2012 2nd semester
MIMO Communication Systems

#4: Array Signal Processing

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Agenda

Aim of today

Derive SNR and BER performance of maximal ratio diversity combining

Contents

- **Review of Characteristics of Propagation Channels in Land Mobile Communications**
- **Plane wave signal model**
  - **Beamforming & interference cancellation**
- **Multi-path signal model**
  - **Diversity combining**
  - **Spatial correlation & its effect**

1. Review of Characteristics of Propagation Channels in Land Mobile Communications

Characteristics of Propagation Channel for Land Mobile Communications (1)

- **Variations in propagation channel for land mobile communications are characterized by the following 3 variations assuming frequency spectrum of VHF band (30 MHz – 300 MHz) or UHF band (300 MHz – 3 GHz)**

■ **Distance-dependent path loss (large-scale propagation effects):**

- **Attenuation in the received signal level at a given distance between a transmitter and a receiver.**
- **Received signal variation due to distance-dependent path loss occurs over long distance (100 – 1000 m).**

■ **Shadowing (short-scaled propagation effects):**

- **Variation caused by obstacles between a transmitter and receiver that attenuate received signal power through absorption, reflection, scattering, and diffraction.**
- **When attenuation due to obstacle is strong, the signal is blocked.**
- **Received signal variation due to shadowing occurs over distances that are proportional to the length of obstructing object (10 – 100 m in outdoor environments and less in indoor environments)**

■ **Instantaneous fading variation:**

- **Variation caused by obstacles surrounding a user equipment (UE).**

Characteristics of Propagation Channel for Land Mobile Communications (2)

- **Distance-dependent path loss**
  Variation in received signal power according to increasing distance from a BS

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- **Shadowing**
  Variation caused by obstacles such as large buildings between a transmitter and receiver

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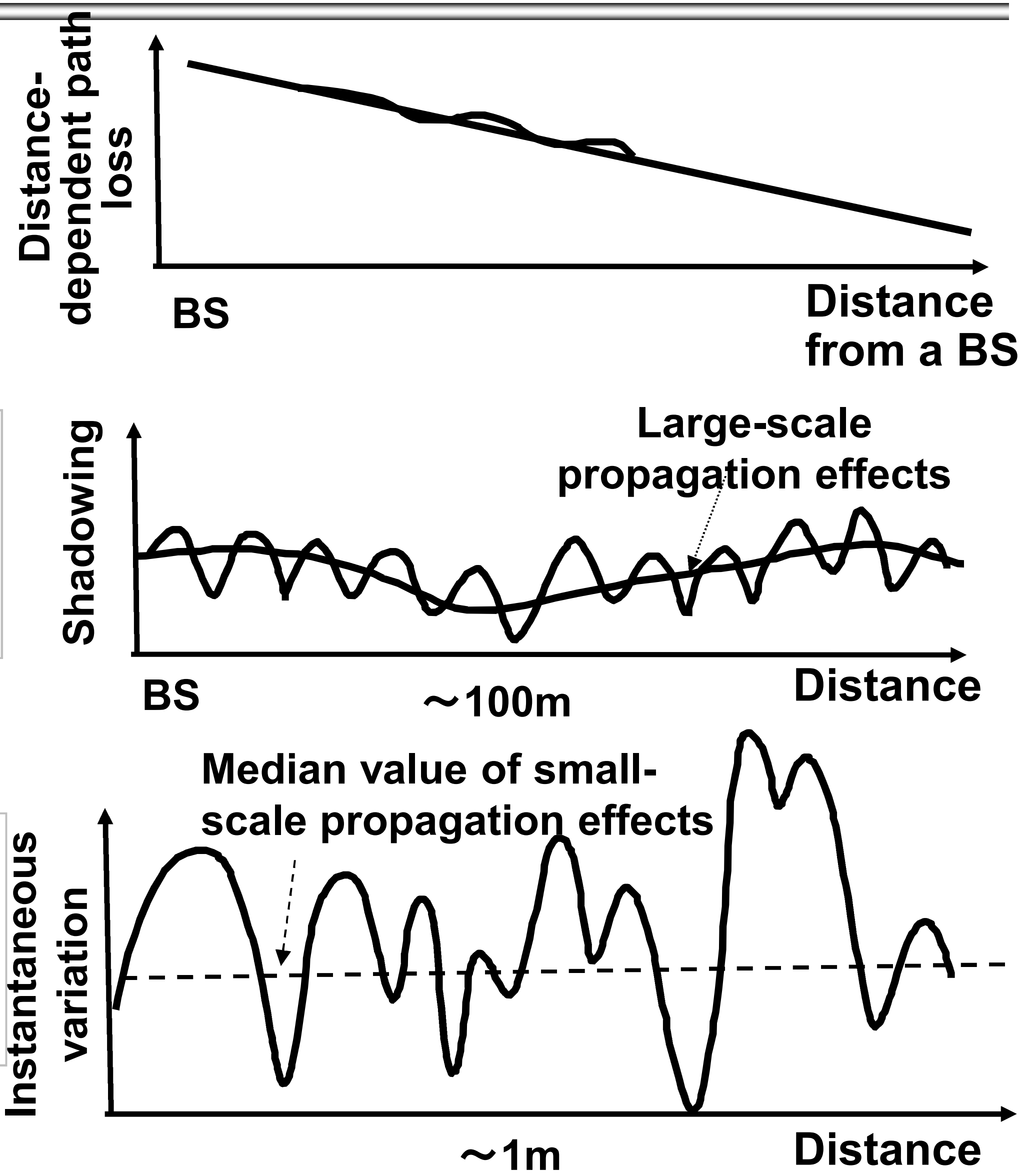
- **Fading (multipath fading for wideband transmission)**
  Variation caused by obstacles surrounding a user equipment.



Distance-Dependent Path Loss

- **A number of distance-dependent path loss models have been development to predict path loss in typical wireless environments such as large urban macro cells, urban micro cells, and inside buildings.**
- **Empirical path loss model in urban areas ➔ Okumura-Hata model**

$$L_p = 65.25 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$+ (44.9 - 6.55\log(h_b))\log_{10}(r)$$

where f_c is carrier frequency, h_b is height of BS antenna, h_m is height of a antenna of a UE, and "r" is distance between a MS and a UE

- **Empirical path loss formula for 2GHz-freqyency band**

$$\boxed{L_p = 128.1 + 37.6\log_{10}(r)}$$

Log-Normal Shadowing

- **Signal transmitted through a wireless channel typically experiences random variation due to blockage from objects in signal path ➔ random variation of the received signal power**
- **But, in general, the location size, and dielectric properties, changes in reflecting surfaces etc. of the blocking objects are unknown ➔ statistical models are used to characterize the attenuation due to shadowing**
- **The most common model is for the additional attenuation is "log-normal shadowing" ➔ accuracy of the model has been empirically confirmed in outdoor and indoor radio propagation environments.**

Narrowband Rayleigh Fading (1)

- UE moves in the direction of 2 degree at a speed of ν (m/s)
- Path *i* of plane wave is received from the direction of ξ_i degree with amplitude of a_i, and phase of ϕ_i ($i = 1, \ldots, N$).
- Received composite signal $a_r(t)$ is represented as next equation assuming carrier angular frequency of ω_c.

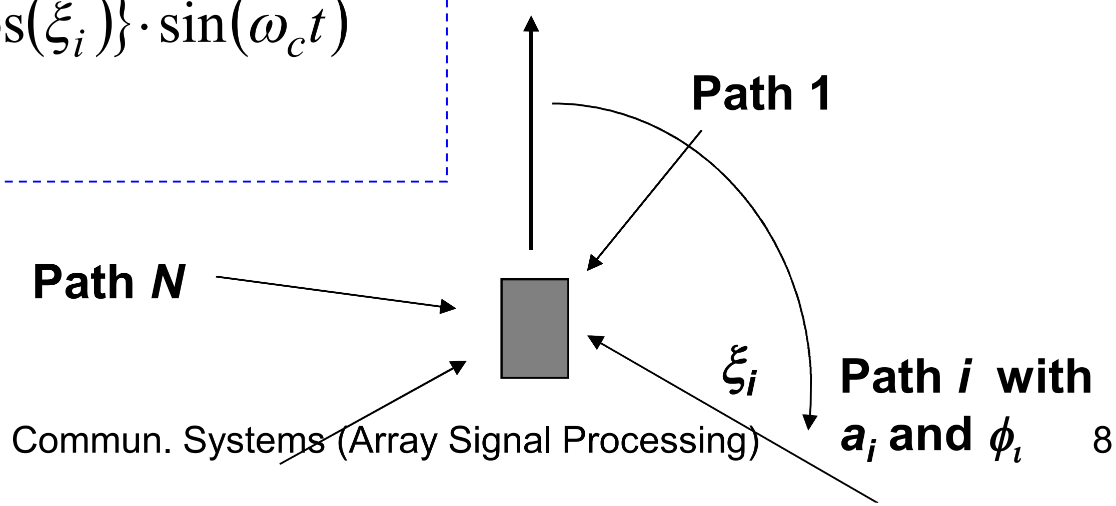
$$a_r(t) = \sum_{i=1}^{N} a_i \cdot \cos\{\omega_c t + \phi_i + 2\pi v t/\lambda \cdot \cos(\xi_i)\}$$

$$= \sum_{i=1}^{N} a_i \cdot \cos\{\phi_i + 2\pi v t/\lambda \cdot \cos(\xi_i)\} \cdot \cos(\omega_c t)$$

$$- \sum_{i=1}^{N} a_i \cdot \sin\{\phi_i + 2\pi v t/\lambda \cdot \cos(\xi_i)\} \cdot \sin(\omega_c t)$$

Phase shift (frequency shift) caused by the relative movement to the direction of a transmitter ➔ Doppler shift

Direction of movement (speed of v (m/s))



Narrowband Rayleigh Fading (2)

- **Assuming that *N* is large and amplitude a_i and phase ϕ_i are independently different, the composite signal is approximated as Gaussian random process with zero mean and equal variance based on the central limit theorem**

$$\sum_{i=1}^{N} a_i \cdot \cos\{\phi_i + 2\pi v t/\lambda \cdot \cos(\xi_i)\}$$

$$\sum_{i=1}^{N} a_i \cdot \sin\{\phi_i + 2\pi v t/\lambda \cdot \cos(\xi_i)\}$$

- **Composite signal is represented as**

$$a_r(t) = R(t) \cdot \cos\{\omega_c t + \theta(t)\}$$

- **Amplitude *R(t)* is Rayleigh distributed that follows probability density function (PDF) of Rayleigh distribution based on the next equation**

$$p(R) = 2R/\sigma \cdot \exp\left(-R^2/\sigma\right)$$

where $\sigma\left(=\sum_{i=1}^{N} a_i^2\right)$ **is mean squared value of Rayleigh variable**

- **Phase $\phi_i(t)$ is uniformly distributed on [0, 2π]**

Fading Maximum Doppler Frequency

$v/\lambda \cdot \cos(\xi_i)$	Doppler frequency shift due to the relative movement of a UE
v/λ	Maximum Doppler frequency that indicates variation in amplitude or power of a received signal suffering Rayleigh fading

■ **Maximum Doppler frequency**

$$f_D = v/\lambda = {v \cdot f_c}/{c}$$

- *c*: light velocity (= 3×10^8 m/second)
- Assuming carrier frequency of 2 GHz and speed f 3 km/h, f_D becomes 5.55 Hz
- Maximum Doppler frequency is proportional to carrier frequency and to a speed of a UE (MS)

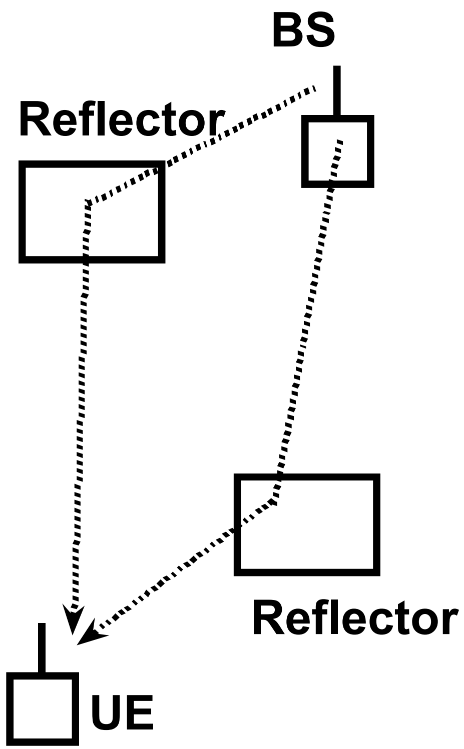
Frequency-Selective Fading (1)

■ **Frequency-selective fading (multipath fading)**

- **In wideband transmission with short symbol time, delayed paths due to obstacles between a BS and a UE influence on the performance.**
- **Delayed paths interfere mutually when the delay time of the paths is distinct compared to the symbol time (e.g., the delay time is longer than approximately 10 % of symbol time)**

- **Different amplitude and phase variations at respective frequency components**

➔ **Frequency-selective fading (multipath fading)**



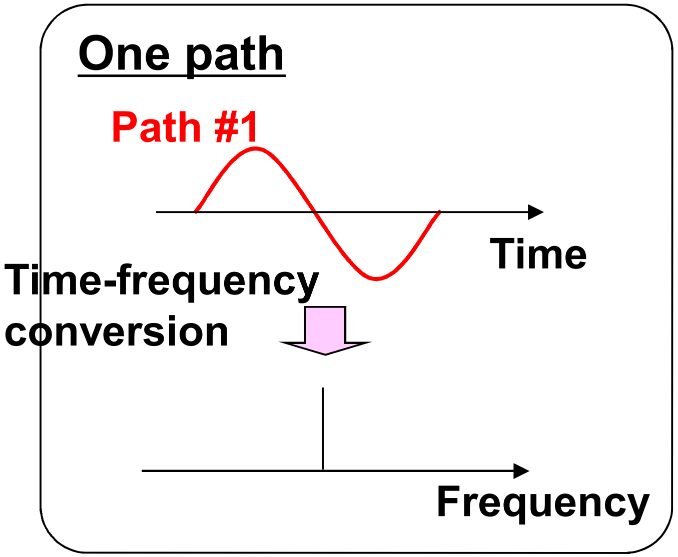
Frequency-Selective Fading (2)

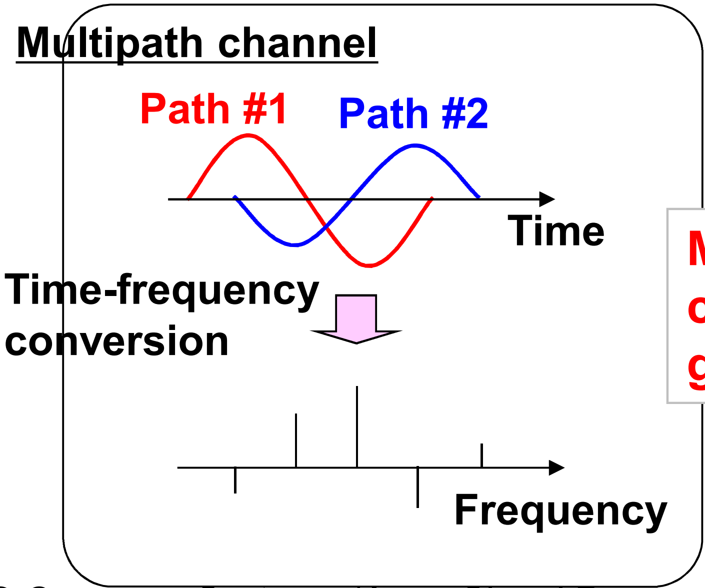
■ One-path channel

- Single frequency component, i.e., delay time of paths is negligible compare to symbol time (narrowband signal)
- Frequency-flat fading and varies in time domain.

■ Multipath fading ➔

- Delay times of paths are distinct compared to symbol time
- Frequency-selectivity appears from the composite signal of multipath signals





Multiple frequency components are generated

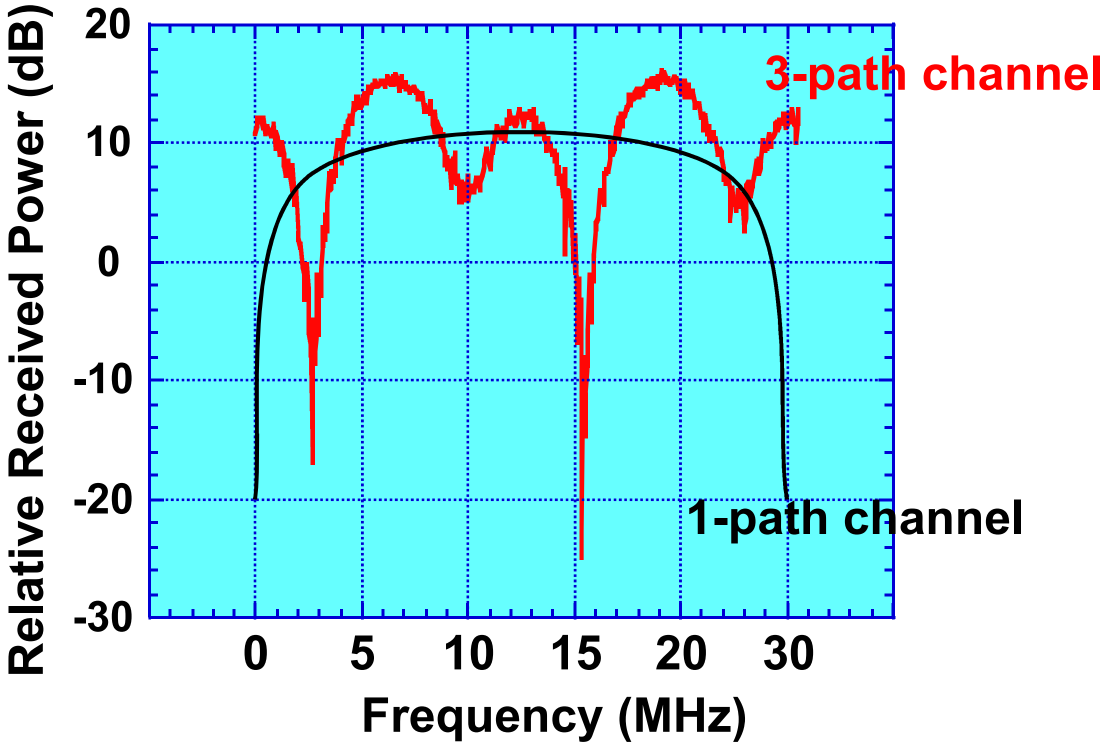
Frequency-Selective Fading (3)

■ Narrowband signal

- Frequency-flat fading
- PDC (symbol rate of 21 kbps) ➔ almost flay fading

■ Wideband signal (W-CDMA, HSPA, LTE)

- Delay time of paths is much longer compared to chip or symbol time ➔ frequency-selectivity appears



Measures of Frequency Selectivity

- Assume complex Gaussian wide-sense stationary uncorrelated scattering (WSSUS) channels that arre specified by their scattering functions.

- Measures of frequency selectivity
- Root mean square (r.m.s.) delay spread : r.m.s. delay spread of a channel is defined as the variance in the time delay with the normalized delay power profile of the channel as the probability density function.
- Coherence bandwidth:
  - Coherence bandwidth with level *k* is defined as the widest bandwidth in which the correlation of the channel frequency response is greater than *k*, where *k* is selected to be close to one between zero and one
  - Coherence bandwidth means the bandwidth over which the channel frequency response is highly correlated
  - Coherence bandwidth is represented using the r.m.s. delay spread as , $B_c = \frac{1}{h \cdot \tau_{rms}}$ where *h* is a constant value

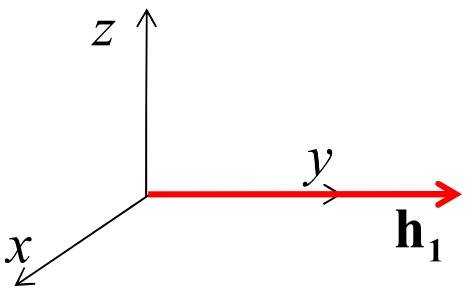
2. Beamforming & Interference Cancellation Using Array Antennas

Review of Eigen-Value Decomposition

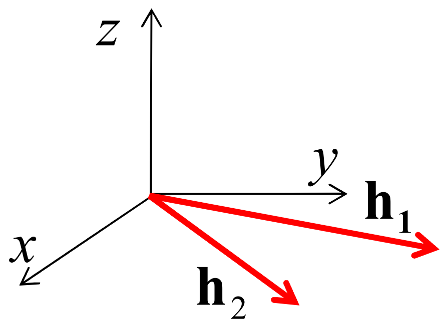
Question

Illustrate eigen-vectors of correlation matrix $\mathbf{R} = \mathbf{H}\mathbf{H}^H$

(A) $\mathbf{H} = [\mathbf{h}_1] \in C^{3\times 1}$



(B) $\mathbf{H} = [\mathbf{h}_1 \quad \mathbf{h}_2] \in C^{3\times 2}$



Eigen-value decomposition

Channel matrix: $\mathbf{H}$

Correlation matrix: $\mathbf{R} = \mathbf{H}\mathbf{H}^H$

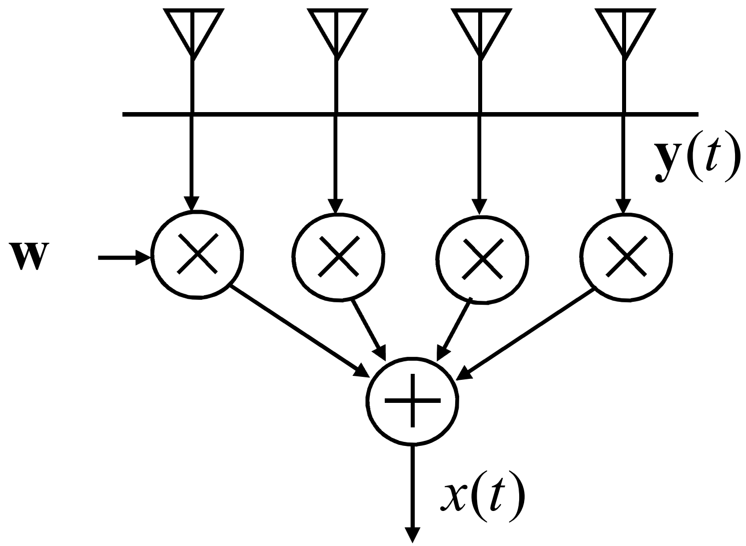
Eigen-decomposition: $\mathbf{R} = \mathbf{E}\,\mathbf{\Lambda}\,\mathbf{E}^H$

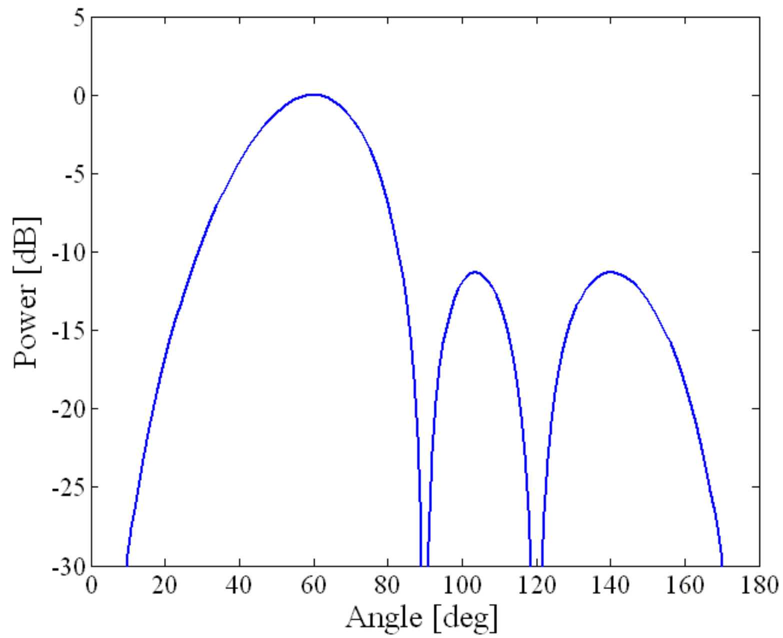
Eigen vectors: $\mathbf{E} = [\mathbf{e}_1 \quad \mathbf{e}_2 \quad \mathbf{e}_3]$

Eigen values: $\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

Classification of Array Processing

- Static or Mobile corresponds to Fixed or Adaptive
- RF control or BB control
- Beamforming, diversity, or Interference cancellation
- Average SNR, outage SNR, or SIR

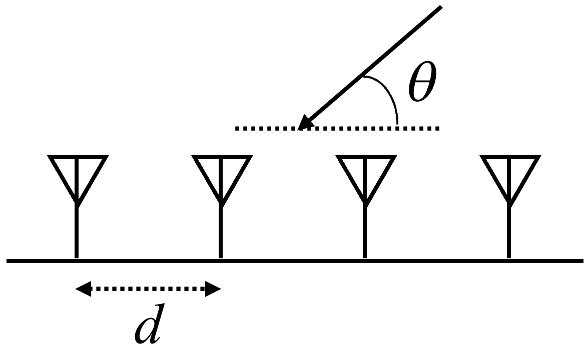




Plane Wave Signal Model

Plane wave signal model

$$\mathbf{y}(t) = \mathbf{h}(\theta)s(t) + \mathbf{n}(t)$$



Channel response

$$\mathbf{h}(\theta) = \beta d(\theta)\mathbf{a}(\theta)$$

Array manifold

$$\mathbf{a}(\theta) = \left[1, e^{jkd\cos\theta}, e^{jk2d\cos\theta}, \cdots, e^{jk(M-1)d\cos\theta}\right] \qquad k = \frac{2\pi}{\lambda}$$

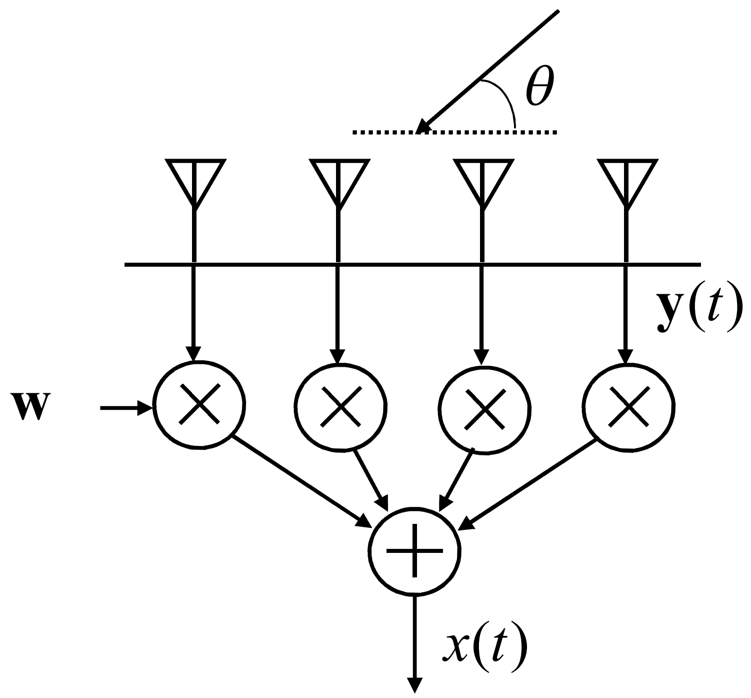
Antenna element directivity

$$d(\theta) \cong 1$$ Omni-directional pattern

Beamforming

Received signal

$$\mathbf{y}(t) = \beta \mathbf{a}(\theta) s(t) + \mathbf{n}(t)$$



Array combining

$$x(t) = \mathbf{w}^H \mathbf{y}(t)$$

Retro directive beamforming

$$\mathbf{w} = \beta \mathbf{a}(\theta)$$

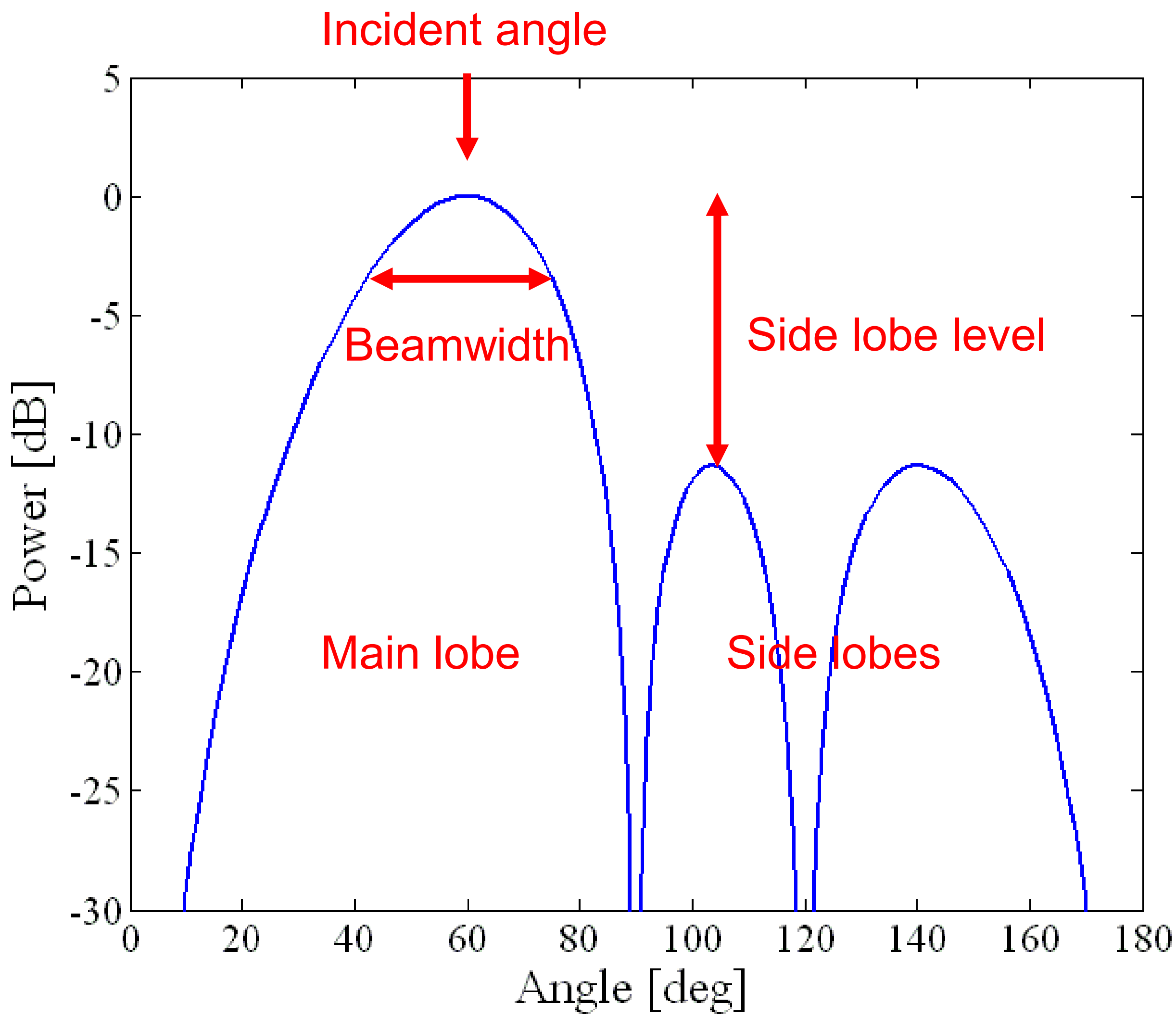
$$x(t) = M\beta^2 s(t) + \sum_{i=1}^{M} \beta n_i(t)$$

Output SNR

Array gain

$$\gamma_o = \frac{\mathrm{E}[|s_o(t)|^2]}{\mathrm{E}[|n_o(t)|^2]} = \frac{M^2 \beta^4 \mathrm{E}[|s(t)|^2]}{\beta^2 \sum_{i=1}^{M} \mathrm{E}[|n_i(t)|^2]} = \frac{M^2 \beta^2 P}{M\sigma^2} = \frac{M\beta^2 P}{\sigma^2}$$

Beamforming



Interference Cancellation

Subspace decomposition

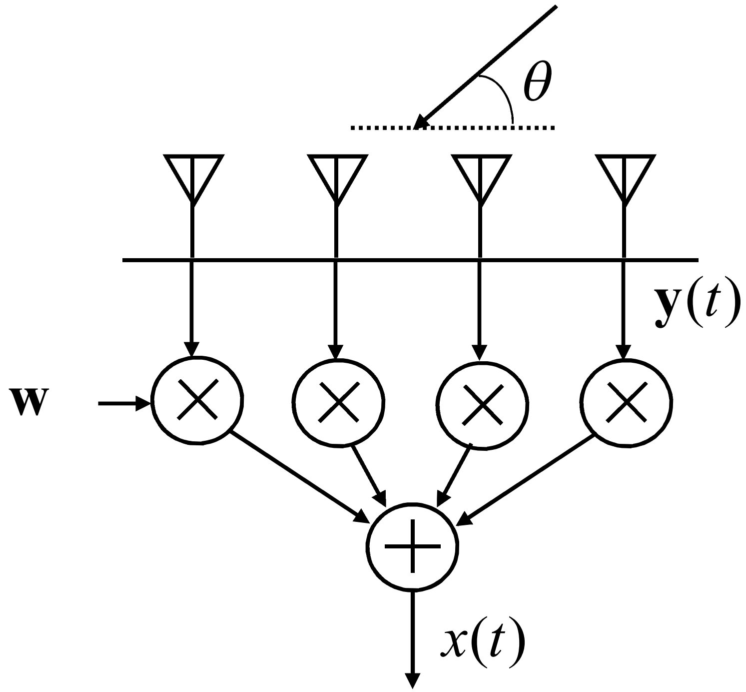
$$\mathbf{R} = \mathbf{h}\mathbf{h}^H = \mathbf{E}\Lambda\mathbf{E}^H$$

$$\Lambda = \mathrm{diag}[\lambda_1, \lambda_2, \lambda_3, \lambda_4]$$

$$\lambda_1 > \lambda_2 = \lambda_3 = \lambda_4 = 0$$

$$\mathbf{E} = [\underbrace{\mathbf{e}_1}_{\text{Signal space}}, \underbrace{\mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4}_{\text{Null space}}]$$

$$\mathbf{e}_i^H \mathbf{R} \mathbf{e}_i = 0 \quad (i = 2,3,4)$$



Interference cancellation

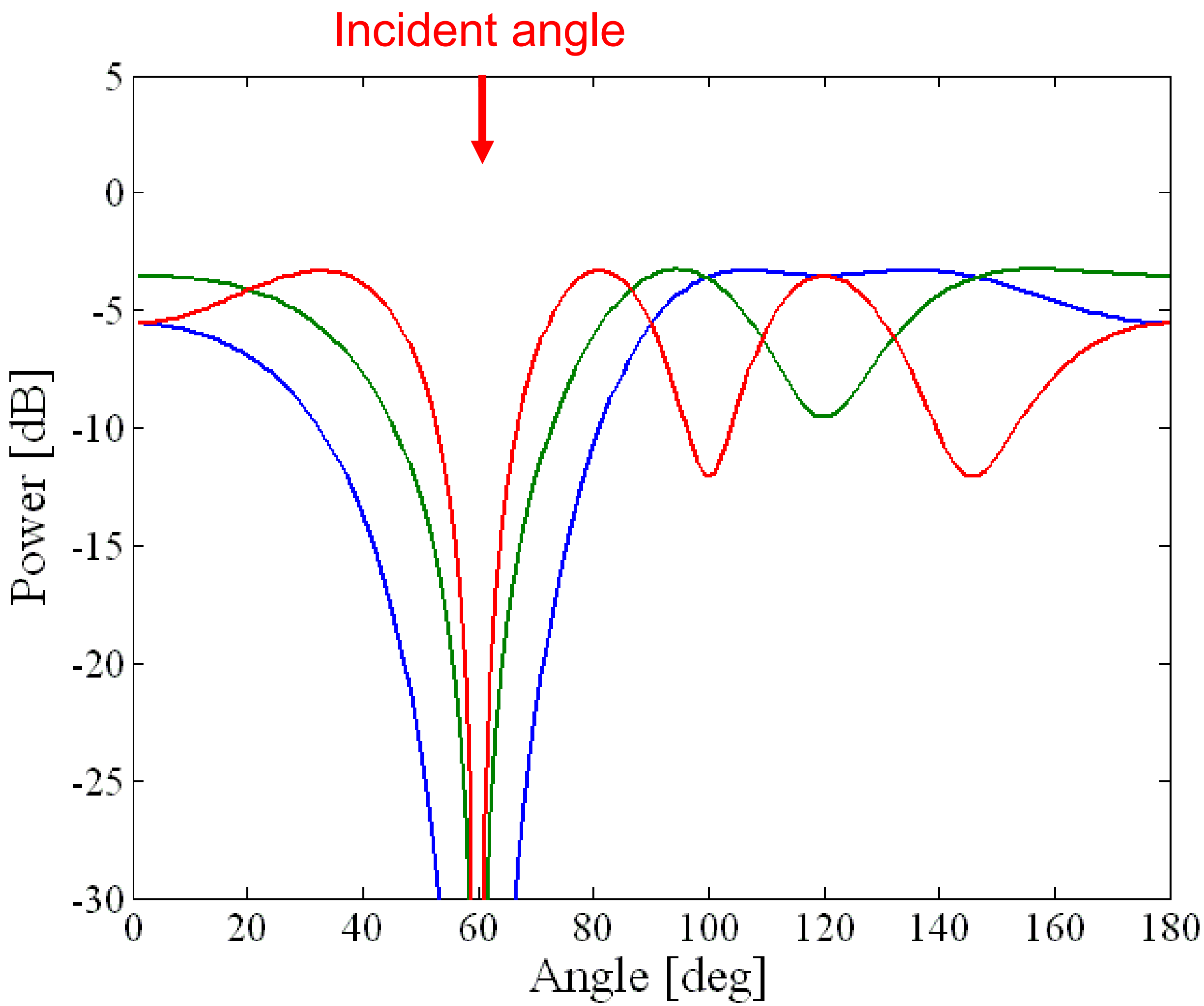
$$x(t) = \mathbf{w}^H \mathbf{y}(t)$$

$$\mathbf{w} = \mathbf{e}_i \quad (i = 2,3,4)$$

Output SNR

$$\gamma_o = \frac{\mathbf{w}^H \mathbf{h}\mathbf{h}^H \mathbf{w} P}{\mathbf{w}^H \mathbf{w} \sigma^2} = \frac{0}{\sigma^2}$$

Interference Cancellation



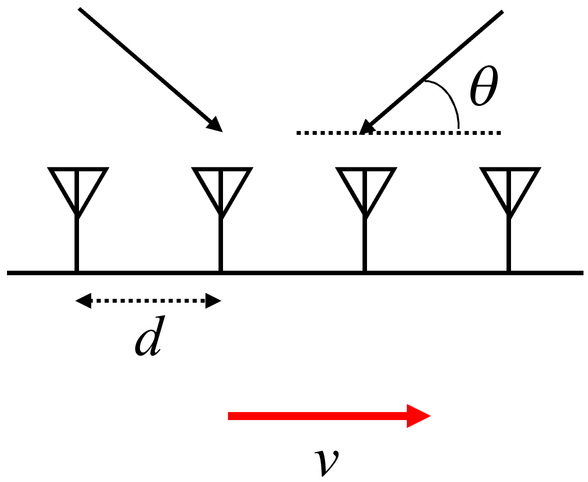
3. Diversity Combining

Multipath Signal Model

Multi-path signal model

$$\mathbf{y}(t) = \mathbf{h}s(t) + \mathbf{n}(t)$$

$$\mathbf{h}(\theta) = \sum_{l=1}^{L} \mathbf{h}(\theta_l)$$



Time variant channel response

$$\mathbf{h}(\theta_l) = \beta_l(t) d(\theta_l) \mathbf{a}(\theta_l)$$

$$\beta_l(t) = \beta_l e^{jkvt\cos\theta_l}$$

Diversity Combining

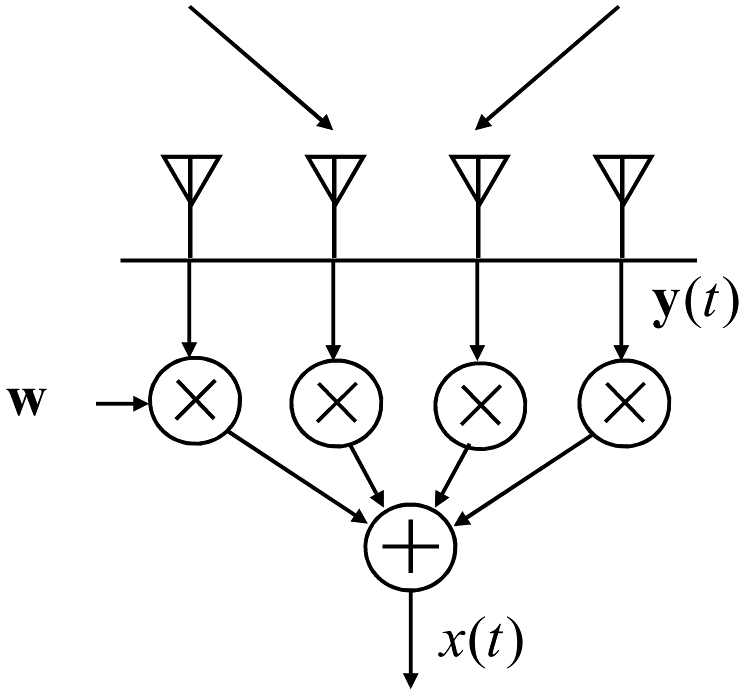
Multi-path signal model

$$\mathbf{y}(t) = \mathbf{h}s(t) + \mathbf{n}(t)$$

Maximum ratio diversity combining

$$x(t) = \mathbf{w}^H \mathbf{y}(t)$$

$$\mathbf{w} = \mathbf{h}$$



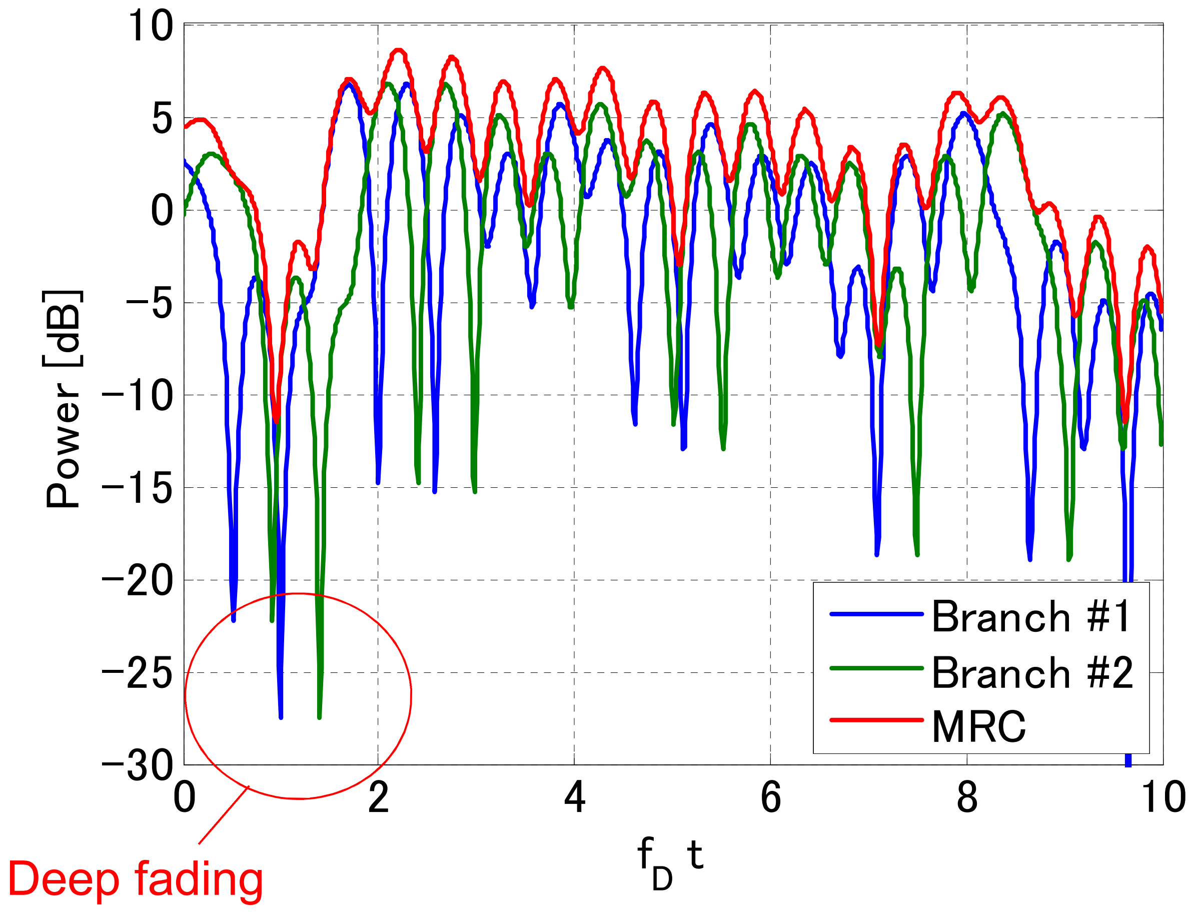
Output SNR

$$\gamma_o = \frac{\mathrm{E}[\left|\mathbf{h}^H \mathbf{h} s(t)\right|^2]}{\mathrm{E}[\left|\mathbf{h}^H \mathbf{n}(t)\right|^2]}$$

$$= \frac{\left|\mathbf{h}^H \mathbf{h}\right|^2 P}{\mathbf{h}^H \mathbf{h} \sigma^2} = \frac{\mathbf{h}^H \mathbf{h} P}{\sigma^2} = \sum_{i=1}^{M} \left|h_i\right|^2 \frac{P}{\sigma^2} = \sum_{i=1}^{M} \gamma_i$$

Sum of each SNR

Diversity Combining



Characteristic Function

PDF of sum of independent random variables

$$f(x) \qquad f(y) \qquad f(x,y) = f(x)f(y)$$

$$z = x + y$$

$$f(z) = \int f(x)f(z-x)\mathrm{d}x$$

Convolution

Characteristic function

$$f(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\varphi(t)\exp(-j\gamma t)\mathrm{d}t \longleftrightarrow \varphi(t) = \int_{0}^{\infty} f(\gamma)\exp(j\gamma t)\mathrm{d}\gamma$$

Characteristic function on convolution

$$\gamma = \sum_i \gamma_i \longleftrightarrow \varphi(t) = \prod_i \varphi_i(t)$$

PDF of Diversity Combining

Output SNR of MRC

$$\gamma = \sum_{i=1}^{M} \gamma_i$$

Characteristic function of each branch

$$f(\gamma_i) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_i}{\bar{\gamma}} \right) \qquad \varphi_i(t) = \frac{1}{1 - j\bar{\gamma}t}$$

PDF of output SNR in MRC

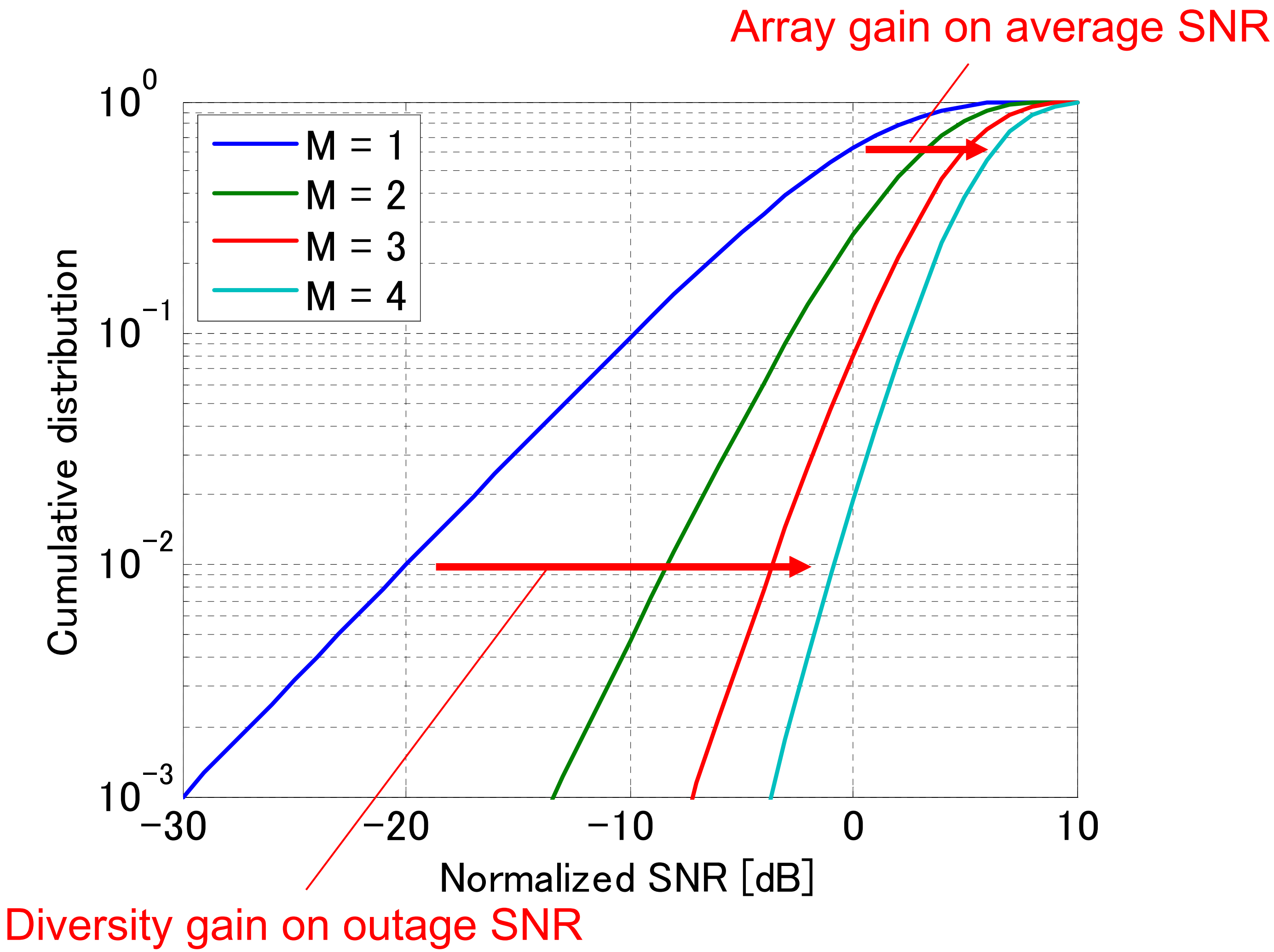
$$\varphi(t) = \prod_{i=1}^{M} \varphi_i(t) = \left(\frac{1}{1 - j\bar{\gamma}t} \right)^M$$

$$f(\gamma) = \frac{1}{(M-1)!\,\bar{\gamma}^M} \gamma^{M-1} \exp\left(-\frac{\gamma}{\bar{\gamma}} \right)$$

Diversity gain

(Xi square distribution)

CDF of Diversity Combining



BER of Diversity Combining

Average BER

$$\overline{P}_e(\overline{\gamma}) = \int P_e(\gamma) f(\gamma) \mathrm{d}\gamma$$

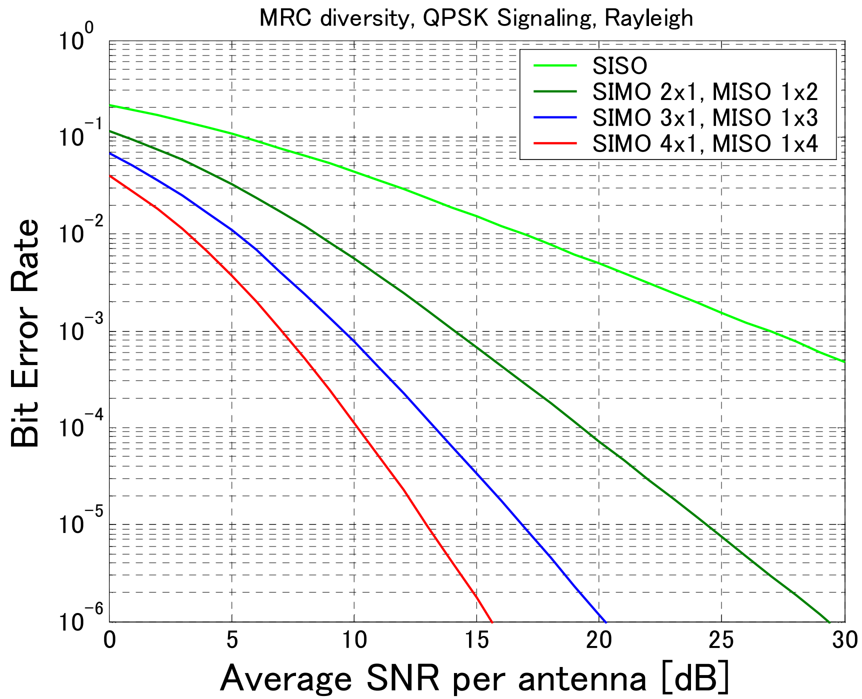
Instantaneous BER for BPSK signaling

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma}\right)$$

PDF of output SNR for MRC

$$f(\gamma) = \frac{1}{(M-1)!\overline{\gamma}^M}\gamma^{M-1}\exp\left(-\frac{\gamma}{\overline{\gamma}}\right)$$

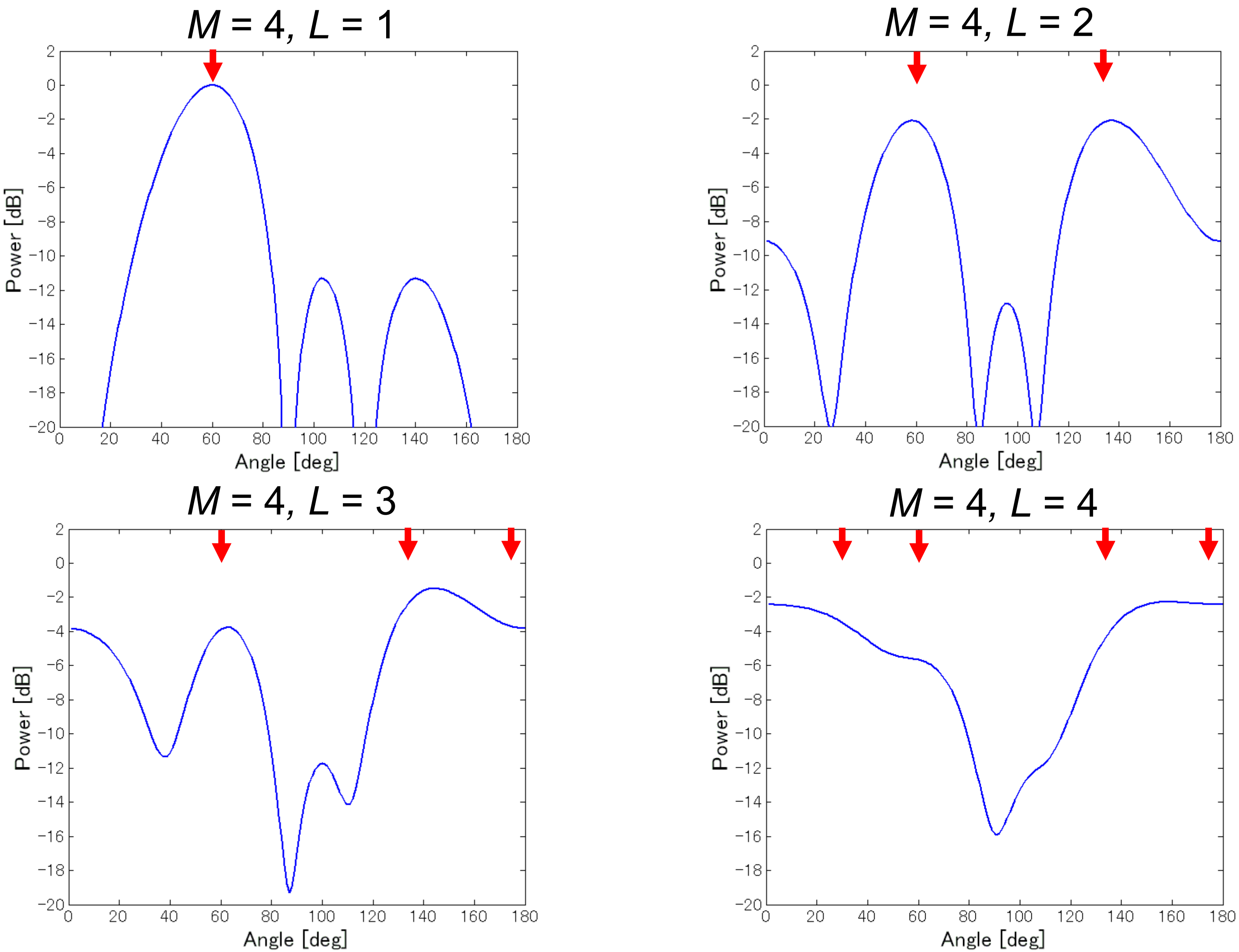
Average BER performance



Average BER for MRC

$$\overline{P}_e(\overline{\gamma}) = \frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right)^M \sum_{i=0}^{M-1}\binom{M-1+i}{i}\left(\frac{1}{2}\left(1-\sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right)\right)^i$$

Beam Pattern Interpretation of Diversity



Diversity with Non-Identical Elements

Non-identical elements

$\bar{\gamma}_i \neq \bar{\gamma}_j$ due to $d_i(\theta) \neq d_j(\theta)$

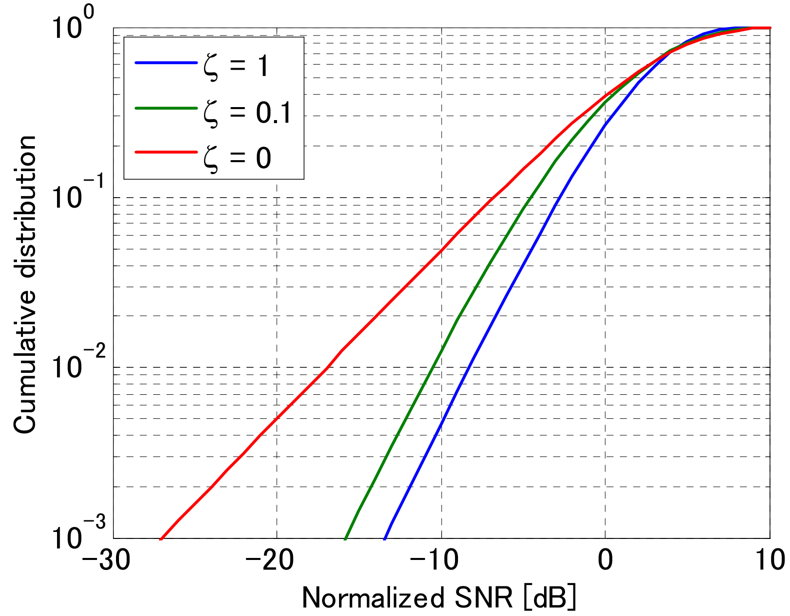
Characteristic function of output SNR with non-identical elements

$$\varphi(t) = \prod_{i=1}^{M} \frac{1}{1 - j\bar{\gamma}_i t}$$

PDF of output SNR in MRC with non-identical elements

$$f(\gamma) = \frac{1}{\prod_{i=1}^{M} \bar{\gamma}_i} \sum_{i=1}^{M} \frac{\exp\left(-\gamma / \bar{\gamma}_i\right)}{\prod_{k=1,k\neq i}^{M} \left(1/\bar{\gamma}_k - 1/\bar{\gamma}_i \right)}$$

CDF of output SNR



$M = 2$

$\bar{\gamma}_1 + \bar{\gamma}_2 = 2 \qquad \zeta = \bar{\gamma}_2 / \bar{\gamma}_1$

Spatial Correlation

Correlation matrix of received signals

$$\mathbf{R}_{\mathrm{y}} = \mathrm{E}[\mathbf{y}\mathbf{y}^H] = P\,\mathrm{E}[\mathbf{h}\mathbf{h}^H] + \sigma^2\mathbf{I}$$

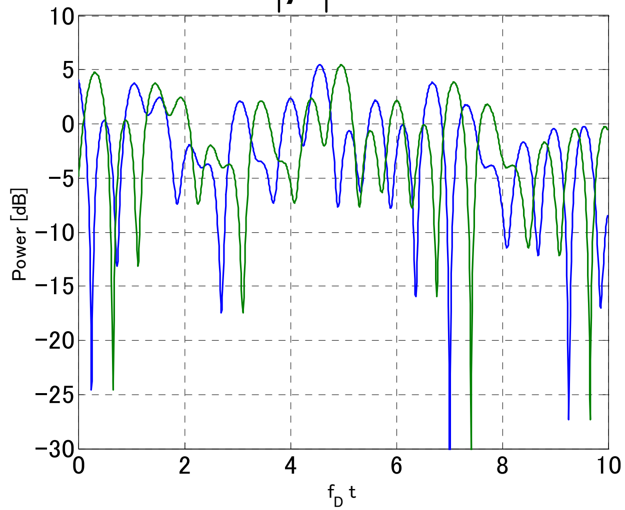
Correlation matrix of channels

$$\mathbf{R}_{\mathrm{h}} = \mathrm{E}[\mathbf{h}\mathbf{h}^H] = \mathrm{E}\begin{bmatrix} h_1 h_1^* & h_1 h_2^* \\ h_2 h_1^* & h_2 h_2^* \end{bmatrix}$$

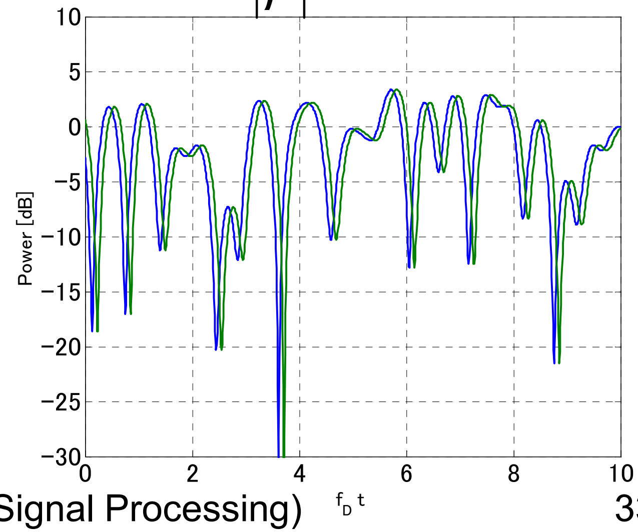
$$\begin{cases} = \begin{bmatrix} \bar{g}_{\mathrm{h}} & 0 \\ 0 & \bar{g}_{\mathrm{h}} \end{bmatrix} & \text{Uncorrelated} \\ = \begin{bmatrix} \bar{g}_{\mathrm{h}} & \bar{g}_{\mathrm{h}}\rho \\ \bar{g}_{\mathrm{h}}\rho^* & \bar{g}_{\mathrm{h}} \end{bmatrix} & \text{Correlated} \end{cases}$$

$|\rho|$: Correlation coefficient between branches

$|\rho| = 0$



$|\rho| = 0.9$



Diversity Combining with Correlation

Eigen decomposition of correlation matrix

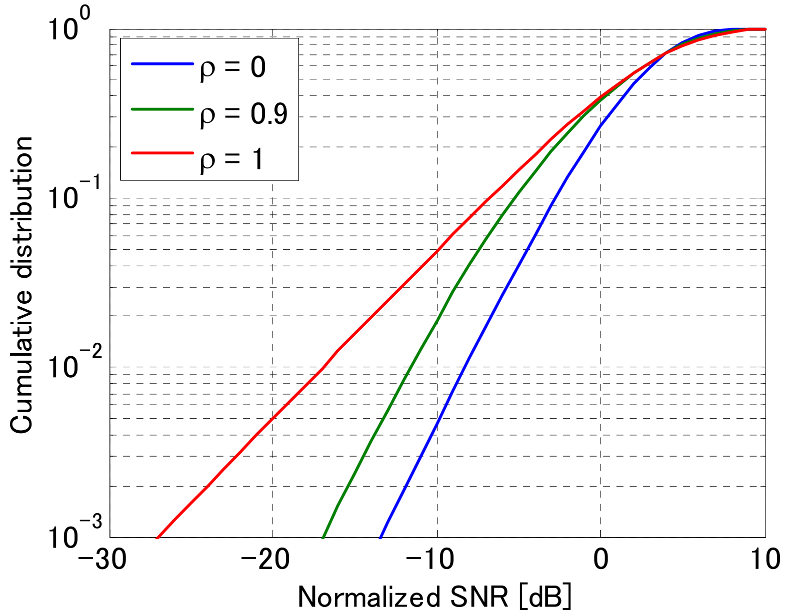
$$\mathbf{R}_{\mathrm{h}} = \begin{bmatrix} \bar{g}_h & \bar{g}_h \rho \\ \bar{g}_h \rho^* & \bar{g}_h \end{bmatrix} = \mathbf{E} \Lambda \mathbf{E}^H$$

$$\lambda_1 = \left(1 + |\rho|\right)\bar{g}_{\mathrm{h}} \qquad \lambda_2 = \left(1 - |\rho|\right)\bar{g}_{\mathrm{h}}$$

Orthogonal conversion

$$\mathbf{E}^H \mathbf{R}_{\mathrm{y}} \mathbf{E} = P \begin{bmatrix} \left(1 + |\rho|\right)\bar{g}_{\mathrm{h}} & 0 \\ 0 & \left(1 - |\rho|\right)\bar{g}_{\mathrm{h}} \end{bmatrix} + \sigma^2 \mathbf{I}$$

CDF of output SNR



Diversity with non-identical elements

$$f(\gamma) = \frac{1}{2|\rho|\bar{\gamma}} \left[\exp\left(-\frac{\gamma}{(1+|\rho|)\bar{\gamma}} \right) - \exp\left(-\frac{\gamma}{(1-|\rho|)\bar{\gamma}} \right) \right]$$

Angular Profile & Spatial Correlation

Correlation matrix

$$\mathbf{R}_{\mathrm{h}} = \mathrm{E}\left[\mathbf{h}\mathbf{h}^H\right] = \begin{bmatrix} \bar{g}_{\mathrm{h}} & \bar{g}_{\mathrm{h}}\rho \\ \bar{g}_{\mathrm{h}}\rho^* & \bar{g}_{\mathrm{h}} \end{bmatrix} \qquad \mathbf{h} = \sum_{l=1}^{L} \beta_l e^{jkvt\cos\theta_l} \mathbf{a}(\theta_l)$$

Uncorrelated scattering

$$\mathrm{E}\left[e^{jkvt\cos\theta_i} e^{-jkvt\cos\theta_j}\right] = 0 \qquad \text{for} \qquad i \neq j$$

Spatial correlation

$$\mathbf{R}_{12} = \mathrm{E}\left[\left(\sum_{l=1}^{L} \beta_l e^{jkvt\cos\theta_l}\right)\left(\sum_{l=1}^{L} \beta_l e^{jkvt\cos\theta_l} e^{jkd\cos\theta_l}\right)^*\right]$$

$$= \sum_{l=1}^{L} \left|\beta_l\right|^2 e^{-jkd\cos\theta_l} = \int_0^{2\pi} \left|\beta(\theta)\right|^2 e^{-jkd\cos\theta} \mathrm{d}\theta$$

Angular profile

$$\rho = \frac{\mathbf{R}_{12}}{\bar{g}_{\mathrm{h}}} = \int_0^{2\pi} P(\theta) e^{-jkd\cos\theta_l} \mathrm{d}\theta \qquad P(\theta) = \frac{\left|\beta(\theta)\right|^2}{\int_0^{2\pi} \left|\beta(\theta)\right|^2 \mathrm{d}\theta}$$

Angular Profile & Spatial Correlation

Uniform distribution

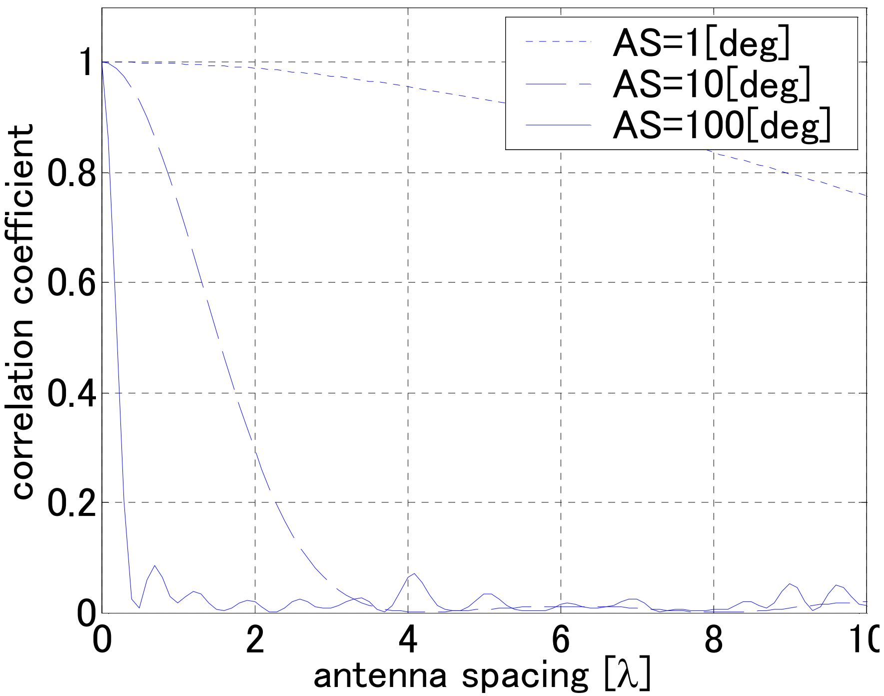
$$P(\theta) = \frac{1}{2\pi} \quad \longleftrightarrow \quad \rho(d) = \mathrm{J}_0(kd)$$

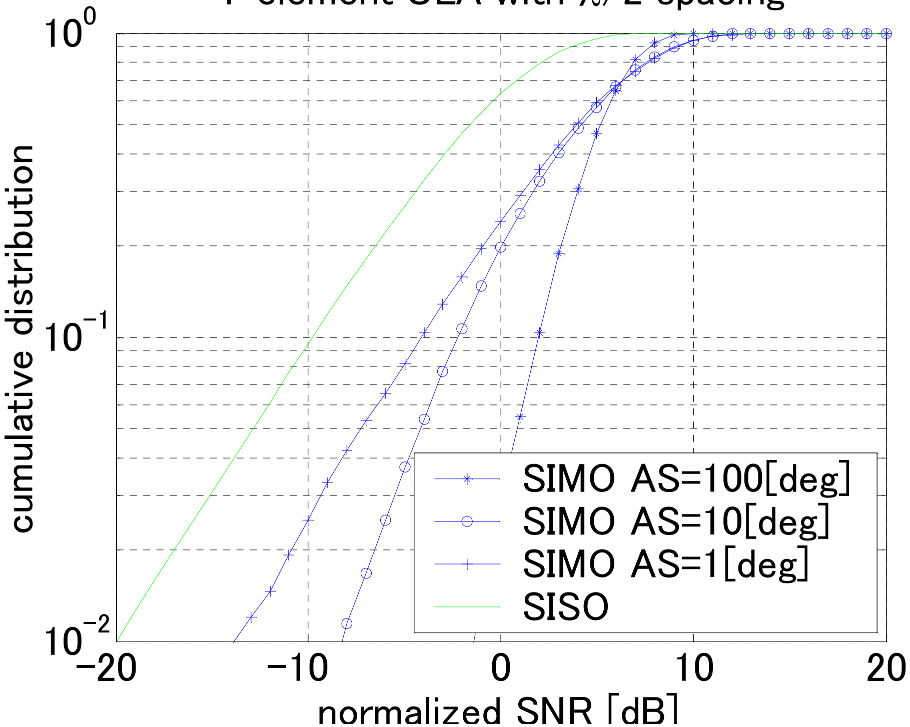
0th order Bessel function

Gaussian distribution

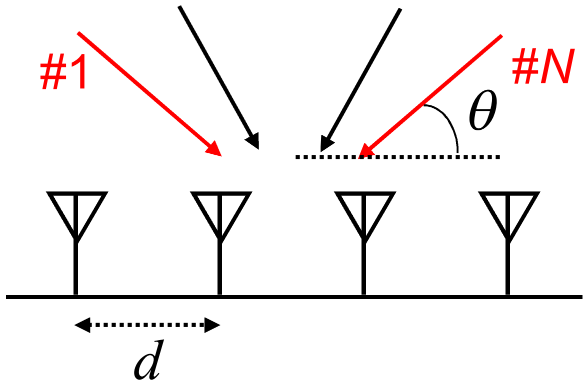
$$P(\theta) = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\left(-\frac{(\theta-\theta_0)^2}{2\sigma_\theta^2}\right)$$

$$\longleftrightarrow \quad \rho(d) \cong \exp\left(-\frac{(\sigma_\theta kd \sin\theta_0)^2}{2}\right)$$





Diversity with Interference Cancellation



Received signal with interference

$$\mathbf{y}(t) = \mathbf{h}_{\mathrm{D}} s_{\mathrm{D}}(t) + \sum_{i=1}^{N} \mathbf{h}_{\mathrm{I}i} s_{\mathrm{I}i}(t) + \mathbf{n}(t)$$

Interference cancellation

$$\mathbf{Q} = \left[\mathbf{e}_{N+1}, \cdots, \mathbf{e}_M\right] \in C^{M \times (M-N)}$$

$$\begin{aligned}\mathbf{Q}^H \mathbf{y} &= \mathbf{Q}^H \mathbf{h}_{\mathrm{D}} s(t) + \mathbf{Q}^H \mathbf{n}(t) \\ &= \tilde{\mathbf{h}}_{\mathrm{D}} s(t) + \tilde{\mathbf{n}}(t) \in C^{M-N}\end{aligned}$$

Diversity combining

$$\mathbf{x} = \mathbf{w}^H \mathbf{Q}^H \mathbf{y}$$

$$\mathbf{w} = \tilde{\mathbf{h}}_{\mathrm{D}}$$

Interference cancellation (N_{th} order)

Diversity combining (M-N_{th} order)

Subspace decomposition

$$\mathbf{H}_{\mathrm{I}} = \left[\mathbf{h}_{\mathrm{I}1}, \mathbf{h}_{\mathrm{I}2}, \cdots, \mathbf{h}_{\mathrm{I}N}\right]$$

$$\mathbf{H}_{\mathrm{I}} \mathbf{H}_{\mathrm{I}}^H = \mathbf{E} \Lambda \mathbf{E}^H$$

$$\mathbf{E} = \left[\mathbf{e}_1, \cdots, \mathbf{e}_N, \underbrace{\mathbf{e}_{N+1}, \cdots, \mathbf{e}_M}_{\text{Null space}}\right]$$

Summary

- Array signal processing
  - Beamforming & interference cancellation for plane wave signal
  - Diversity combining for multi-path signal
  - Diversity combining with interference cancellation

$\Downarrow$

Improvement on SNR, SIR, and outage SNR

- Further revolution

What happen if array antennas are employed both at Tx and Rx

MIMO communication system