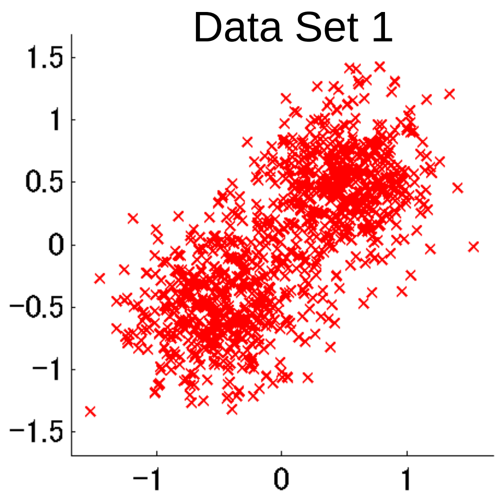
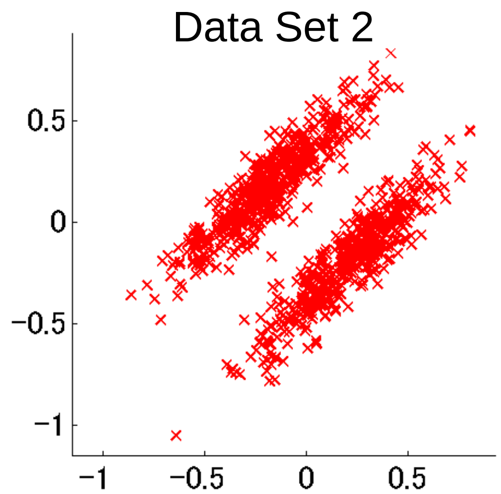
Homework

1. Implement PCA and reproduce the 2-dimensional examples shown in the class.
   - Data sets 1 and 2 are available from

http://sugiyama-www.cs.titech.ac.jp/~sugi/data/DataAnalysis





   - Test PCA with your own (artificial or real) data and analyze the characteristics of PCA.

Homework (cont.)

2. Let

- $\boldsymbol{B}$: $m \times d, (1 \leq m \leq d)$
- $\boldsymbol{C}, \boldsymbol{D}$: $d \times d$, positive definite, symmetric
- $\{\lambda_i, \boldsymbol{\psi}_i\}_{i=1}^{m}$: Sorted generalized eigenvalues and normalized eigenvectors of $\boldsymbol{C}\boldsymbol{\psi} = \lambda \boldsymbol{D}\boldsymbol{\psi}$

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d \qquad \langle \boldsymbol{D}\boldsymbol{\psi}_i, \boldsymbol{\psi}_j \rangle = \delta_{i,j}$$

Prove that a solution of

$$\boldsymbol{B}_{min} = \underset{\boldsymbol{B} \in \mathbb{R}^{m \times d}}{\operatorname{argmin}} \left[\operatorname{tr}(\boldsymbol{B}\boldsymbol{C}\boldsymbol{B}^\top) \right]$$

$$\text{subject to } \boldsymbol{B}\boldsymbol{D}\boldsymbol{B}^\top = \boldsymbol{I}_m$$

is given by

$$\boldsymbol{B}_{min} = (\boldsymbol{\psi}_d | \boldsymbol{\psi}_{d-1} | \cdots | \boldsymbol{\psi}_{d-m+1})^\top$$

Homework (cont.)

3. Prove that PCA uncorrelates the samples; more specifically, prove that the covariance matrix of the PCA-embedded samples is the following diagonal matrix:

$$\sum_{i=1}^{n} \overline{\boldsymbol{z}}_i \overline{\boldsymbol{z}}_i^\top = \mathrm{diag}\left(\lambda_1, \lambda_2, \ldots, \lambda_m\right)$$

$$\overline{\boldsymbol{z}}_i = \boldsymbol{B}_{PCA} \overline{\boldsymbol{x}}_i$$

$$\boldsymbol{B}_{PCA} = (\boldsymbol{\psi}_1 | \boldsymbol{\psi}_2 | \cdots | \boldsymbol{\psi}_m)^\top$$

Suggestion

- Read the following article for upcoming classes:
  - X. He & P. Niyogi: Locality preserving projections, In *Advances in Neural Information Processing Systems 16*, MIT Press, Cambridge, MA, 2004.

http://books.nips.cc/papers/files/nips16/NIPS2003_AA20.pdf