2011 1st semester
MIMO Communication Systems

#10: Multi-User MIMO

Kei Sakaguchi
<sakaguchi@mobile.ee.titech.ac.jp>

June 21, 2011

Schedule (2^{nd} half)

	Date	Text	Contents
#7	May 31	A-5	MIMO receiver
#8	June 7	A-3, 4	MIMO transmitter
#9	June 14	B-9	Adaptive commun. system
#10	June 21	A-6, B-14	Multi-user MIMO
#11	June 28	B-15, 16	Distributed MIMO networks
#12	July 5		Standardization of MIMO
	July 12		Final Examination

Agenda

■ Aim of today

Derive throughput performances
of multi-user MIMO communication systems

■ Contents

- Multi-user communications
- MIMO Multiple Access (MA)
- MIMO Broadcast (BC)
- Adaptive multi-user MIMO
- User & antenna scheduling
- Measurement experiment

Warming Up

Question

Given a matrix $\mathbf{H} = [\mathbf{h}_1 \quad \mathbf{h}_2] \in C^{2\times 2}$, calculate a matrix $\mathbf{Q}_{\mathrm{D}}$ to diagonalize $\mathbf{H}$ and unitary matrices $\mathbf{Q}_{\mathrm{R}}, \mathbf{Q}_{\mathrm{L}}$ to upper and lower triangulize $\mathbf{H}$ by using subspace decomposition of the vectors $\mathbf{h}_1, \mathbf{h}_2$.

$$\mathbf{Q}_{\mathrm{D}}^H \mathbf{H} = \begin{bmatrix} \Omega_{11} & 0 \\ 0 & \Omega_{22} \end{bmatrix} \qquad \mathbf{Q}_{\mathrm{R}}^H \mathbf{H} = \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \qquad \mathbf{Q}_{\mathrm{L}}^H \mathbf{H} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}$$

Subspace decomposition using eigenvalue decomposition

Correlation matrix: $\mathbf{R}_x = \mathbf{h}_x \mathbf{h}_x^H$ Eigen vectors: $\mathbf{E}_x = [\mathbf{h}_x^{\parallel} \quad \mathbf{h}_x^{\perp}]$

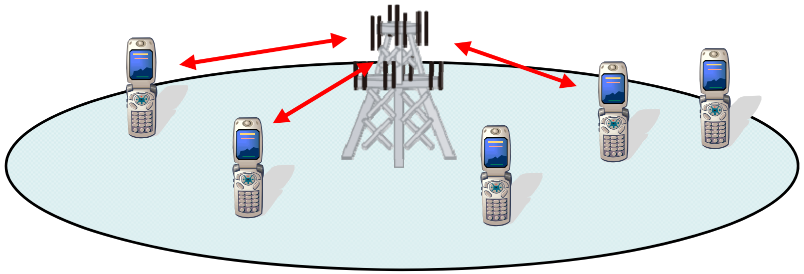
Eigen-decomposition: $\mathbf{R}_x = \mathbf{E}_x \mathbf{\Lambda}_x \mathbf{E}_x^H$ Eigen values: $\mathbf{\Lambda}_x = \begin{bmatrix} \lambda_x & 0 \\ 0 & 0 \end{bmatrix}$

Multi-User Communications

Multi-user scenario

Up-link = Multiple Access (MA)

Down-link = Broadcast (BC)



Access schemes

	Identifier	Orthogonality	Spectrum efficiency	Standard
FDMA	Frequency	Yes	Low	1G
TDMA	Time	Yes if synchronous	Low	2G
CDMA	Code	Yes if synchronous	Low	3G
SDMA	Space	No	High	PHS, iBurst, LTE, 4G

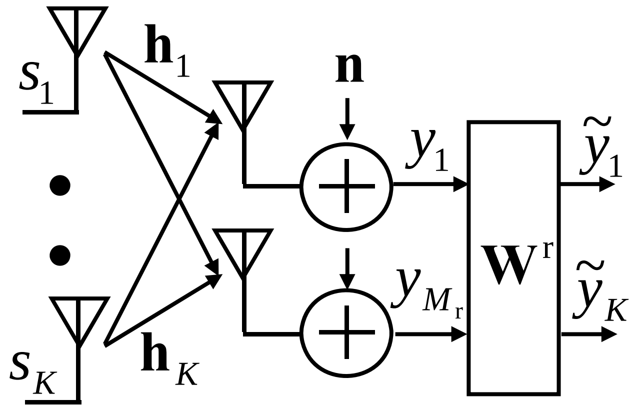
MIMO-MA System Model

Signal model

$$\mathbf{y} = \sum_{i=1}^{K} \mathbf{h}_i s_i + \mathbf{n} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \cdots & \mathbf{h}_K \end{bmatrix}$$

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}_1 & \cdots & \mathbf{s}_K \end{bmatrix}^T$$



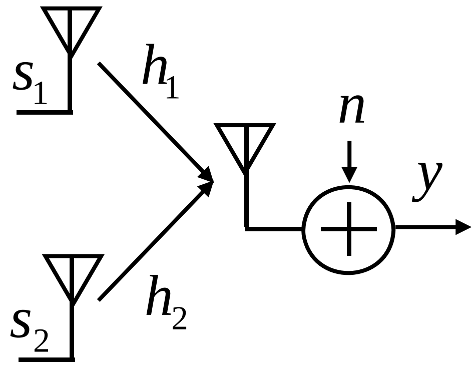
- Equivalent to single-user MIMO with all MIMO processing at receiver
- Per transmitter power constraint $P_i = \mathrm{E}[|s_i|^2] \le P$
- The number of receive antennas assumed to be $M_{\mathrm{r}} \ge K$

Capacity Region of MA Channel

Single antenna receiver

$$y = h_1 s_1 + h_2 s_2 + n$$

$$P_1 = \mathrm{E}[|s_1|^2] \quad P_2 = \mathrm{E}[|s_2|^2]$$

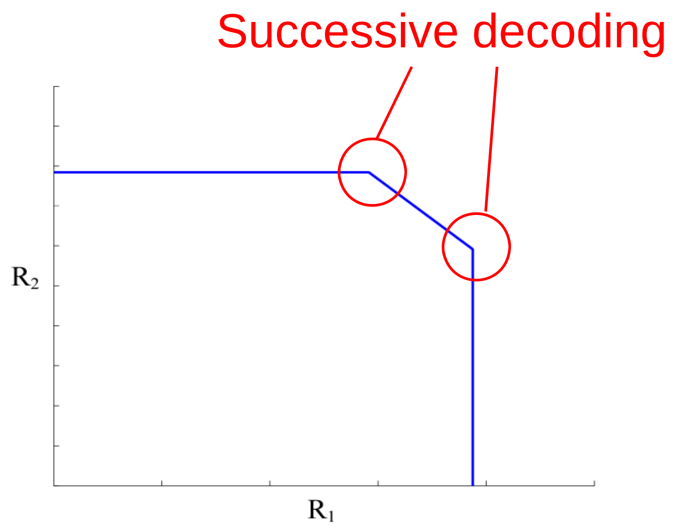


Capacity region

$$R_1 \le \log_2\left(1 + \frac{|h_1|^2 P_1}{\sigma^2}\right)$$

$$R_2 \le \log_2\left(1 + \frac{|h_2|^2 P_2}{\sigma^2}\right)$$

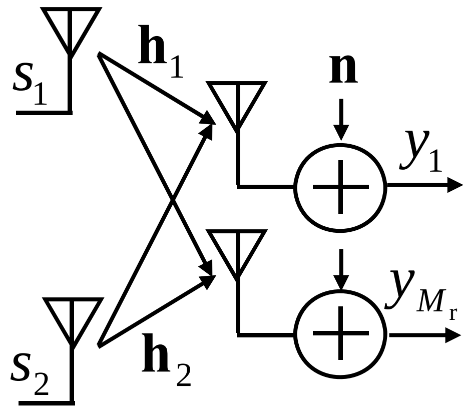
$$R_1 + R_2 \le \log_2\left(1 + \frac{|h_1|^2 P_1 + |h_2|^2 P_2}{\sigma^2}\right)$$



Capacity of MIMO-MA

Multiple antenna receiver

$$\mathbf{y} = \mathbf{h}_1 s_1 + \mathbf{h}_2 s_2 + \mathbf{n} = \mathbf{H}\mathbf{s} + \mathbf{n}$$



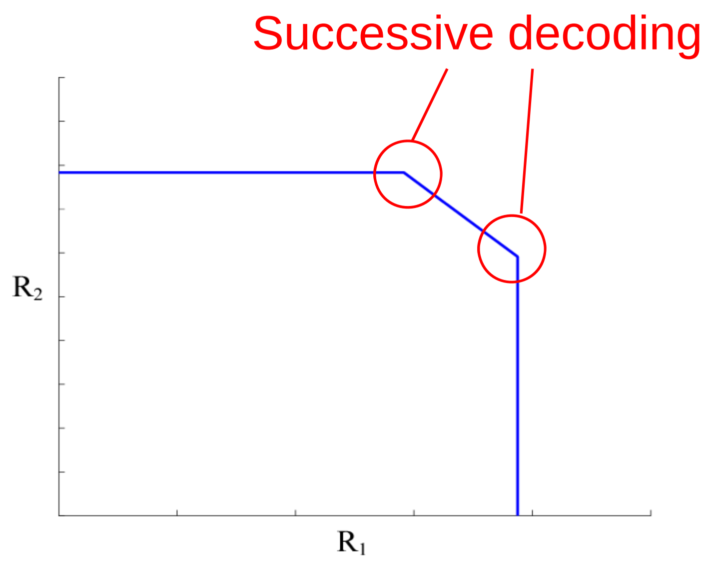
Capacity region

$$C_{\mathrm{MA}} = \bigcup_{P_i \le P} \sum_{i \in S} R_i \le \log \left| \mathbf{I} + \sum_{i \in S} \frac{P_i}{\sigma^2} \mathbf{h}_i \mathbf{h}_i^H \right|$$

$$= \bigcup_{P_i \le P} \sum_{i \in S} R_i \le \log \left| \mathbf{I} + \mathbf{H}\mathbf{P}\mathbf{H}^H \right|$$

$$S \subseteq \{1,2\}$$

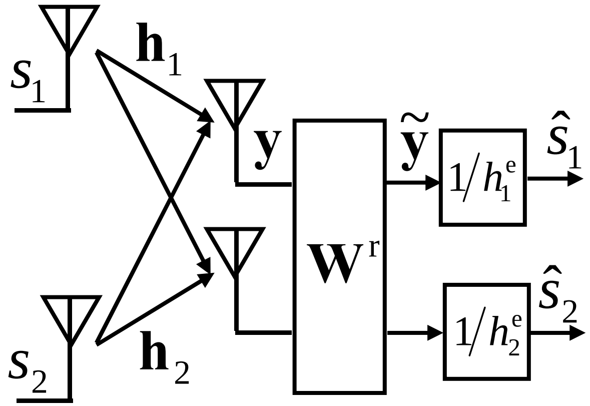
Scales lineary with Rank *H*



MIMO-MA Receiver (ZF)

Received signal

$$\mathbf{y} = \mathbf{h}_1 s_1 + \mathbf{h}_2 s_2 + \mathbf{n} = \mathbf{H}\mathbf{s} + \mathbf{n}$$



Receiver Zero Forcing

$$\mathbf{w}_1^{\mathrm{r}} = (\mathbf{h}_2)^{\perp}$$

$$\mathbf{w}_2^{\mathrm{r}} = (\mathbf{h}_1)^{\perp}$$

Mutually orthogonal channel

$$\tilde{\mathbf{y}} = \begin{bmatrix} \mathbf{w}_1^{\mathrm{r}} & \mathbf{w}_2^{\mathrm{r}} \end{bmatrix}^H \mathbf{y} = \begin{bmatrix} h_1^{\mathrm{e}} & 0 \\ 0 & h_2^{\mathrm{e}} \end{bmatrix} \mathbf{s} + \tilde{\mathbf{n}}$$

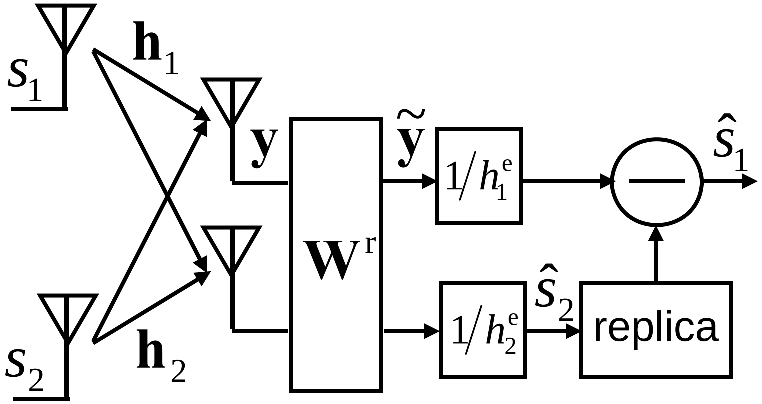
Coherent detection

$$\hat{\mathbf{s}} = \begin{bmatrix} 1/h_1^{\mathrm{e}} & 0 \\ 0 & 1/h_2^{\mathrm{e}} \end{bmatrix} \tilde{\mathbf{y}}$$

MIMO-MA Receiver (SIC)

Received signal

$$\mathbf{y} = \mathbf{h}_1 s_1 + \mathbf{h}_2 s_2 + \mathbf{n} = \mathbf{H}\mathbf{s} + \mathbf{n}$$



Successive Interference Cancellation

$$\mathbf{w}_1^{\mathrm{r}} = (\mathbf{h}_1)^{\parallel}$$

Diversity gain

$$\mathbf{w}_2^{\mathrm{r}} = (\mathbf{h}_1)^{\perp}$$

Remaining interference

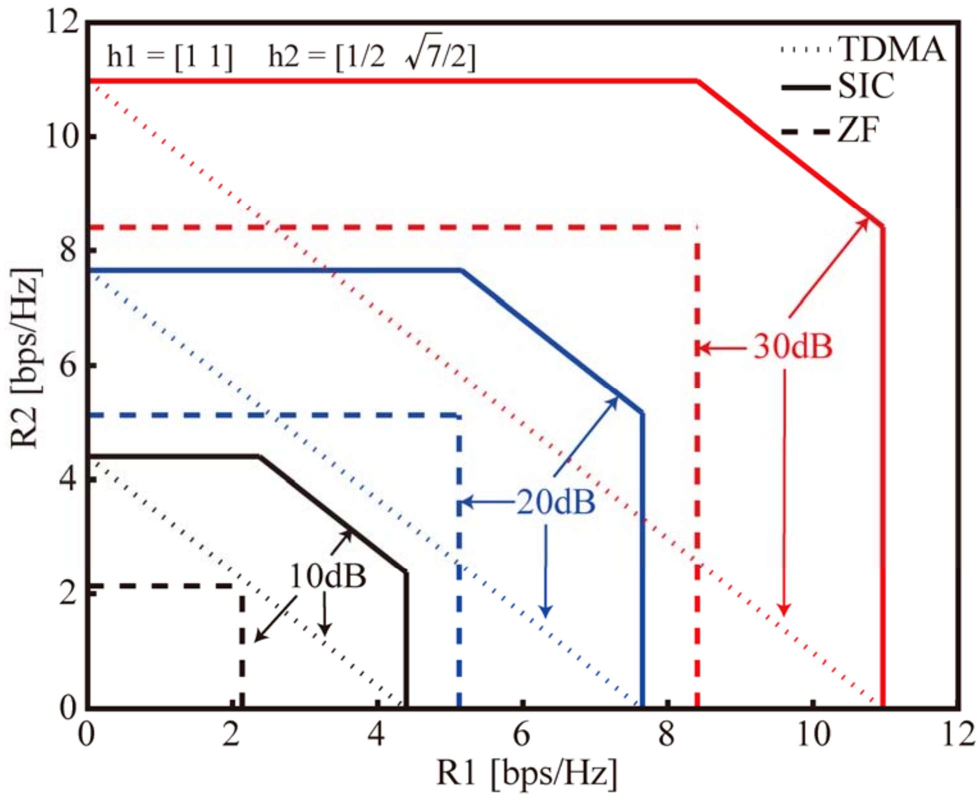
$$\tilde{\mathbf{y}} = \left[\mathbf{w}_1^{\mathrm{r}} \quad \mathbf{w}_1^{\mathrm{r}}\right]^H \mathbf{y} = \begin{bmatrix} h_1^{\mathrm{e}} & h_2^{\mathrm{i}} \\ 0 & h_2^{\mathrm{e}} \end{bmatrix} \mathbf{s} + \tilde{\mathbf{n}}$$

Interference cancellation

$$\hat{\mathbf{s}} = \begin{bmatrix} 1/h_1^{\mathrm{e}} & -h_2^{\mathrm{i}} / \left(h_1^{\mathrm{e}} h_2^{\mathrm{e}}\right) \\ 0 & 1/h_2^{\mathrm{e}} \end{bmatrix} \tilde{\mathbf{y}}$$

Achievable Rates of MIMO-MA

- Higher throughput due to user multiplexing gain
- SIC always outperforms TDMA due to diversity gain



MIMO-BC System Model

Signal model

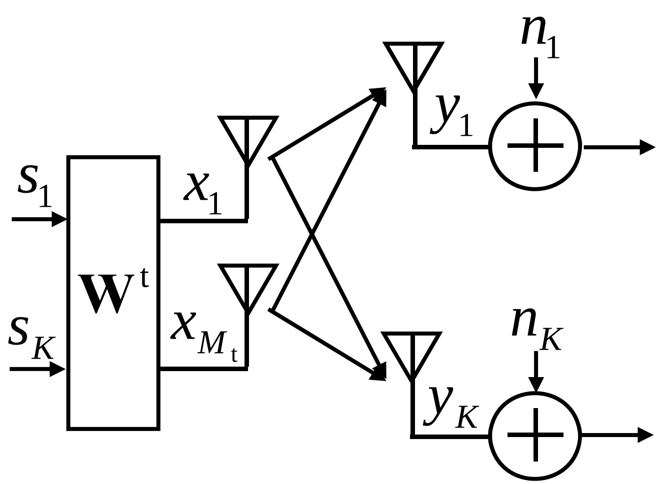
$$y_1 = \mathbf{h}_1^T \mathbf{x} + n_1$$

$$\vdots$$

$$y_K = \mathbf{h}_K^T \mathbf{x} + n_K$$

$$\mathbf{y} = \mathbf{H}^T \mathbf{x} + \mathbf{n}$$

$$\mathbf{H} = [\mathbf{h}_1 \quad \cdots \quad \mathbf{h}_K]$$



- Equivalent to single-user MIMO with all MIMO processing at transmitter
- Total power constraint $\sum_{i=1}^{K} P_i \le P$
- The number of transmit antennas assumed to be $M_t \ge K$

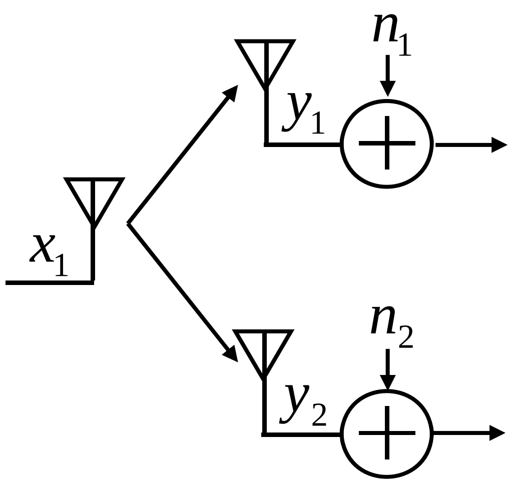
Capacity Region of BC Channel

Single antenna transmitter

$$y_1 = h_1 x + n_1 \qquad y_2 = h_2 x + n_2$$

$$x = s_1 + s_2$$ — Superposition coding

$$\alpha P = \mathrm{E}[|s_1|^2] \qquad (1-\alpha)P = \mathrm{E}[|s_2|^2]$$



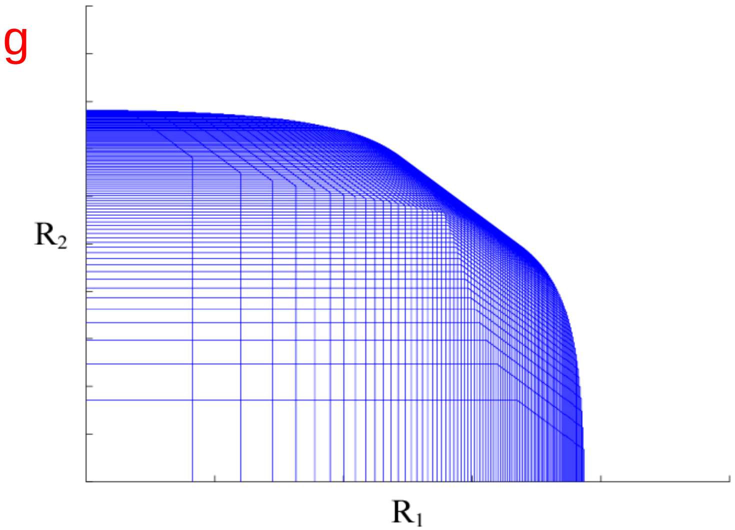
Capacity region

Assuming $|h_1|^2 > |h_2|^2$

Joint decoding

$$R_1 \le \log\left(1 + \frac{\alpha |h_1|^2 P}{\sigma^2}\right)$$

$$R_2 \le \log\left(1 + \frac{(1-\alpha)|h_2|^2 P}{\alpha |h_1|^2 P + \sigma^2}\right)$$

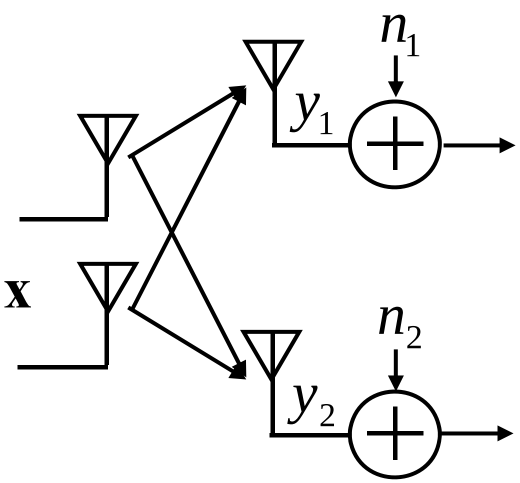


Capacity of MIMO-BC

Multiple antenna transmitter

$$y_1 = \mathbf{h}_1^T \mathbf{x} + n_1 \qquad y_2 = \mathbf{h}_2^T \mathbf{x} + n_2$$

$$\mathbf{y} = \mathbf{H}^T \mathbf{x} + \mathbf{n}$$



MA-BC duality

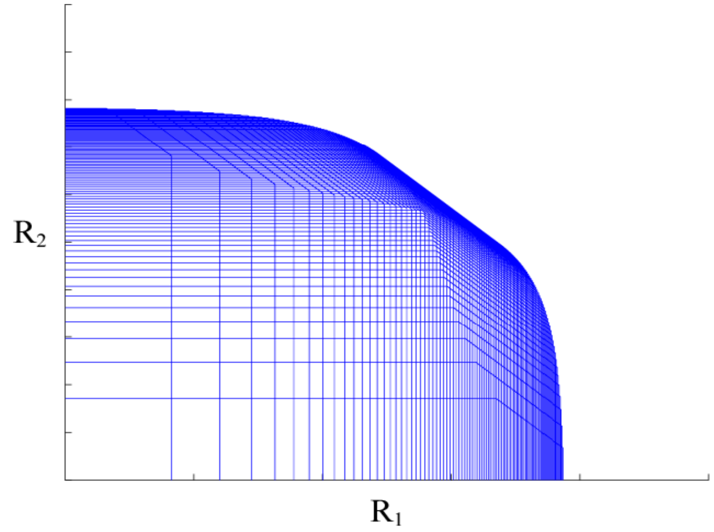
Union of capacity region of dual MA

$$C_{\mathrm{BC}} = \bigcup_{P_1+P_2=P} C_{\mathrm{MA}}(P_1, P_2, \mathbf{H})$$

Power optimization

$$\max_{P_1,P_2} \mu_1 R_1 + \mu_2 R_2$$
$$\text{s.t.} \{P_1 + P_2 = P, \mu_1 + \mu_2 = 1\}$$

Convex optimization, KKT method if $\mu_1 = \mu_2$



MIMO-BC Transmitter (ZF)

Received signal

$$y_1 = \mathbf{h}_1^T \mathbf{x} + n_1 \qquad y_2 = \mathbf{h}_2^T \mathbf{x} + n_2$$

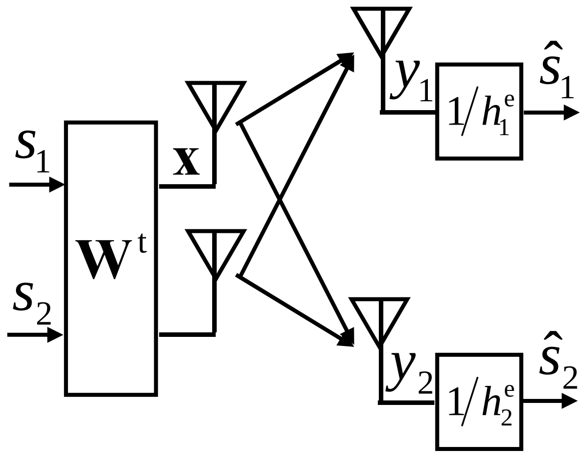
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Transmitter Zero Forcing

$$\mathbf{w}_1^{\mathrm{t}} = \left(\mathbf{h}_2^*\right)^{\perp}$$

$$\mathbf{w}_2^{\mathrm{t}} = \left(\mathbf{h}_1^*\right)^{\perp}$$

$$\mathbf{x} = \left[\mathbf{w}_1^{\mathrm{t}} \quad \mathbf{w}_2^{\mathrm{t}}\right]\mathbf{s}$$



Mutually orthogonal channel

$$\mathbf{y} = \begin{bmatrix} h_1^{\mathrm{e}} & 0 \\ 0 & h_2^{\mathrm{e}} \end{bmatrix} \mathbf{s} + \mathbf{n}$$

Coherent detection

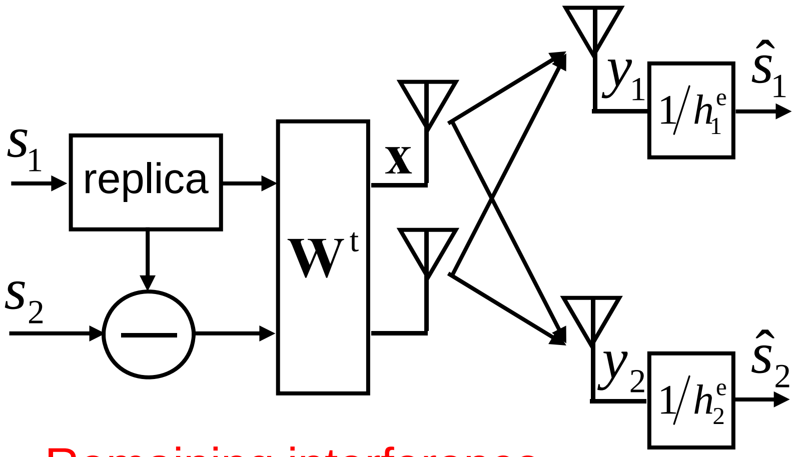
$$\widehat{\mathbf{s}} = \begin{bmatrix} 1/h_1^{\mathrm{e}} & 0 \\ 0 & 1/h_2^{\mathrm{e}} \end{bmatrix} \tilde{\mathbf{y}}$$

MIMO-BC Transmitter (ZF-DPC)

Received signal

$$y_1 = \mathbf{h}_1^T \mathbf{x} + n_1 \qquad y_2 = \mathbf{h}_2^T \mathbf{x} + n_2$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$



Dirty paper coding

Diversity gain

$$\mathbf{w}_1^{\mathrm{t}} = \left(\mathbf{h}_1^*\right)^{\parallel}$$

$$\mathbf{w}_2^{\mathrm{t}} = \left(\mathbf{h}_1^*\right)^{\perp}$$

Remaining interference

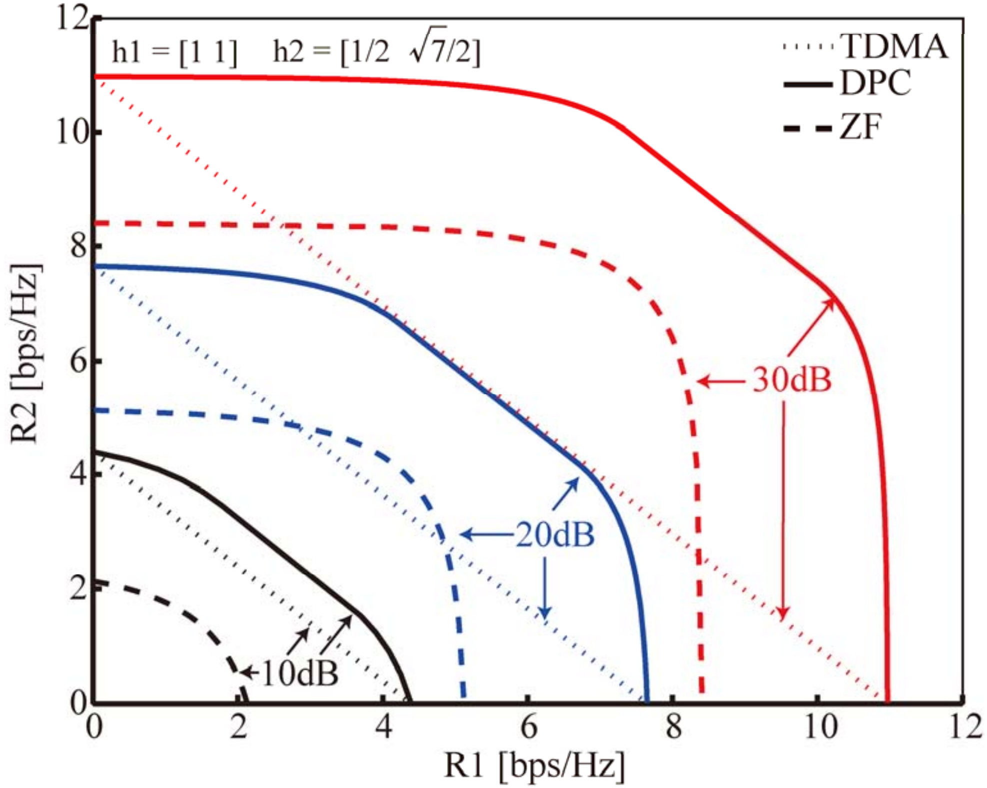
$$\mathbf{y} = \mathbf{H}\begin{bmatrix}\mathbf{w}_1^{\mathrm{t}} & \mathbf{w}_2^{\mathrm{t}}\end{bmatrix}\mathbf{s} + \mathbf{n} = \begin{bmatrix} h_1^{\mathrm{e}} & 0 \\ h_1^{\mathrm{i}} & h_2^{\mathrm{e}} \end{bmatrix}\mathbf{s} + \mathbf{n}$$

Pre-interference cancellation

$$\tilde{\mathbf{y}} = \begin{bmatrix} h_1^{\mathrm{e}} & 0 \\ h_1^{\mathrm{i}} & h_2^{\mathrm{e}} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -h_1^{\mathrm{i}}/h_2^{\mathrm{e}} & 1 \end{bmatrix}\mathbf{s} + \mathbf{n} \qquad \hat{\mathbf{s}} = \begin{bmatrix} 1/h_1^{\mathrm{e}} & 0 \\ 0 & 1/h_2^{\mathrm{e}} \end{bmatrix}\tilde{\mathbf{y}}$$

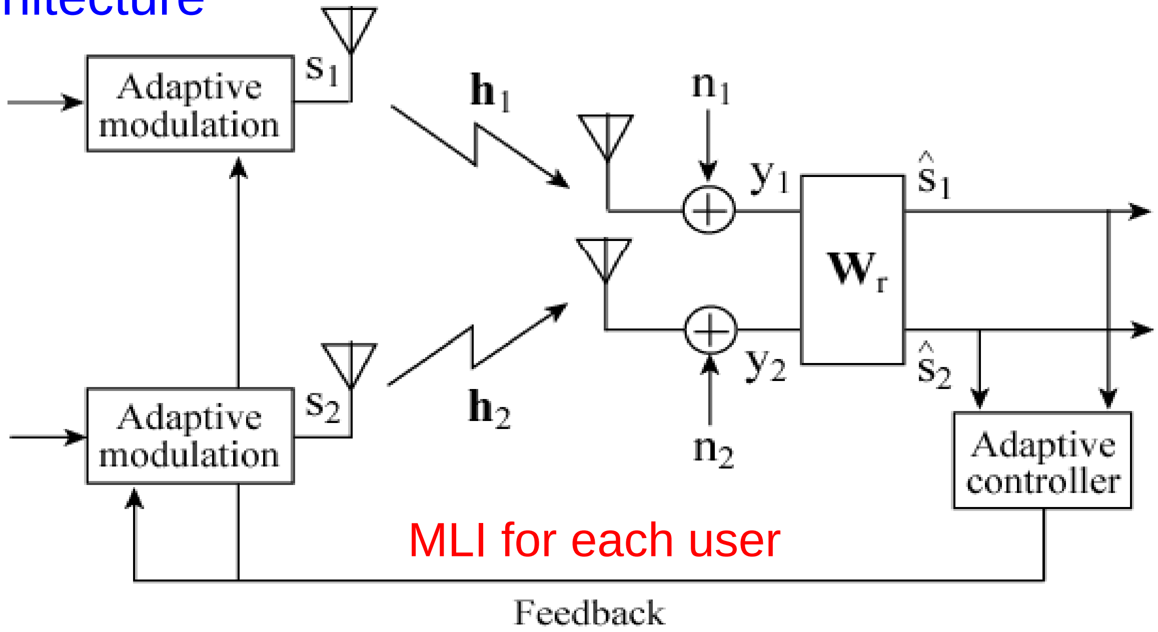
Achievable Rates of MIMO-BC

- Higher throughput due to user multiplexing gain
- DPC always outperforms TDMA due to diversity gain



AMC for Multi-User MIMO

System architecture

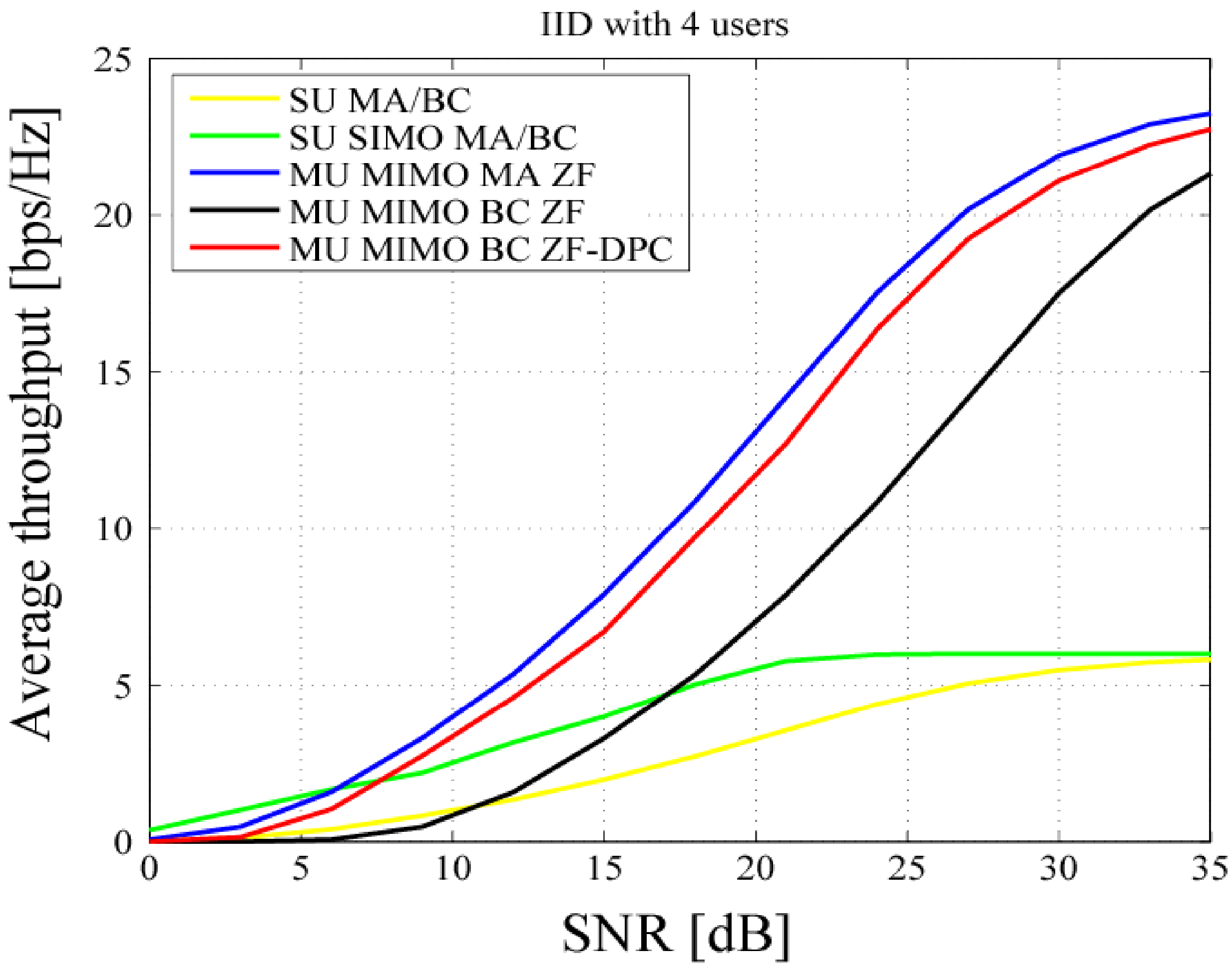


Adaptive algorithm

$$\gamma_i = \frac{P_i \left| \mathbf{w}_i^H \mathbf{h}_i \right|^2}{\sum_{j \neq i} P_j \left| \mathbf{w}_i^H \mathbf{h}_j \right|^2 + \left| \mathbf{w}_i \right|^2 \sigma^2} \qquad M_i^{\text{ary}} = \arg \max_{M^{\text{ary}}} TP(\gamma_i, M^{\text{ary}})$$

Throughput Performance of MU-MIMO

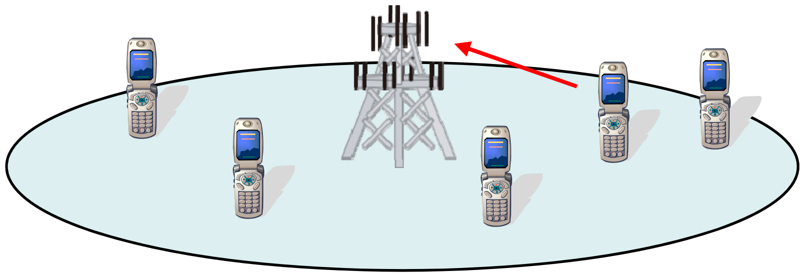
- Higher throughput due to multiplexing gain of different users



User & Antenna Scheduling

Single antenna receiver

$$y = \sum_{i=1}^{K} h_i s_i + n$$



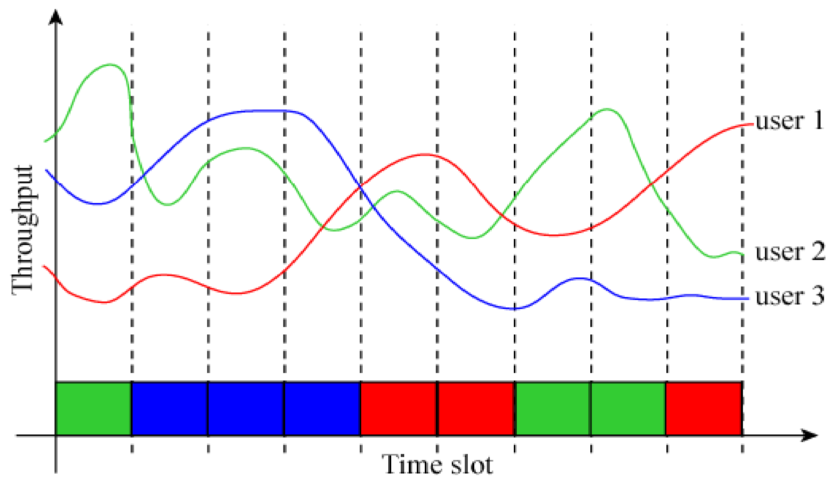
Use & antenna scheduling

Round robin

$$i(n) = \left(i(n-1)+1\right) \bmod K$$

Proportional fair

$$i = \arg\max_j \frac{\log_2\left(1 + \frac{P_j |h_j|^2}{\sigma^2}\right)}{\mathrm{E}\left[TP_j\right]}$$



Maximum throughput

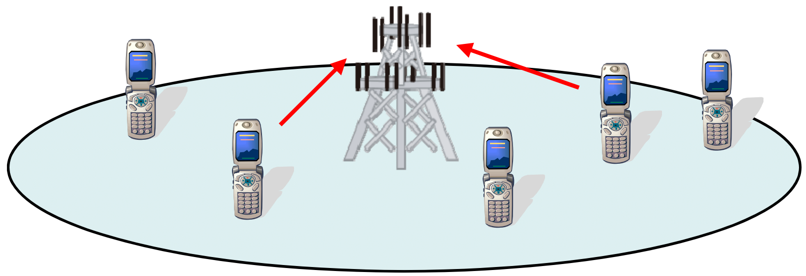
$$i = \arg\max_j \log_2\left(1 + \frac{P_j |h_j|^2}{\sigma^2}\right)$$

Exhaustive Search ZF

Use scheduling

If $K > M_{\mathrm{r}} = 2$

Select 2 users out of *K* users



$$\begin{bmatrix}\mathbf{e}_j^{\perp} & \mathbf{e}_i^{\perp}\end{bmatrix}^H \begin{bmatrix}\mathbf{h}_i & \mathbf{h}_j\end{bmatrix} = \begin{bmatrix} h_i^{\mathrm{e}} & 0 \\ 0 & h_j^{\mathrm{e}} \end{bmatrix}$$

$$\hat{i}, \hat{j} = \arg\max_{i,j} \log_2\left(1 + \frac{P_i \left|h_i^{\mathrm{e}}\right|^2}{\sigma^2}\right) + \log_2\left(1 + \frac{P_j \left|h_j^{\mathrm{e}}\right|^2}{\sigma^2}\right)$$

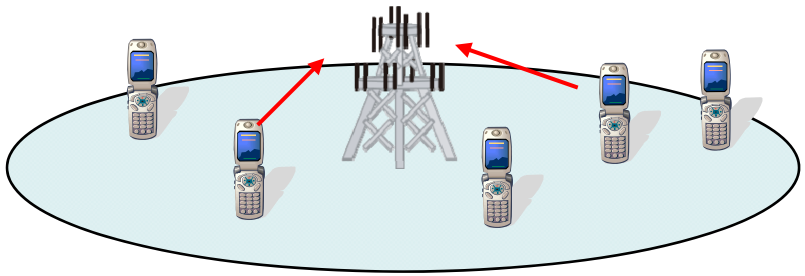
${}_K C_2$ times search

User selection diversity order of ${}_K C_2$

Greedy ZF-DPC (SIC)

Use scheduling

If $K > M_{\mathrm{r}} = 2$



1) Initialization

$$s(1) = \arg\max_i \left|\mathbf{h}_i\right|^2$$

$$\mathbf{e}_{s(1)} = \mathbf{h}_{s(1)}^{\|}$$

2) Iterative ordering based on Gram-Schmidt decomposition

for $i = 2, \cdots, M_{\mathrm{r}}$

$$\mathbf{H}_s = [\mathbf{h}_{s(1)}, \cdots, \mathbf{h}_{s(i-1)}]$$

$$\tilde{\mathbf{h}}_i = \mathbf{H}_s^{\perp} \mathbf{h}_i$$

$$s(i) = \arg\max_i \left|\tilde{\mathbf{h}}_i\right|^2$$

$$\mathbf{e}_{s(i)} = \tilde{\mathbf{h}}_{s(i)}^{\|}$$

end

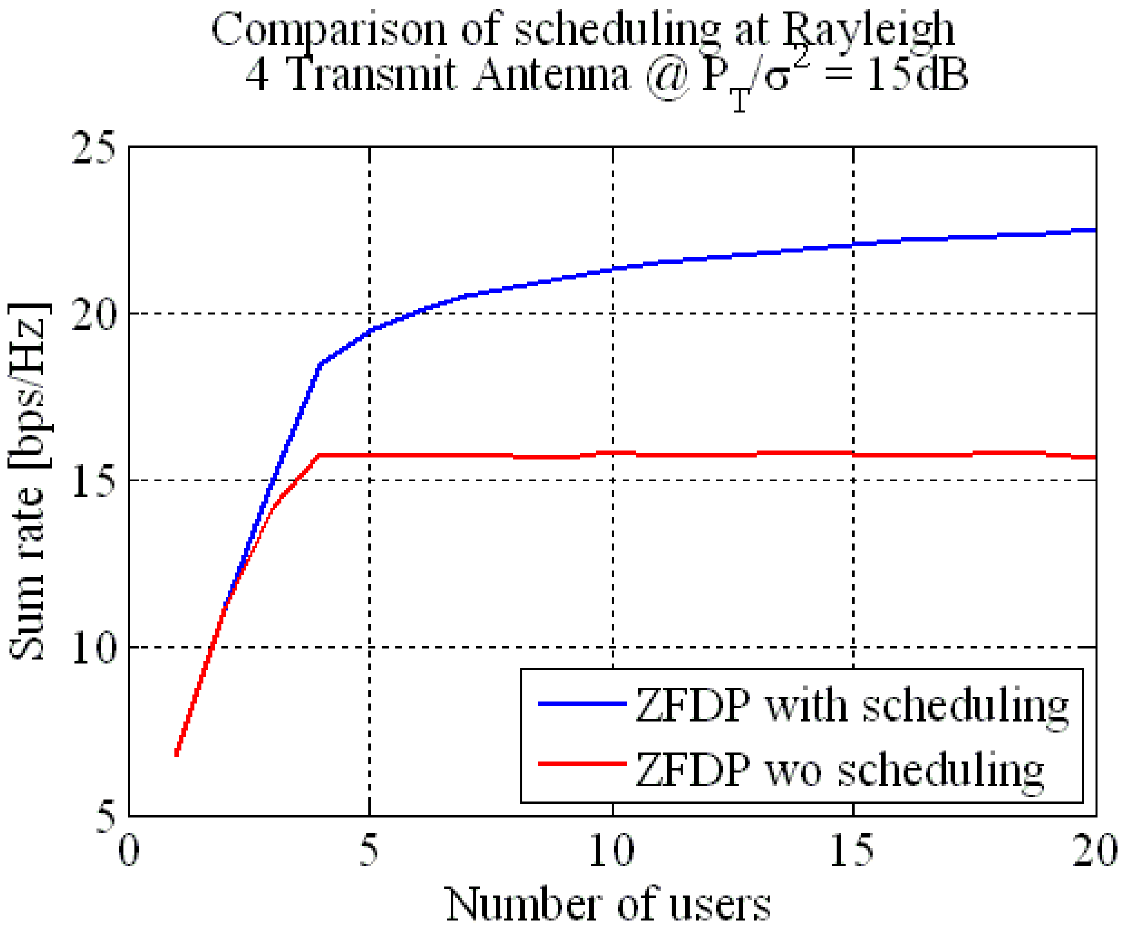
Maximizing diagonal components by using greedy search

$$\mathbf{H}_s = \left[\mathbf{e}_{s(1)} \quad \mathbf{e}_{s(2)}\right] \begin{bmatrix} h_{s(1)} & h_2^{\mathrm{i}} \\ 0 & \tilde{h}_{s(2)} \end{bmatrix}$$

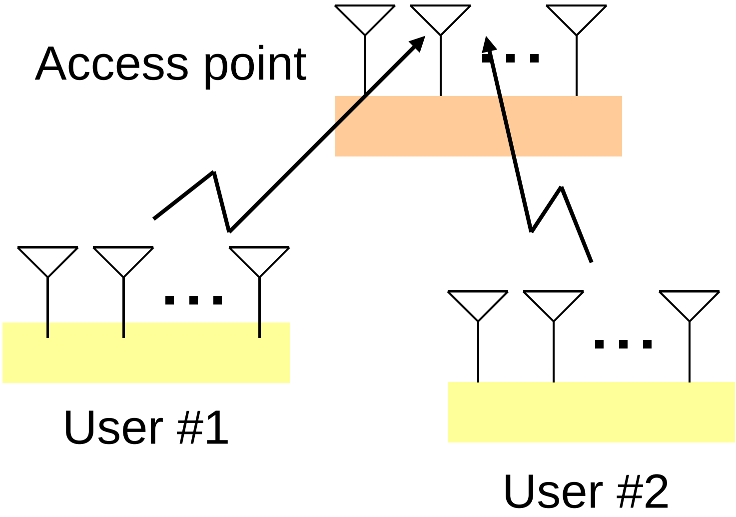
$2K - 1$ times search

Performance on User Scheduling

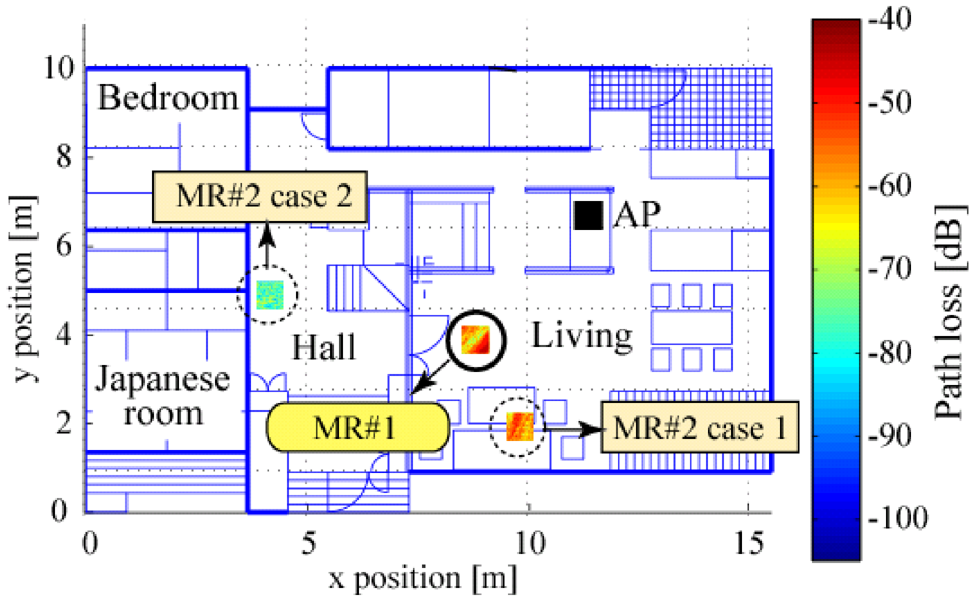
- Capacity scales linearly up to 4 users as in single user (SU)-MIMO
- Capacity scales logarithmically above 4 users due to user selection diversity



Measurement Experiment



MIMO-MA with antenna scheduling

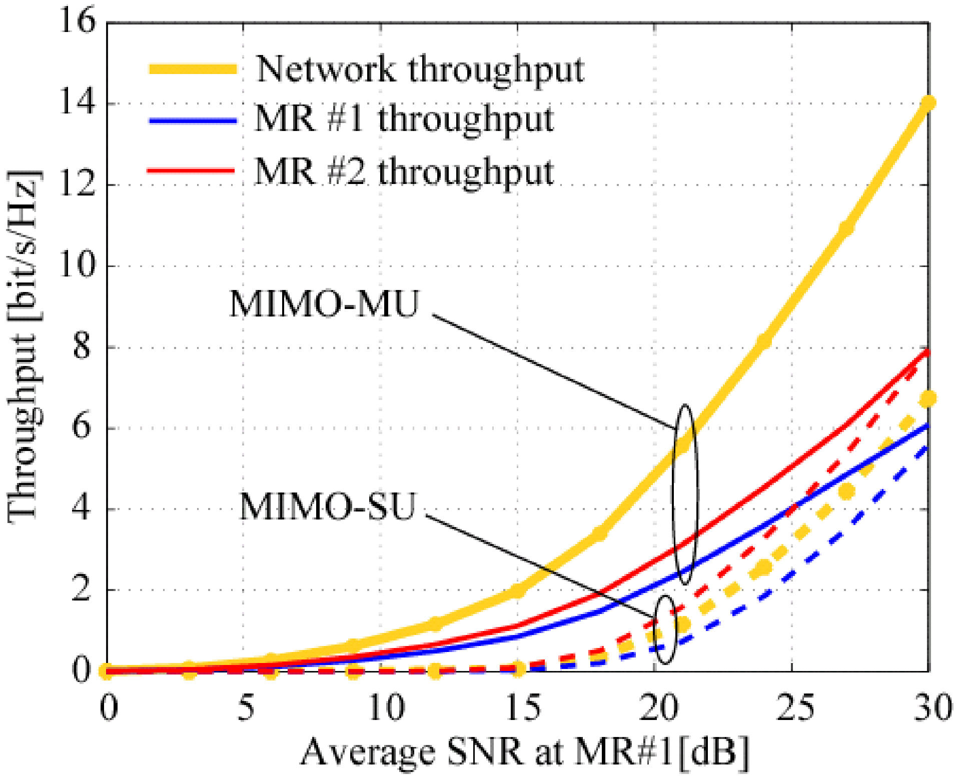


System	MU-MIMO / SU-MIMO (MMSE receiver)
# of antennas	Tx(UT) #1: 4, Tx(UT) #2: 4, Rx(AP): 4
Adaptive scheme	BPSK, QPSK, 16QAM, 64QAM (with antenna scheduling)
UT #1	In living room (Average pathloss -51dB)
UT #2	Case #1: in living room (Average pathloss -54dB) Case #2: in hallway (Average pathloss -71dB)

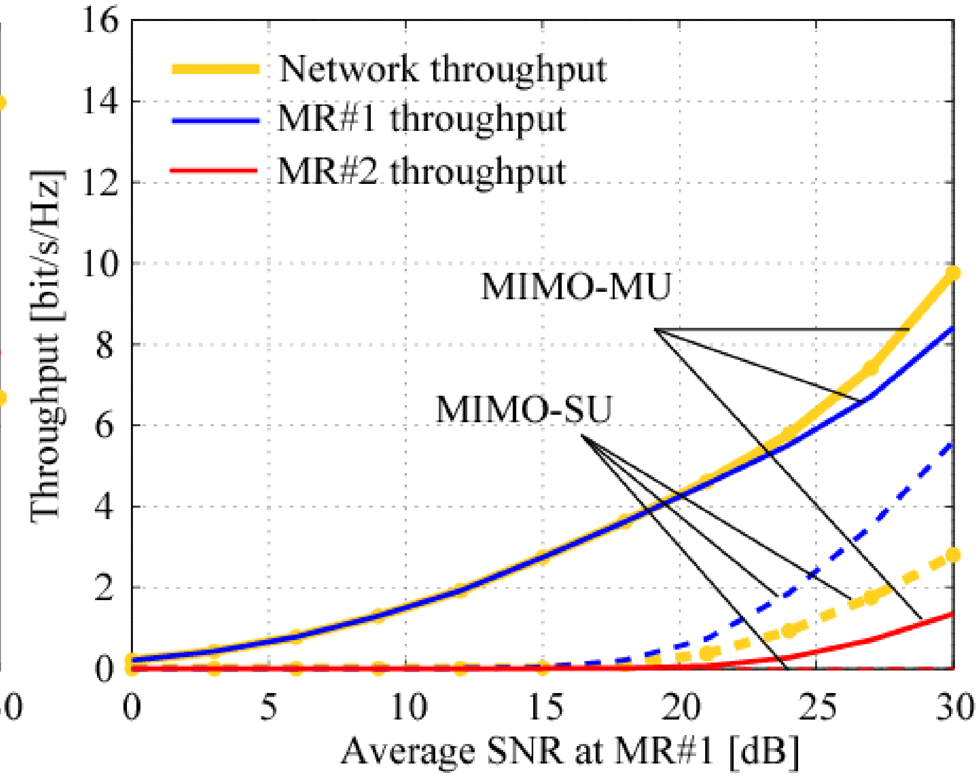
Throughput Performance

- MU-MIMO outperforms SU-MIMO significantly due to user multiplexing
- Antenna scheduling is effective against spatial correlation & bit overloading

Case #1

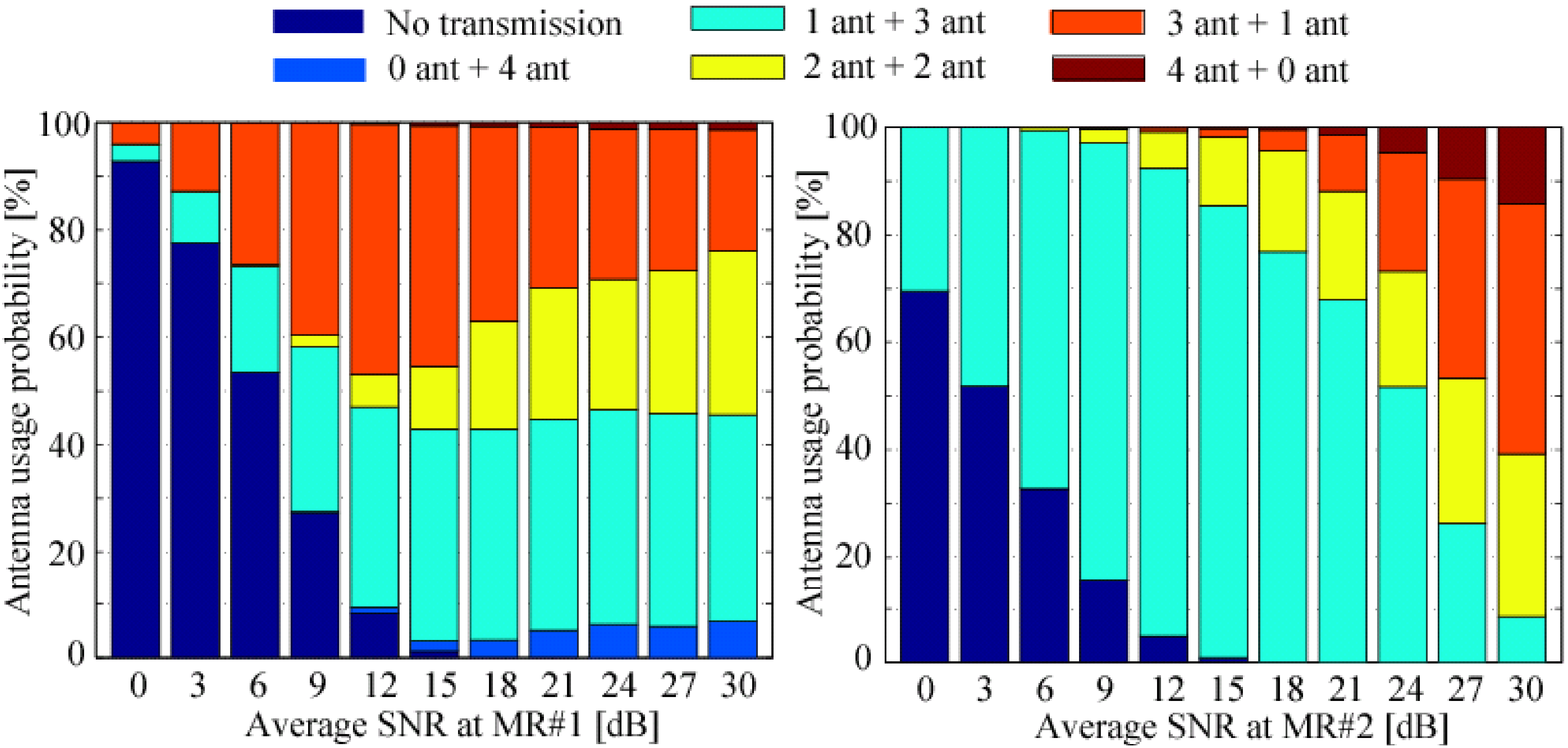


Case #2



Performance on Adaptive Algorithm

- Antenna scheduling works as rank adaptation in low SNR region
- Antenna scheduling works as selection diversity in high SNR region



Case #1 Case #2

Summary

- Multi-User MIMO communications
  - MIMO Multiple Access (MA) for up-link
    - Capacity region based on cut set bound
    - MIMO-MA receiver (ZF, SIC)
  - MIMO Broadcast (BC) for down-link
    - Capacity region based on duality with MA
    - MIMO-BC transmitter (ZF, DPC)
  - Adaptive algorithm for multi-user MIMO
  - User scheduling to achieve user selection diversity

⇩

Combinations of MIMO-MA & MIMO-BC

Distributed MIMO Network