2011 1st semester
MIMO Communication Systems

#8: MIMO Transmitter

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Schedule (2^{nd} half)

	Date	Text	Contents
#7	May 31	A-5	MIMO receiver
#8	June 7	A-3, 4	MIMO transmitter
#9	June 14	B-9	Adaptive commun. system
#10	June 21	A-6, B-14	Multi-user MIMO
#11	June 28	B-15, 16	Distributed MIMO networks
#12	July 5		Standardization of MIMO
	July 12		Final Examination

Agenda

■ Aim of today

Derive BER & throughput performances
of MIMO systems with transmitter pre-processing

■ Contents

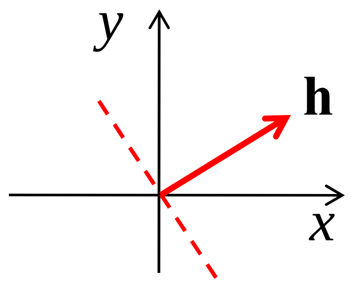
- MIMO transmitter architecture
- MIMO transmit beamforming
- SVD-MIMO (MIMO eigenmode)
- (Quasi-)Orthogonal STBC
- Measurement experiment

Warming Up

Question 1

Given a real vector of $\mathbf{h}$, calculate an unitary matrix of $\mathbf{U}$

$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \qquad \mathbf{U} = \frac{1}{?}\begin{bmatrix} h_1 & ? \\ h_2 & ? \end{bmatrix}$$

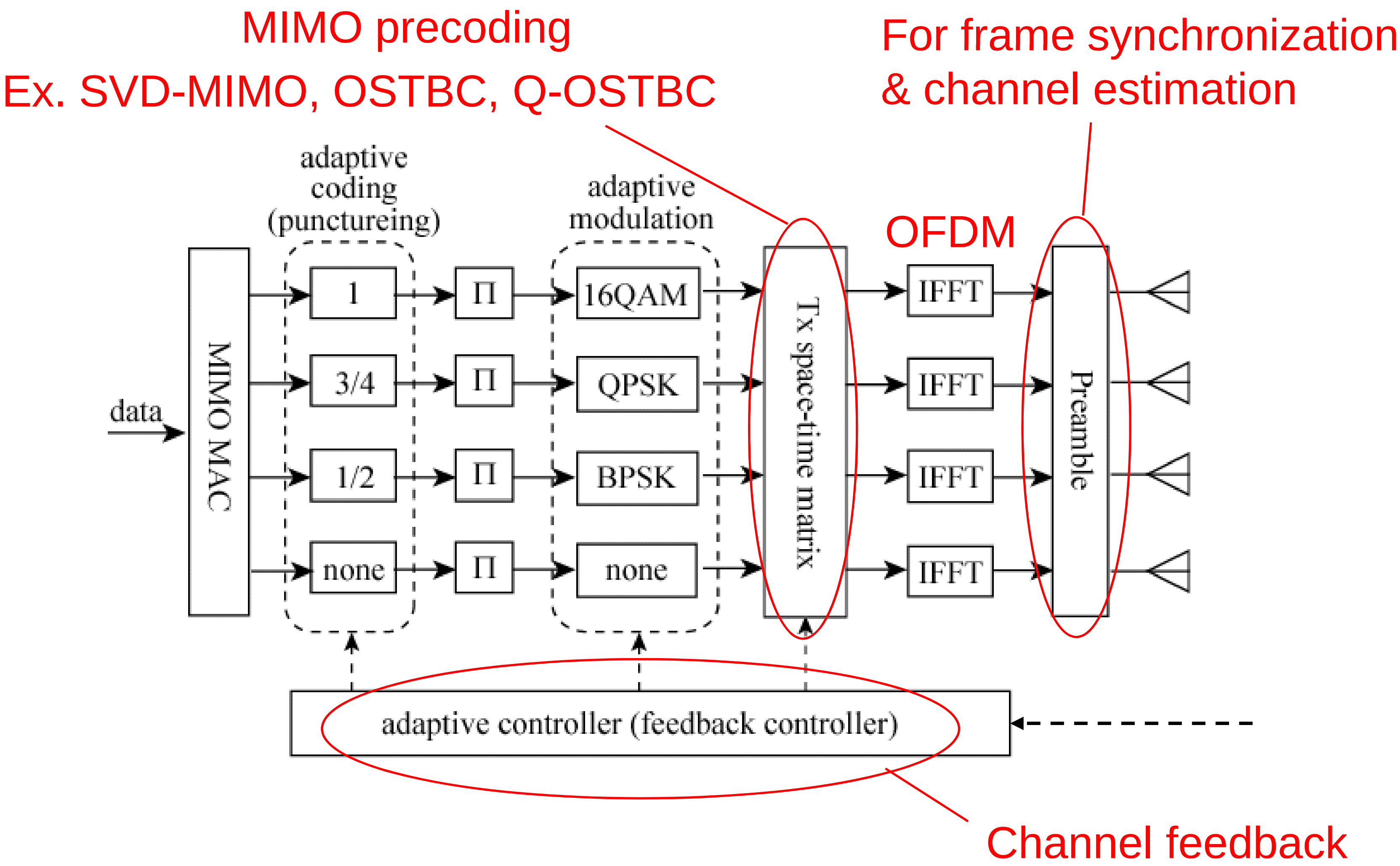


Question 2

Given a complex vector of $\mathbf{h}$, calculate an unitary matrix of $\mathbf{U}$

$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \qquad \mathbf{U} = \frac{1}{?}\begin{bmatrix} h_1 & ? \\ h_2 & ? \end{bmatrix}$$

MIMO Transmitter Architecture



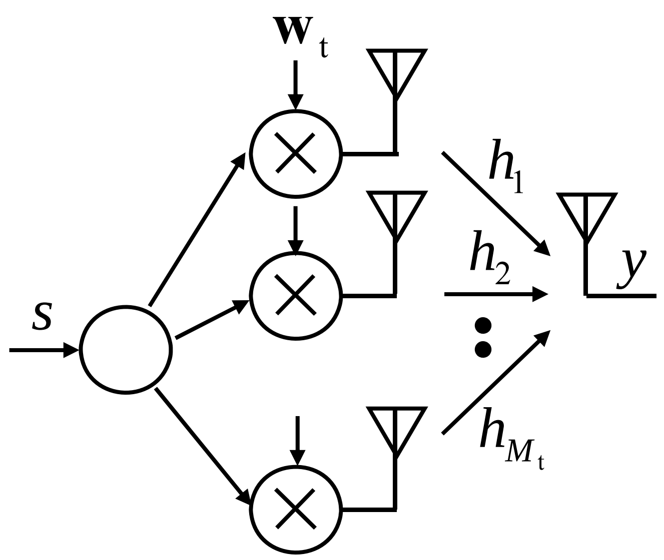
Classification of MIMO Transmitter

	Diversity	Multiplexing	Channel feedback
SVD-MIMO	Full	Full	Required
OSTBC	Full	Single	Not required
Q-OSTBC	Partial	Partial	Not required

MIMO Transmit Beamforming

MISO signal model

$$y(t) = \mathbf{h}^T \mathbf{w}_t s(t) + n(t)$$



Retro directive beamforming

$$\mathbf{w}_t = \mathbf{h}^*$$

Output SNR

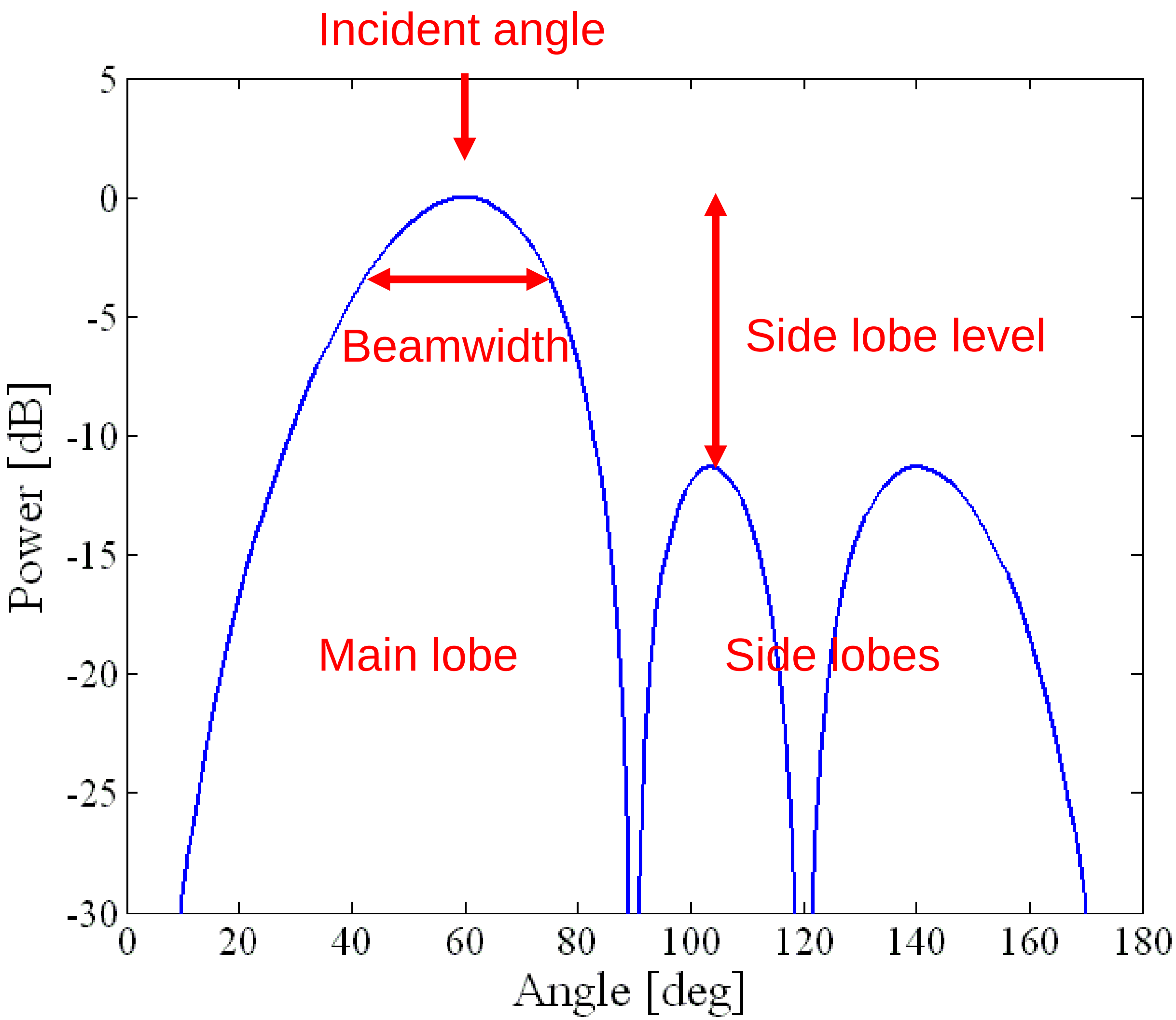
$$\gamma = \max_{\mathbf{w}_t} \frac{E[\left|\mathbf{h}^T \frac{\mathbf{w}_t}{|\mathbf{w}_t|} s\right|^2]}{E[|n|^2]} = \sum_{i=1}^{M_t} |h_i|^2 \frac{P}{\sigma^2}$$

Transmit diversity with order M_t

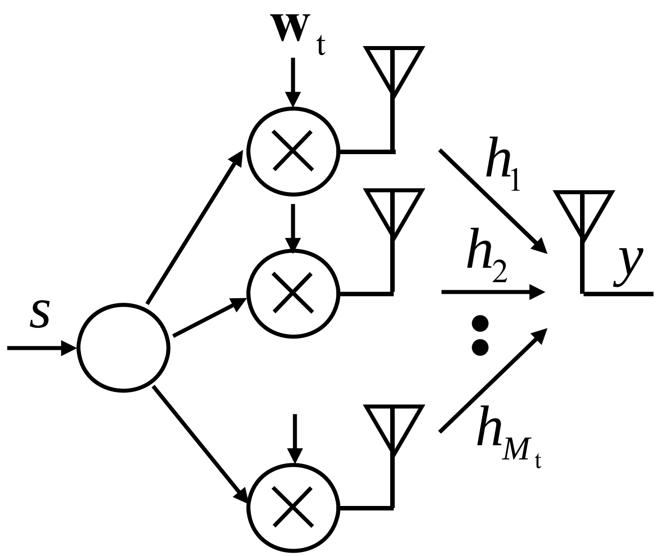
PDF of OSNR in IID environment

$$f(\gamma) = \frac{1}{(M-1)!\,\bar{\gamma}^{M_t}} \gamma^{M_t - 1} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

Transmit beamforming



MIMO Transmit Interference Cancellation



Subspace decomposition

$$\mathbf{R} = \left(\mathbf{h}\mathbf{h}^H\right)^* = \left(\mathbf{E}\Lambda\mathbf{E}^H\right)^*$$

$$\Lambda = \mathrm{diag}[\lambda_1, \lambda_2, \lambda_3, \lambda_4]$$

$$\lambda_1 > \lambda_2 = \lambda_3 = \lambda_4 = 0$$

$$\mathbf{E} = [\underbrace{\mathbf{e}_1}_{\text{Signal space}}, \underbrace{\mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4}_{\text{Null space}}]$$

$$\mathbf{e}_i^T \mathbf{R} \mathbf{e}_i^* = 0 \longrightarrow \mathbf{h}^T \mathbf{e}_i^* = 0 \quad (i = 2,3,4)$$

Interference cancellation

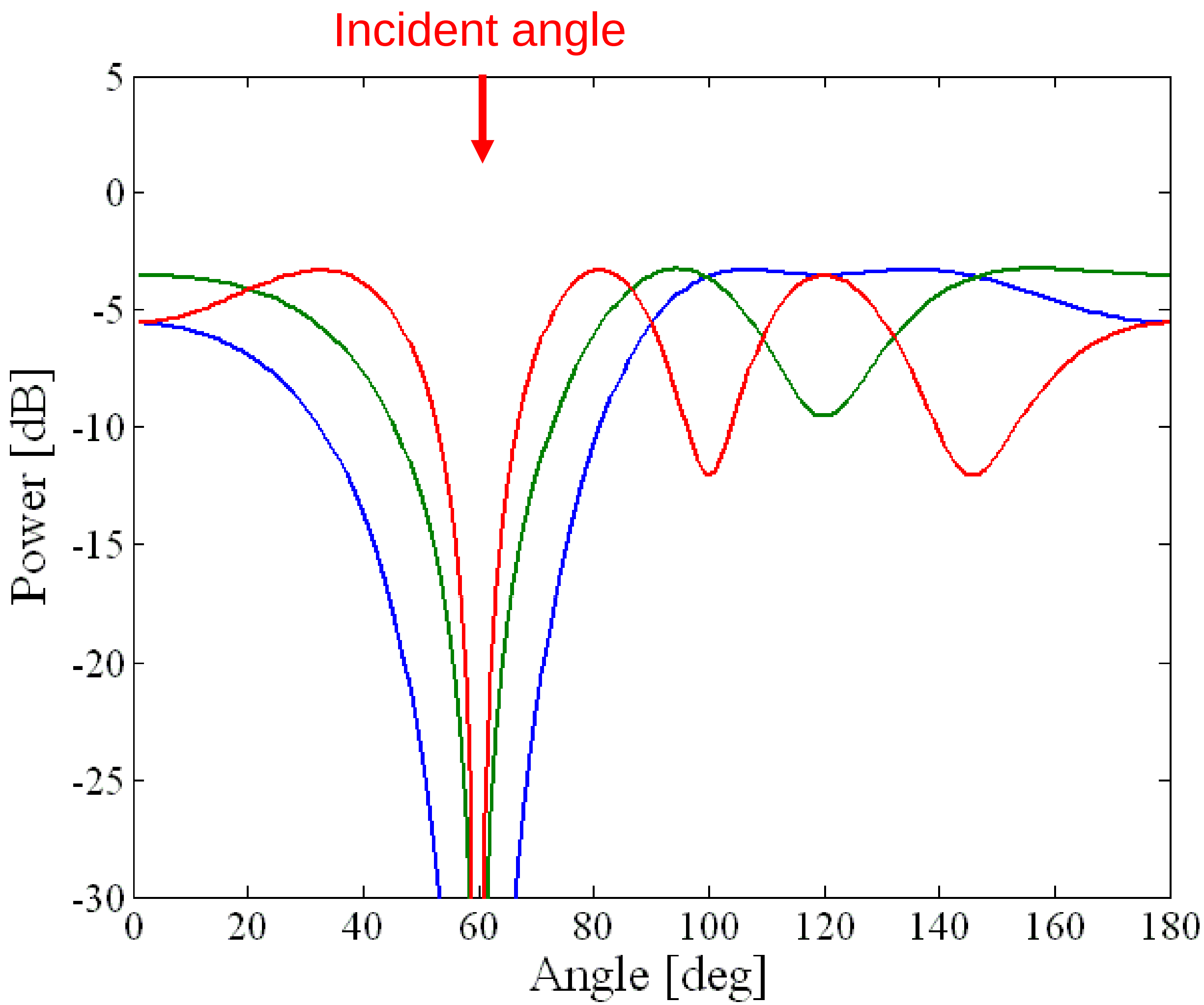
$$y(t) = \mathbf{h}^T \mathbf{w}_{\mathrm{t}} s(t) + n(t)$$

$$\mathbf{w}_{\mathrm{t}} = \mathbf{e}_i^* \quad (i = 2,3,4)$$

Output SNR

$$\gamma = \frac{\mathbf{w}_{\mathrm{t}}^H \mathbf{h}^* \mathbf{h}^T \mathbf{w}_{\mathrm{t}} P}{\sigma^2} = \frac{0}{\sigma^2}$$

Transmit interference cancellation



SVD-MIMO (Theory)

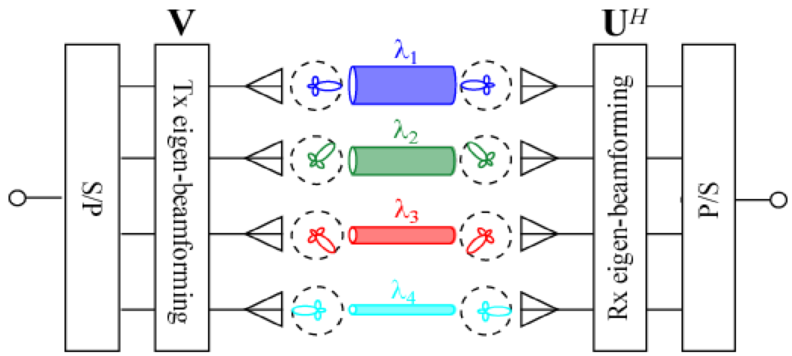
Singular value decomposition

$$\lambda_1 = \max_{\mathbf{u}_1, \mathbf{v}_1} \left| \mathbf{u}_1^H \mathbf{H} \mathbf{v}_1 \right|^2$$

$$\mathbf{H} = \sqrt{\lambda_1} \mathbf{u}_1 \mathbf{v}_1^H + \tilde{\mathbf{H}}$$

$$\Downarrow$$

$$\mathbf{H} = \mathbf{U} \Sigma \mathbf{V}^H$$



Eigenmodes

$$\Sigma = \mathrm{diag}\left[\sqrt{\lambda_1}, \cdots, \sqrt{\lambda_m}\right] \qquad \mathbf{U} = [\mathbf{u}_1, \cdots, \mathbf{u}_m] \qquad \mathbf{V} = [\mathbf{v}_1, \cdots, \mathbf{v}_m]$$

$$m = \min(M_{\mathrm{t}}, M_{\mathrm{r}})$$

SVD-MIMO

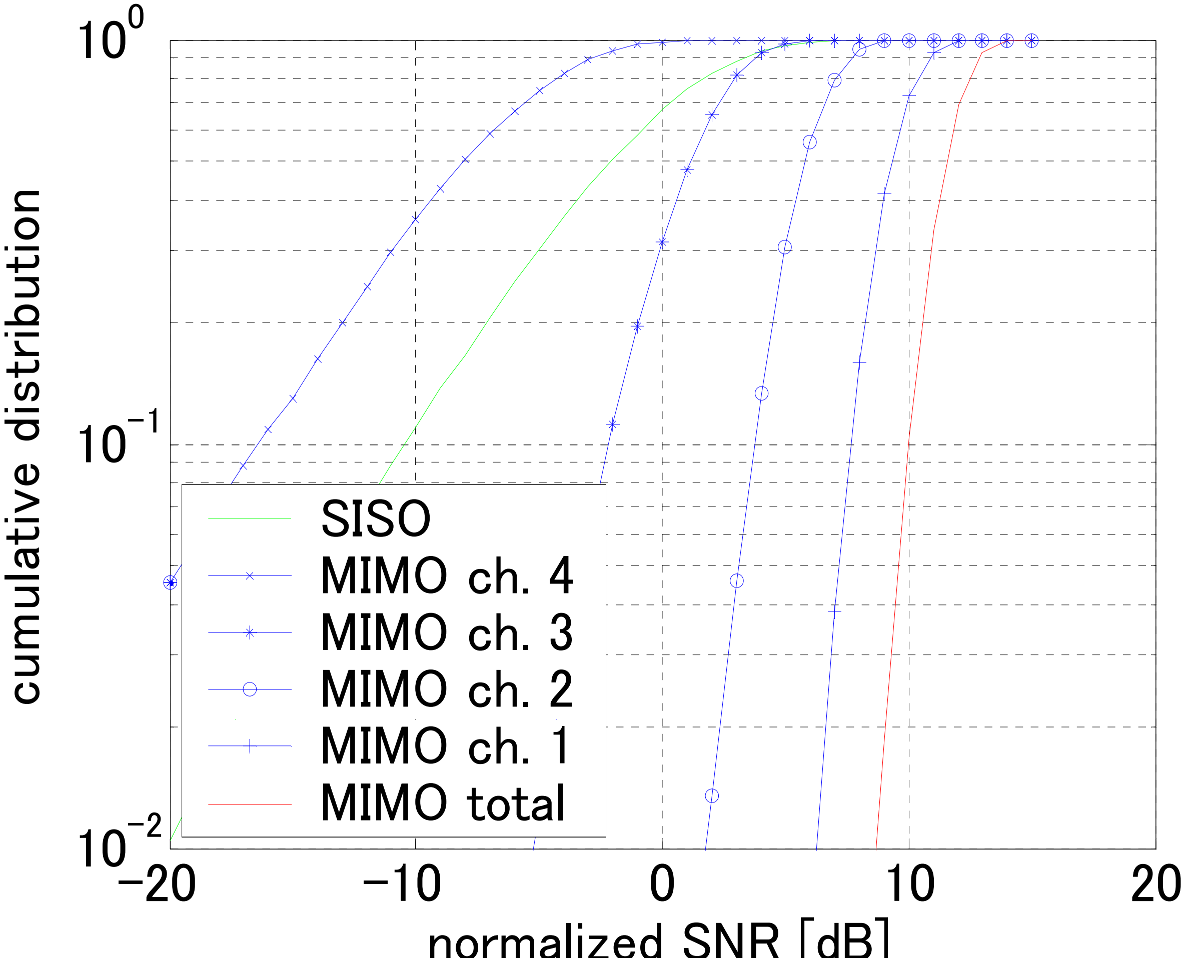
Parallel SISO channel
with different diversity order

$$\mathbf{y}(t) = \mathbf{W}_{\mathrm{r}}^H \mathbf{H} \mathbf{W}_{\mathrm{t}} \mathbf{s}(t) + \mathbf{W}_{\mathrm{r}}^H \mathbf{n}(t)$$

$$= \Sigma \mathbf{s}(t) + \tilde{\mathbf{n}}(t) \qquad \text{if} \qquad \mathbf{W}_{\mathrm{t}} = \mathbf{V} \qquad \mathbf{W}_{\mathrm{r}} = \mathbf{U}$$

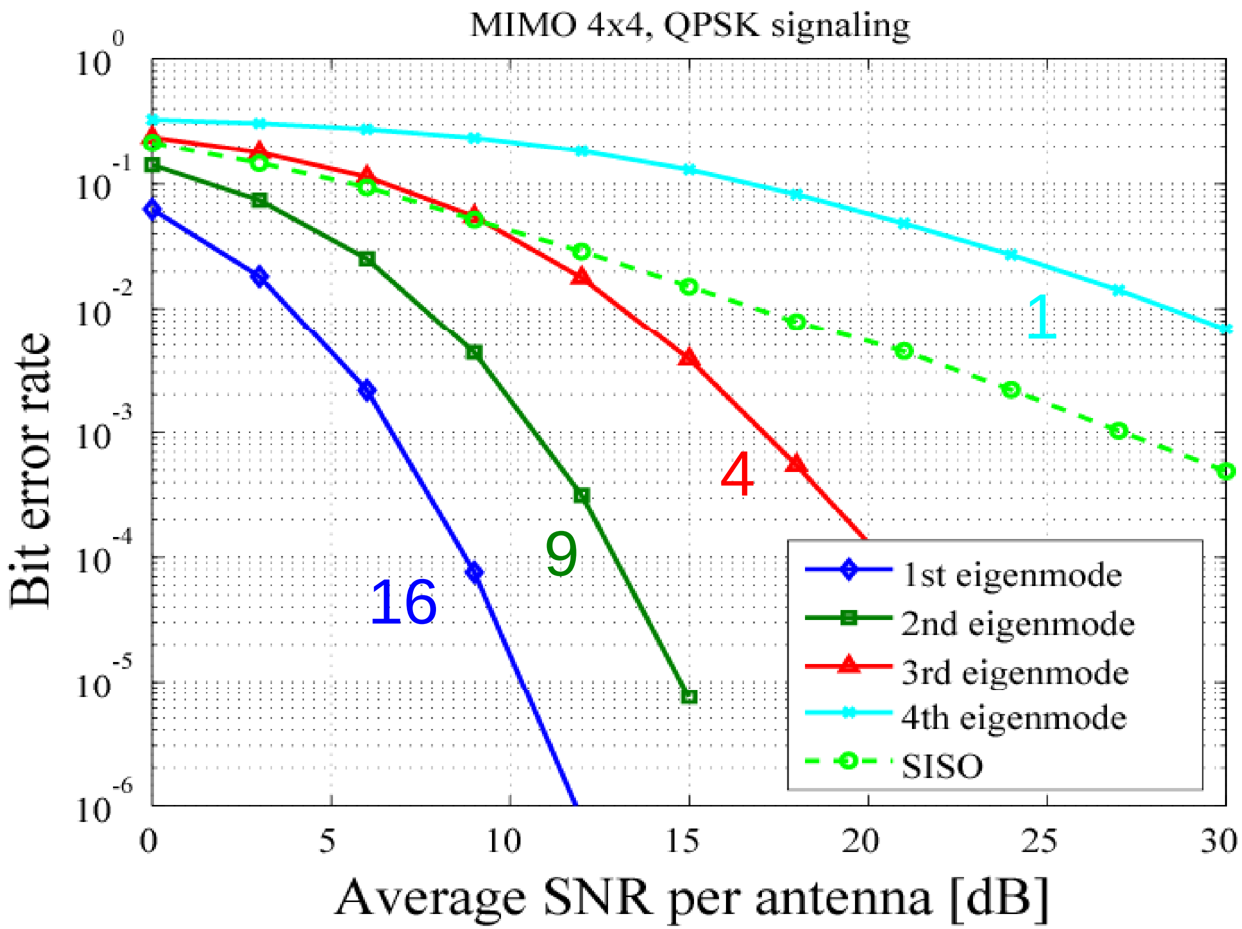
SVD-MIMO (Effective SNR)

- Different diversity order for each eigenmode



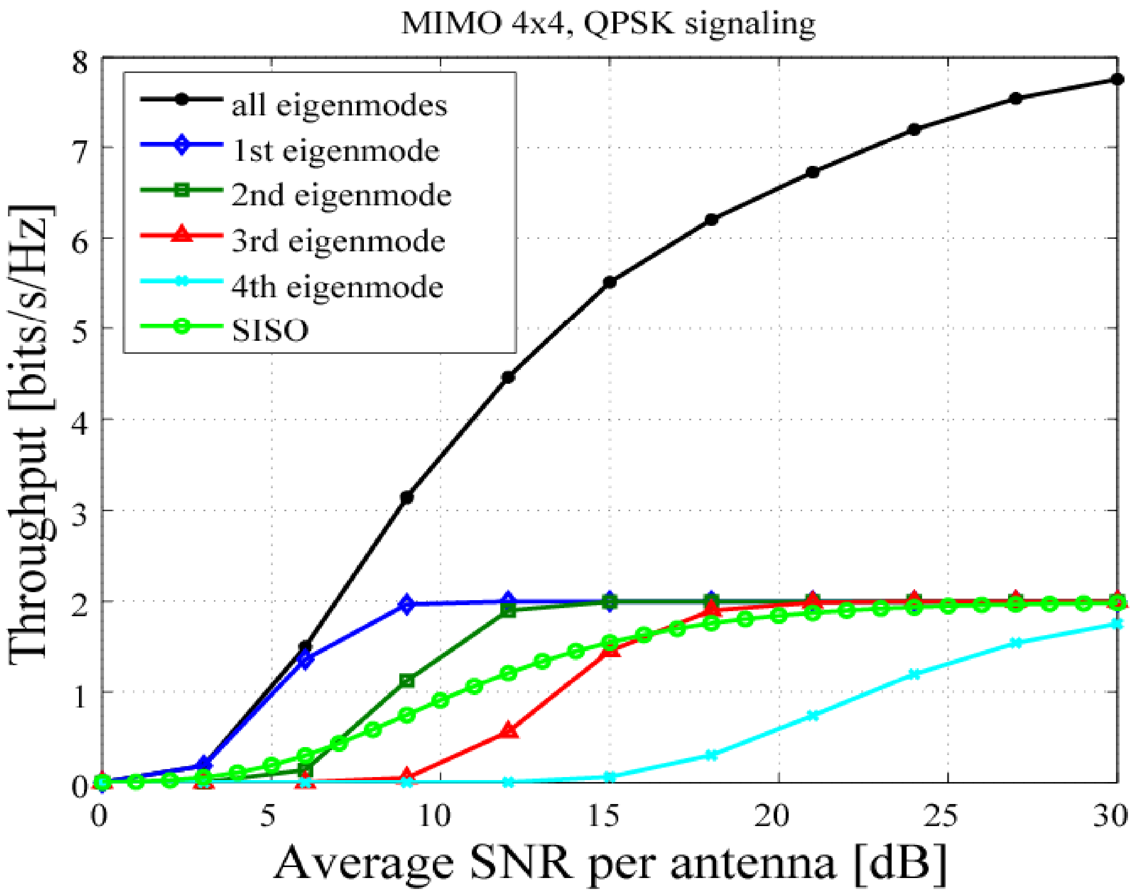
SVD-MIMO (BER)

- Different diversity order for each eigenmode



SVD-MIMO (Throughput)

- Higher order eigenmodes don't work effectively with fixed modulation order



Orthogonal STBC (Coding)

2 transmit antennas (rate: 1)

$$\mathbf{C} = \left[\mathbf{s}(k) \quad \mathbf{s}(k+1)\right] = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$

Orthogonal sequence

$$\mathbf{C}\mathbf{C}^H = \begin{bmatrix} P & 0 \\ 0 & P \end{bmatrix}$$

4 transmit antennas (rate: ½)

$$\mathbf{C} = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & s_3 & -s_2 & s_1 & s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix}$$

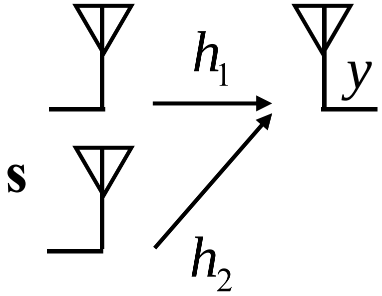
Orthogonal STBC (1-Rx Decoding)

Receive signal

$$\begin{bmatrix} y(k) & y(k+1) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + \begin{bmatrix} n(k) & n(k+1) \end{bmatrix}$$

Equivalent STBC MIMO channel

$$\begin{bmatrix} y(k) \\ y^*(k+1) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n(k) \\ n^*(k+1) \end{bmatrix}$$



Equivalent channel

$$\tilde{\mathbf{y}} = \mathbf{H}_e \mathbf{s} + \tilde{\mathbf{n}}$$

$$\mathbf{H}_e^H \mathbf{H}_e = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} = \begin{bmatrix} |\mathbf{h}|^2 & 0 \\ 0 & |\mathbf{h}|^2 \end{bmatrix}$$

ZF detection

Virtually orthogonal

$$\hat{\mathbf{s}} = \mathbf{H}_e^{-1} \tilde{\mathbf{y}} = \frac{1}{|h_1|^2 + |h_2|^2} \mathbf{H}_e^H \tilde{\mathbf{y}} = \frac{1}{|\mathbf{h}|^2} \mathbf{H}_e^H \tilde{\mathbf{y}}$$

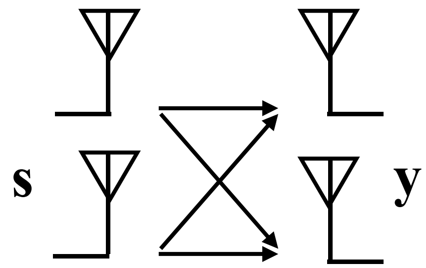
Orthogonal STBC (2-Rx Decoding)

Receive signal

$$[\mathbf{y}(k) \quad \mathbf{y}(k+1)] = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + [\mathbf{n}(k) \quad \mathbf{n}(k+1)]$$

Equivalent STBC MIMO channel

$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$



$$\begin{bmatrix} y_1(k) \\ y_1^*(k+1) \\ y_2(k) \\ y_2^*(k+1) \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{12}^* & -H_{11}^* \\ H_{21} & H_{22} \\ H_{22}^* & -H_{21}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1(k) \\ n_1^*(k+1) \\ n_2(k) \\ n_2^*(k+1) \end{bmatrix}$$

$$\tilde{\mathbf{y}} = \mathbf{H}_{\mathrm{e}}\mathbf{s} + \tilde{\mathbf{n}}$$

$$|\mathbf{H}|_{\mathrm{F}}^2 = |H_{11}|^2 + |H_{12}|^2 + |H_{21}|^2 + |H_{22}|^2$$

$$\mathbf{H}_{\mathrm{e}}^H \mathbf{H}_{\mathrm{e}} = \begin{bmatrix} |\mathbf{H}|_{\mathrm{F}}^2 & 0 \\ 0 & |\mathbf{H}|_{\mathrm{F}}^2 \end{bmatrix}$$

Virtually orthogonal

ZF detection

$$\hat{\mathbf{s}} = \mathbf{H}_{\mathrm{e}}^{-1}\tilde{\mathbf{y}} = \frac{1}{|\mathbf{H}|_{\mathrm{F}}^2} \mathbf{H}_{\mathrm{e}}^H \tilde{\mathbf{y}}$$

Orthogonal STBC (Effective SNR)

Effective SNR

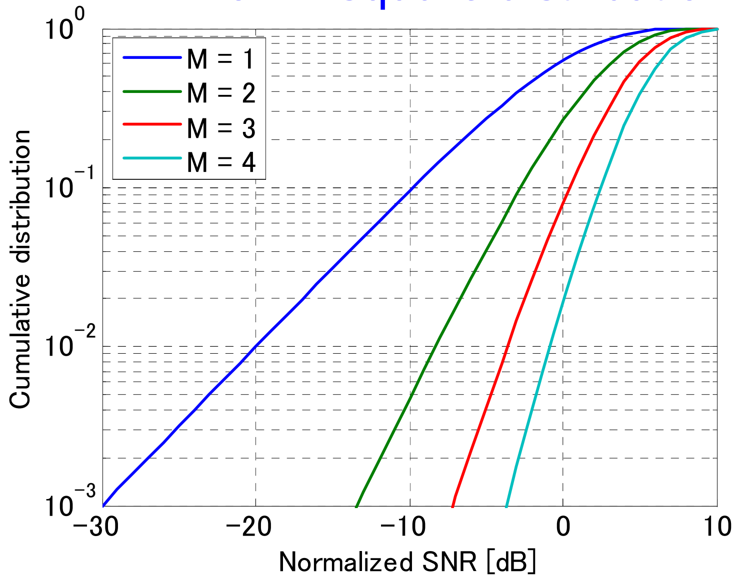
$$\hat{\mathbf{s}} = \frac{1}{\|\mathbf{H}\|_F^2}\mathbf{H}_e^H\tilde{\mathbf{y}} = \mathbf{s} + \frac{1}{\|\mathbf{H}\|_F^2}\mathbf{H}_e^H\tilde{\mathbf{n}} = \mathbf{s} + \hat{\mathbf{n}}$$

$$\mathrm{E}\left[\hat{\mathbf{n}}\hat{\mathbf{n}}^H\right] = \frac{\sigma^2}{\|\mathbf{H}\|_F^4}\mathbf{H}_e^H\mathbf{H}_e = \frac{\sigma^2}{\|\mathbf{H}\|_F^2}\mathbf{I}_4$$

$$\gamma^{\mathrm{STBC}} = \frac{P\|\mathbf{H}\|_F^2}{M_t\sigma_2} = \frac{P\sum_{i=1}^{2}\sum_{j=1}^{2}\left|H_{ij}\right|^2}{M_t\sigma_2}$$

Transmit & receive full diversity

PDF of Xi-square distribution

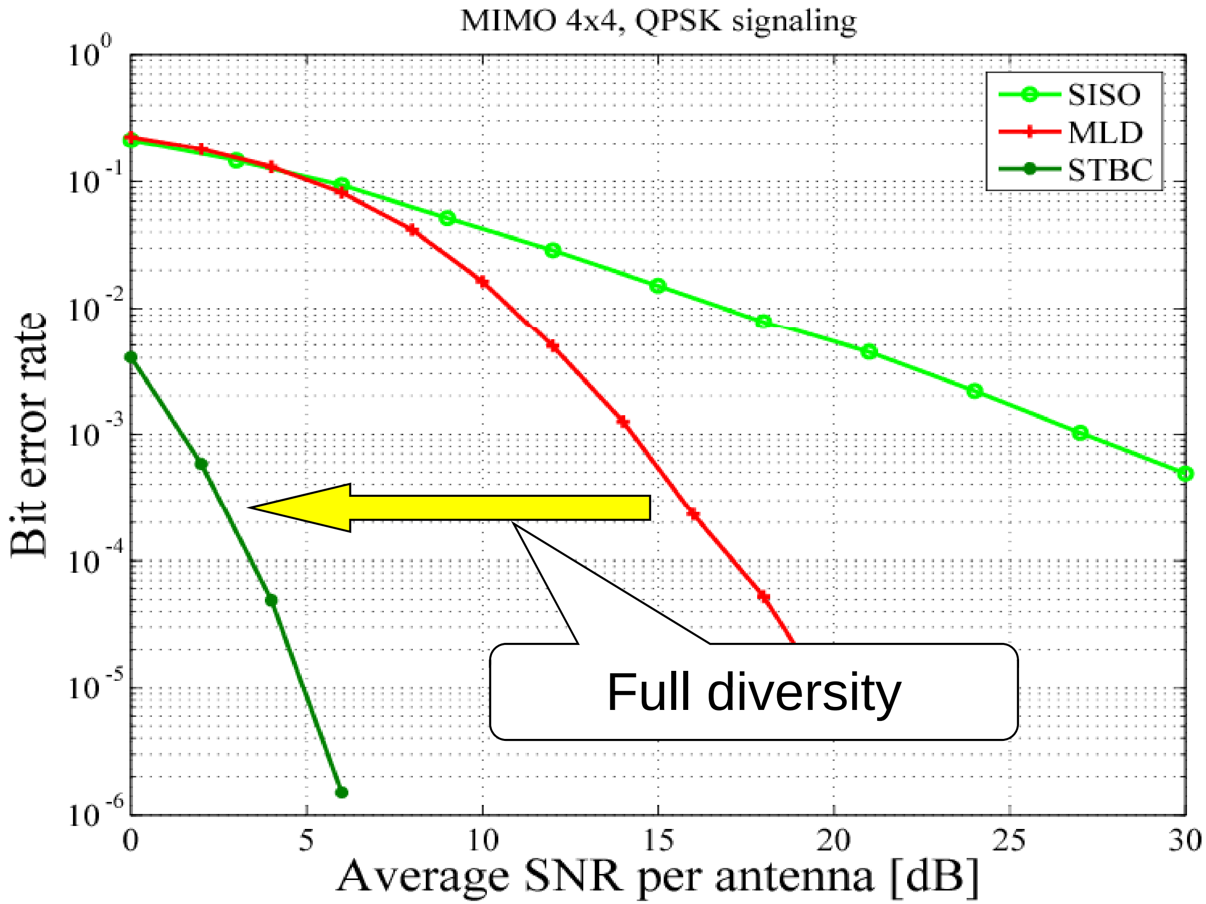


PDF of effective SNR

$$f(\gamma) = \frac{1}{(M-1)!\bar{\gamma}^M}\gamma^{M-1}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \qquad M = M_t M_r$$

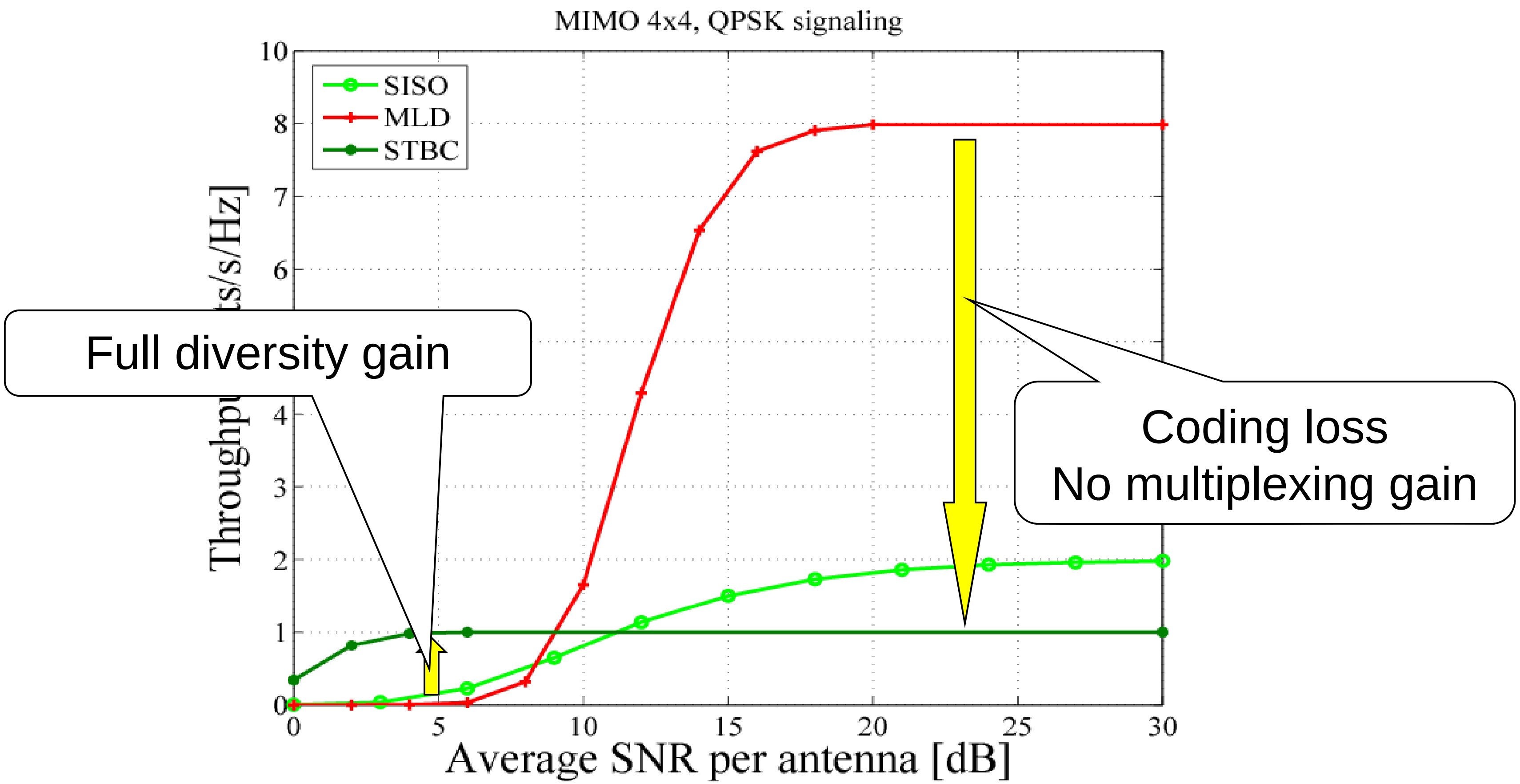
Orthogonal STBC (BER)

- OSTBC achieves full diversity gain (4 x 4 = 16)



Orthogonal STBC (Throughput)

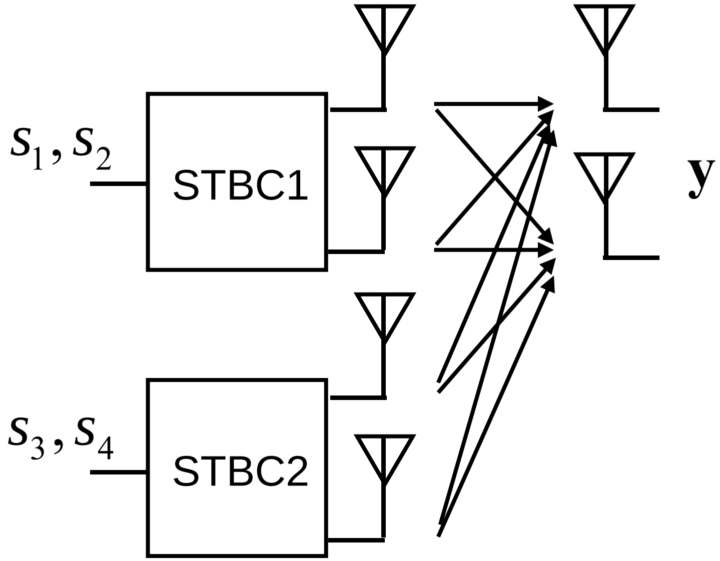
- OSTBC sacrifices data rate to coding rate (1/2)



Quasi-Orthogonal STBC (Coding)

4 transmit antennas (rate: 2)

$$\mathbf{C} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \\ s_3 & -s_3^* \\ s_4 & s_4^* \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix}$$



Quasi-orthogonal

$$\mathbf{C}\mathbf{C}^H = \begin{bmatrix} \frac{P}{2}\mathbf{I}_2 & \mathbf{C}_1\mathbf{C}_2^* \\ \mathbf{C}_2\mathbf{C}_1^* & \frac{P}{2}\mathbf{I}_2 \end{bmatrix}$$

Quasi-Orthogonal

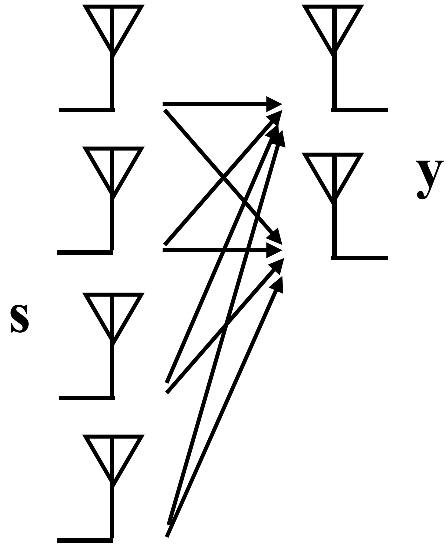
Q-OSTBC (Linear Detection)

Receive signal

$$[\mathbf{y}(k) \quad \mathbf{y}(k+1)] = \mathbf{H}\begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \\ s_3 & -s_4^* \\ s_4 & s_3^* \end{bmatrix} + [\mathbf{n}(k) \quad \mathbf{n}(k+1)]$$

$$\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & H_{23} & H_{24} \end{bmatrix} = [\mathbf{H}_1 \quad \mathbf{H}_2]$$

Equivalent STBC MIMO channel



$$\begin{bmatrix} y_1(k) \\ y_1^*(k+1) \\ y_2(k) \\ y_2^*(k+1) \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{12}^* & -H_{11}^* & H_{14}^* & -H_{13}^* \\ H_{21} & H_{22} & H_{23} & H_{24} \\ H_{22}^* & -H_{21}^* & H_{24}^* & -H_{23}^* \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_1(k) \\ n_1^*(k+1) \\ n_2(k) \\ n_2^*(k+1) \end{bmatrix}$$

$$\tilde{\mathbf{y}} = \mathbf{H}_{\mathrm{e}}\mathbf{s} + \tilde{\mathbf{n}} \qquad \mathbf{H}_{\mathrm{e}}^H\mathbf{H}_{\mathrm{e}} = \begin{bmatrix} |\mathbf{H}_1|_{\mathrm{F}}^2\mathbf{I}_2 & \mathbf{H}_{\mathrm{e}12}^H\mathbf{H}_{\mathrm{e}34} \\ \mathbf{H}_{\mathrm{e}34}^H\mathbf{H}_{\mathrm{e}12} & |\mathbf{H}_2|_{\mathrm{F}}^2\mathbf{I}_2 \end{bmatrix}$$

Quasi-Orthogonal

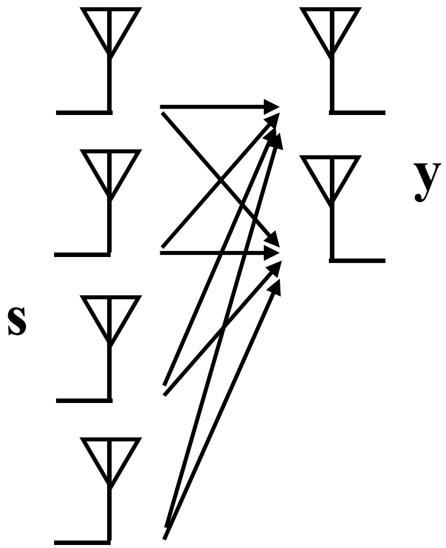
Q-OSTBC (Effective SNR)

ZF detection

$$\hat{\mathbf{s}} = \mathbf{W}\tilde{\mathbf{y}} = \left(\mathbf{H}_{\mathrm{e}}^{H}\mathbf{H}_{\mathrm{e}}\right)^{-1}\mathbf{H}_{\mathrm{e}}^{H}\tilde{\mathbf{y}}$$

MMSE detection

$$\hat{\mathbf{s}} = \mathbf{W}\tilde{\mathbf{y}} = \left(\mathbf{H}_{\mathrm{e}}^{H}\mathbf{H}_{\mathrm{e}} + \frac{M_{\mathrm{t}}\sigma^{2}}{P}\mathbf{I}_{M_{\mathrm{t}}}\right)^{-1}\mathbf{H}_{\mathrm{e}}^{H}\tilde{\mathbf{y}}$$



Effective SNR

$$\gamma_{i}^{\mathrm{ZF}} = \frac{P}{M_{t}\sigma^{2}\left(\mathbf{H}_{\mathrm{e}}^{H}\mathbf{H}_{\mathrm{e}}\right)_{ii}^{-1}}$$

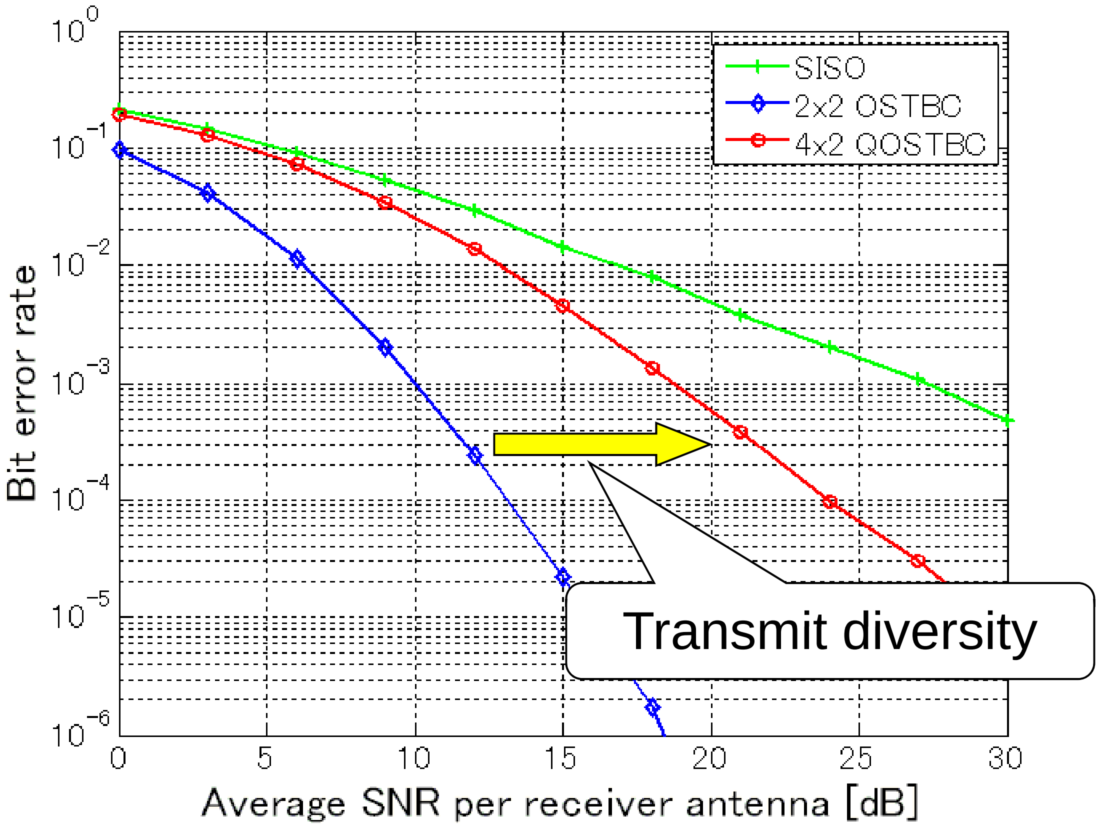
$$\gamma_{i}^{\mathrm{MMSE}} = \frac{1}{\left(\frac{P}{M_{\mathrm{t}}\sigma^{2}}\mathbf{H}_{\mathrm{e}}^{H}\mathbf{H}_{\mathrm{e}} + \mathbf{I}_{M_{\mathrm{t}}}\right)_{ii}^{-1}} - 1$$

Diversity order

$$g_{\mathrm{d}} = \underbrace{2\times 2}_{\text{STBC}} - \underbrace{2}_{\text{Interference cancellation}} = 2$$

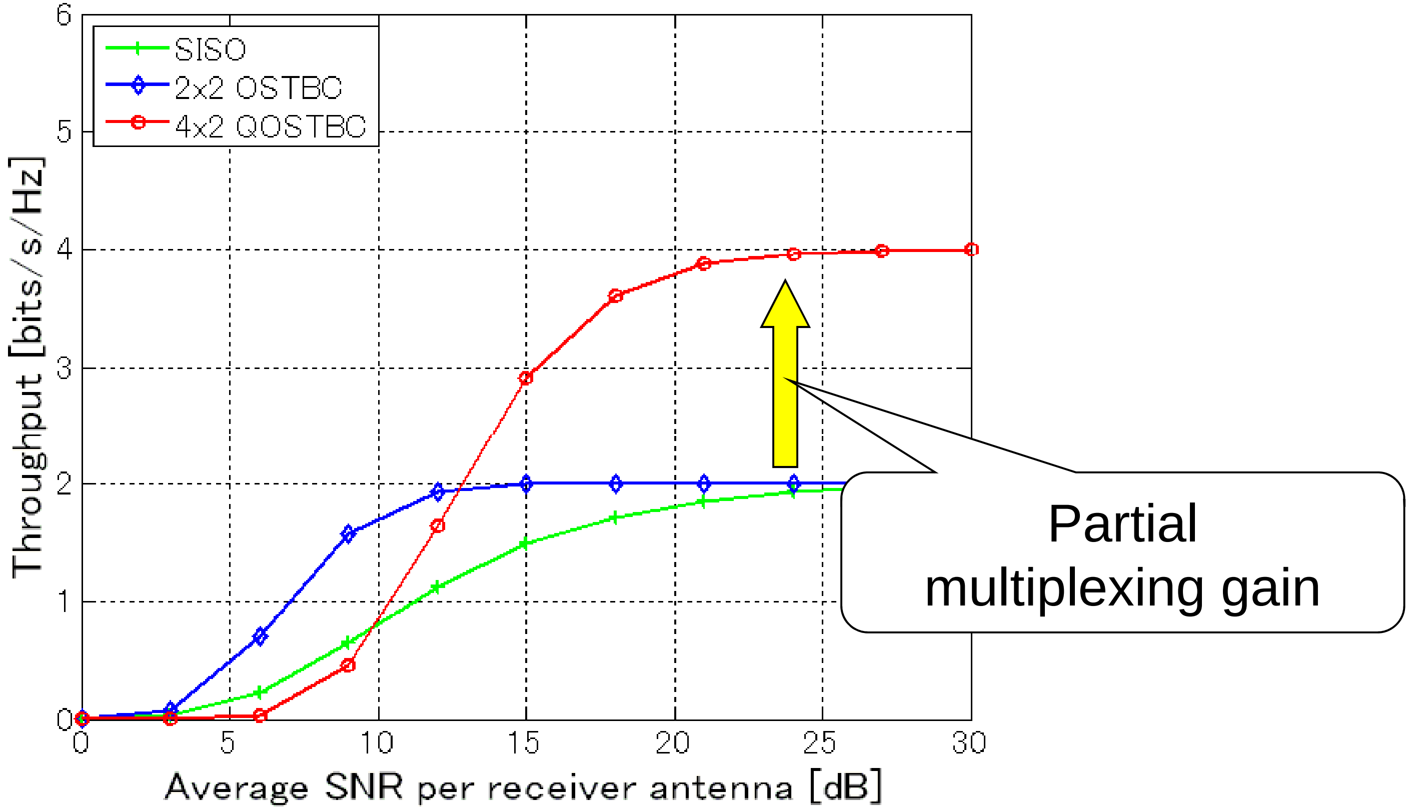
Quasi-Orthogonal STBC (BER)

- QOSTBC achieves blind transmit diversity even with spatial multiplexing



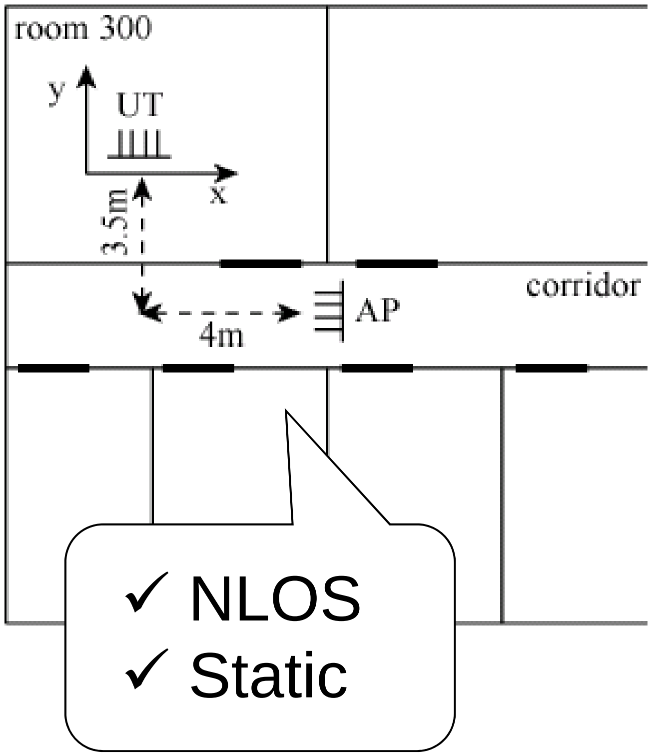
Quasi-Orthogonal STBC (Throughput)

- QOSTBC achieves twice throughput than that of normal OSTBC



Measurement Condition

Environment

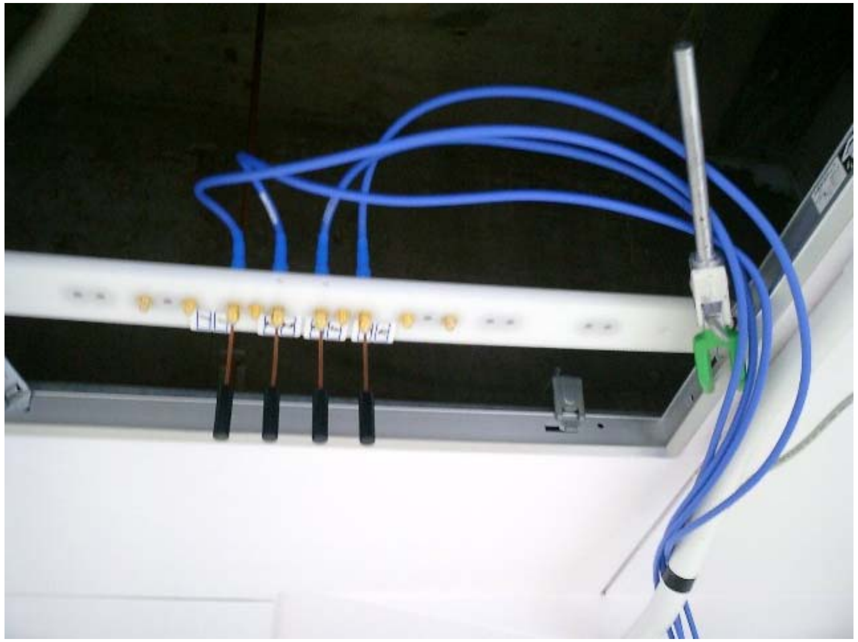


Parameters

Center freq.	5.2[GHz]
Array structure	Half a wavelength spacing 4-element linear array
Symbol rate	125[ksps]
Modulation	(BPSK), QPSK, (16QAM)
Frame structure	128(31:training, 97:data)
Measurement points	441 2x2[cm^2] step in 40x40[cm^2]

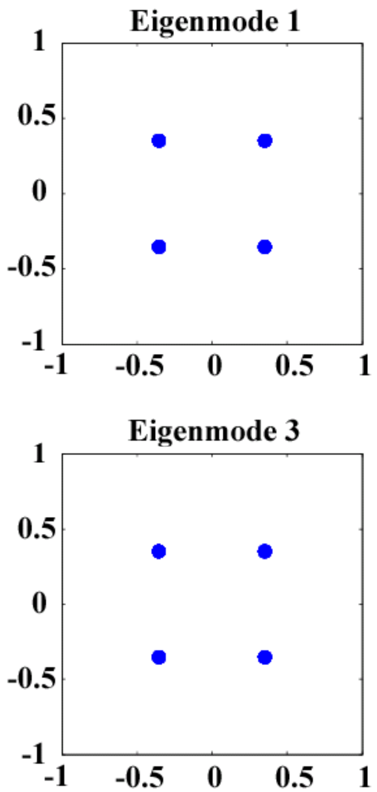
Measurement Environment

Transmit array antenna（AP） Receive array antenna（UT）

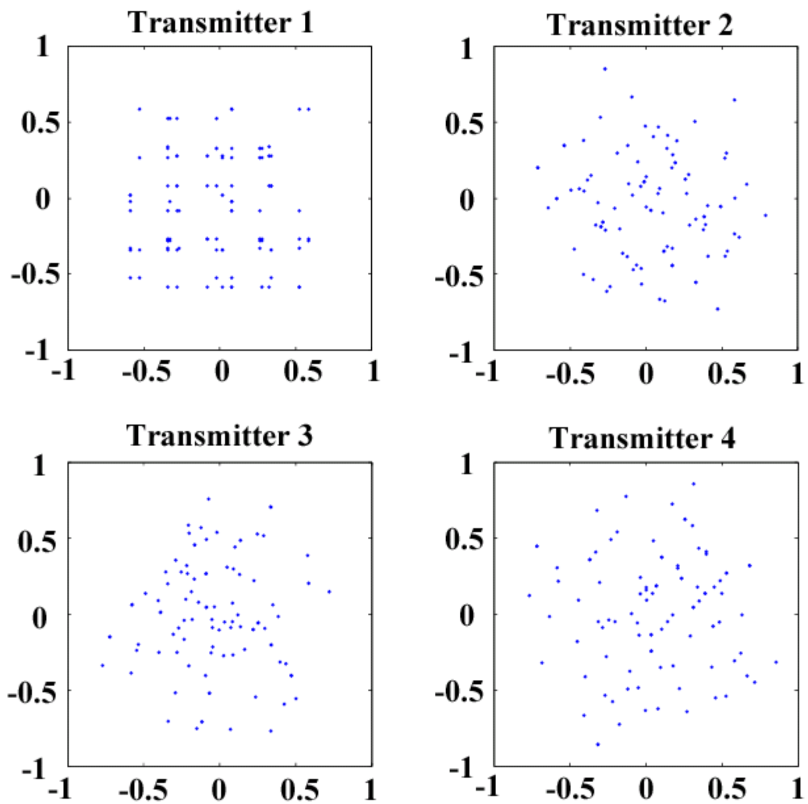


Transmit Signal Constellation

Before eigen-beamforming

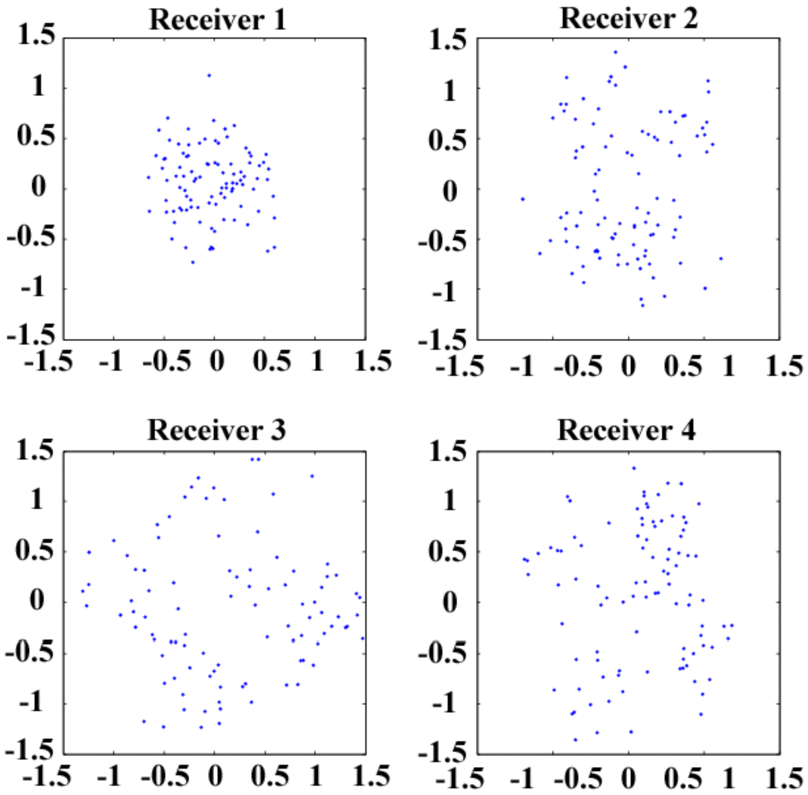


After eigen-beamforming

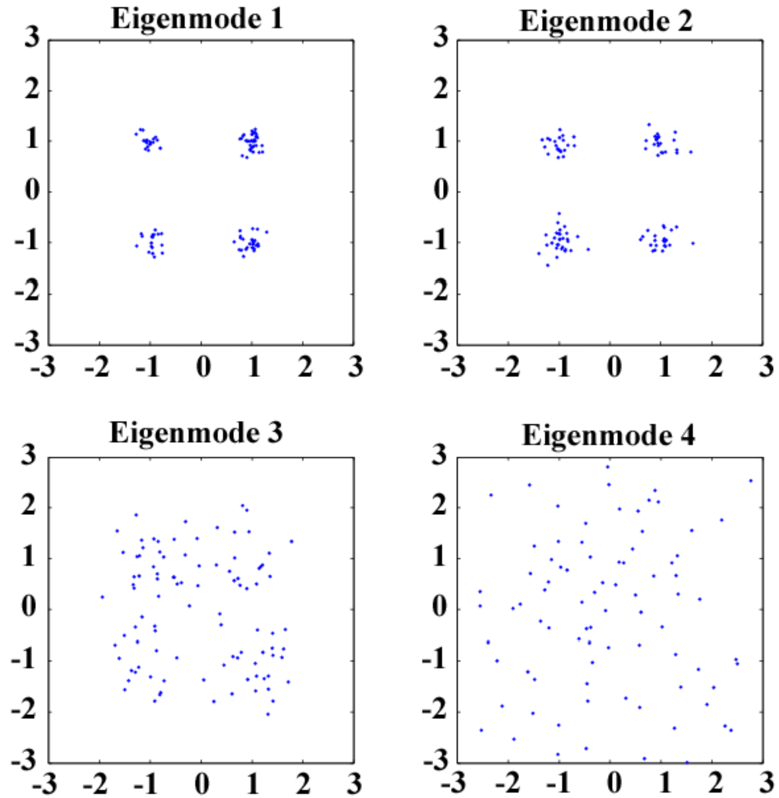


Receive Signal Constellation

Before eigen-beamforming

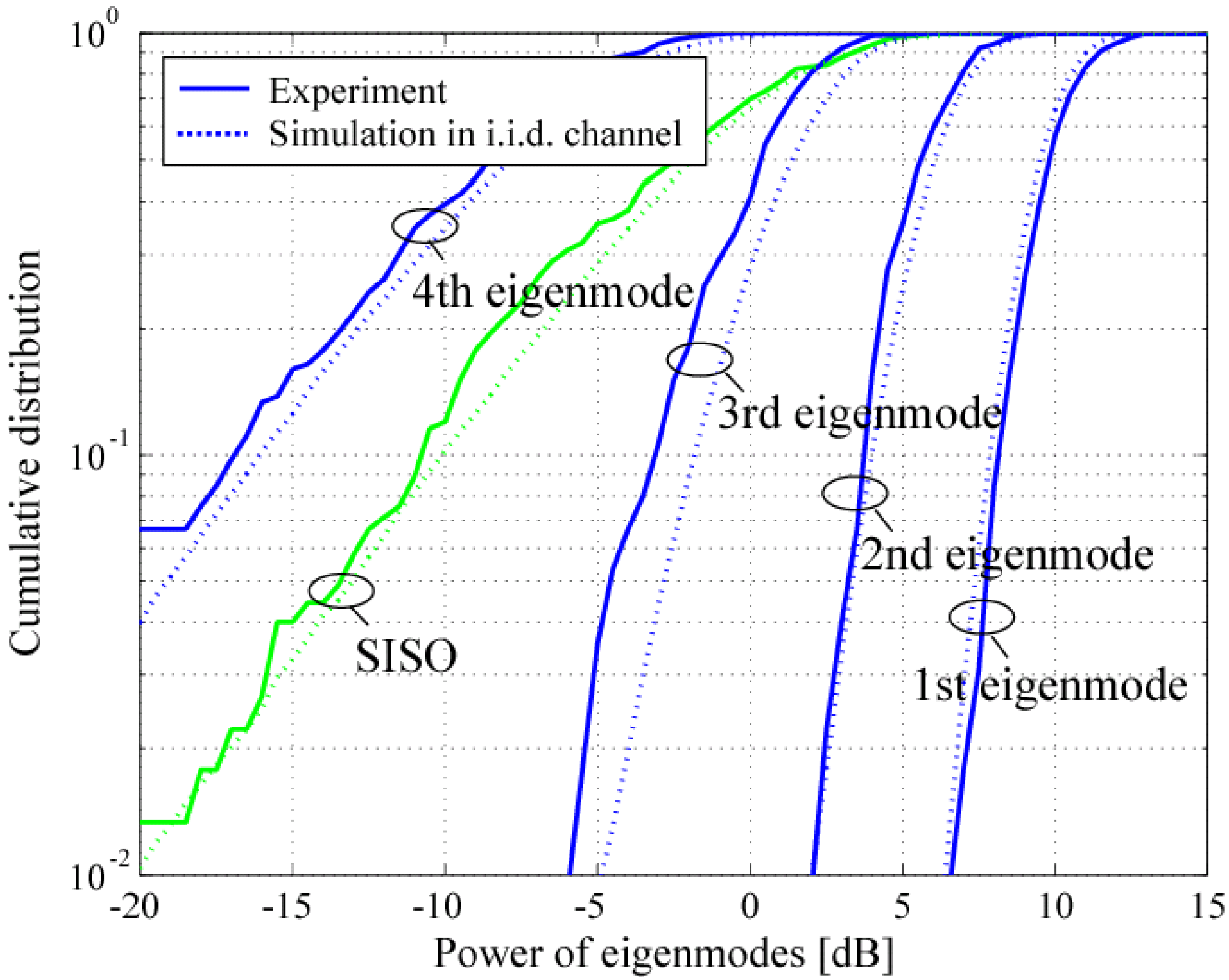


After eigen-beamforming



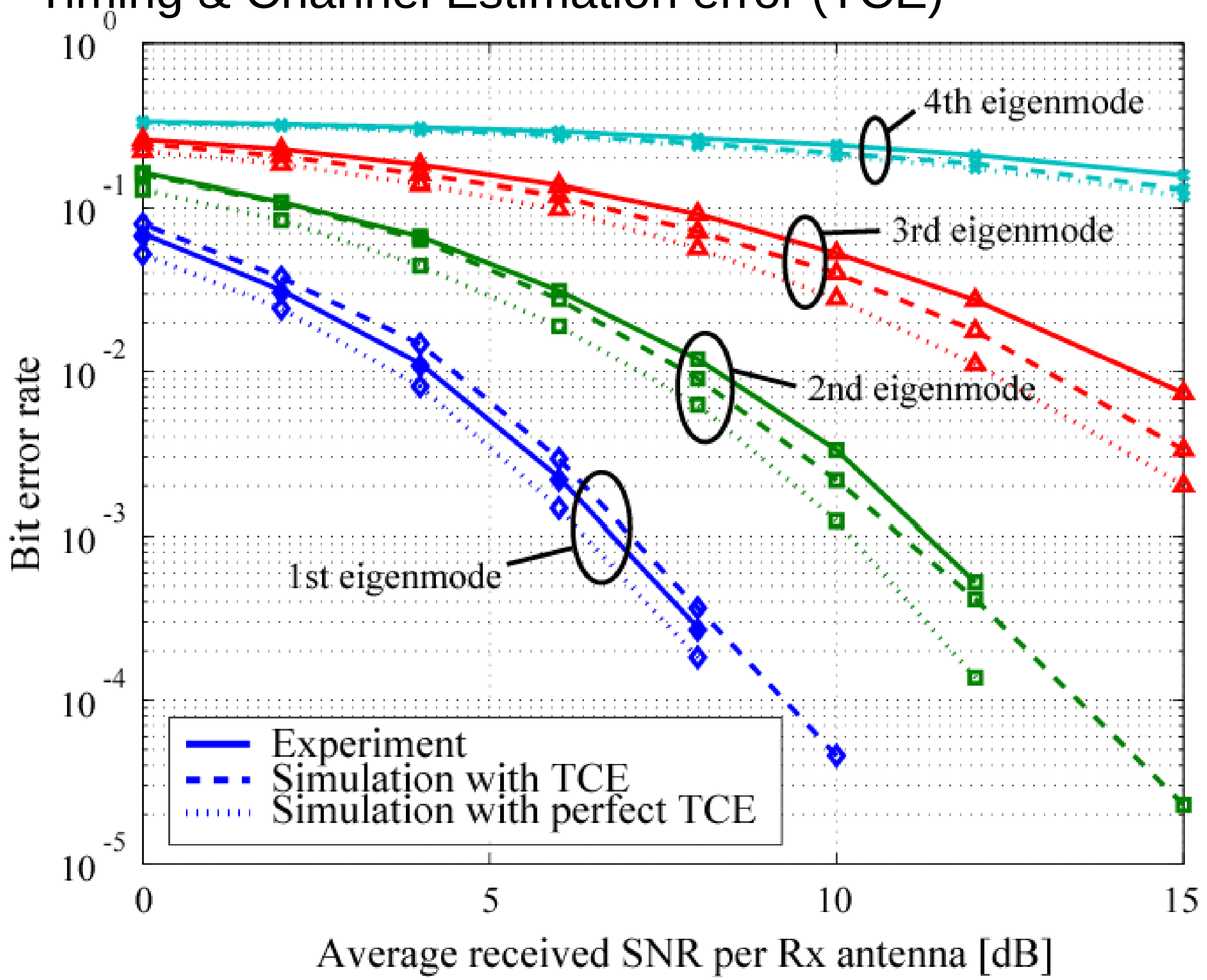
Measured Eigen-values

- Measured eigen-values agree well with theoretical values in IID environment



Measured BER

- Measured BER agree well with theoretical values with Timing & Channel Estimation error (TCE)

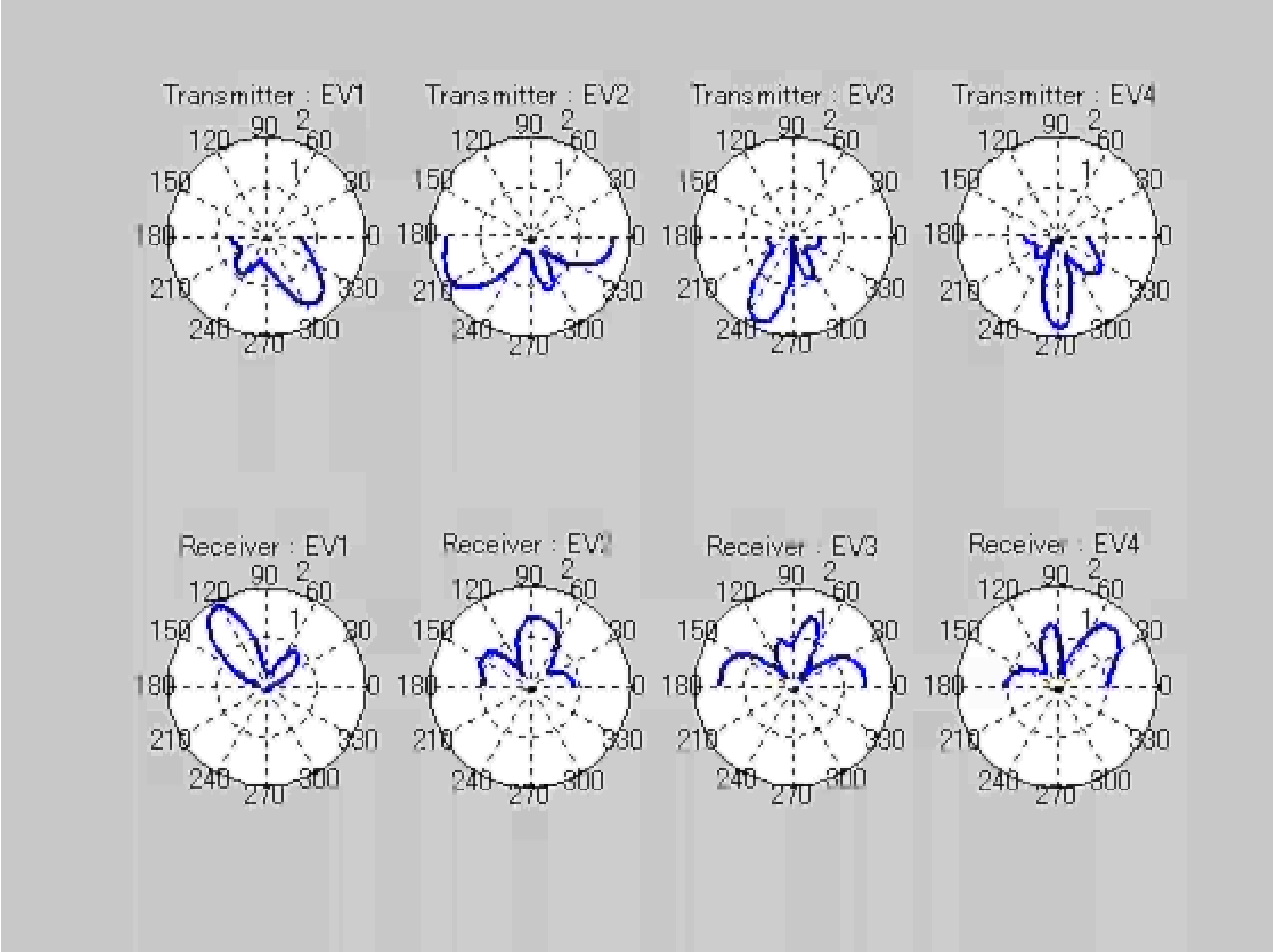


Measured Beam Pattern

#1 #2 #3 #4

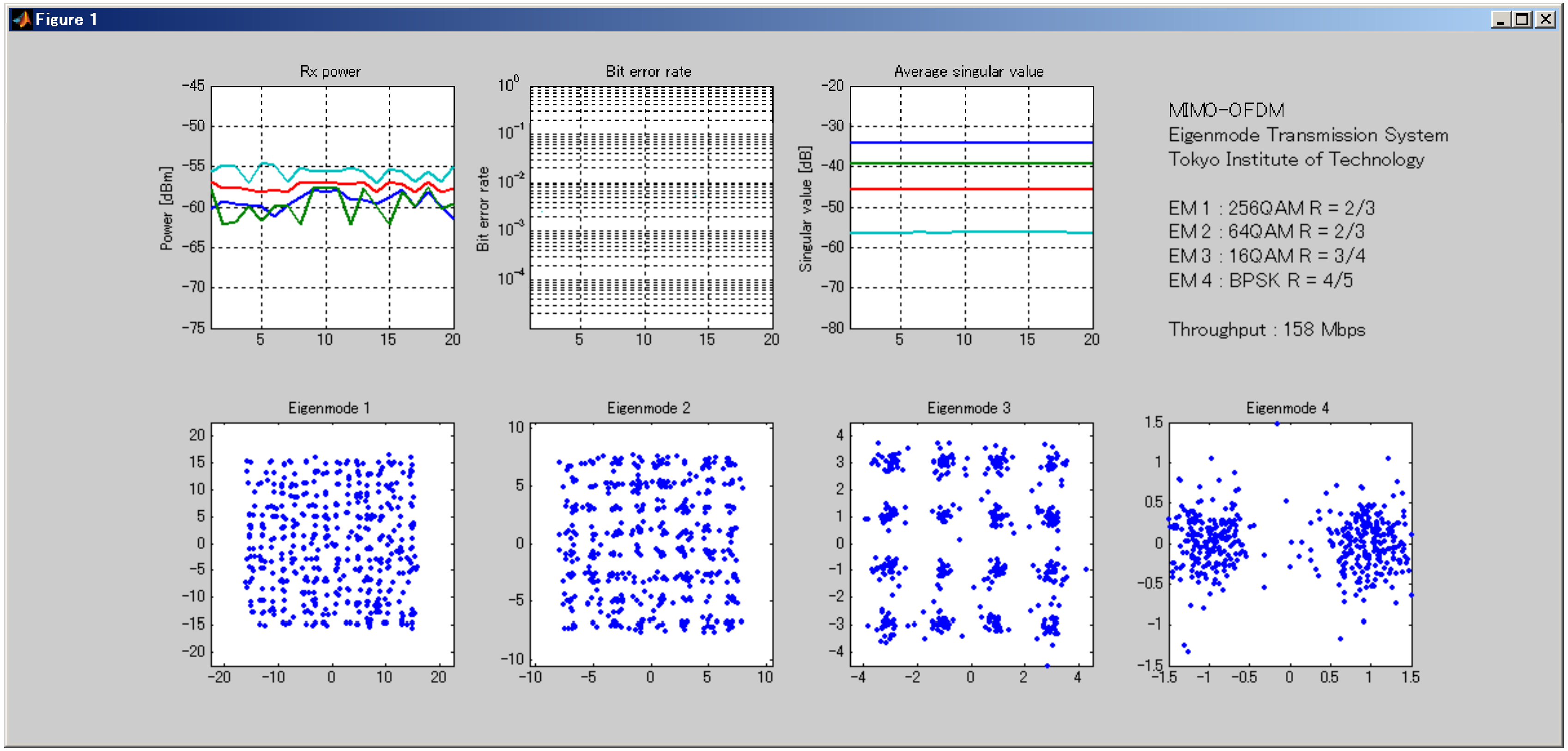
Tx

Rx



Demonstration Video

- 4x4 SVD-MIMO achieves 8 + 6 + 4 + 1 = 19 [bps/Hz]



Summary

- MIMO transmitter
  - OSTBC achieves blind full diversity gain while sacrificing multiplexing gain
  - QOSTBC achieves blind transmit diversity gain with partial spatial multiplexing
  - SVD-MIMO provides spatial multiplexing with different diversity gains by utilizing channel feedback

Effective utilization of different diversity order in SVD-MIMO

Adaptive MIMO communication system