2012 2nd semester
MIMO Communication Systems

#2: Fundamentals of Wireless Communication

Mamoru Sawahashi
<sawahasi@tcu.ac.jp>
Oct. 16, 2012

Agenda

■ Aim of today

Derive throughput performance of basic SISO system

■ Contents

- **Review of Capacity of SISO Channel**
- **Review of Digital Modulation and Detection**
- **Review of Derivation for BER**
- **Review of average BER Performance in Fading Channel**

1. Review of Capacity of SISO Channel

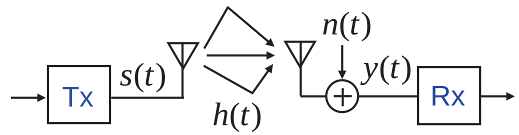
Narrow Band System

Received signal model

Received signal, Additive noise

$$y(t) = h(t)s(t) + n(t)$$

Channel response, Transmit signal



Propagation channel

$$h(t) \approx \mathrm{const}$$

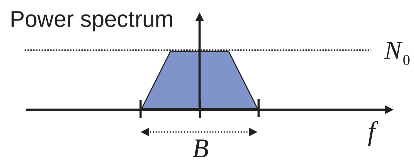
over unit signal packet

Transmit power

$$P = \mathrm{E}[|s(t)|^2]$$

Noise power

$$\sigma^2 = \mathrm{E}[|n(t)|^2] = N_0 B$$



Channel Capacity

Channel capacity of real number system

$$C_R = \frac{1}{T}\max_{E[s^2]\le P} I(S;Y) = \frac{1}{2T}\log_2\left(1+\frac{P|h|^2}{\sigma^2}\right) = \frac{B}{2}\log_2\left(1+\frac{P|h|^2}{\sigma^2}\right)$$

Mutual information

$$I(S;Y) = H(Y) - H(Y\mid S) = H(Y) - H(S+N\mid S) = H(Y) - H(N)$$

Entropy of Gaussian signal

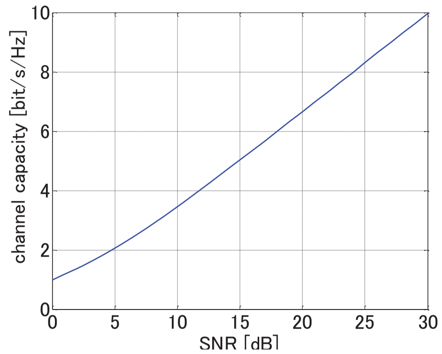
$$H(N) = \frac{1}{2}\log_2 2\pi e\sigma^2 \qquad H(Y) \le \frac{1}{2}\log_2 2\pi e(|h|^2 P + \sigma^2)$$

Channel capacity of complex number system

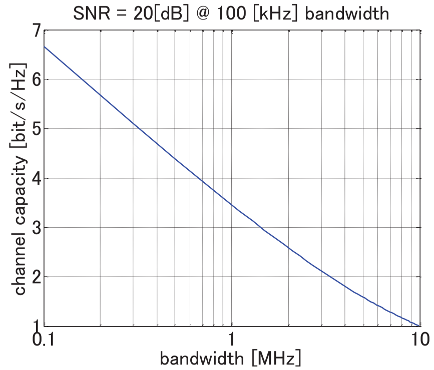
$$C_C = 2\times\frac{B}{2}\log_2\left(1+\frac{P/2\,|h|^2}{\sigma^2/2}\right) = B\log_2\left(1+\frac{P|h|^2}{\sigma^2}\right) \quad \text{[bits/s]}$$

Achievable Data Rate of SISO

Power dependency



Bandwidth dependency



2. Review of Digital Modulation and Detection

Passband Modulation Principles

- **Modulated carrier signals encode information bit in the amplitude *$\alpha(t)$*, frequency *$f(t)$*, or *$\phi(t)$* of a carrier signal**
- **Modulated signal is represented as**

$$s(t) = \alpha(t)\cos[2\pi(f_c + f(t))t + \theta(t) + \phi_0] = \alpha(t)\cos(2\pi f_c t + \phi(t) + \phi_0)$$

where

$\phi(t) = 2\pi f(t)t + \theta(t)$ **and *$\phi_0(t)$* is the carrier offset of a carrier.**

- **We rewrite the right-hand side of the above equation in terms of in-phase and quadrature components as**

$$s(t) = \alpha(t)\cos(\phi(t)+\phi_0)\cos(2\pi f_c t) - \alpha(t)\sin(\phi(t)+\phi_0)\sin(2\pi f_c t)$$
$$= s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

where $s_I(t) = \alpha(t)\cos(\phi(t)+\phi_0)$ ➔ **in-phase component of *s(t)***

$s_Q(t) = \alpha(t)\sin(\phi(t)+\phi_0)$ ➔ **quadrature component of *s(t)***

- **We rewrite *s(t)* in terms of its equivalent lowpass representation as**

$$s(t) = \mathrm{Re}\left[u(t)e^{j2\pi f_c t}\right]$$

where

$u(t) = s_I(t) + js_Q(t)$ ⇨ **Complex envelope**

Amplitude and Phase Modulation

- Information bit stream is encoded in the amplitude and/or phase of the transmitted signal
- Let T_s and $K = \log_2 M$ be symbol duration and the number of information bits which are encoded in the amplitude and/or phase of the transmitted signal *s(t)*, $0 \le t \le T_s$.
- We rewrite transmitted signal over one symbol duration by using basis function $\phi_1(t) = g(t)\cos(2\pi f_c t + \phi_0)$ and $\phi_2(t) = -g(t)\sin(2\pi f_c t + \phi_0)$ as

$$s(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$$

  where *g(t)* is a shaping pulse
- The *i*-th message is sent over $kT \le t \le (k+1)T$. Then, we set

$$s_I(t) = s_{i1}g(t),\ s_Q(t) = s_{i2}g(t)$$

- The in-phase and quadrature signal components are baseband signals with the spectral characteristics determined by the pulse shape, *g(t)*.
  - Bandwidth of in-phase or quadrature component, *B*, is equal to that of *g(t)*
  - Bandwidth of the transmitted signal *s(t)* becomes *2B*.

Amplitude and Phase Modulation

- Signal constellation for amplitude and phase modulation is defined based on the constellation points $\{(s_{i1}, s_{i2}) \in \Re^2, i = 1, \dots, M\}$
- Equivalent lowpass signal of *s(t)* is represented as

$$s(t) = \mathrm{Re}\left[x(t)e^{j\phi_0}e^{j2\pi f_c t}\right]$$

  where

$$x(t) = (s_{i1} + js_{i2})g(t)$$

  - $s_i = (s_{i1}, s_{i2})$ ➔ symbol associated with the $\log_2 M$ bits
  - T_s ➔ symbol duration (symbol time)
  - Bit rate: $K = \log_2 M$ bits per symbol

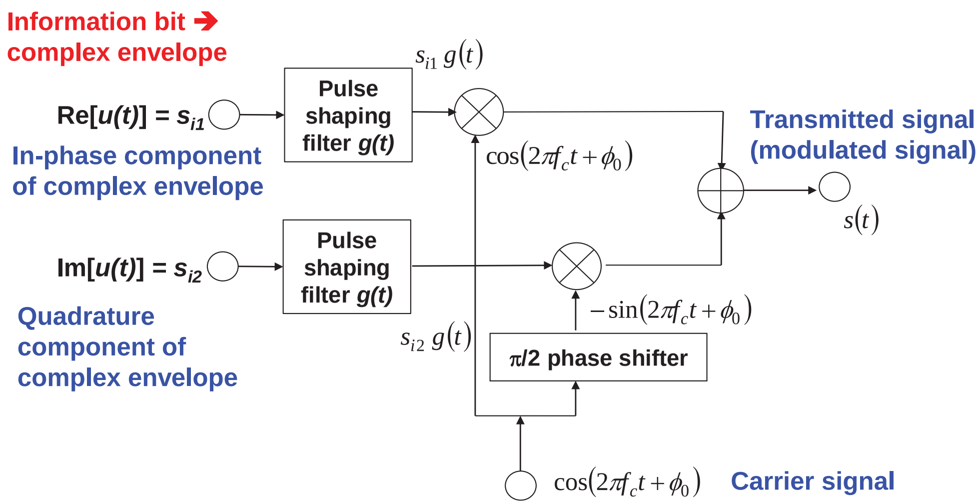
- Number of bits per symbol: $K = \log_2 M$
- Signal constellation: $\{s_i, i = 1, \dots, M\}$
- Pulse shaping filter: g(t)

➔ Digital modulation design

Decided to improve spectral efficieny and conbat inter-symbol interference (ISI)

Amplitude/Phase Modulator

Amplitude/phase modulator with the structure below is called "quadrature modulator "

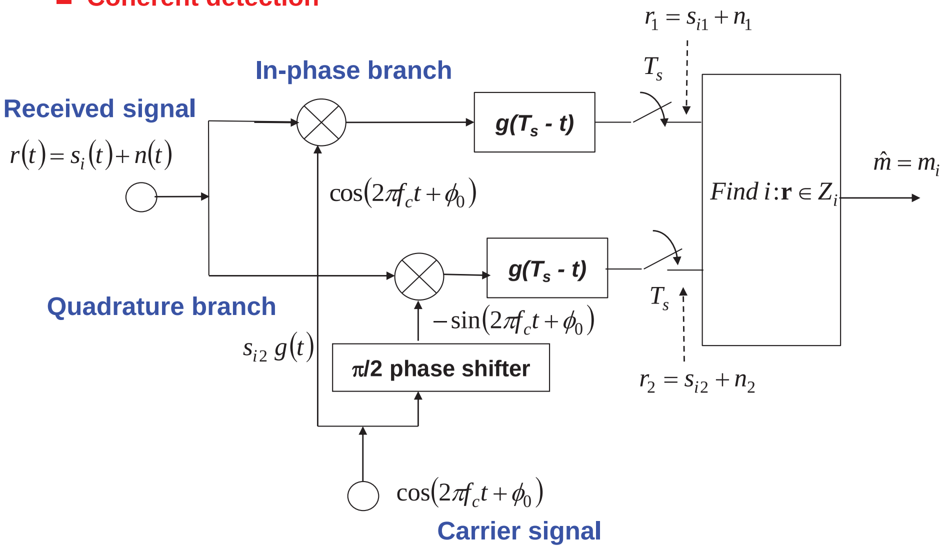


Digital Modulation Schemes

Radio parameters carrying information bits	Analog modulation	Digital modulation
Carrier wave amplitude: information encoded in amplitude only	AM (Amplitude Modulation)	ASK (Amplitude Shift Keying) or PAM (pulse Amplitude Modulaiton)
Carrier wave frequency: information encoded in frequency only	FM (Frequency Modulation)	FSK (Frequency Shift Keying)
Carrier wave phase: information encoded in phase only	PM (Phase Modulation)	PSK (Phase Shift Keying)
Carrier wave amplitude and phase: information encoded in both amplitude and phase		QAM (Quadrature Amplitude Modulation) ➔ ASK + PSK

Amplitude/Phase Demodulator

■ Coherent detection



Pulse Amplitude Modulation (1)

- Linear modulation, i.e., one-dimensional pulse amplitude modulation (PAM)
- All the information is encoded into the signal amplitude, A_i.
- Transmitted signal for one-dimensional PAM signal over one symbol duration is given as

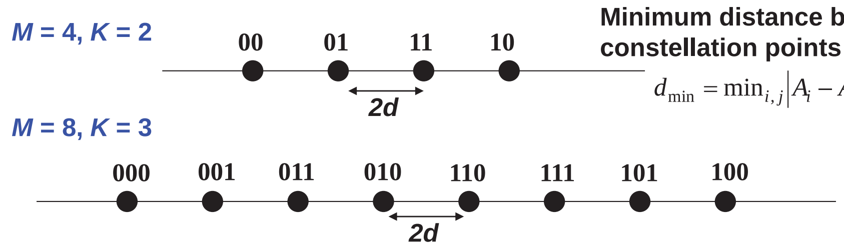
$$s_i(t)=\mathrm{Re}\left[A_i g(t)e^{j2\pi f_c t}\right]=A_i g(t)\cos(2\pi f_c t),\quad 0\le t\le T_s$$

where

$$A_i=(2i-1-M)d,\ i=1,2,\ldots,M$$

- Signal constellation is $\{A_i,\ i=1,2,\ldots,M\}$.

Minimum distance between constellation points ➔ $d_{\min}=\min_{i,j}|A_i-A_j|=2d$



Pulse Amplitude Modulation (2)

- Each pulse carries $K=\log_2 M$ bits per symbol duration T_s.
- Signal energy with the *i*-th constellation becomes as

$$E_{si}=\int_0^{T_s} s_i^2(t)dt=\int_0^{T_s} A_i^2 g^2(t)\cos^2(2\pi f_c t)dt=A_i^2$$

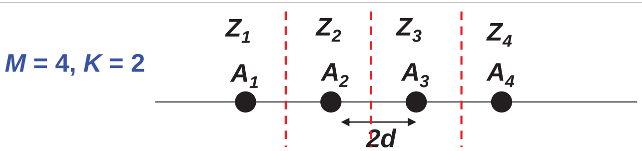
- Hence, average signal energy per symbol becomes as

$$\bar{E}_s=\frac{1}{M}\sum_{i=1}^{M}A_i^2$$

■ Decision method

- Decision region $Z_i,\ i=1,2,\ldots,M$ associated with pulse amplitude $A_i=(2i-1-M)d,\ i=1,2,\ldots,M$ becomes as

$$Z_i=\begin{cases}(-\infty, A_i+d)\ for\ i=1\\ [A_i-d, A_i+d)\ for\ 2\le i\le M-1\\ [A_i-d,\infty)\ for\ i=M\end{cases}$$



Phase Shift Keying (1)

- In PSK, all the information is encoded into the phase of the transmitted signal
- Transmitted signal using PSK over one symbol duration is given as

$$s_i(t)=\mathrm{Re}\left[Ag(t)e^{\frac{j2\pi(i-1)}{M}}e^{j2\pi f_c t}\right]=Ag(t)\cos\left[2\pi f_c t+\frac{2\pi(i-1)}{M}\right]$$

$$=Ag(t)\cos\left[\frac{2\pi(i-1)}{M}\right]\cos 2\pi f_c t-Ag(t)\sin\left[\frac{2\pi(i-1)}{M}\right]\sin 2\pi f_c t,\quad for\ 0\le t\le T_s$$

- Constellation points or symbols (s_{i1}, s_{i2}) are gives as

$$s_{i1}=A\cos\left[\frac{2\pi(i-1)}{M}\right],\quad s_{i2}=A\sin\left[\frac{2\pi(i-1)}{M}\right]\quad for\ i=1,\ldots,M$$

- Parameters $\theta_i=\frac{2\pi(i-1)}{M}\ for\ i=1,\ldots,M=2^K$ are the different phases in the signal constellation points that carry information bits.

Phase Shift Keying (2)

Decision method

- We represent the received signal $\mathbf{r} = r_1 + jr_2 = re^{j\theta} \in \Re^2$ in polar coordinates.
- Decision region for PSK signal is given as

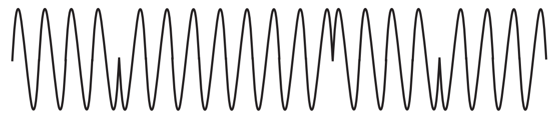
$$Z_i(t) = \left\{ re^{j\theta} : \frac{2\pi(i-1.5)}{M} \le \theta \le \frac{2\pi(i-0.5)}{M} \right\}$$

BPSK

Binary PSK (BPSK)

- In BPSK, one symbol carries 1 bit.
- Since amplitude is constant, we set $A_i(t)= A$とすると,

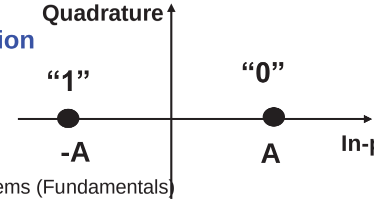
$$s_i(t) = \begin{cases} Ag(t)\cos(2\pi f_c t + 0) = Ag(t)\cos(2\pi f_c t) \; for\, bit\, "0" \\ Ag(t)\cos(2\pi f_c t + \pi) = -Ag(t)\cos(2\pi f_c t) \; for\, bit\, "1" \end{cases}$$



Carrier frequency such as 800 MHz, 2 GHz

0	π	π	*0*	π	Modulated carrier phase

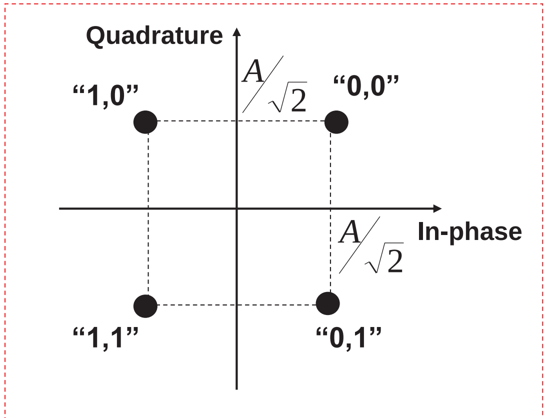
Signal constellation for BPSK



QPSK (1)

In general, initial phase is offset by $\pi/4$.

Signal constellation which is used in commercial system including W-CDMA, LTE.



In QPSK, in-phase component *Re[u(t)]* and quadrature component *Im[u(t)]* become $1/\sqrt{2}$ times compared to the amplitude of complex envelope ➔ average transmission power is identical regardless of modulation scheme.

QPSK (2)

- Transmitted signal for a combination of bits "0,0"
  ➔ amplitude of complex envelope is constant $a_m(t) = A$

$s(t) = A\cos(2\pi f_c t + \pi/4)$
$= A\cos(\pi/4)\cos(2\pi f_c t) - A\sin(\pi/4)\sin(2\pi f_c t)$
$= \frac{A}{\sqrt{2}}\cos(2\pi f_c t) - \frac{A}{\sqrt{2}}\sin(2\pi f_c t)$

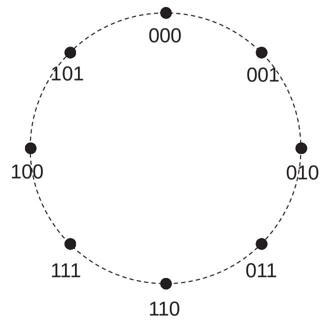
Complex envelope for QPSK is given as

- $Re[u(t)] = A/\sqrt{2}$, $Im[u(t)] = A/\sqrt{2}$ for bits "0,0"
- $Re[z(t)] = -A/\sqrt{2}$, $Im[z(t)] = A/\sqrt{2}$ for bits "1,0"
- $Re[z(t)] = -A/\sqrt{2}$, $Im[z(t)] = -A/\sqrt{2}$ for bits "1,1"
- $Re[z(t)] = A/\sqrt{2}$, $Im[z(t)] = -A/\sqrt{2}$ for bits "0,1"

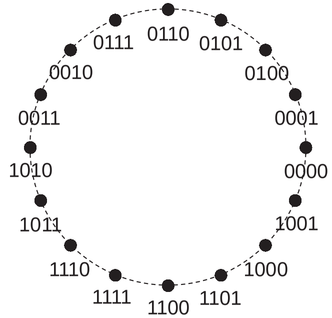
MPSK

- In general, *M*-phase PSK (MPSK) carriers $K = \log_2 M$ information bits.
- *M* constellation points are mapped with equal phase difference of $2\pi/M = 2\pi/2^K$ (radian)

- 8PSK
  ➔ carry 3 bits per symbol



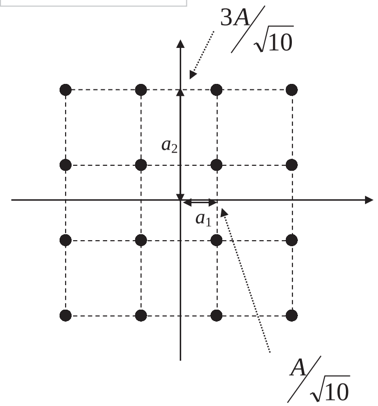
- 16PSK
  ➔ carry 4 bits per symbol



QAM (1)

- Combination of ASK and PSK
- Square QAM is used since Euclidean distance between constellation points is longer compared to that of other constellations ➔ provide better bit error rate (BER).

- 16QAM

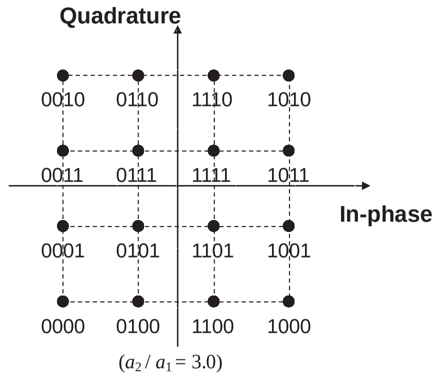


- Since $a_2 = 3a_1$, we set $a_1 = x$.
- Summation of squared values of amplitudes of the 4 constellation points in the 1st quadrant is given as
  ($2x^2 + 10x^2 + 10x^2 + 18x^2$).
- The average signal power should be identical to that of the signal power of PSK including 16PSK
  ($2x^2 + 10x^2 + 10x^2 + 18x^2$)/4 = A^2
- *$X^2 = A^2/10$*

$$x = A/\sqrt{10}$$

QAM (2)

- Gray mapping is used
- 16QAM carries 4 bits per one symbol duration.



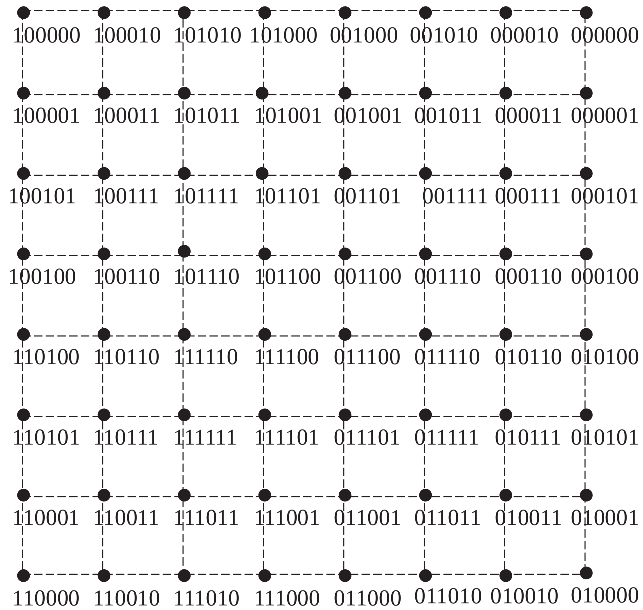
16QAM signal constellation with Gray mapping

- Minimum distance between constellation points
  - 16QAM ➔ $2A/\sqrt{10}$
  - 16PSK ➔ $2A\sin(\pi/16)$

- 16QAM is used in commercial systems including HSDPA, LTE and WiMAX.

QAM (3)

- Signal constellations of square 64QAM scheme.



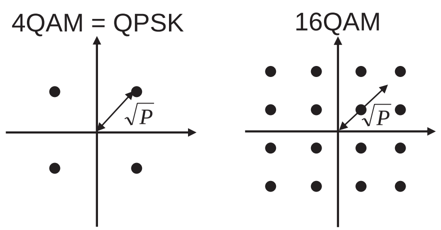
QAM Modulation

QAM modulation

- Message

$m_R = \{0,1,\cdots,\sqrt{M_{ary}}-1\} \quad m_I = \{0,1,\cdots,\sqrt{M_{ary}}-1\}$

M_{ary} QAM $\longrightarrow$ $r = \log_2 M_{ary}$ [bits/symbol]



- Symbol

$s = s_R + js_I$

$= (2m_R - \sqrt{M_{ary}} + 1) + j(2m_I - \sqrt{M_{ary}} + 1)$

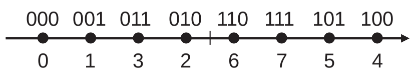
Power normalization

$$P = 2\left(\frac{2}{\sqrt{M_{ary}}}\sum_{i=1}^{\sqrt{M_{ary}}/2}(2i-1)^2\right) = \frac{2(M_{ary}-1)}{3} \longrightarrow s = s/\sqrt{P}$$

Gray coding

Adjacent symbols differ by one binary digit

$s_R = \text{gray}_{\sqrt{M_{ary}}}(m_R)$ Ex. $\text{gray}_8(m_R)$



Modulation Schemes in Cellular Systems

2.0G	3.0G	3.5G	3.9G	4.0G
PDC	W-CDMA	HSDPA/ HSUPA	LTE	LTE-Advanced

Dedicated channel
(voice traffic with constant rate)

Shared Channel
(focus on data traffic with variable rate)

QPSK

- "Release" means version of specification

■ HSDPA
- Rel. 5: QPSK, 16QAM
- Rel. 8: 64QAM is added

■ HSUPA
- Rel. 6: QPSK
- Rel. 8: 16QAM is added

■ Rel. 8: LTE
- DL: QPSK, 16QAM, 64QAM
- UL: QPSK, 16QAM, (64QAM is optional)

■ Rel. 10: LTE-Advanced

➔ Identical to those in Rel. 8 LTE

3. Review of Derivation for BER

E_b/N_0 (1)

■ Definition of E_b/N_0
- E_b: signal energy per information bit
- N_0: noise power per Hertz, i.e., noise power spectrum density.

➔ E_b/N_0 indicates signal energy per bit-to-noise power spectrum density ratio

● Reason why we use E_b/N_0
- As explained, the number of information bits per symbol duration (i.e., per Hertz) is different according to the modulation scheme used.
- Hence, in some times, we want to investigate system performance focusing on the applied technique regardless of modulation scheme ➔ normalized SNR carrying one information bit, i.e., E_b/N_0

E_b/N_0 (2)

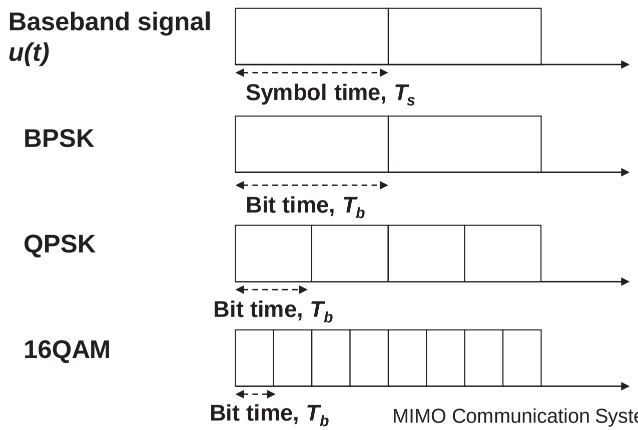
■ Relation between SNR and E_b/N_0
- $E_b = S / R_b = ST_b$
  - R_b: information bit rate
  - T_b: Bit time (bit duration)
- $N_0 = N / B$
  - B: Signal bandwidth (note that we do not need consider noise outside signal bandwidth)

● $E_b/N_0 = (B/R_b) \times SNR$

Relationship Between Bit and Symbol

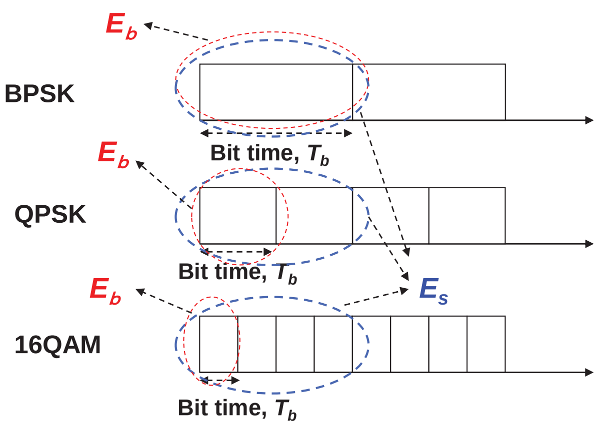
■ Assuming symbol rate of $f_s = 1/T_s$ (T_s is symbol time), signal transmission bandwidth is represented as $(1+\alpha) f_s = (1+\alpha) / T_s$ (α indicates roll-off factor of raise root cosine Nyquist filter)

■ Assuming bit rate of $f_b = 1/T_b$ (T_b is bit time), relationship between symbol time and bit time of modulation scheme is given below.



Signal Energy per Bit

- E_b: signal energy per information bit
- E_s: signal energy per information symbol



■BPSK
Carry 1 bit per symbol
$E_s = E_b$

■QPSK
Carry 2 bits per symbol
$E_s = 2E_b$

■16QAM
Carry 4 bits per symbol
$E_s = 4E_b$

Maximum Likelihood Detection (1)

● Receive signal $r(t)$ is represented as

$$r(t) = hs(t) + n(t)$$

- $s(t)$: transmitted signal
- h: fading complex envelope, channel response ➔ channel variation of amplitude and phase in propagation channel
- $n(t)$: background noise

■ Consider the case when we transmit bit"1" or "0 (-1)" (bit "1" and "0" occur with equal probability)

- Signal pulse $s_1(t)$ for signal S_1 which corresponds to bit "1" and signal pulse $s_0(t)$ for signalS_0 which corresponds to bit "0".
- Either of pulse signals is transmitted in the T_b duration (T_b : pulse duration)

● Maximum Likelihood Detection (MLD) ➔ either of S_0 or S_1, which provides smaller difference between the received signal $r(t)$ and hS_0 or hS_1,i.e., high correlation, is more likely.

$$\int_0^{T_b} (r(t) - hs_0(t))^2 dt < \int_0^{T_b} (r(t) - hs_1(t))^2 dt \Rightarrow S_0$$

$$\int_0^{T_b} (r(t) - hs_0(t))^2 dt > \int_0^{T_b} (r(t) - hs_1(t))^2 dt \Rightarrow S_1$$

Maximum Likelihood Detection (2)

- MLD ➔ either of S_0 or S_1, which provides smaller difference between the received signal *r(t)* and hS_0 or hS_1, i.e., high correlation, is more likely.

$$\int_0^{T_b}(r(t)-hs_0(t))^2dt<\int_0^{T_b}(r(t)-hs_1(t))^2dt\Rightarrow S_0$$
$$\int_0^{T_b}(r(t)-hs_0(t))^2dt>\int_0^{T_b}(r(t)-hs_1(t))^2dt\Rightarrow S_1$$

Decision variable

$$v=\int_0^{T_b}r(t)h[s_0(t)-s_1(t)]dt>\frac{1}{2}\int_0^{T_b}h^2\left[s_0^2(t)-s_1^2(t)\right]dt\Rightarrow S_0$$
$$v=\int_0^{T_b}r(t)h[s_0(t)-s_1(t)]dt<\frac{1}{2}\int_0^{T_b}h^2\left[s_0^2(t)-s_1^2(t)\right]dt\Rightarrow S_1$$

BER in AWGN Channel (1)

- Consider bit error rate (BER) when transmitting 1 bit in the duration of $[0,T_b]$ in additive white Gaussian noise (AWGN) channel ➔ corresponds to BER for BPSK.

- Transmit signal $s_1(t)$ for bit "1" or signal $s_0(t)$ for bit "0 (-1)".
- Signal energy per information bit E_b is represented as

$$E_b=h^2\int_0^{T_b}s_0^2(t)dt=h^2\int_0^{T_b}s_1^2(t)dt$$

where E_b indicates signal energy required for transmitting 1 information bit.

- Receiver decides which signal is transmitted between $s_1(t)$ and $s_0(t)$

➔ The decision criterion is to decide plus or minus for the decision variable, i.e., difference of respective correlations between *r(t)* and $s_0(t)$ or *r(t)* and $s_1(t)$.

$$v=\int_0^{T_b}r(t)hs_0(t)dt-\int_0^{T_b}r(t)hs_1(t)dt>0\Rightarrow S_0$$
$$v=\int_0^{T_b}r(t)hs_0(t)dt-\int_0^{T_b}r(t)hs_1(t)dt<0\Rightarrow S_1$$

BER in AWGN Channel (2)

- Decision variable *v* is Gaussian variable with the average of $u=\bar{v}$ and variance of $\sigma^2=\langle(v-\bar{v})^2\rangle$.
- Assume the case when bit "0" is transmitted ➔ $r(t)=hs_0(t)+n(t)$

$$\bar{v}=\overline{\int_0^{T_b}r(t)hs_0(t)dt-\int_0^{T_b}r(t)hs_0(t)dt}$$
$$=\int_0^{T_b}\overline{r(t)}\cdot\overline{h(s_0(t)-s_1(t))}dt$$
$$=\int_0^{T_b}hs_0(t)h(s_0(t)-s_1(t))dt=E_b(1-\rho)$$

$$\overline{r(t)}=hs_0(t)\ \because\overline{n(t)}=0$$

$$v-\bar{v}=\int_0^{T_b}r(t)hs_0(t)dt-\int_0^{T_b}r(t)hs_1(t)dt-\int_0^{T_b}hs_0(t)h(s_0(t)-s_1(t))dt$$
$$=\int_0^{T_b}[hs_0(t)+n(t)]hs_0(t)dt-\int_0^{T_b}[hs_0(t)+n(t)]hs_1(t)dt-\int_0^{T_b}hs_0(t)h(s_0(t)-s_1(t))dt$$
$$=\int_0^{T_b}\{[hs_0(t)+n(t)]hs_0(t)-[hs_0(t)+n(t)]hs_1(t)-hs_0(t)h(s_0(t)-s_1(t))\}dt$$
$$=\int_0^{T_b}\{n(t)hs_0(t)-n(t)hs_1(t)\}dt=\int_0^{T_b}n(t)h(s_0(t)-s_1(t))dt$$

BER in AWGN Channel (3)

- Hence, the variance is derived as

$$\sigma^2=\left\langle(v-\bar{v})^2\right\rangle=\left\langle\left[\int_0^{T_b}n(t)h(s_0(t)-s_1(t))dt\right]^2\right\rangle$$
$$=\int_0^{T_b}\int_0^{T_b}\langle n(t)n(t')\rangle h^2(s_0(t)-s_1(t))(s_0(t')-s_1(t'))dtdt'$$
$$=\frac{N_0}{2}\int_0^{T_b}h^2(s_0(t)-s_1(t))^2dt=N_0E_b(1-\rho)$$

where

$$\rho=\frac{1}{E_b}h^2\int_0^{T_b}s_0(t)s_1(t)dt$$

$$\langle n(t)n(t')\rangle=\frac{N_0}{2}\delta(t-t')$$

➔ Noise components at different times are uncorrelated

- Probability density function (PDF) for decision variable *v* when transmitting bit "0" is given as

$$p(v|S_0)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(v-\bar{v})^2}{2\sigma^2}}$$

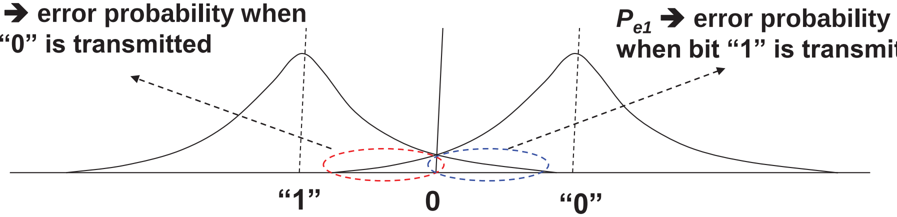
BER in AWGN Channel (4)

- Error occurs when decision variable ν becomes minus value ➔ BER P_{e0} is given as

$$P_{e0} = \int_{-\infty}^{0} p(\nu|S_0)d\nu = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{0} \exp\left[-\frac{(\nu-u)^2}{2\sigma^2}\right]d\nu = \frac{1}{2}\mathrm{erfc}\frac{u}{\sqrt{2}\sigma}$$

where erfc(x) indicates complementary error function

$$\mathrm{erfc}(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} \exp(-t^2)dt$$



Probability distribution function in Gaussian channel

BER in AWGN Channel (5)

$$P_{e0} = \frac{1}{2}\mathrm{erfc}\frac{u}{\sqrt{2}\sigma} = \frac{1}{2}\mathrm{erfc}\sqrt{\frac{E_b(1-\rho)}{2N_0}}$$

- BER is decided from the 3 factors: average signal energy per bit E_b, noise power spectrum density N_0, cross correlation between transmitted signals ρ.
- Assuming constant E_b and N_0 values, BER is decided only by the correlation factor ρ between $s_1(t)$ and $s_0(t)$. ➔ BER becomes minimum when ρ = -1, i.e., $s_1(t) = - s_0(t)$. ➔ antipodal signal.
- BPSK is typical example of antipodal signal.
- Considering the symmetry, BER for transmitting bit "1" becomes $P_{e1} = P_{e0}$.
- BER for BPSK in AWGN channel (non-fading channel) becomes as.

$$P_e = \frac{1}{2}(P_{e0} + P_{e1}) = \frac{1}{2}\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}$$

BER of Coherent Detection for BPSK in AWGN Channel

- BER for BPSK in Gaussian channel is givens as

$$P_e = \frac{1}{2}\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}$$

$$p_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{SNR}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_s}{N_0}}\right)$$

In BPSK, $E_s = E_b$ and SNR = E_s/N_0

BER of Coherent Detection for QPSK in AWGN Channel

- In QPSK, coherent detection is performed for in-phase and quadrature components individually.
- Amplitude of symbol, i.e., constellation point of QPSK becomes $1/\sqrt{2}$ that of BPSK.
- Hence, BER is given as

$$P_e = \frac{1}{2}\mathrm{erfc}\sqrt{\frac{E_s}{2N_0}} = \frac{1}{2}\mathrm{erfc}\sqrt{\frac{SNR}{2}}$$

- In QPSK, $E_s = 2E_b$. BER using E_b is given as

$$p_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- BER using coherent detection as a function of E_b/N_0 is identical between BPSK and QPSK.
- But, BER as a function of E_s/N_0 for QPSK is degraded by 3 dB compared to that for BPSK.

BER on QPSK Signaling

Channel estimation

$$\hat{h} = \int \frac{y_T(t)}{s_T(t)} dt$$

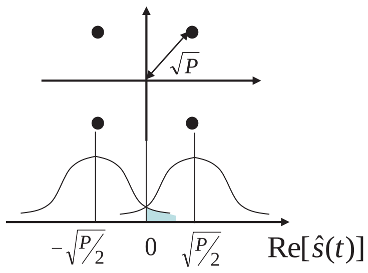
Coherent detection

$$\hat{s}(t) = \frac{y(t)}{\hat{h}} = s(t) + \frac{n(t)}{\hat{h}}$$

Bit error rate (BER)

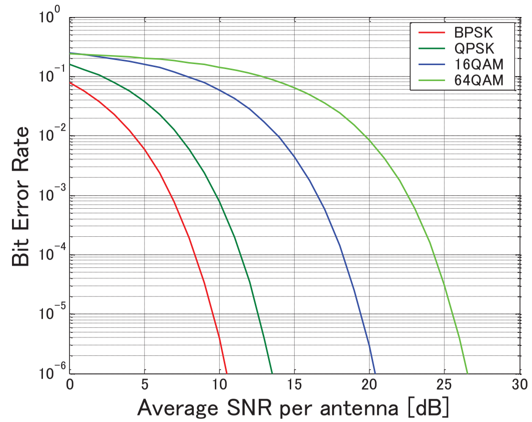
$$P_{eb}(\gamma) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{\gamma}{2}}\right) \qquad \gamma = \frac{P|\hat{h}|^2}{\sigma^2}$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-z^2) dz$$



BER Performance (AWGN Channel)

QAM signaling



Throughput Performance

Frame structure

Preamble	Data

Data length L

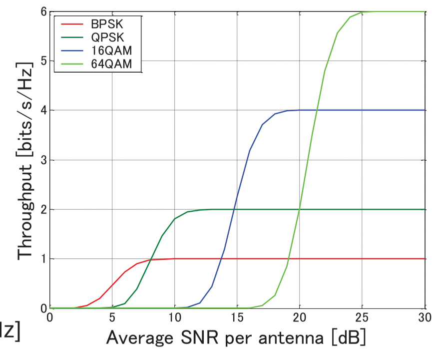
Packet error rate

$$P_{ep}(\gamma) = 1 - (1 - P_{eb}(\gamma))^L$$

Throughput

$$TP(\gamma) = \log_2 M_{ary} (1 - P_{eb}(\gamma))^L \quad \text{[bits/s/Hz]}$$

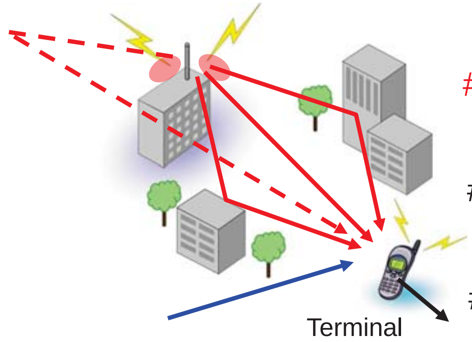
Throughput performance



4. Review of Average BER Performance in Fading Channel

Wireless Communication Channel

Wireless is vulnerable!



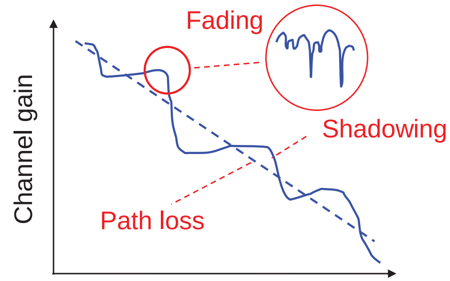
#1: Path loss due to distance
→ Lower average SNR
#2: Multi-path fading
→ Deep SNR dip
#3: Time dispersive fading
→ Inter symbol interference
#4: Multi-access interference
→ Co-channel interference

Issue #1: Path loss

- Channel gain decreases in accordance with distance between Tx & Rx
- It results in lower SNR, higher BER, and lower throughput

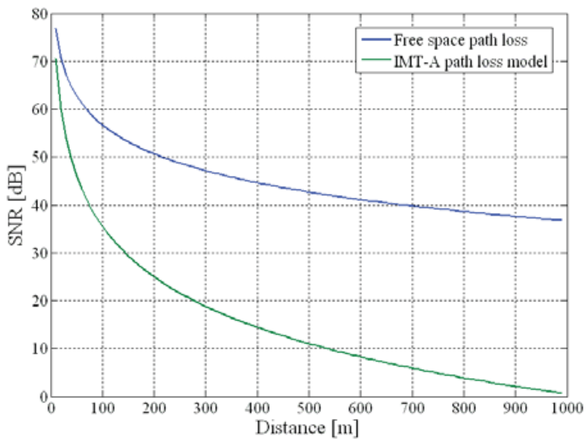
Free space path loss

$$G_{pl} = 10\log_{10}\left(\frac{\lambda}{4\pi d}\right)^2$$
$$= -20\log_{10} d - 20\log_{10} f + 28$$



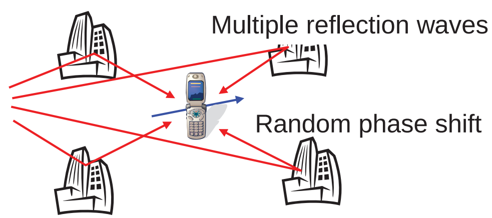
Received SNR

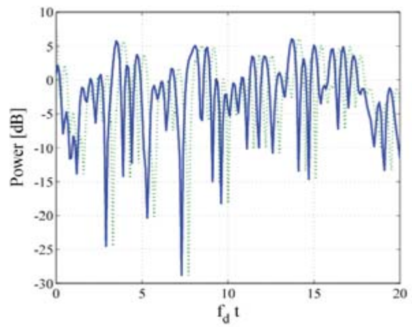
Transmit power = 40dBm, Noise power = -100dBm, Frequency = 3.5GHz



Issue #2: Multipath Fading

- Instantaneous SNR variation due to multi-path fading
- Deep SNR dip results in serious BER performance

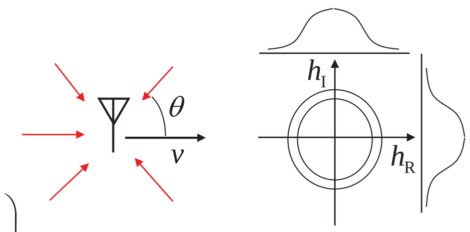




Multipath Rayleigh Fading

Multi-path channel

$$h(t)=\sum_l \beta_l \exp\left(j2\pi\frac{vt}{\lambda}\cos\theta\right)$$



Central limit theorem

$$f(h_R)=f(h_I)=\frac{1}{\sqrt{2\pi\alpha}}\exp\left(-\frac{x^2}{2\alpha^2}\right)$$

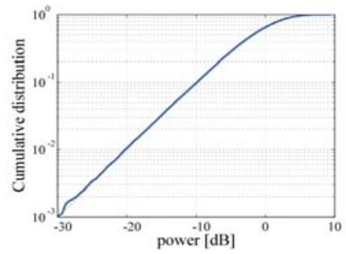
$$\alpha^2=E[|h_R|^2]=E[|h_I|^2]$$

PDF of channel power or SNR

$$f(\gamma)=\frac{1}{\bar\gamma}\exp\left(-\frac{\gamma}{\bar\gamma}\right)$$

$\gamma=|h|^2$ or $\gamma=\frac{P|h|^2}{\sigma^2}$

Cumulative distribution



BER Performance in Fading Environment

Instantaneous BER

$$P_{eb}(\gamma)=\frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\gamma}{2}}\right)\qquad \gamma=\frac{P|h(t)|^2}{\sigma^2}$$

PDF of SNR

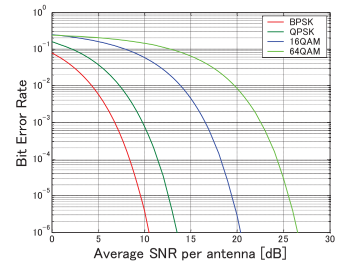
$$f(\gamma)=\frac{1}{\bar\gamma}\exp\left(-\frac{\gamma}{\bar\gamma}\right)\qquad \bar\gamma=\mathrm{E}\left[\frac{P|h(t)|^2}{\sigma^2}\right]$$

Average BER & TP

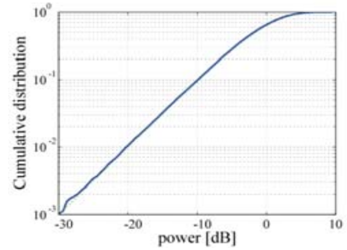
$$\bar P_{eb}(\bar\gamma)=\int f(\gamma)P_{eb}(\gamma)\mathrm{d}\gamma=\frac{1}{2}\left(1-\sqrt{\frac{\bar\gamma}{2+\bar\gamma}}\right)$$

$$\overline{TP}(\bar\gamma)=\int f(\gamma)TP(\gamma)\mathrm{d}\gamma$$

BER performance

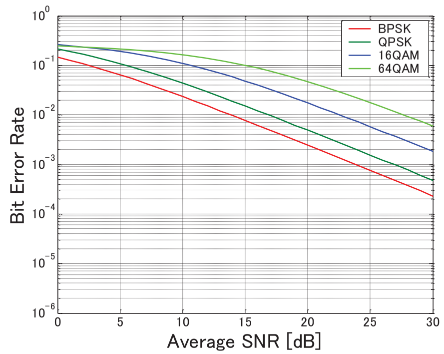


Cumulative distribution

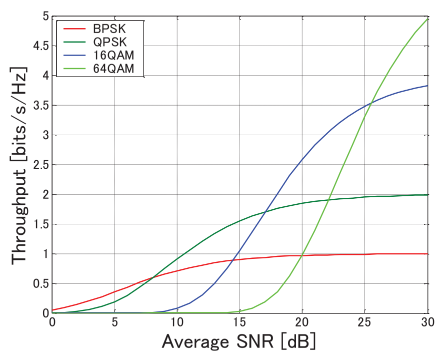


BER & Throughput Performance

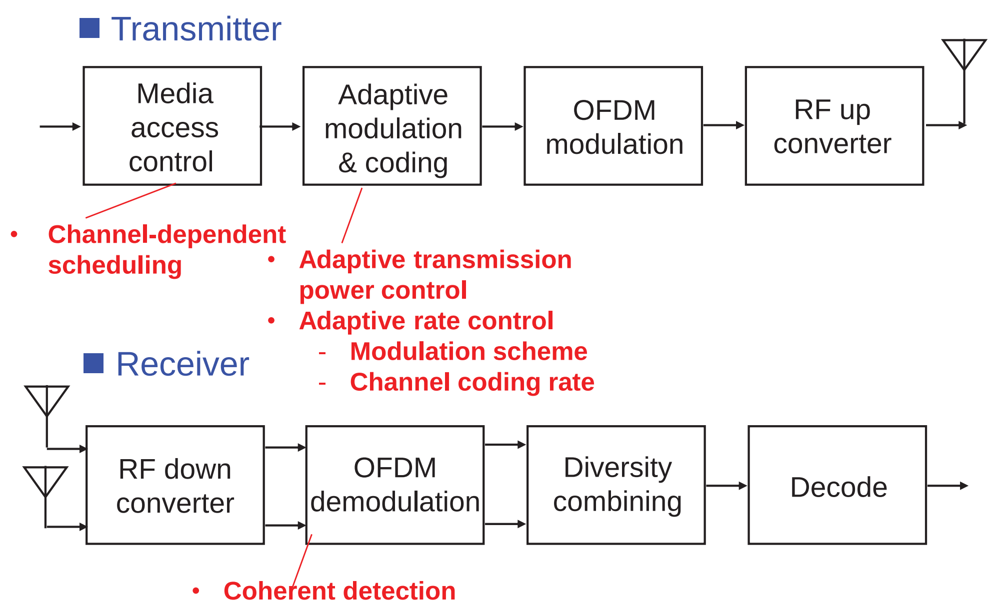
Average BER performance



Average TP performance



Modern Wireless Transceiver



Summary

- In narrow system
  - Channel capacity is achieved by Gaussian signaling
  - Path loss & multi-path fading are major issues in wireless communication channel
  - QAM for variable rate modulation with limited constellation
  - Derive throughput performance of QAM signal via BER calculation
  - Throughput performance degrades severely due to multi-path fading

For wideband system

OFDM for wireless broadband