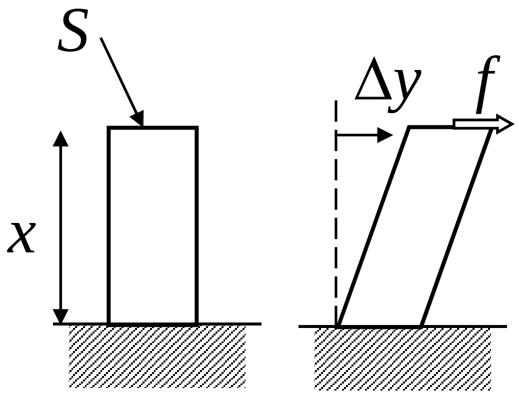
円柱のねじり変形　Torsional deformation in a column

せん断変形
Shear strain



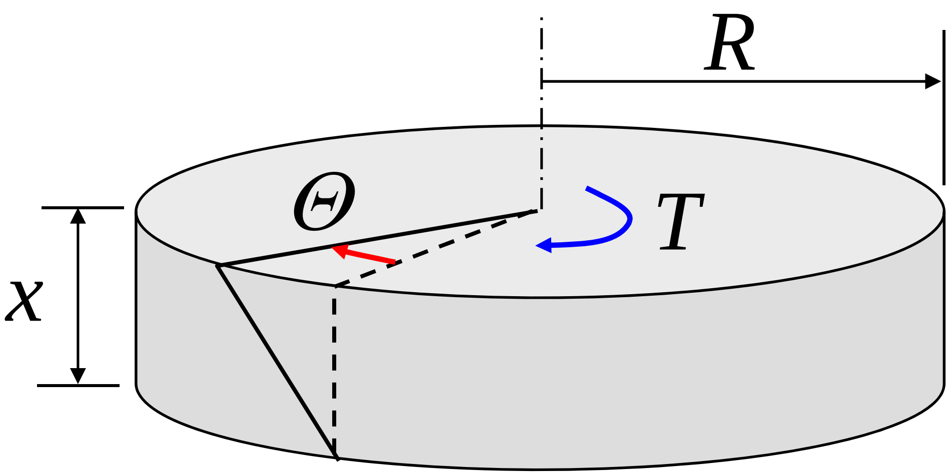
$$\frac{f}{S} = G\frac{\Delta y}{x}$$

剛性率
rigidity

$$T_{xy} = GS_{xy}$$

$$S_{xy} = S_{yx}$$

円筒のねじり変形　Torsional deformation in a column



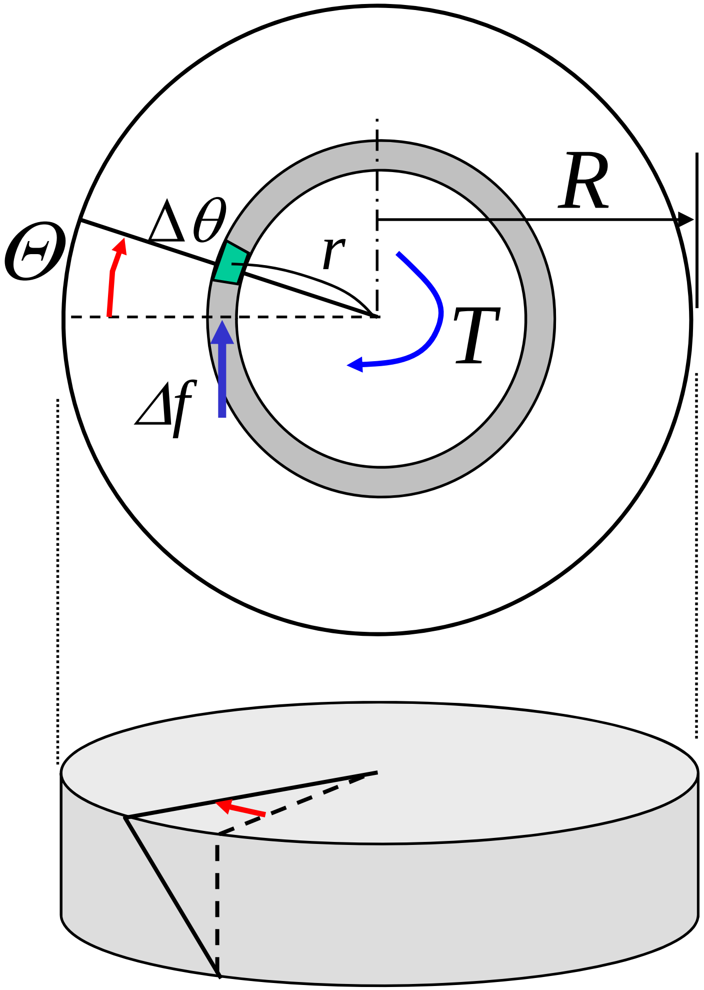
厚さx、半径Rの円板にトルクTを与え、角度Θ変形させる。

Appling torque T to a disk with radius R and thickness x, angular displacement Θ appears.

円柱のねじり変形

トルクTとねじれ角Θの関係を求める

Relationship between the torque T and the angular displacement Θ



$$\frac{\Delta f}{S} = G\frac{\Delta y}{x}$$

$$\begin{cases} S = \Delta r \cdot r\Delta\theta \\ \Delta y = r\Theta \end{cases}$$

$$\Delta f = G\frac{r^2 \Delta r \Delta\theta}{x}\Theta$$

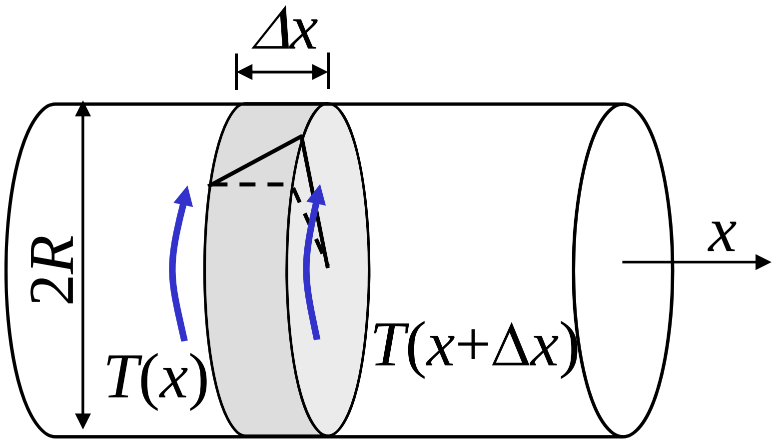
$$T = \int_0^{2\pi}\int_0^R r \cdot df = \frac{2\pi G\Theta}{x}\int_0^{2\pi} r^3 dr = \frac{\pi}{2}R^4 G\frac{\Theta}{x}$$

$$T = IG\frac{\Theta}{x}$$

断面2次モーメント

円柱 $I \equiv \iint_S r^2 dS = \int_0^{2\pi}\int_0^R r^2 \cdot rdrd\theta = \frac{\pi}{2}R^4$

円柱のねじり波　Torsional waves in a rod



弾性の式　Equation of elasticity

$$T = -IG\frac{\partial \Theta}{\partial x}$$

$$J = \frac{1}{2}MR^2 = \frac{1}{2}\rho\Delta x \cdot \pi R^2 \cdot R^2 = \rho I \Delta x$$

運動方程式

Equation of motion

$$J\frac{\partial \Omega}{\partial t} = -\left(T(x+\Delta x) - T(x)\right)$$

$$\rho I\frac{\partial \Omega}{\partial t} = -\frac{\partial T}{\partial x}$$

振動角速度

Angular velocity

$$\Omega = \frac{\partial \Theta}{\partial t}$$

円柱のねじり波 Torsional waves in a rod

<u>波動方程式</u> Wave equations

$$\frac{\partial^2 \Omega(x)}{\partial x^2} = \frac{\rho}{G}\frac{\partial^2 \Omega(x)}{\partial t^2}$$

ねじり波音速

Sound speed for torsional waves

$$c = \sqrt{\frac{G}{\rho}}$$

１次元横波 One-dimensional transverse waves

$$\left.\begin{aligned}
\rho\frac{\partial^2 u}{\partial t^2} &= (\lambda+\mu)\frac{\partial \Delta}{\partial x} + \mu\nabla^2 u\\
\rho\frac{\partial^2 v}{\partial t^2} &= (\lambda+\mu)\frac{\partial \Delta}{\partial y} + \mu\nabla^2 v\\
\rho\frac{\partial^2 w}{\partial t^2} &= (\lambda+\mu)\frac{\partial \Delta}{\partial z} + \mu\nabla^2 w
\end{aligned}\right\} \Longrightarrow \rho\frac{\partial^2 v}{\partial t^2} = \mu\frac{\partial^2 v}{\partial x^2}$$

$$c_t = \sqrt{\frac{\mu}{\rho}}$$

$\mu = G$ SH波

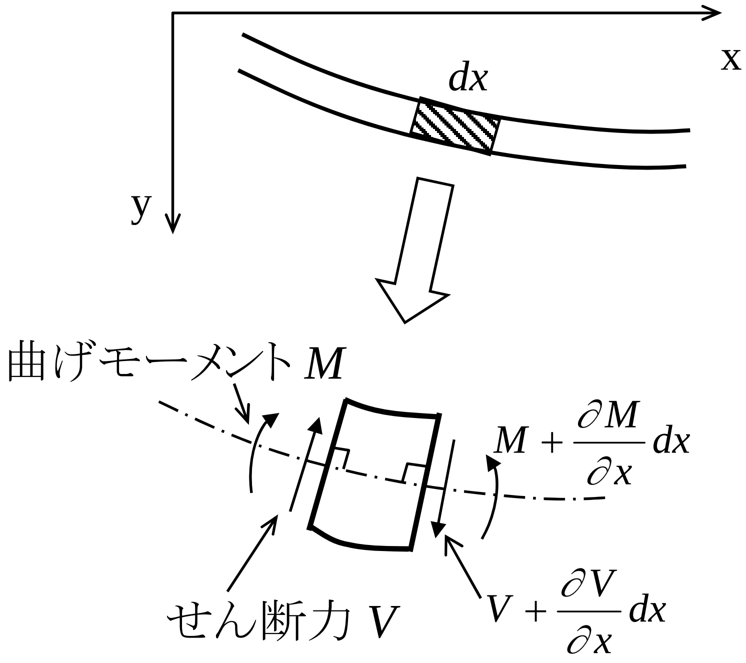
$$G = \frac{E}{2(1+\sigma)}$$

よって、$c_t = \dfrac{c_0}{\sqrt{2(1+\sigma)}}$

ねじり波音速は細棒の縦波音速の0.61倍（アルミニウム）、0.62倍（鋼）

Torsional sound speed is approximately 60% of that of longitudinal waves in aluminum or steel.

細棒のたわみ振動（1） Bending vibration in a beam



Bernoulli-Euler approximation

- 断面は平面のまま

  Cross section remains plane

  - せん断変形無し　No shear strain
  - 軸方向伸びがy方向に直線的に変化

  Linear change of axial extension in thickness

$$\begin{cases} \dfrac{\partial^2 y}{\partial x^2} = -\dfrac{M}{EI} \\[2ex] \dfrac{\partial V}{\partial x} = \rho A \dfrac{\partial^2 y}{\partial t^2} \end{cases}$$

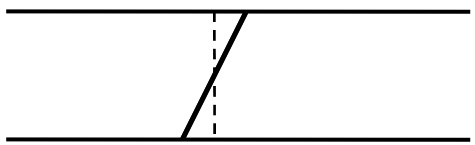
断面2次モーメント: I

断面積: A

- 回転慣性無視

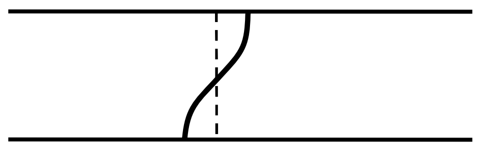
  Rotational inertia is ignored

$$V = \frac{\partial M}{\partial x}$$



Bernoulli-Euler approx.

低周波　Low frequency



高周波

High frequency

細棒のたわみ振動（２） Bending vibration in a beam

$$\frac{\partial^4 y}{\partial x^4} + \frac{\rho A}{EI}\frac{\partial^2 y}{\partial t^2} = 0$$

$y = Ae^{j(kx-\omega t)}$ とすると、

$$k = \pm\sqrt{\frac{\omega}{a}}, \quad \pm j\sqrt{\frac{\omega}{a}}$$

分散性： 波数（伝搬速度）が周波数に依存

Dispersion: wave number (sound speed) depends on frequency

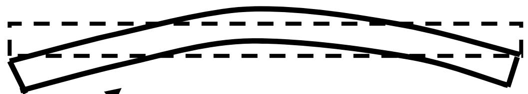
調和振動解

Harmonic solution

$$Y = Ae^{jkx} + Be^{-jkx} + Ce^{kx} + De^{-kx}$$

例：両端自由の細い棒（板）の振動 （長さ l） free-free beam of length l

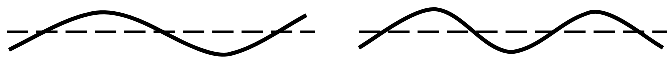
$$\frac{d^2Y(0)}{\partial x^2} = \frac{d^3Y(0)}{\partial x^3} = 0, \quad \frac{d^2Y(l)}{\partial x^2} = \frac{d^3Y(l)}{\partial x^3} = 0$$



⟹ 周波数方程式 $\cos\beta l \cosh\beta l = 1$

Frequency equation

$$\Rightarrow \beta_0 l = 0,\ \beta_1 l = 4.730,\ \beta_2 l = 7.853,\ \beta_3 l = 10.996, \cdots$$



たわみ振動　角棒に伝搬するたわみ波

Bending vibration propagating along a rectangular beam

曲げ剛性　$D = EI = E\dfrac{bh^3}{12}$

$$k = \omega^{\frac{1}{2}}\left(\frac{\rho S}{EI}\right)^{\frac{1}{4}} = 4.67\sqrt{\frac{f}{h}}C_0^{-\frac{1}{2}}$$

$$C_B = 1.35\sqrt{C_0 fh}$$