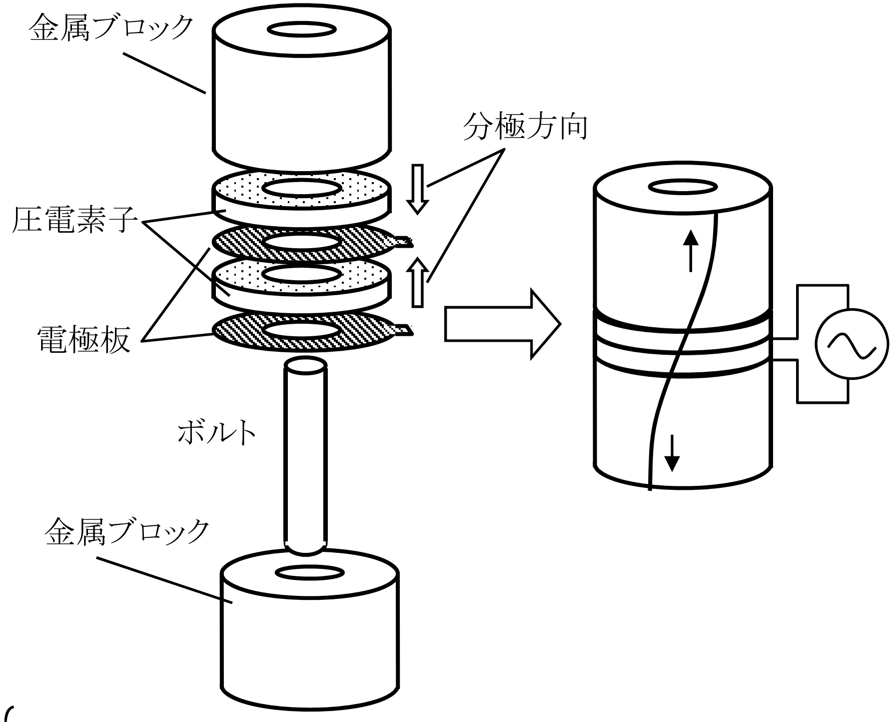
最大運動エネルギ $$U_{K\max} = \frac{1}{2} m v_{max}^2 = \frac{\rho V}{4} \frac{T_{max}^2}{\rho E} = \frac{T_{max}^2}{4E} V$$

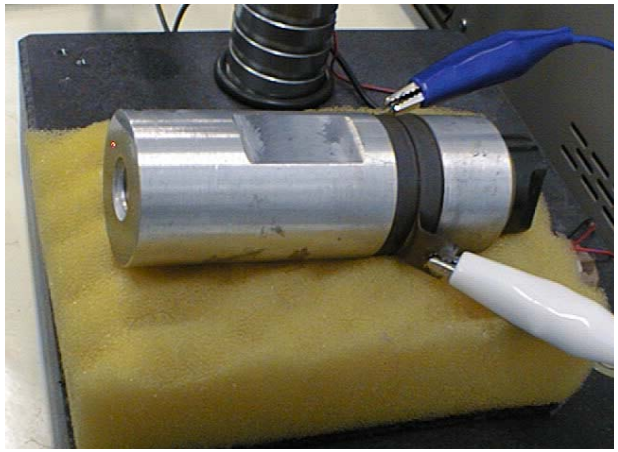
<u>最大パワー密度</u> $$P_{max} = \frac{f \cdot U_{K\max}/2}{V} = \frac{f \cdot T_{max}^2}{8E} \times 10^{-6}\ [\mathrm{W/cc}]$$

20kHz で 約60 W/cc

高周波化すれば高パワー密度化

ボルト締めランジュバン型振動子



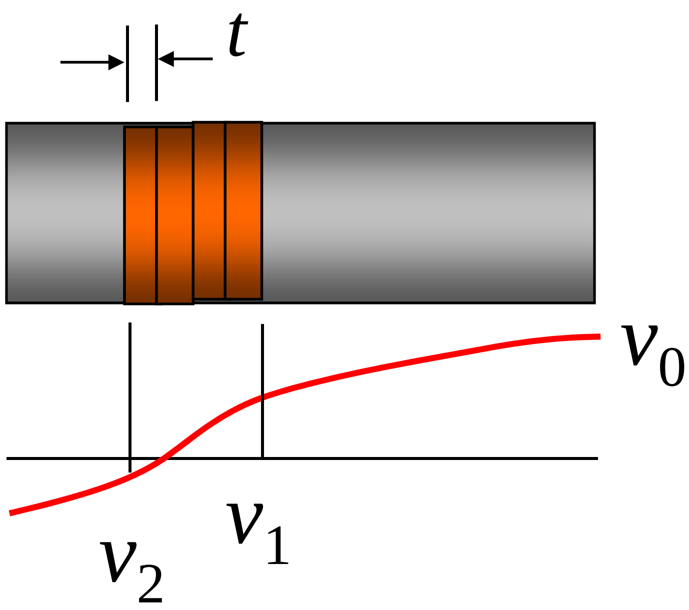


実際のボルト締め振動子

- 金属ブロックにより長さをかせぐ＝低周波化
- ボルトで締める＝予圧により強度増大

セラミックスは引っ張り強度が低い

ランジュバン振動子の力係数　$A = A_{PZT} \times \phi$



変成比　$\phi = (v_1 - v_2) / v_0$

PZT1枚の力係数　A_{PZT}

$$A_{PZT} = eS / t$$

- e,　圧電応力定数
- S,　面積
- t,　PZT1枚の厚さ

PZTは振動の節（応力最大点）に入れる