2012 2nd semester
MIMO Communication Systems

#3: OFDM Wireless Access

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Oct. 23, 2012

Contents

■ Aim of today

Principles, operations, and performance of OFDM

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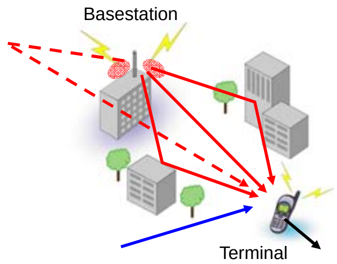
- **OFDM Performance**

1. Frequency Selective & Time Dispersive Fading

Wireless Communication Channel

Wireless is vulnerable!



#1: Path loss due to distance
→ Lower average SNR

#2: Multi-path fading
→ Deep SNR dip

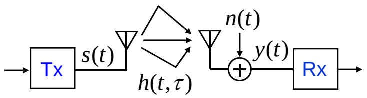
#3: Time dispersive fading
→ Inter symbol interference

#4: Multi-access interference
→ Co-channel interference

Wideband Signal Model

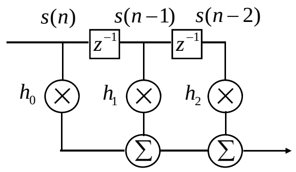
Received signal

$$y(t)=\int h(\tau)s(t-\tau)\mathrm{d}\tau+n(t)$$



Discrete representation

$$y(n\Delta t)=\sum_{l} h_l s(n\Delta t-l\Delta\tau)+n(n\Delta t)$$



Frequency domain

$$\tilde{y}(f)=\tilde{h}(f)\tilde{s}(f)+\tilde{n}(f)$$

Uncertainty theorem

$$\tilde{y}(k\Delta f)=\tilde{h}(k\Delta f)\tilde{s}(k\Delta f)+\tilde{n}(k\Delta f) \quad \Delta F=1/\Delta t \quad \Delta f=1/\Delta T$$

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Frequency Selective Fading

Two-path model

$$y(t)=h_0 s(t)+h_1 s(t-\Delta\tau)+n(t)$$

Impulse response

$$h(\tau)=h_0\delta(\tau)+h_1\delta(\tau-\Delta\tau)$$

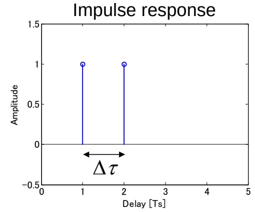
Frequency response

$$\tilde{h}(f)=h_0+h_1\exp(-j2\pi f\Delta\tau)$$

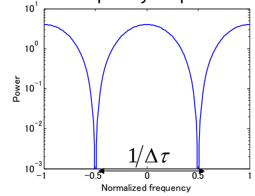
Condition for narrow band signal

Bandwidth $\Delta F << 1/\Delta\tau$

Impulse response



Frequency response



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Frequency Spectrum

Received signal

$$y(t)=\int h(\tau)s(t-\tau)\mathrm{d}\tau+n(t)$$

Auto correlation

$$R_s(\tau)=\mathrm{E}\left[s^*(t)s(t+\tau)\right]$$

Power spectrum of transmit signal

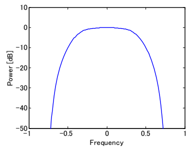
$$S_s(f)=\int R_s(\tau)\exp(-j2\pi f\tau)\mathrm{d}\tau$$

Power spectrum of received signal

$$S_y(f)=\left|\tilde{h}(f)\right|^2 S_s(f)$$

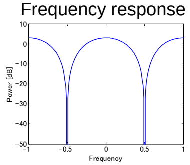
$$\tilde{h}(f)=\int h(t)\exp(-j2\pi ft)\mathrm{d}t$$

Transmit signal

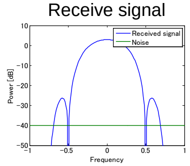


$\Delta F > 1/\Delta\tau$

Frequency response



Receive signal



Distortion in power spectrum

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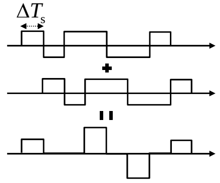
Time Dispersive Fading & Inter Symbol Interference (ISI)

Two-path model

$$y(t)=\sum h(\Delta\tau)s(t-\Delta\tau)+n(t)$$

$$=h_0 s(t)+h_1 s(t-\Delta T_s)+n(t)$$

if $\Delta\tau=\Delta T_s$

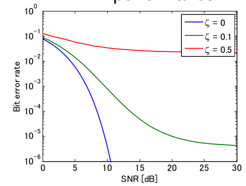


Signal to Interference & Noise Ratio (SINR)

$$\gamma_1=\frac{|h_0|^2 P}{|h_1|^2 P+\sigma^2}=\frac{|h_0|^2}{|h_1|^2+\sigma^2/P}$$

$$P_{eb}=\frac{1}{2}\mathrm{erfc}\left(\sqrt{\gamma_1}\right) \quad \text{for BPSK}$$

BER performance



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Channel Capacity

Two-path model

$$y(t) = h_0 s(t) + h_1 s(t - \Delta\tau) + n(t)$$

Transmit signal with pulse shaping

$$s(t) = \sum_i s_i g(t - iT_s)$$

Receive SINR

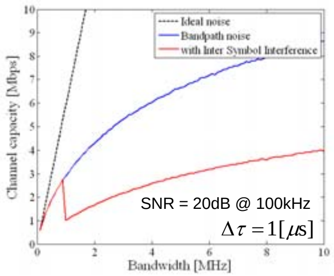
$$\gamma_1 = \frac{|h_0 g(0) + h_1 g(-\Delta\tau)|^2 P}{\sum_{i \neq 0} |h_1 g(iT_s - \Delta\tau)|^2 P + \sigma^2}$$

Channel capacity

$$C = B\log_2(1+\gamma_1) \approx \frac{1}{T_s}\log_2(1+\gamma_1)$$

$$\sigma^2 = BN_0$$

Average channel capacity



2. Features of OFDM

Multicarrier Modulation

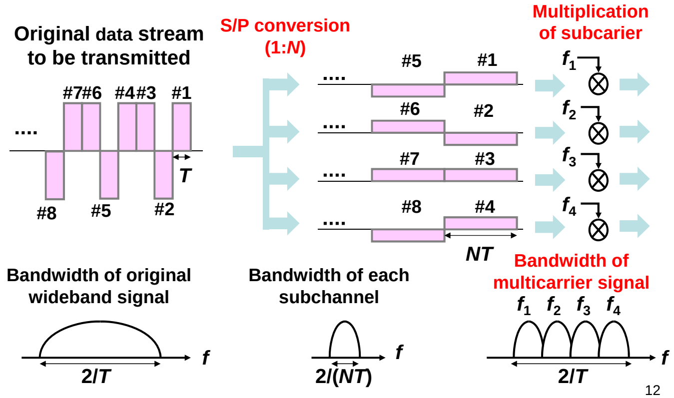
- **Multicarrier modulation divides high-speed data stream into multiples substreams to be transmitted over different orthogonal subchannels centered at different subcarrier frequencies**
- **Let *R* and *B* be the data rate and bandwidth of high-speed data stream (wideband) signal**
- **Coherence bandwidth for the channel is assumed to be $B_c < B$**

➔ The wideband signal experiences multi-path fading, i.e., frequency-selective fading

- **Multicarrier modulation divides wideband signal into N linearly modulated subchannels in parallel**
  - **Subchannel bandwidth: $B_N = B / N$**
  - **Data rate:** $R_N \approx R/N$
- **Symbol time (symbol duration) is much longer than the delay spread of the channel ➔ subchannel experiences little ISI (inter-symbol interference) degradation**
  - **$B_N << B_c$ ➔** $T_N \approx 1/B_N >> 1/B_c \approx T_m$ **(T_m is delay spread of the channel)**
- **Subchannel bandwidth is narrower than coherence bandwidth, i.e., $B_N = B / N << B_c$ ➔ each subchannel experiences flat fading**

Principle of Multicarrier Modulation (1)

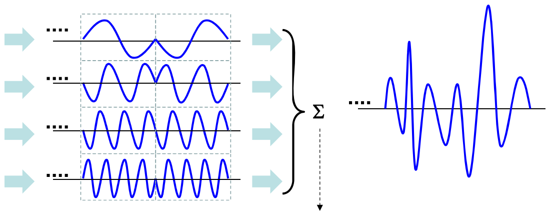
- **Symbol time after serial-to-parallel (S/P) conversion becomes *N* times longer compared to original symbol time**



Principle of Multicarrier Modulation (2)

- Subchannel signal in time domain after multiplied with subcarrier frequency

■ Multicarrier signal in time domain
➔ Increasing fluctuation in amplitude which leads to large peak-to-average power ration (PAPR)

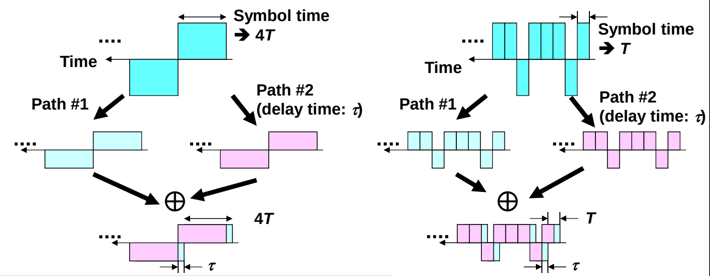


◆ Multiplexing for subchannels ➔ Multicarrier signal

Robustness for Multipath Fading (Time Domain)

Multicarrier signal (for instance 4 subchannels)

Wideband signal-carrier signal



■ Symbol time becomes 4 times longer compared to that for original single-carrier signal
➔ Relative delay time for symbol time decreases
➔ Decreasing influence of ISI ($4T >> \tau$)

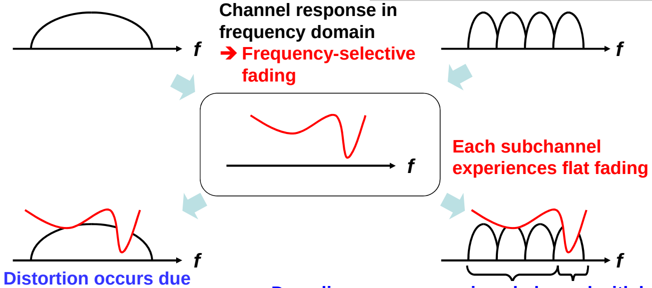
■ Relative delay time for symbol time increases according to the increasing data rate
➔ Increasing ISI

Robustness for Multipath Fading (Frequency Domain)

● Wideband signal-carrier signal

● Multicarrier signal (for instance 4 subchannels)

Channel response in frequency domain
➔ Frequency-selective fading



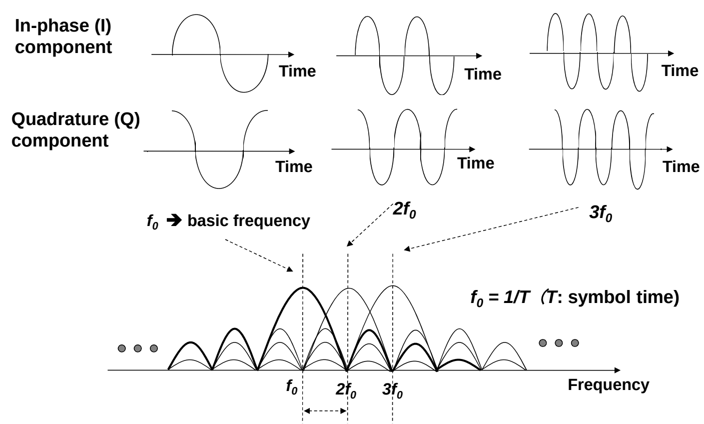
Each subchannel experiences flat fading

- Distortion occurs due to inter-symbol interference (ISI)

- Decoding error occurs in subchannel with low received SNR (or received signal level)
- But, transmitted bits are successfully decoded

➔ Erroneously decoded bits are corrected by FEC (channel coding).

Frequency Spectrum for OFDM



3. OFDM Transmitter and Receiver Structures

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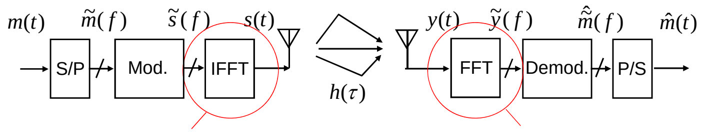
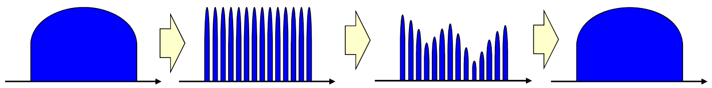
Outline of OFDM Transmitter and Receiver



- Block transmission system using S/P & P/S converter
- Convert wide band signal to super position of narrow band signals satisfying $\Delta f << 1/\Delta\tau$



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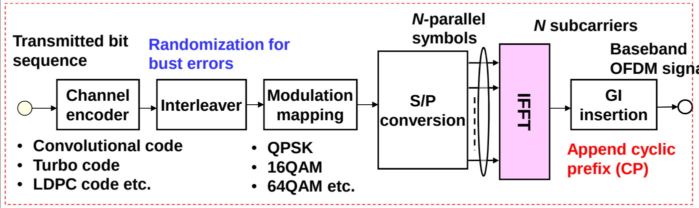
Structure of OFDM Transmitter



- **Information bit is channel-encoded using turbo code or LDPC code etc.**
- **Coded bit sequence is bit-interleaved to randomize burst error.**
- **Bit sequence after bit-interleaving is mapped to constellation point according to modulation scheme including QPSK, 16QAM, 64QAM etc.**
- **Modulated symbol sequence is serial-to-parallel-converted to *N*-parallel symbol sequences.**
- ***N*-parallel data symbols are fed into IFFT to generate OFDM signal.**
- **Finally, cyclic prefix is appended at the beginning of each FFT block to avoid inter-symbol interference.**

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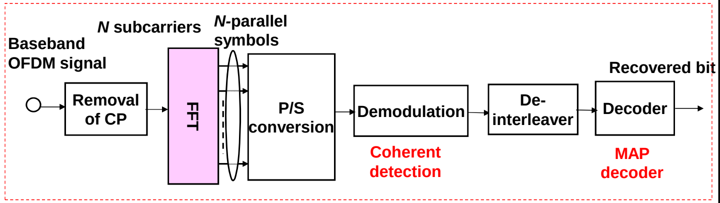
Structure of OFDM Receiver



- **After removing Cyclic prefix (CP), OFDM signal is converted into the N-parallel symbols by FFT.**
- **Parallel symbols are converted into serial symbol sequence.**
- **In general, channel response at each subcarrier position is estimated using reference signal (or pilot signal). Then, coherent detection is performed using the estimated channel response.**
- **In general, log-likelihood ratio (LLR) of each bit is computed.**
- **The LLR is fed into channel decoder**
- **At the last iteration of channel decoder, LLR is hard-decided to recover transmitted bits.**

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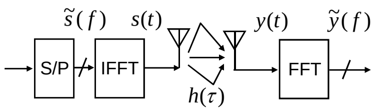
OFDM Modulation



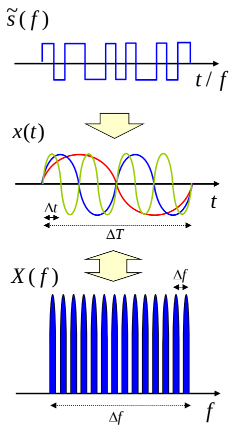
OFDM modulation

$$s(t)=\frac{1}{\sqrt{K}}\sum_{k=0}^{K-1}\tilde{s}(k\Delta f)\exp(j2\pi k\Delta f t)$$

Number of subcarriers

Subcarrier interval

$$0\le t\le \Delta T = 1/\Delta f$$



OFDM parameters

Subcarrier interval

$$\Delta f << 1/\Delta\tau$$

Bandwidth

$$\Delta F = K\Delta f$$

Sampling period

$$\Delta t \le 1/\Delta F$$

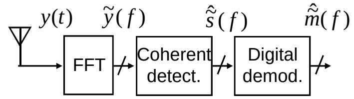
OFDM symbol period

$$\Delta T = 1/\Delta f$$

OFDM Demodulation

Receive signal

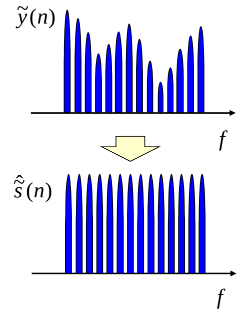
$$y(t)=\int h(\tau)s(t-\tau)dt+n(t)$$



OFDM demodulation

$$\tilde{y}(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}y(n)\exp\left(-j2\pi\frac{nk}{N}\right)$$

$$=\tilde{h}(k)\tilde{s}(k)+\tilde{n}(k)$$



Frequency Domain Equalizer（FDE）

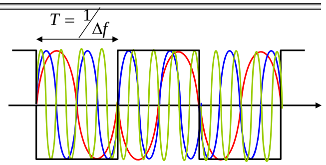
$$\hat{\tilde{s}}(k)=\tilde{y}(k)/\tilde{h}(k)$$

Coherent detection in frequency domain

Frequency Spectrum for OFDM Signal

OFDM signal

$$s(t)=\frac{1}{\sqrt{K}}\sum_{k=0}^{K-1}\tilde{s}(k)\exp(j2\pi k\Delta f t)$$



Auto correlation

$$R_s(\tau)=\mathrm{E}\left[s^*(t)s(t+\tau)\right]$$

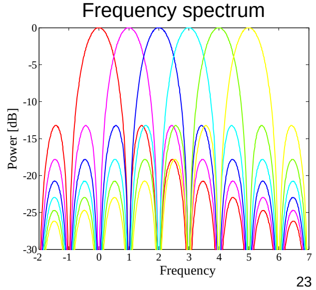
$$=\frac{1}{K}\sum_{k=0}^{K}\left(1-\frac{|\tau|}{T}\right)\exp(j2\pi k\Delta f\tau)$$

Auto correlation function of rectangular pulse

Power spectrum

$$S_S(f)=\frac{1}{K}\sum_{n=0}^{K-1}\mathrm{sinc}^2(T(f-n\Delta f))$$

Frequency spectrum



4. DFT

Definition of DFT

- **DFT (Discrete Fourier Transform)**
- **Let *f(k)* (*k = 0, 1, ..., N-1*) denote a discrete time sequence, *N*-point DFT of *f(k)* is defined as**

$$\mathrm{DFT}[f(k)] = F(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} f[k] \cdot W_N^{kn} \qquad (n = 0, 1, \ldots, N-1)$$

where W_N is twiddle factor which is represented as

$$W_N = \exp\left(-j\frac{2\pi}{N}\right)$$

- ✓ ***N* denotes the number of samples in one symbol duration and that of subcarriers. In DFT, *N* takes ant integer.**
- ✓ ***F(n)* characterized the frequency component of the time samples *f(k)* associated with the original signal *f(t)***

Definition of IDFT

- **IDFT (Inverse Discrete Fourier Transform)**
- **The sequence *f(k)* is recovered from its DFT *F(n)* using inverse DFT (IDFT) *as***

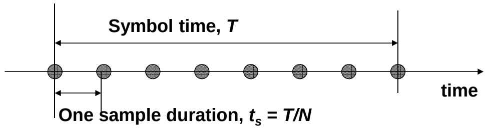
$$\mathrm{IDFT}[F(n)] = f(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} F[n] \cdot W_N^{-kn} \qquad (k = 0, 1, \ldots, N-1)$$

where

$$W_N^{-1} = \exp\left(j\frac{2\pi}{N}\right)$$

- ✓ ***N* indicates the number of samples over one symbol time and that of subcarriers.**
- ✓ ***f(k)* is called as inverse discrete Fourier transform (IDFT) of *F(n)***
- ✓ **In OFDM, *F(n)* represents *N* parallel data symbols corresponds to each subcarrier component. The frequency components are converted into time samples by performing inverse DFT on these *N* samples.**

DFT Operation in OFDM



- **Sample point over OFDM symbol in time domain**
  ➔ Multiply subcarrier to each subchannel

IDFT (IFFT)

$$x_k = x(k \cdot t_s) = \sum_{n=0}^{N-1} X_n \cdot e^{j\frac{2\pi kn}{N}} = \sum_{n=0}^{N-1} X_n \cdot W^{-kn} \quad (k = 0, 1, \ldots, N-1)$$

- **Data symbol at each subcarrier after removing subcarrier signal**

DFT (FFT)

$$X_n = \frac{1}{N} \sum_{k=0}^{N-1} x_k \cdot e^{-j\frac{2\pi kn}{N}} = \frac{1}{N} \sum_{k=0}^{N-1} x_k \cdot W^{kn} \quad (n = 0, 1, \ldots, N-1)$$

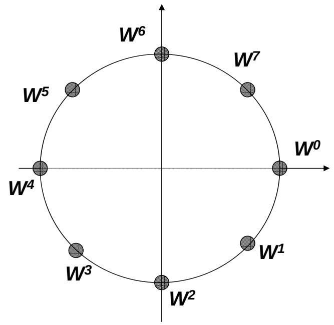
$$W_N = \exp\left(-j\frac{2\pi}{N}\right)$$

Twiddle Factor, W_N

- **Twiddle factor W_N denotes points which are divided into *N* equal phases for a unit circle in the complex plane.**

Twiddle factor in case of *N* = 8

$$W_N = \exp\left(-j\frac{2\pi}{N}\right)$$



5. Generation of Baseband OFDM Signal

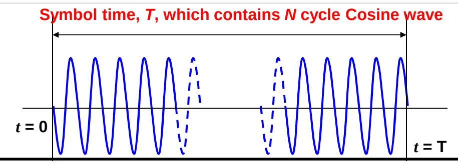
Subcarrier Component in OFDM Signal

- Let f_0 be a basic subcarrier frequency, i.e., lowest subcarrier frequency which corresponds to subcarrier spacing.
- Then, OFDM signal comprises multicarrier signals.
  - Symbol time of $T = 1/f_0$
  - Subcarrier frequency of $n \times f_0$
- OFDM signal over one symbol is represented as

$$a_n \cos(2\pi n f_0 t) - b_n \sin(2\pi n f_0 t)$$

where a_n and b_n are in-phase and quadrature components of complex envelop of data symbol

- OFDM symbol length is from $t = 0$ to $t = T$.
- One OFDM symbol duration contains Cosine wave with n cycles
- Amplitude and phase components at n-th subcarrier component varies according to complex envelop, a_n and b_n



Generation of Baseband OFDM Signal (1)

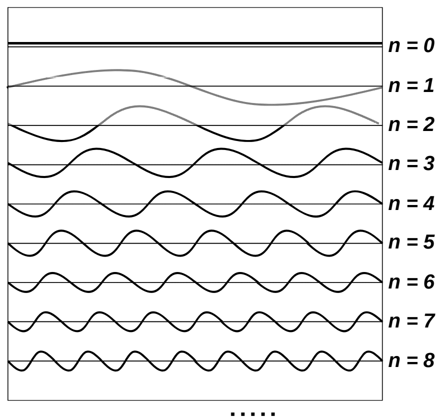
- Let $s_B(t)$ be the summation of N subcarrier components of OFDM signal in which n indicates subcarrier index.
- $s_B(t)$ is called baseband OFDM signal which is given as

$$S_B(t) = \sum_{n=0}^{N-1} \{a_n \cdot \cos(2\pi n f_0 t) - b_n \cdot \sin(2\pi n f_0 t)\}$$

- $s_B(t)$ is multicarrier signal which add N modulated symbols with different subcarriers.
- f_0 is subcarrier spacing (or subcarrier separation).
- Parameters, a_n and b_n are the in-phase and quadrature components of complex envelope of data modulation at the n-th subcarrier
- One OFDM symbol duration contains N sets of data symbols.

Generation of Baseband OFDM Signal (2)

- Details of baseband OFDM signal



- $s_B(t)$ comprises multicarrier signals
- Basic subcarrier frequency f_0 repeats n cycles at the n-th subcarrier

Generation of Baseband OFDM Signal (3)

- Baseband OFDM signal $s_B(t)$ is represented as

$$S_B(t)=\sum_{n=0}^{N-1}\{a_n\cdot\cos(2\pi n f_0 t)-b_n\cdot\sin(2\pi n f_0 t)\}\ .$$

- We express the above equation in complex notation as

$$S_B(t)=\mathrm{Re}\left[\sum_{n=0}^{N-1} d_n\cdot e^{j2\pi n f_0 t}\right]\ .$$

  - d_n is complex envelope of data symbol which modulates *n*-th subcarrier ➔ $d_n = a_n + j \times b_n$
- We define *u(t)* as in the next equation.

$$u(t)=\sum_{n=0}^{N-1} d_n\cdot e^{j2\pi n f_0 t}$$

- $s_B(t)$ is real part of *u(t)*.

➔ $s_B(t)$ is generated from *u(t)*.

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Generation of Baseband OFDM Signal (4)

- We consider sampled value of *u(t)* with the sampling interval of $1/(Nf_0)$.
- When sampling is performed over one symbol duration, $T= 1/f_0$, *N* sampled values are computed as

$$u\left(\frac{k}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\cdot e^{j2\pi n f_0\frac{k}{Nf_0}}$$

$$=\sum_{n=0}^{N-1} d_n\cdot e^{j\frac{2\pi nk}{N}}$$

$$=\sum_{n=0}^{N-1} d_n\cdot\left(e^{j\frac{2\pi}{N}}\right)^{nk}\quad (k=0,1,2,\ldots,N-1)$$

- *N* sampled values in *u(t)* are generated by performing inverse DFT (IDFT) for *N* complex data symbols, d_n.
- Note that only the frequency component f_0 is necessary to generate multicarrier signal.

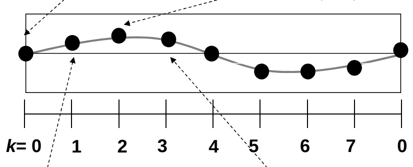
Example of Baseband OFDM Signal Generation (1)

- *N* = 8

n = 1

$$u\left(\frac{0}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\quad (k=0)$$

$$u\left(\frac{2}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\cdot\left(e^{j\frac{2\pi}{8}}\right)^2\quad (k=2)$$



$$u\left(\frac{1}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\cdot\left(e^{j\frac{2\pi}{8}}\right)\quad (k=1)$$

$$u\left(\frac{3}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\cdot\left(e^{j\frac{2\pi}{8}}\right)^3\quad (k=3)$$

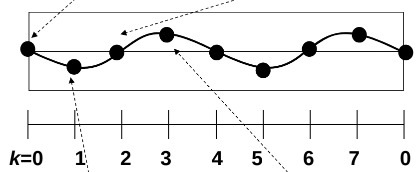
Example of Baseband OFDM Signal Generation (2)

- *N* = 8

n = 2

$$u\left(\frac{0}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\quad (k=0)$$

$$u\left(\frac{2}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\cdot\left(e^{j\frac{2\pi}{N}}\right)^4\quad (k=2)$$



$$u\left(\frac{1}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\cdot\left(e^{j\frac{2\pi}{N}}\right)^2\quad (k=1)$$

$$u\left(\frac{3}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n\cdot\left(e^{j\frac{2\pi}{N}}\right)^6\quad (k=3)$$

6. De-multiplexing of Baseband OFDM Signal

De-multiplexing of Baseband OFDM Signal (1)

- De-multiplex (or demodulate) data symbols from complex baseband OFDM signal *u(t)* which is given as

$$u(t)=\sum_{n=0}^{N-1} d_n \cdot e^{j2\pi n f_0 t}$$

- Sampled signal for u(t) with the sampling interval of $1/(Nf_0)$ over one OFDM symbol duration is given as

$$u\left(\frac{k}{Nf_0}\right)=\sum_{n=0}^{N-1} d_n \cdot e^{j\frac{2\pi nk}{N}}$$

$$=\sum_{n=0}^{N-1} d_n \cdot \left(e^{j\frac{2\pi}{N}}\right)^{nk} \quad (k=0,1,2,....,N-1)$$

- Since $u(k/Nf_0)$ (k = 0, 1, 2, …, N-1) is IDFT for data symbol d_n (n = 0, 1, 2,…,N-1) ➔ de-multiplex (or demodulate) complex data symbol d_n by performing DFT to $u(k/Nf_0)$.

De-multiplexing of Baseband OFDM Signal (2)

- De-multiplex data symbol d_n from OFDM signal by performing DFT to $u(k/Nf_0)$ as

$$d_l=\frac{1}{N}\sum_{k=0}^{N-1} u\left(\frac{k}{Nf_0}\right)\cdot e^{-j\frac{2\pi kl}{N}}$$

$$=\frac{1}{N}\sum_{k=0}^{N-1} u\left(\frac{k}{Nf_0}\right)\cdot\left(e^{-j\frac{2\pi}{N}}\right)^{kl} \quad (l=0,1,2,....,N-1)$$

Coefficient of DFT using W_N

7. FFT

Matrix Notation of DFT

$$F(k)=\sum_{n=0}^{N-1} f[n]\cdot W_N^{kn} \qquad (k=0,1,\ldots\ldots,N-1)$$

■Matrix notation of *N*-point DFT when *N* = 8

$$\begin{bmatrix}F(0)\\F(1)\\F(2)\\F(3)\\F(4)\\F(5)\\F(6)\\F(7)\end{bmatrix}=\begin{bmatrix}W^0&W^0&W^0&W^0&W^0&W^0&W^0&W^0\\W^0&W^1&W^2&W^3&W^4&W^5&W^6&W^7\\W^0&W^2&W^4&W^6&W^8&W^{10}&W^{12}&W^{14}\\W^0&W^3&W^6&W^9&W^{12}&W^{15}&W^{18}&W^{21}\\W^0&W^4&W^8&W^{12}&W^{16}&W^{20}&W^{24}&W^{28}\\W^0&W^5&W^{10}&W^{15}&W^{20}&W^{25}&W^{30}&W^{35}\\W^0&W^6&W^{12}&W^{18}&W^{24}&W^{30}&W^{36}&W^{42}\\W^0&W^7&W^{14}&W^{21}&W^{28}&W^{35}&W^{42}&W^{49}\end{bmatrix}\cdot\begin{bmatrix}f(0)\\f(1)\\f(2)\\f(3)\\f(4)\\f(5)\\f(6)\\f(7)\end{bmatrix}$$

- For *N*-point DFT, N^2 times complex multiplications and *N(N-1)* times complex additions are necessary for DFT processing ➔ Huge computational complexity when *N* is large!

FFT Processing (Only Result)

◆ FFT processing

Number of complex multiplications is decreased to 8 (64 for DFT)

$$\begin{bmatrix}F(0)\\F(4)\\F(2)\\F(6)\\F(1)\\F(5)\\F(3)\\F(7)\end{bmatrix}=\begin{bmatrix}1&1&0&0&0&0&0&0\\1&-1&0&0&0&0&0&0\\0&0&1&1&0&0&0&0\\0&0&1&-1&0&0&0&0\\0&0&0&0&1&1&0&0\\0&0&0&0&1&-1&0&0\\0&0&0&0&0&0&1&1\\0&0&0&0&0&0&1&-1\end{bmatrix}\cdot\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&W^0&0&0&0&0&0\\0&0&0&W^2&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&W^0&0\\0&0&0&0&0&0&0&W^2\end{bmatrix}$$

$$\times\begin{bmatrix}1&0&1&0&0&0&0&0\\0&1&0&1&0&0&0&0\\1&0&-1&0&0&0&0&0\\0&1&0&-1&0&0&0&0\\0&0&0&0&1&0&1&0\\0&0&0&0&0&1&0&1\\0&0&0&0&1&0&-1&0\\0&0&0&0&0&1&0&-1\end{bmatrix}\cdot\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&W^0&0&0&0\\0&0&0&0&0&W^1&0&0\\0&0&0&0&0&0&W^2&0\\0&0&0&0&0&0&0&W^3\end{bmatrix}\cdot\begin{bmatrix}1&0&0&0&1&0&0&0\\0&1&0&0&0&1&0&0\\0&0&1&0&0&0&1&0\\0&0&0&1&0&0&0&1\\1&0&0&0&-1&0&0&0\\0&1&0&0&0&-1&0&0\\0&0&1&0&0&0&-1&0\\0&0&0&1&0&0&0&-1\end{bmatrix}\begin{bmatrix}f(0)\\f(1)\\f(2)\\f(3)\\f(4)\\f(5)\\f(6)\\f(7)\end{bmatrix}$$

FFT Algorithm

■ **FFT algorithm**

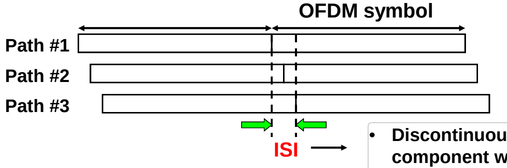
- FFT algorithm contains (*N/2*) sets of butterfly operations by the (log_2N) stages
- Most of computations in FFT are butterfly operations without complex multiplications ➔ significant decrease in computational complexity
- Resultant number of complex multiplications become as *(log_2N-1)x(N/2)*.

➔ Number of complex multiplications is decreased to approximately 1/220 for *N* = 1024.

8. Cyclic Prefix

ISI and ICI in Time Dispersive Channel

- In land mobile communications, multipath fading occurs which brings about time dispersive channel
- ISI (Inter-Symbol Interference):
  - In radio access system using OFDM, multiple OFDM symbol are transmitted in a series ➔ time dispersive channel causes ISI between successive OFDM symbols
- ICI (Inter-Carrier Interference):
  - Time dispersive channel destroys orthogonality between subcarriers ➔ causes ICI



- Discontinuous subcarrier component when ISI occurs ➔ orthogonality between subcarriers is destroyed (ICI)

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Matrix Representation

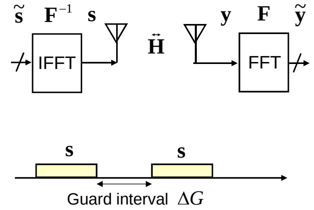
OFDM modulation

Transmit signal block

$\tilde{\mathbf{s}} = [\tilde{s}_0 \quad \tilde{s}_1 \quad \cdots \quad \tilde{s}_{K-1}]^T$

$\mathbf{s} = [s_0 \quad s_1 \quad \cdots \quad s_{N-1}]^T$

$\mathbf{s} = \mathbf{F}^{-1}\tilde{\mathbf{s}}$ Inverse DFT



Convolution in matrix form

If $\Delta G > \Delta\tau_{\max}$

L-path model

$\mathbf{h} = [h_0 \quad h_1 \quad \cdots \quad h_{L-1} \quad 0 \quad 0]^T$

Received signal block

$\mathbf{y} = [y_0 \quad y_1 \quad \cdots \quad y_{N-1}]^T$

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & \cdots & 0 & 0 \\ h_1 & h_o & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-2} \\ n_{N-1} \end{bmatrix}$$

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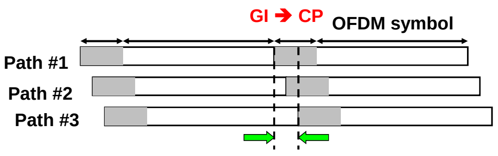
Insertion of Guard Interval

- In time dispersive channel due to multipath fading, guard interval (GI) is inserted at the beginning of each OFDM symbol
- Insertion of a silent guard period between successive OFDM symbols (i.e., zero padding) would avoid ISI, but does not avoid the destruction of subcarrier orthogonality.
- Cyclic prefix (CP) is used in GI
  - CP preserves the orthognality of subcarriers and prevents ISI between successive OFDM symbols ➔ very simple 1-tap equalizer (i.e., conventional coherent detection) is applicable



No ISI and ICI

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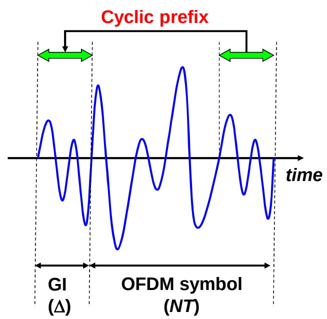
Cyclic Prefix

- Guard interval is inserted that contains a cyclic extension of the OFDM symbol

➔ continuity of each subcarrier signal is maintained as long as length of channel response (maximum delay time of paths) is less than CP length



- The use of a cyclic prefix in the transmitted signal has the disadvantage of requiring more signal energy.
- The loss in transmit energy due to CP insertion is $E_{loss} = NT / (NT + \Delta)$

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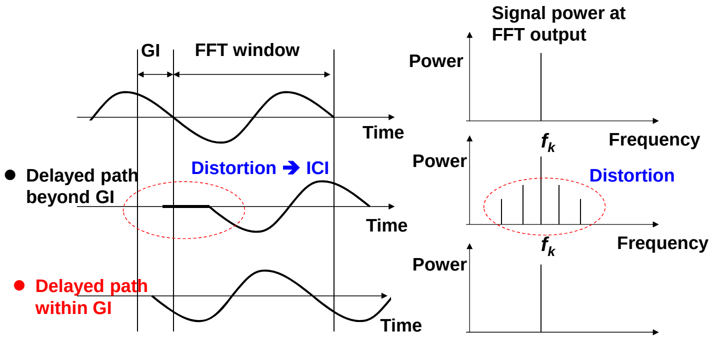
Effect of Cyclic Prefix

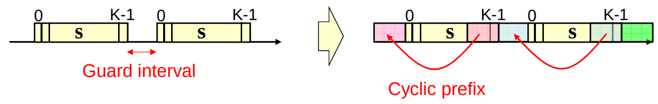


Subcarrier frequency does not change (only the phase is shifted) ➔ does not cause ICI

Cyclic Prefix

Block transmission

Cyclic prefix



Matrix representation of received signal block

Cyclic shift matrix

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & \cdots & 0 & 0 \\ h_1 & h_o & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix} \Rightarrow \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & h_{L-1} & \cdots & h_1 \\ h_1 & h_o & \cdots & h_{L-1} & \vdots \\ \vdots & \vdots & \ddots & \vdots & h_{L-1} \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix}$$

Psudo-periodical transmission

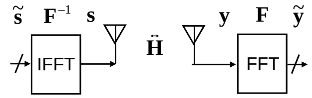
9. OFDM Performance

OFDM Transmission

Diagonalization of cyclic shift matrix

Cyclic shift matrix

$$\ddot{\mathbf{H}} = \begin{bmatrix} h_o & 0 & h_{L-1} & \cdots & h_1 \\ h_1 & h_o & \cdots & h_{L-1} & \vdots \\ \vdots & \vdots & \ddots & \vdots & h_{L-1} \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix}$$



OFDM transmission

$$\mathbf{y} = \ddot{\mathbf{H}}\mathbf{s} + \mathbf{n}$$

$$\tilde{\mathbf{y}} = \mathbf{F}\ddot{\mathbf{H}}\mathbf{s} + \mathbf{n} = \mathbf{F}\ddot{\mathbf{H}}\mathbf{F}^{-1}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

$$= \mathbf{F}\ddot{\mathbf{H}}\mathbf{F}^{-1}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

$$= \mathrm{diag}[\tilde{\mathbf{h}}]\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

K-parallel transmission

Interesting feature of cyclic shift matrix

$$\mathbf{F}\ddot{\mathbf{H}}\mathbf{F}^{-1} = \begin{bmatrix} \tilde{h}_0 & 0 & \cdots & 0 \\ 0 & \tilde{h}_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{h}_{K-1} \end{bmatrix} = \mathrm{diag}[\tilde{\mathbf{h}}]$$

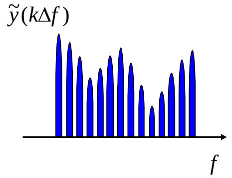
Frequency response

$$\tilde{\mathbf{h}} = \mathbf{F}\mathbf{h}$$

BER Performance

SNR per subcarrier

$$\gamma_k = \frac{P/K\,|\tilde{h}_k|^2}{\sigma^2/K} = \frac{P|\tilde{h}_k|^2}{\sigma^2} \quad \longleftrightarrow \quad \bar{\gamma}_k = \bar{\gamma}$$



PDF of SNR for each subcarrier

$$f(\gamma_k) = \frac{1}{\bar{\gamma}_k}\exp\left(-\frac{\gamma_k}{\bar{\gamma}_k}\right)$$

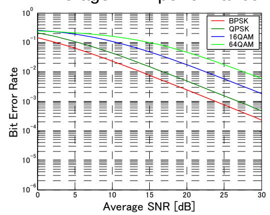
Average BER for each subcarrier

$$\bar{P}_{eb}^{k}(\bar{\gamma}_k) = \int f(\gamma_k) P_{eb}(\gamma_k)\,d\gamma_k$$

Overall average BER

$$\bar{P}_{eb}(\bar{\gamma}) = \frac{1}{K}\sum_{k=0}^{K-1}\bar{P}_{eb}^{k}(\bar{\gamma}_k)$$

Average BER performance



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Channel Capacity

OFDM signal model

$$\tilde{y}_k = \tilde{h}_k \tilde{s}_k + \tilde{n}_k, \quad k = 1, \cdots, K$$

SNR for each subcarrier

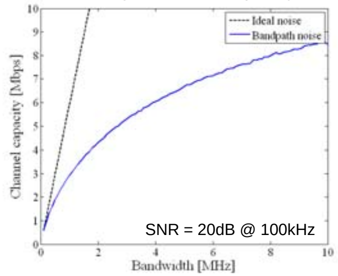
$$\gamma_k = \frac{P/K\,|h_k|^2}{\sigma^2/K} = \frac{P|h_k|^2}{\sigma^2}$$

Channel capacity

$$C = \sum_{k=0}^{K-1}\Delta f \log_2(1+\gamma_k)$$

$$\sigma^2 = BN_0$$

Average channel capacity



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Summary

- In wideband signal
  - Time dispersive fading causes inter-symbol interference
  - Frequency selective fading causes distortion in power spectrum
  - OFDM converts wide band signal to multiple narrow band signals
  - IFFT, FFT, and cyclic prefix creates parallel orthogonal channels
  - Problem of Rayleigh fading still remains even by using OFDM

Some measure for Rayleigh fading

Array signal processing

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