2011 1st semester
MIMO Communication Systems

#7: MIMO Receiver

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Schedule (2^{nd} half)

	Date	Text	Contents
#7	May 31	A-5	MIMO receiver
#8	June 7	A-3, 4	MIMO transmitter
#9	June 14	B-9	Adaptive commun. system
#10	June 21	A-6, B-14	Multi-user MIMO
#11	June 28	B-15, 16	Distributed MIMO networks
#12	July 5		Standardization of MIMO
	July 12		Final Examination

Agenda

Aim of today

Derive BER & throughput performances
of basic linear & non-linear MIMO receivers

Contents

- MIMO receiver architecture
- Frame synchronization & channel estimation
- Linear MIMO detection algorithm
- Non-linear MIMO detection algorithm
- Measurement experiment

Warming Up

- Question 1

Describe generalized matrix inverse of non-square matrix $\mathbf{H}$ by using SVD.

$$\mathbf{H}^{+} =$$

- Question 2

Prove that the following equation holds.

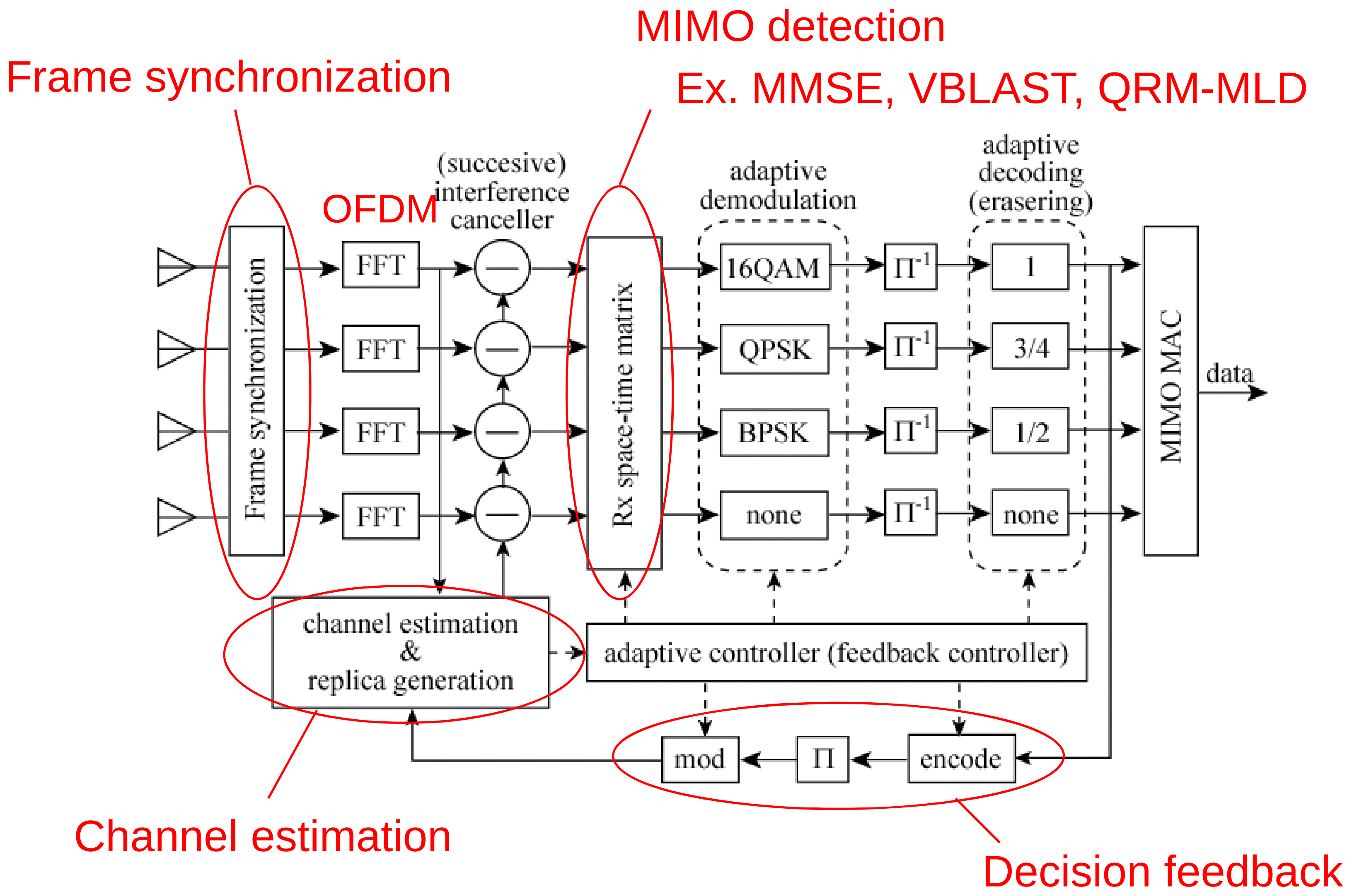
$$\mathbf{H}^{+} = \left(\mathbf{H}^{H}\mathbf{H}\right)^{-1}\mathbf{H}^{H}$$

- Singular Value Decomposition

Non-square matrix: $\mathbf{H} \in C^{n\times m} \quad n > m$

SVD: $\mathbf{H} = \mathbf{U}\,\mathbf{\Sigma}\,\mathbf{V}^{H} \quad \mathbf{U} \in C^{n\times m} \quad \mathbf{\Sigma} \in C^{m\times m} \quad \mathbf{V} \in C^{m\times m}$

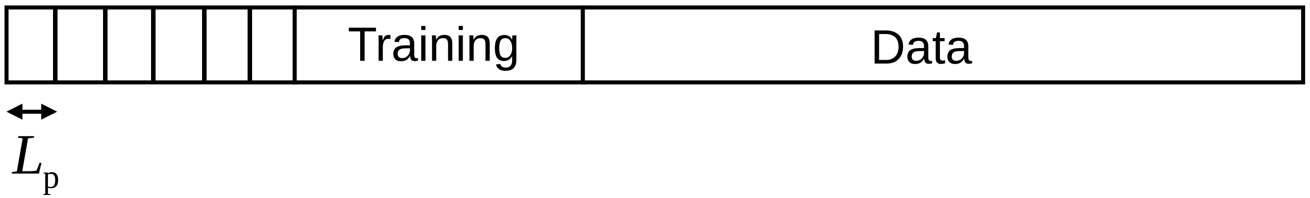
MIMO Receiver Architecture



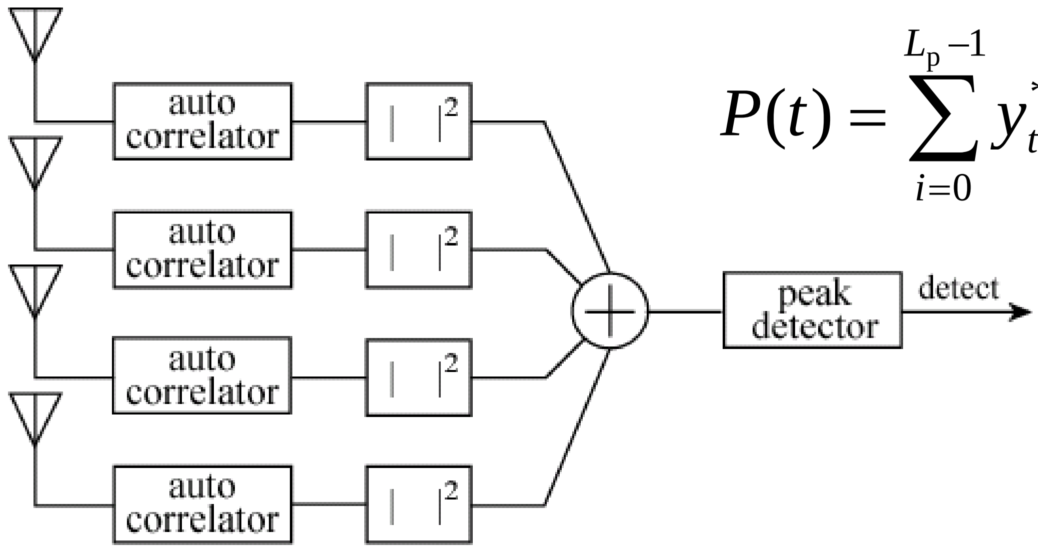
Frame Synchronization

Preamble

Preamble with repetition sequence



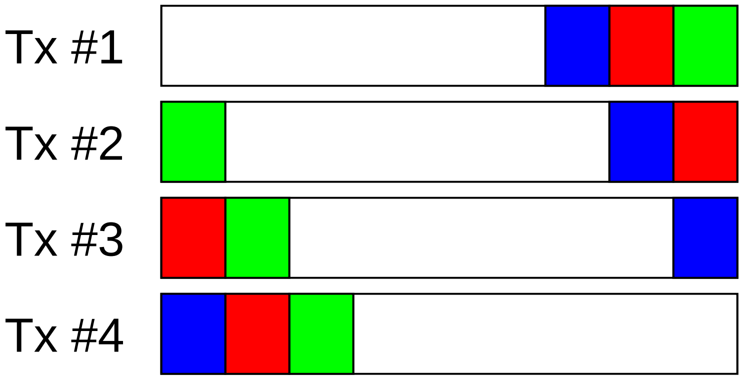
Correlation based frame detector with receive diversity



$$P(t) = \sum_{i=0}^{L_\mathrm{p}-1} y_{t+i}^{*} y_{t+i+L_\mathrm{p}}$$

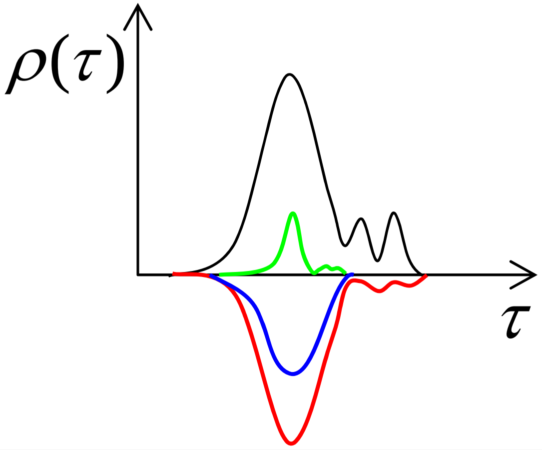
Cyclic Delay Preamble

Cyclic delay preamble



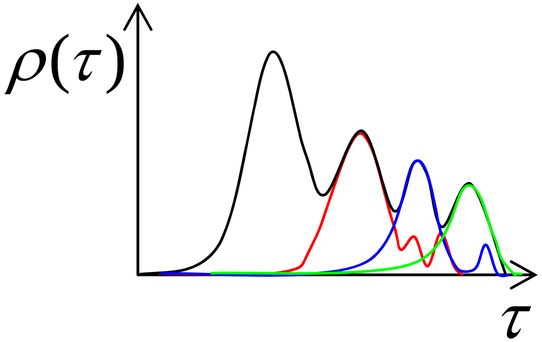
Without cyclic delay

Null beamforming effect



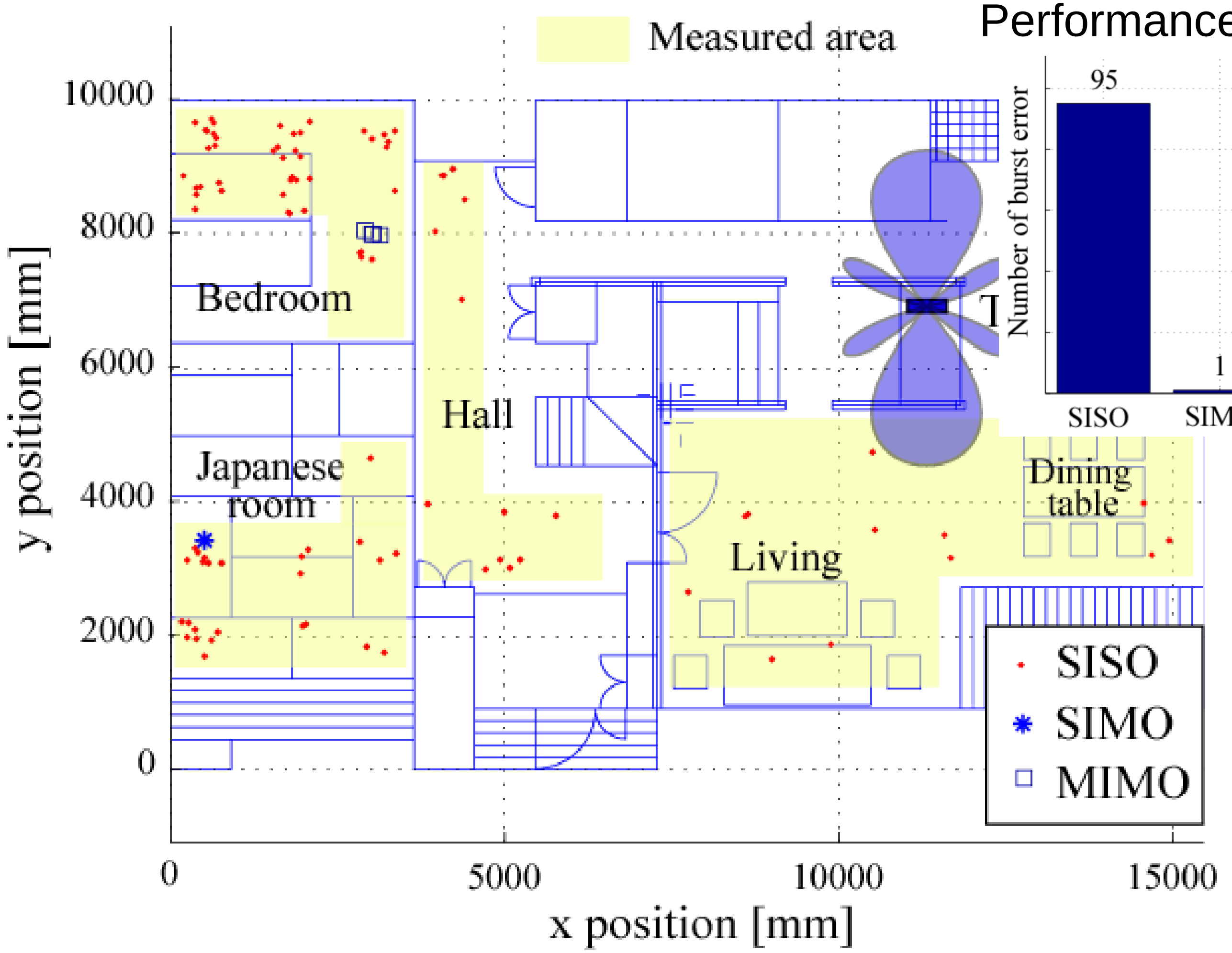
With cyclic delay

Transmit delay diversity

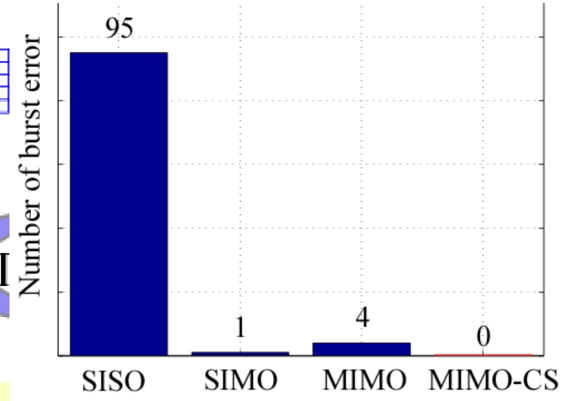


Experiment on MIMO Synchronization

- Performance improvement owing to transmit & receive diversity



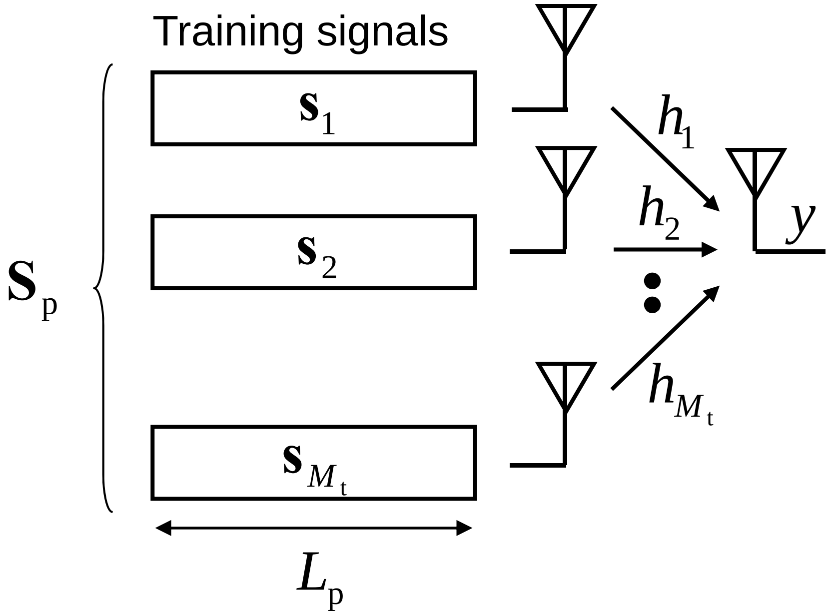
Performance of frame synch.



Channel Estimation

At each receive antenna

$$y = \mathbf{h}^T \mathbf{s} + n$$
$$= \mathbf{s}^T \mathbf{h} + n$$



Training signal

$$\mathbf{y}_\mathrm{p} = \mathbf{S}_\mathrm{p}\mathbf{h} + \mathbf{n}_\mathrm{p}$$
$$\mathbf{S}_\mathrm{p} = \left[\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_{M_\mathrm{t}}\right] \in C^{L_\mathrm{p} \times M_\mathrm{t}}$$

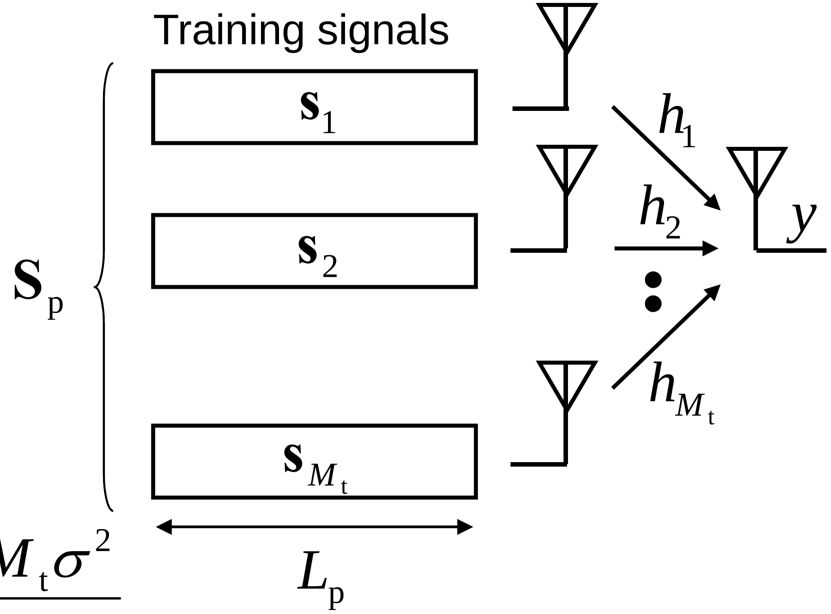
Channel estimation

$$\hat{\mathbf{h}} = \mathbf{S}_\mathrm{p}^{+}\mathbf{y}_\mathrm{p} = \mathbf{h} + \mathbf{S}_\mathrm{p}^{+}\mathbf{n}_\mathrm{p}$$
$$\mathbf{S}_\mathrm{p}^{+} = \left(\mathbf{S}_\mathrm{p}^{H}\mathbf{S}_\mathrm{p}\right)^{-1}\mathbf{S}_\mathrm{p}^{H}$$

Channel estimation error

$$\sigma_\mathrm{ce}^2 = \frac{1}{M_\mathrm{t}} \mathrm{E}\left[\left|\hat{\mathbf{h}} - \mathbf{h}\right|^2\right] = \frac{1}{M_\mathrm{t}} \mathrm{E}\left[\left|\mathbf{S}_\mathrm{p}^{+}\mathbf{n}_\mathrm{p}\right|^2\right]$$
$$= \frac{\sigma^2}{M_\mathrm{t}} \mathrm{Tr}\left[\mathbf{S}_\mathrm{p}^{+}\left(\mathbf{S}_\mathrm{p}^{+}\right)^H\right] = \frac{\sigma^2}{M_\mathrm{t}} \mathrm{Tr}\left[\left(\mathbf{S}_\mathrm{p}^{H}\mathbf{S}_\mathrm{p}\right)^{-1}\right]$$

Optimal Training Sequence



Channel estimation error

$$\sigma_{ce}^2 = \frac{\sigma^2}{M_t}\mathrm{Tr}\left[\left(\mathbf{S}_p^H\mathbf{S}_p\right)^{-1}\right]$$

Optimal training sequence

$\mathrm{Tr}\left[\left(\mathbf{S}_p^H\mathbf{S}_p\right)^{-1}\right]$ is minimal

when $\mathbf{S}_p^H\mathbf{S}_p = \frac{PL_p}{M_t}\mathbf{I}_{M_t} \longrightarrow \sigma_{ce}^2 = \frac{M_t\sigma^2}{L_pP}$

Receive signal with channel estimation error

$$\mathbf{H} = \hat{\mathbf{H}} + \mathbf{N}_{ce} \longrightarrow \mathrm{E}\left[\mathbf{N}_{ce}\mathbf{N}_{ce}^H\right] = \frac{M_t^2\sigma^2}{L_pP}$$

Effective noise

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} = \hat{\mathbf{H}}\mathbf{s} + \mathbf{N}_{ce}\mathbf{s} + \mathbf{n} = \hat{\mathbf{H}}\mathbf{s} + \tilde{\mathbf{n}} \longrightarrow \mathrm{E}\left[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\right] = \sigma^2\left(\frac{M_t}{L_p} + 1\right)\mathbf{I}_{M_r}$$

Classification of MIMO Detector

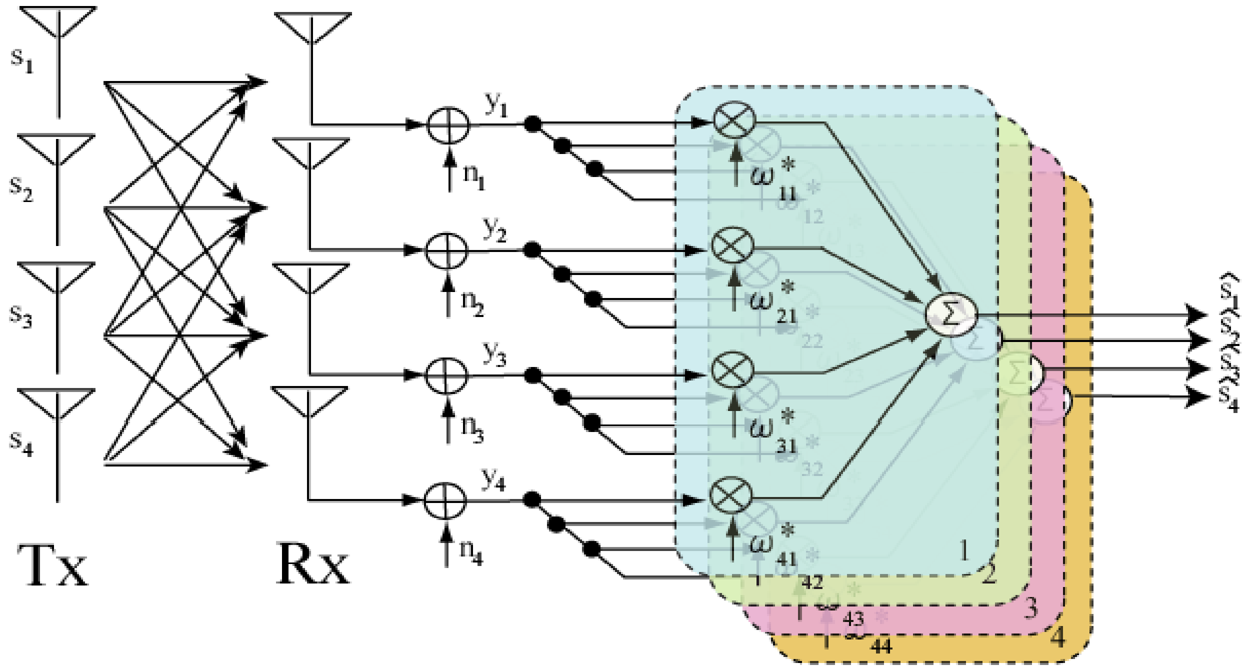
	Schemes	Performance	Complexity
Linear	ZF MMSE	Low	Low
Hybrid	VBLAST QRM-MLD	Medium	Medium
Non-linear	MLD	High	High

Linear MIMO detection

1st stream

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{y} = \begin{bmatrix} w_{11} & \cdots & w_{1m_r} \\ \vdots & \ddots & \vdots \\ w_{m_t 1} & \cdots & w_{m_t m_r} \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_{m_r} \end{bmatrix}$$

M_t-th stream



Interference Cancellation Detection (Zero Forcing)

Received signal

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

ZF weight

$$\mathbf{W} = \mathbf{H}^{+} = \left(\mathbf{H}^{H}\mathbf{H}\right)^{-1}\mathbf{H}^{H}$$

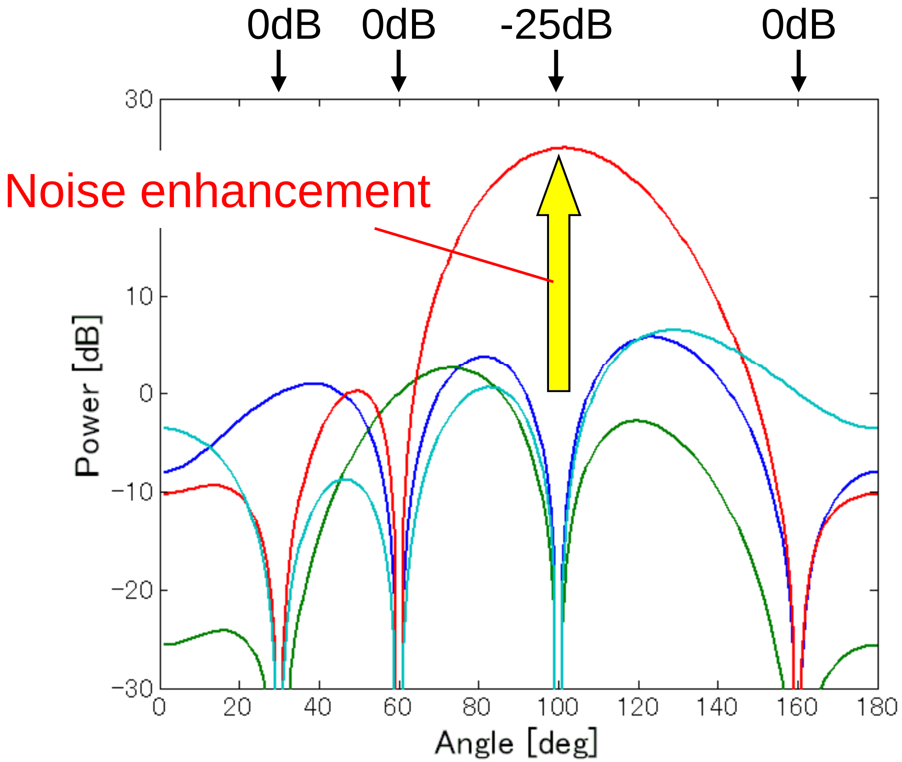
Estimated signal

$$\hat{\mathbf{s}} = \mathbf{H}^{+}\mathbf{y} = \mathbf{s} + \mathbf{H}^{+}\mathbf{n} = \mathbf{s} + \tilde{\mathbf{n}}$$

Effective SNR for *i*-th stream

$$\mathrm{E}\left[\left|\tilde{\mathbf{n}}\tilde{\mathbf{n}}^{H}\right|^{2}\right] = \sigma^{2}\left(\mathbf{H}^{H}\mathbf{H}\right)^{-1}$$

$$\gamma_{i}^{\mathrm{ZF}} = \frac{P}{M_{t}\sigma^{2}\left(\mathbf{H}^{H}\mathbf{H}\right)_{ii}^{-1}}$$



Diversity order

$$g_{\mathrm{d}} = M_{\mathrm{r}} - M_{\mathrm{t}} + 1$$

Minimum Mean Square Error Detection

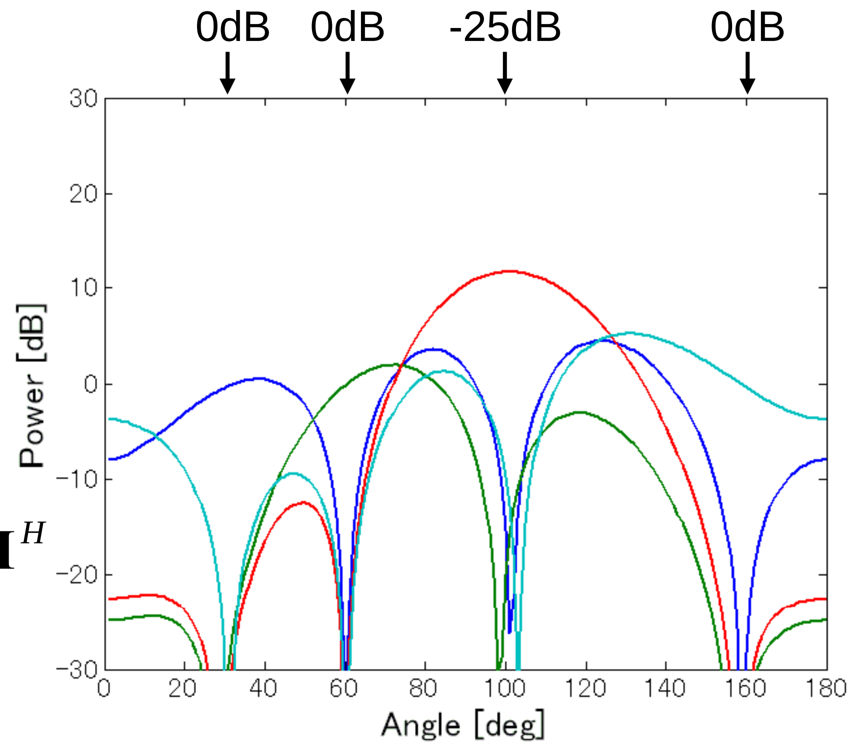
Cost function

$$J(\mathbf{W}) = \mathrm{E}\left[\|\mathbf{s} - \mathbf{W}\mathbf{y}\|^2\right]$$

MMSE weight (Wiener solution)

$$\frac{\partial J(\mathbf{W})}{\partial \mathbf{W}} = 0$$

$$\longrightarrow \quad \mathbf{W} = \left(\mathbf{H}^H\mathbf{H} + \frac{M_{\mathrm{t}}\sigma^2}{P}\mathbf{I}_{M_{\mathrm{t}}}\right)^{-1}\mathbf{H}^H$$



Effective SNR for *i*-th stream

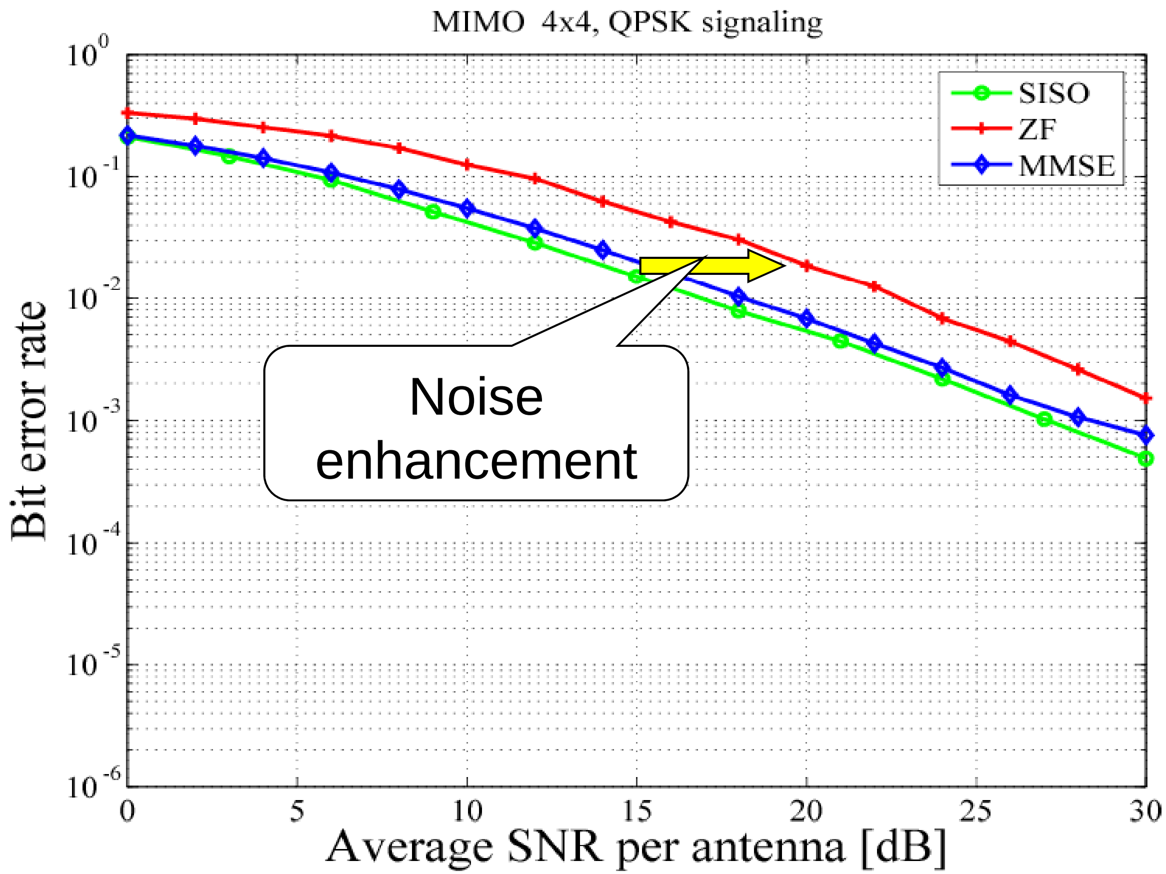
$$\gamma_i^{\mathrm{MMSE}} = \frac{1}{\left(\frac{P}{M_{\mathrm{t}}\sigma^2}\mathbf{H}^H\mathbf{H} + \mathbf{I}_{M_{\mathrm{t}}}\right)^{-1}_{ii}} - 1$$

Diversity order

$$g_{\mathrm{d}} = M_{\mathrm{r}} - M_{\mathrm{t}} + 1$$

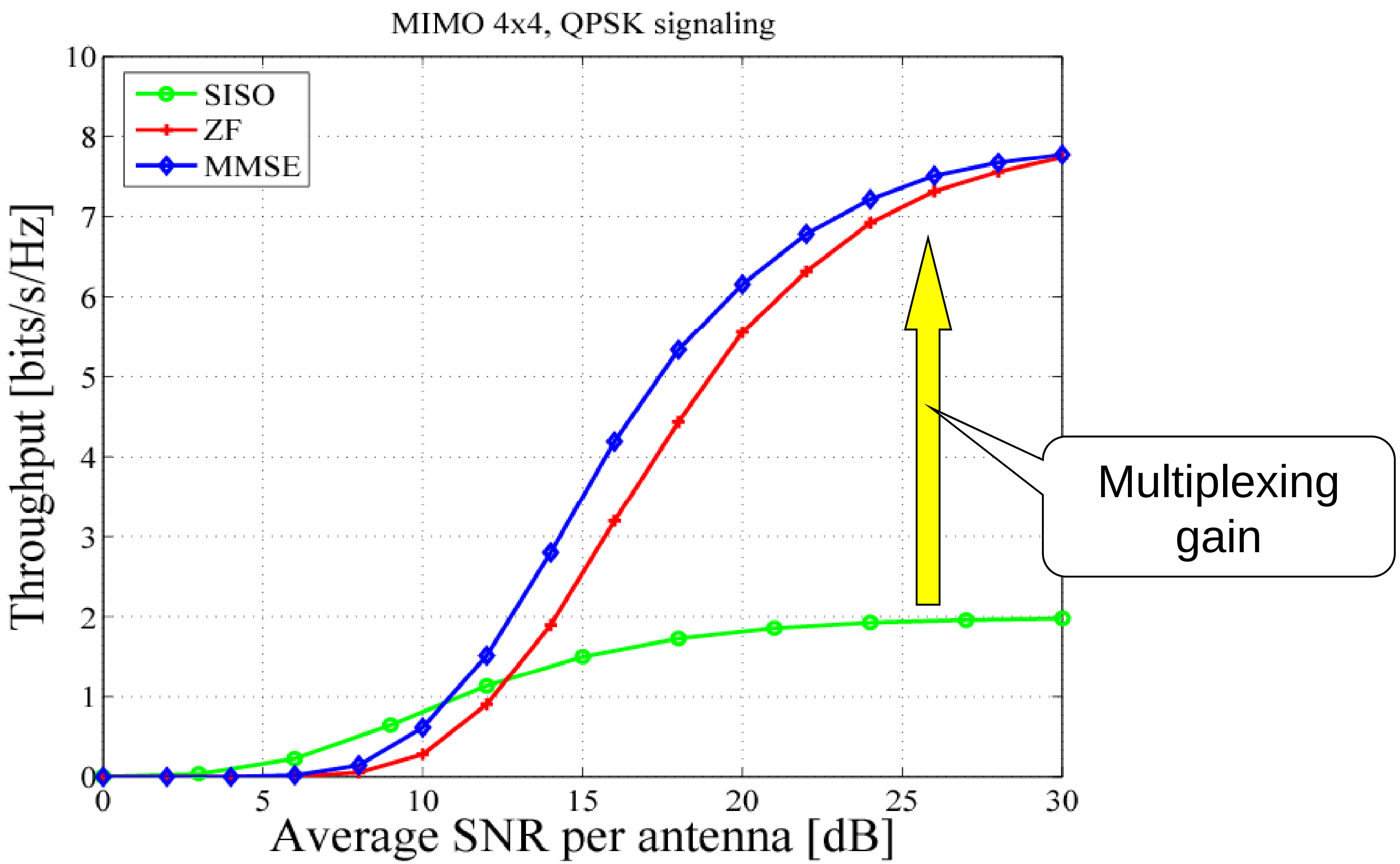
Performance of Linear Schemes

- No diversity gain if $M_r = M_t$
- No performance gain of MIMO (moreover noise enhancement)



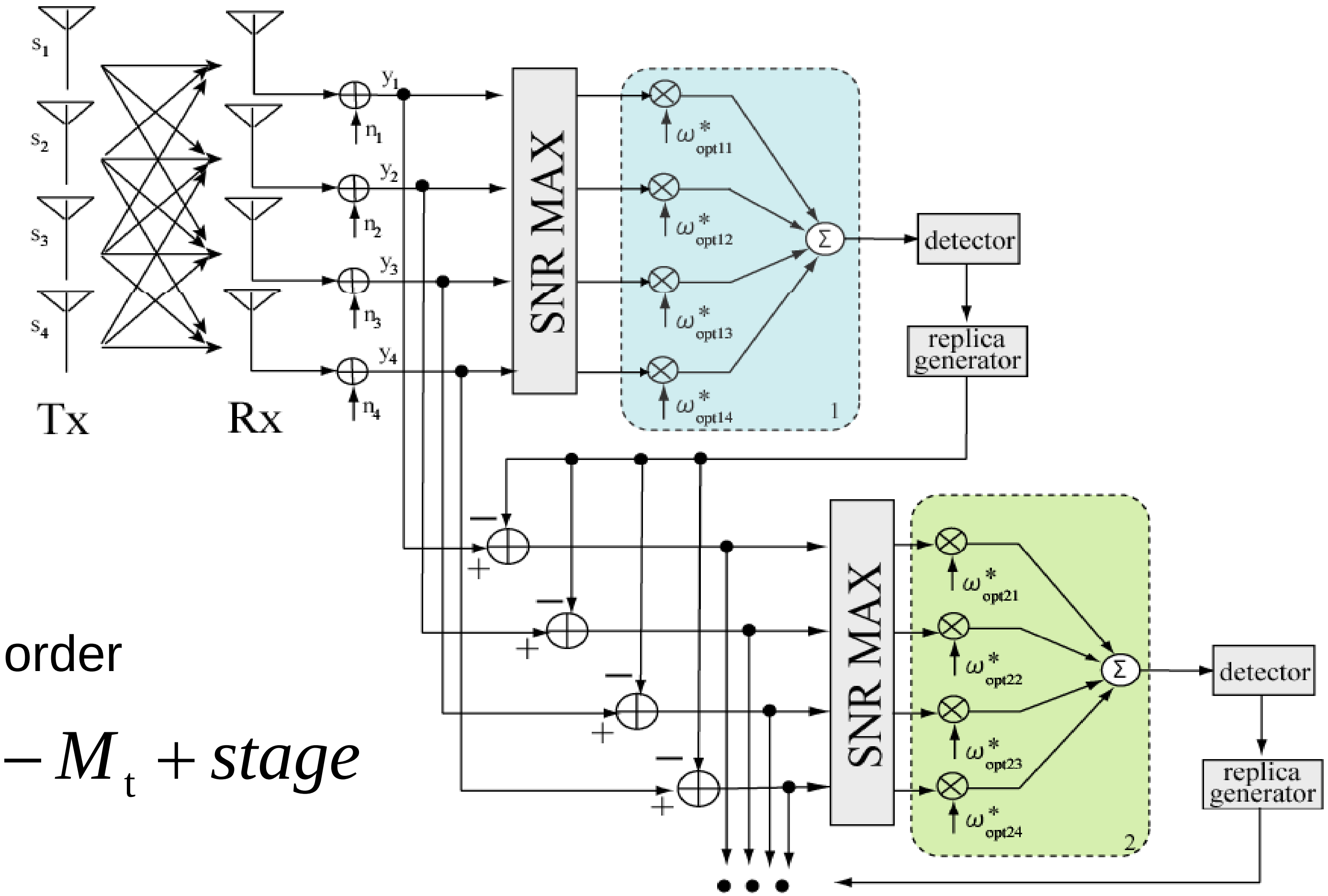
Performance of Linear Schemes

- High performance due to multiplexing gain at high SNR region



Successive Interference Cancellation

- Diversity order increases stage by stage
- M_t times weight calculation

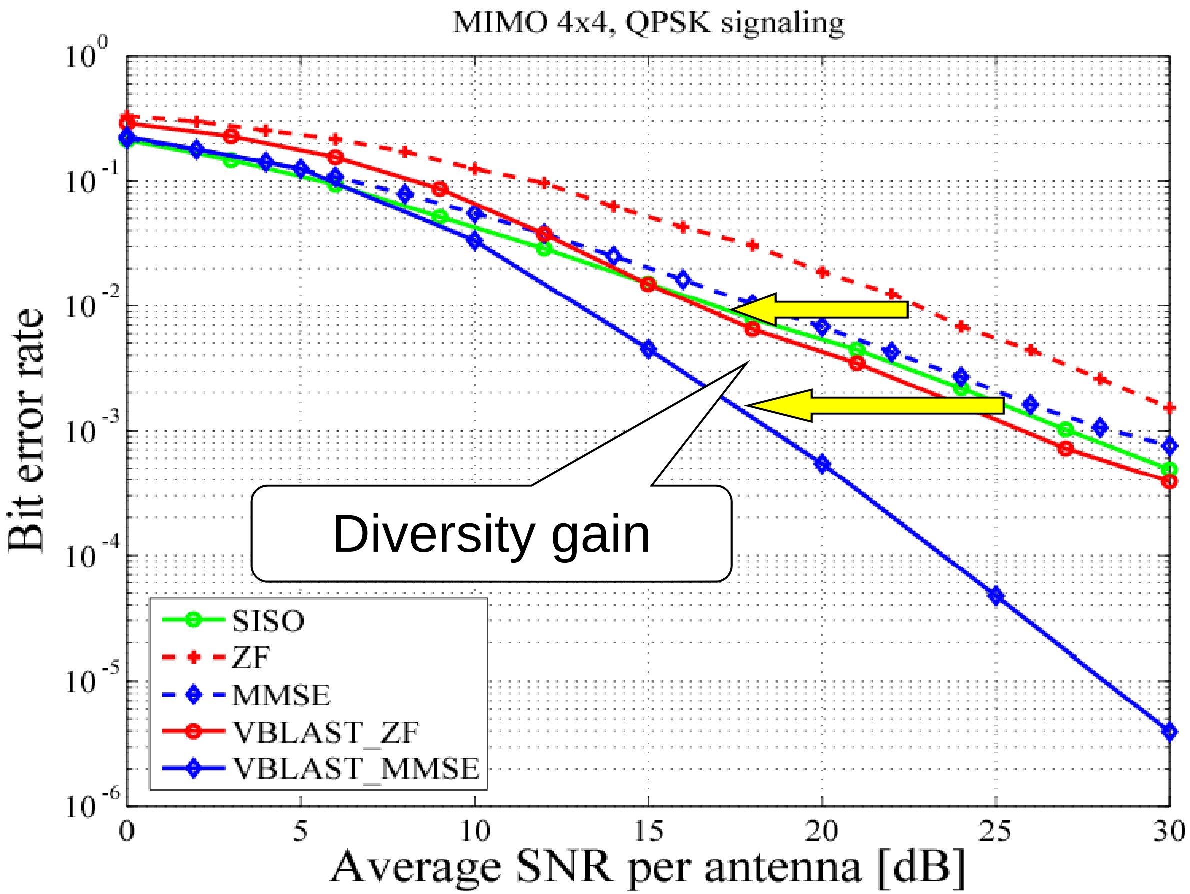


Diversity order

$$= M_r - M_t + stage$$

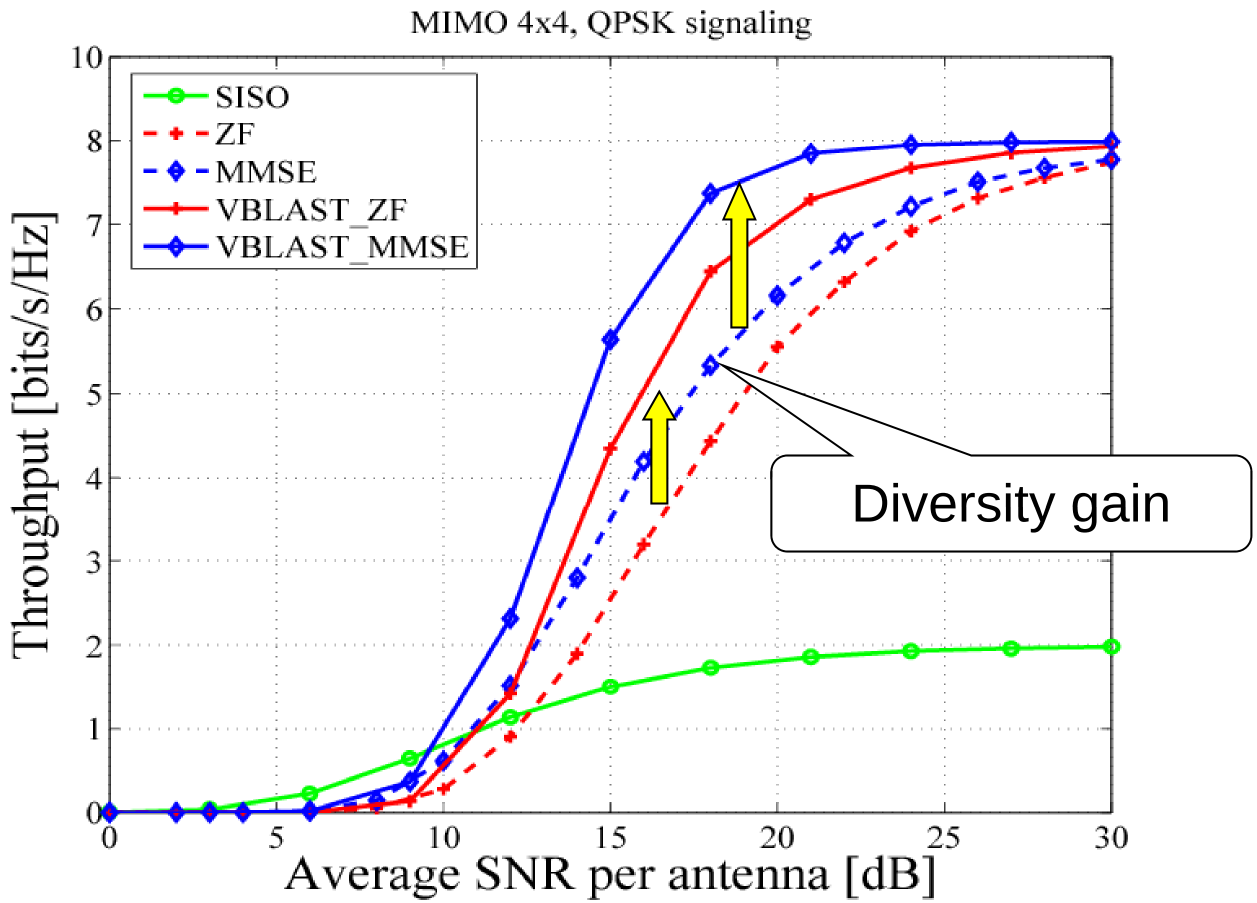
Performance of Hybrid Scheme

- Diversity gain owing to successive interference cancellation



Performance of Hybrid Scheme

- Diversity gain improves the throughput performance at lower SNR region



Maximum Likelihood Detection

Maximum likelihood detection

$$\hat{\mathbf{s}} = \arg\min_{\tilde{\mathbf{s}}} \left|\mathbf{y} - \mathbf{H}\tilde{\mathbf{s}}\right|^2$$

Pair wise error probability

$$p(\mathbf{s}_i \to \mathbf{s}_k) = \frac{1}{2}\operatorname{erfc}\left[\sqrt{\frac{\left|\mathbf{H}(\mathbf{s}_i - \mathbf{s}_k)\right|^2}{2\sigma^2}}\right]$$

$$\leq \left(1 + \frac{P\left|d_{ik}\right|^2}{4\sigma^2 M_{\mathrm{t}}}\right)^{-M_{\mathrm{r}}}$$

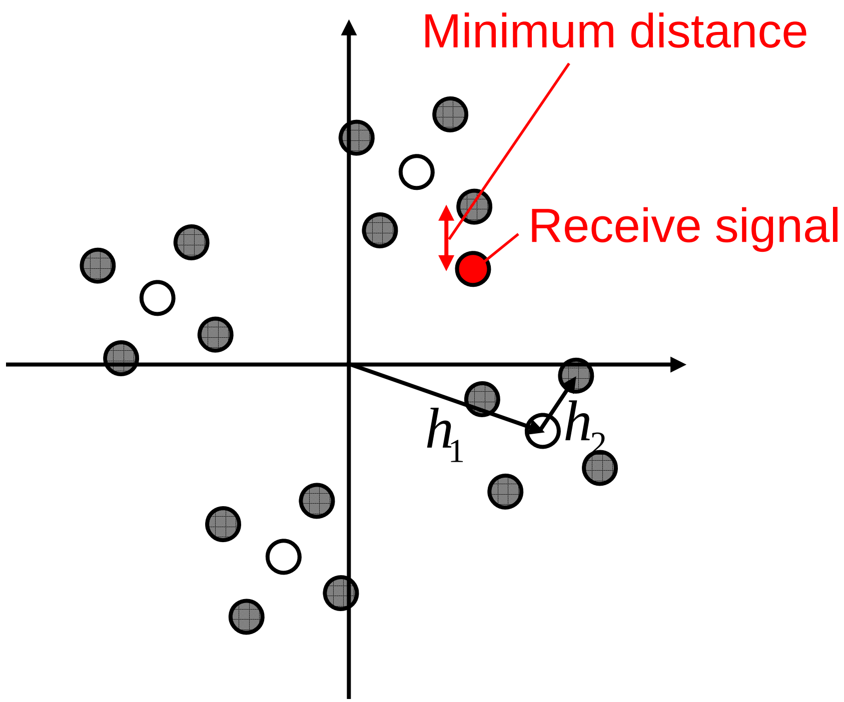
Diversity order

$$g_{\mathrm{d}} = M_{\mathrm{r}}$$

Computational cost

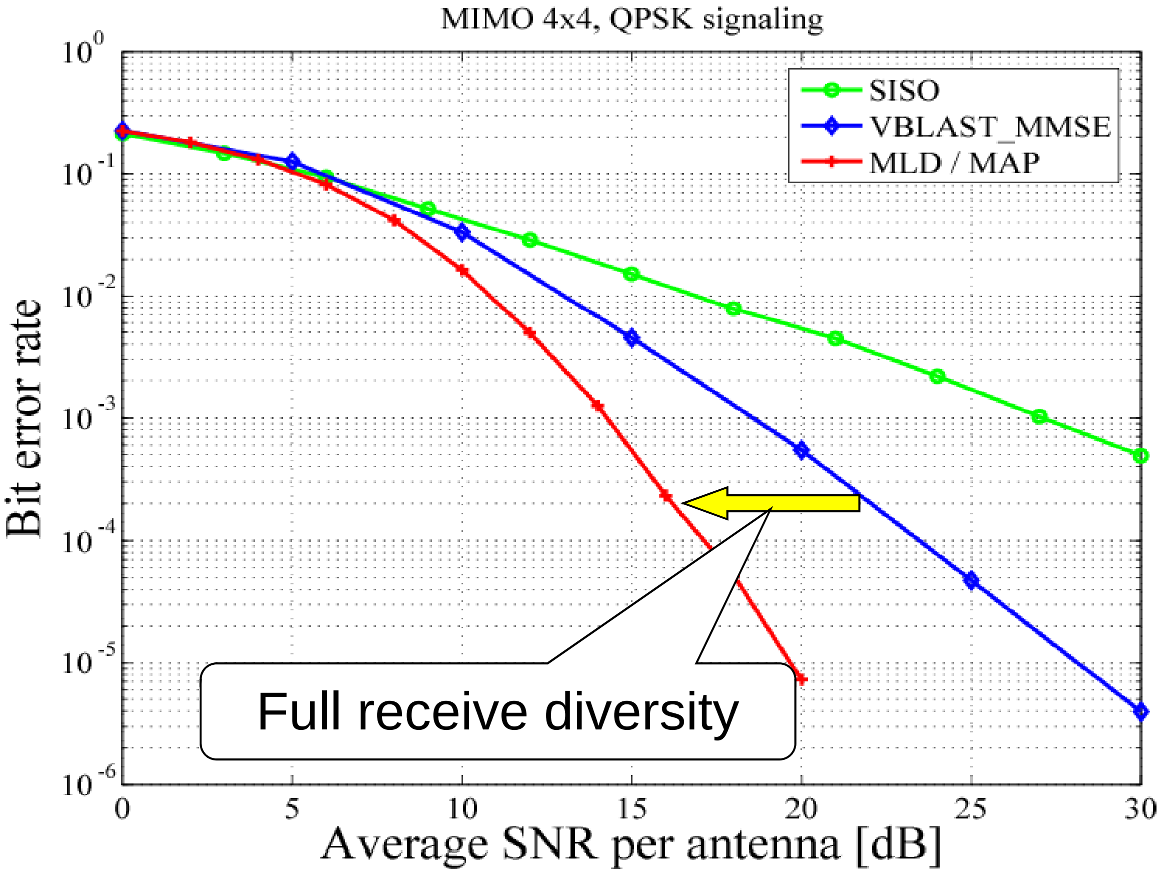
$M_{\mathrm{ary}}^{M_{\mathrm{t}}}$ combinations on M_{r} receiver branches

2 Tx with QPSK signaling



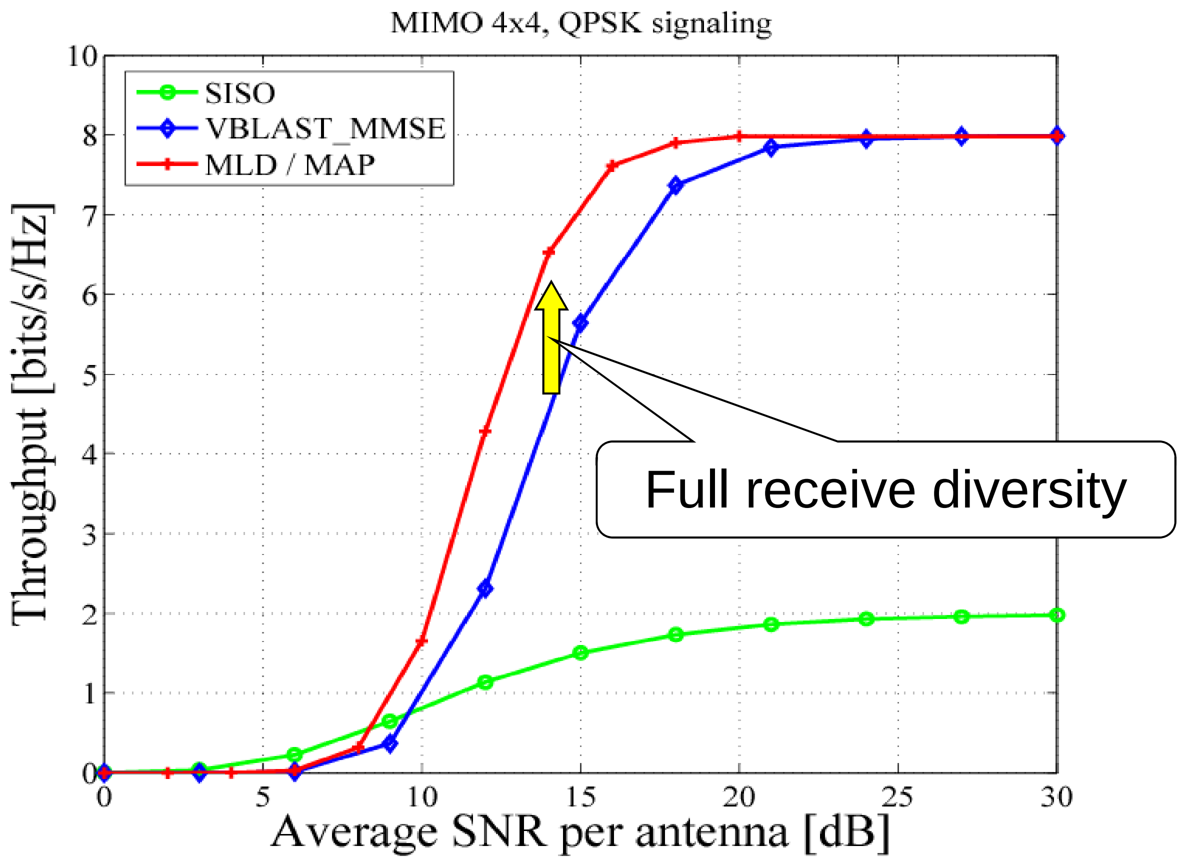
Performance of Non-Linear Scheme

- Full receive diversity due to MLD at all receive branches



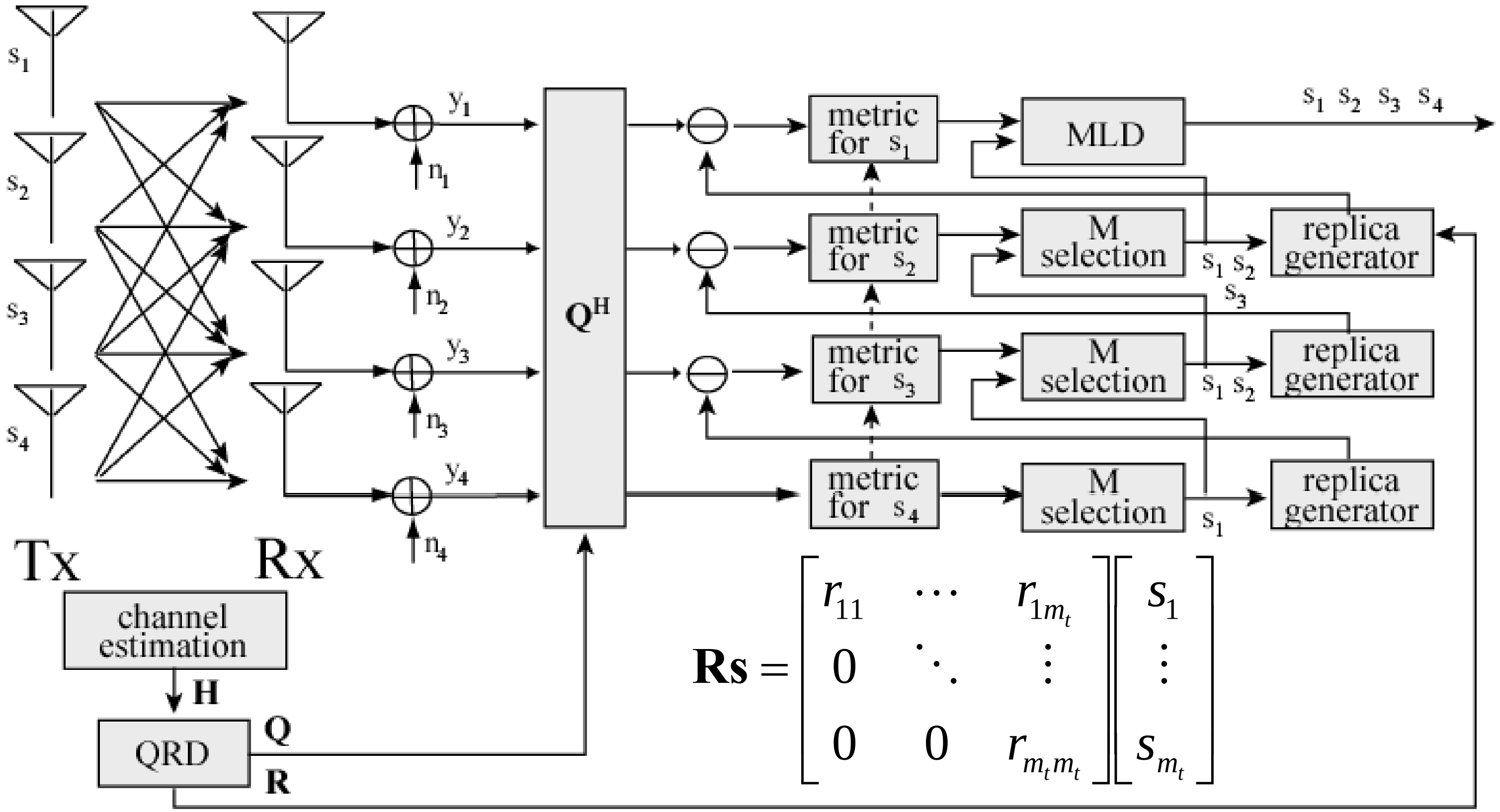
Performance of Non-Linear Scheme

- Maximal available throughput among all MIMO detection schemes
- Its computational complexity is almost prohibited



QRM-MLD

- QRM-MLD approaches the performance of MLD asymptotically
- QRM-MLD decreases the computational complexity drastically

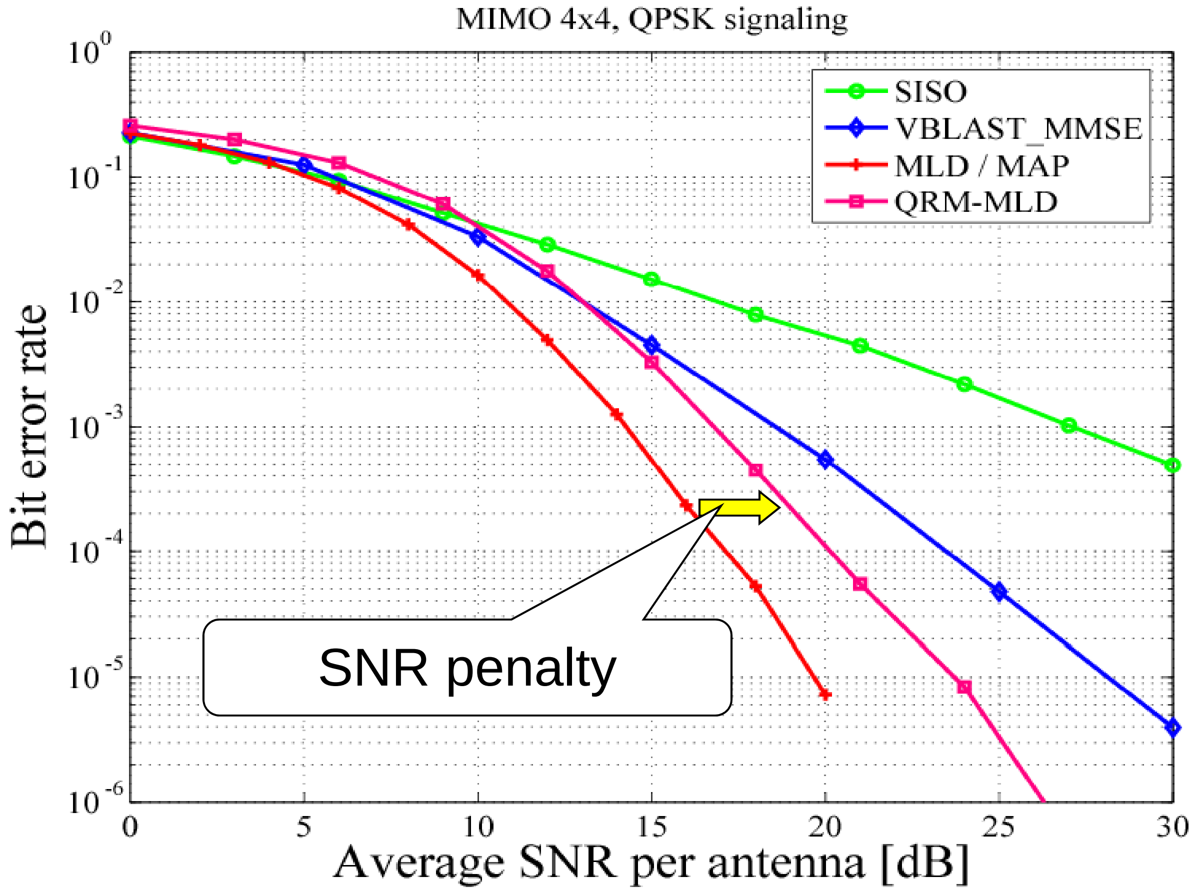


$$\mathbf{Rs} = \begin{bmatrix} r_{11} & \cdots & r_{1m_t} \\ 0 & \ddots & \vdots \\ 0 & 0 & r_{m_t m_t} \end{bmatrix} \begin{bmatrix} s_1 \\ \vdots \\ s_{m_t} \end{bmatrix}$$

Upper triangular decomposition: $\mathbf{H} = \mathbf{QR}$

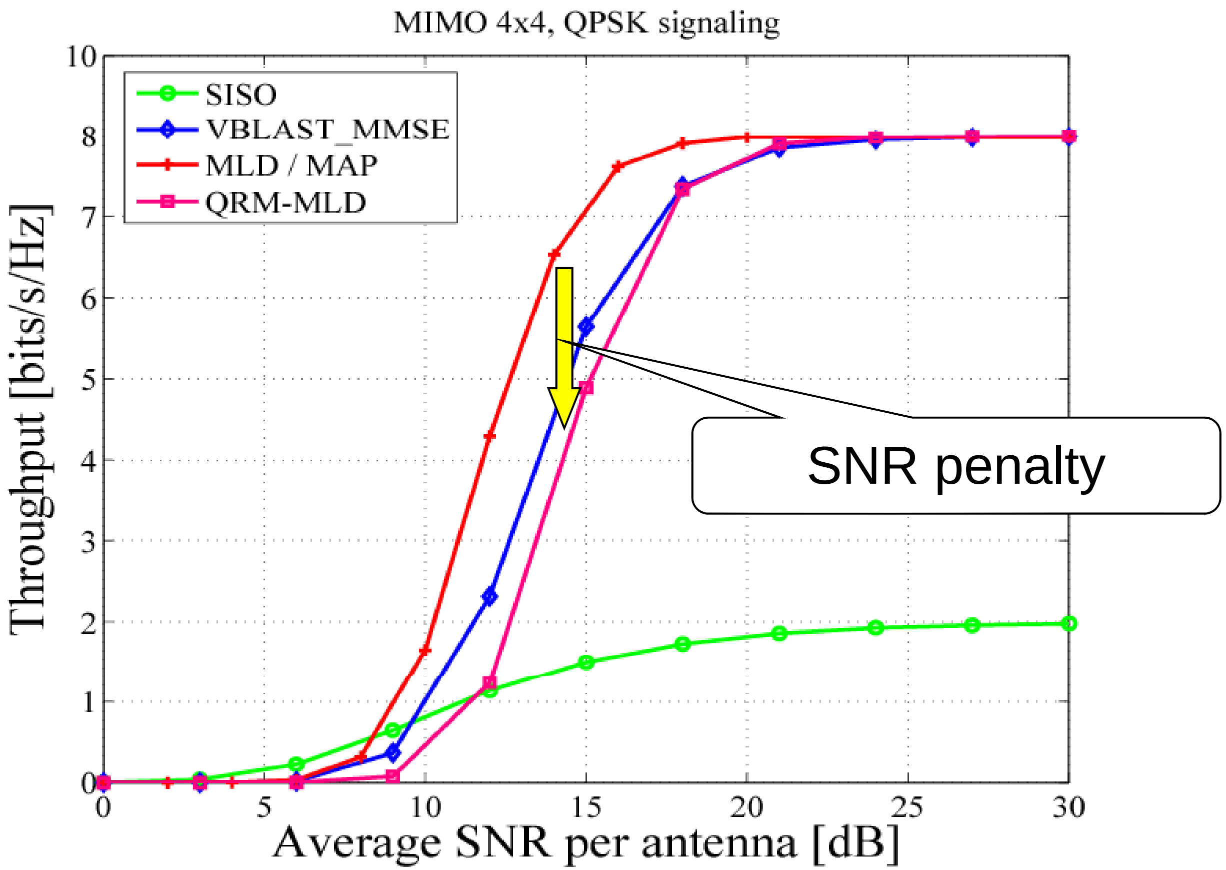
Performance of Hybrid Scheme

- QRM-MLD achieves full receive diversity as with MLD
- There is SNR penalty due to error propagation



Performance of Hybrid Scheme

• In IID, performance of VBLAST & QRM-MLD are almost the same



Comparison of MIMO Detection Schemes

	Multiplexing gain	Diversity gain
MMSE	M_{t}	$M_{\mathrm{r}} - M_{\mathrm{t}} + 1$
VBLAST	M_{t}	$M_{\mathrm{r}} - M_{\mathrm{t}} + stage$
MLD	M_{t}	M_{r}
QRM-MLD	M_{t}	$\leq M_{\mathrm{r}}$

	Complexity
MMSE	$O(\mathrm{INV}) + L \times O(M_{\mathrm{t}} M_{\mathrm{r}})$
VBLAST	$M_{\mathrm{t}} \times O(\mathrm{INV}) + 2L \times O(M_{\mathrm{t}} M_{\mathrm{r}})$
MLD	$O(M_{\mathrm{t}} M_{\mathrm{r}} M_{\mathrm{ary}}^{M_{\mathrm{t}}})$
QRM-MLD	$O(\mathrm{QRD}) + L \times O(M_{\mathrm{t}} M_{\mathrm{r}} + M M_{\mathrm{t}} (M_{\mathrm{t}} - 1) / 2)$

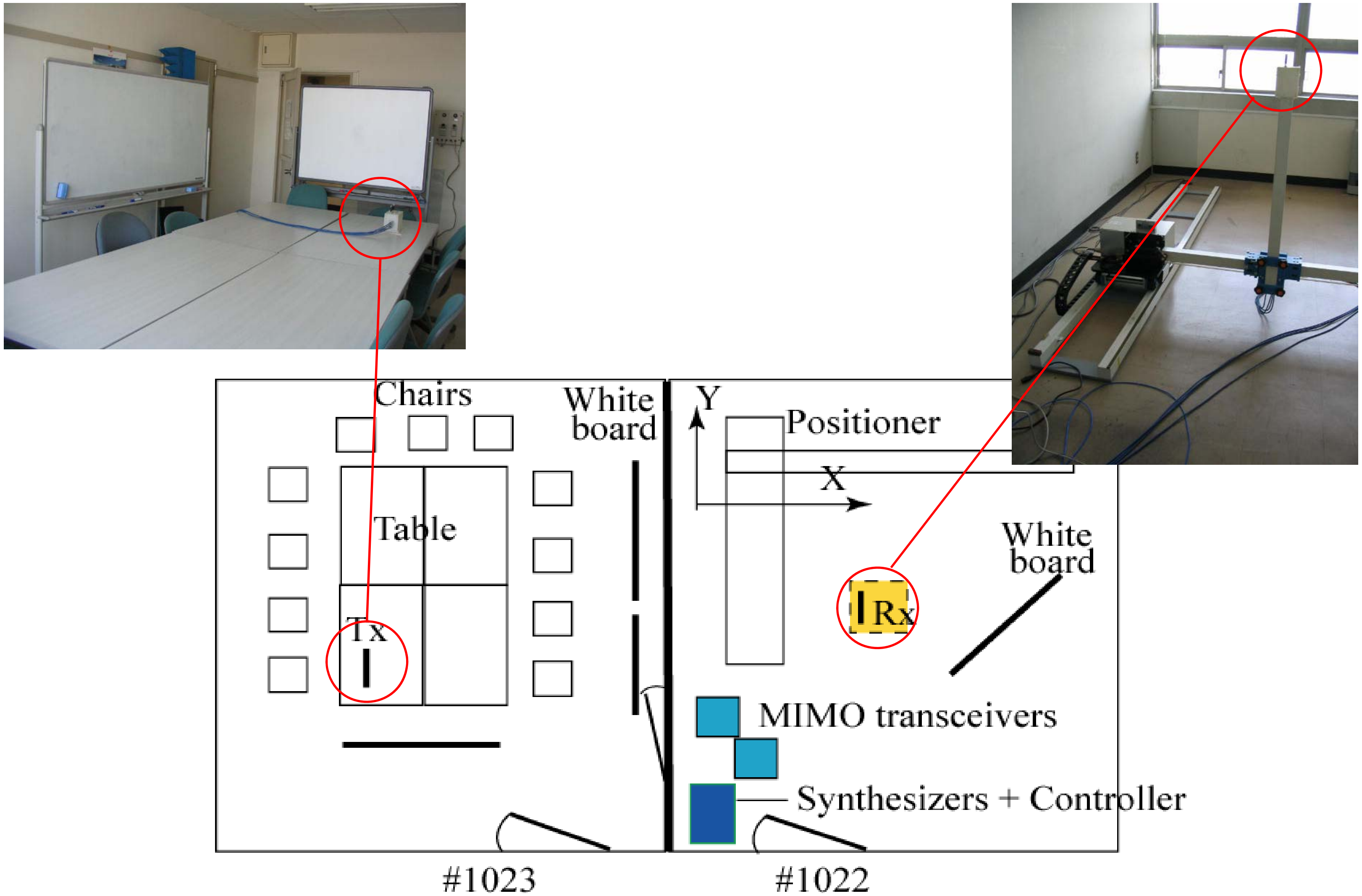
L : Number of symbols

Measurement Experiment

Measurement condition

MIMO structure	4 (Tx) x 4 (Rx)
Array structure	Half a wavelength spacing linear array
MIMO transmit scheme	Spatial multiplexing
MIMO receive scheme	ZF, MMSE, VBLAST, MLD
Modulation	BPSK, QPSK, 16QAM, 64QAM
Frame structure	IEEE802.11a based (without coding)
Packet length	480 bits
Measurement points	40cm x 40cm (2cm step)

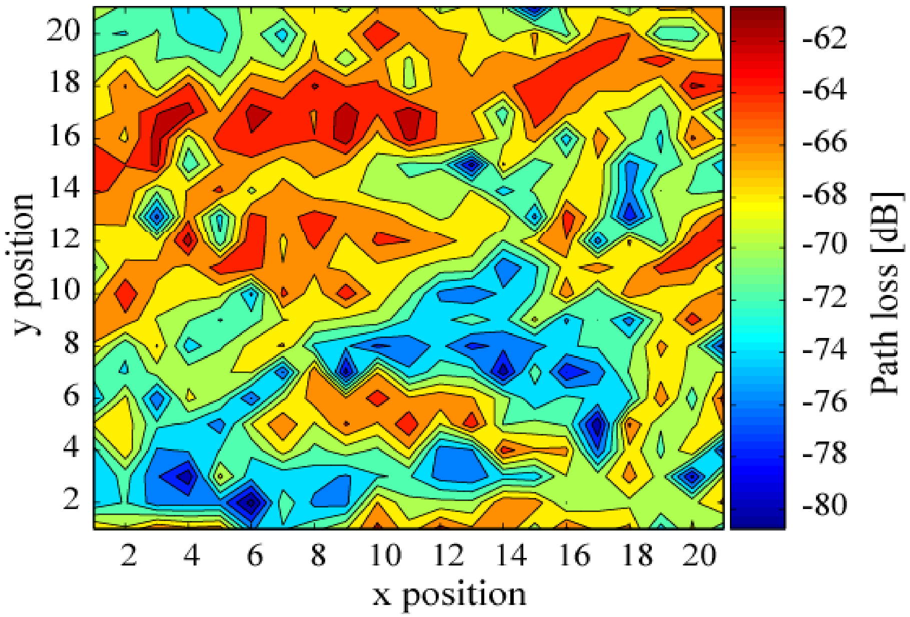
Measurement Environment



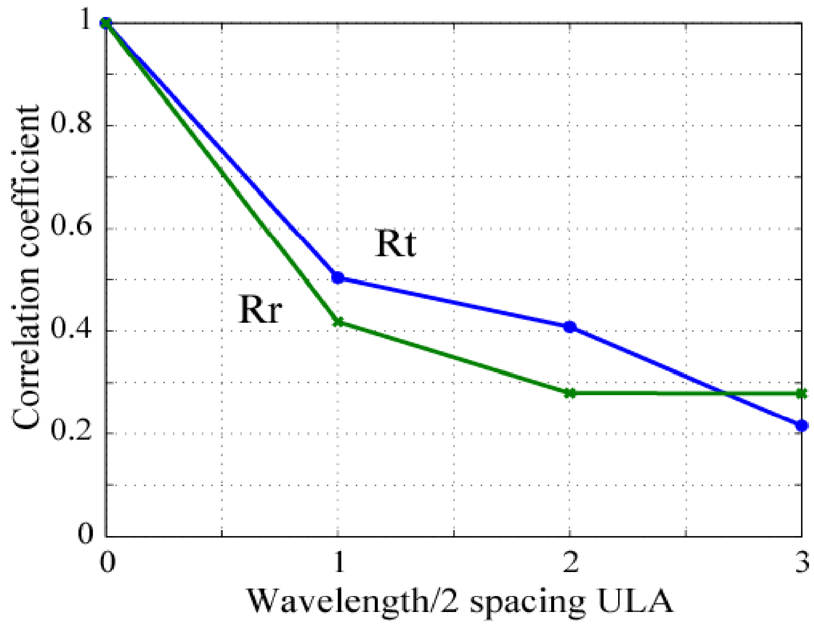
Propagation Characteristics

- Random pathloss distribution in space domain
- Slight spatial correlation will affect MIMO performance

Pathloss distribution



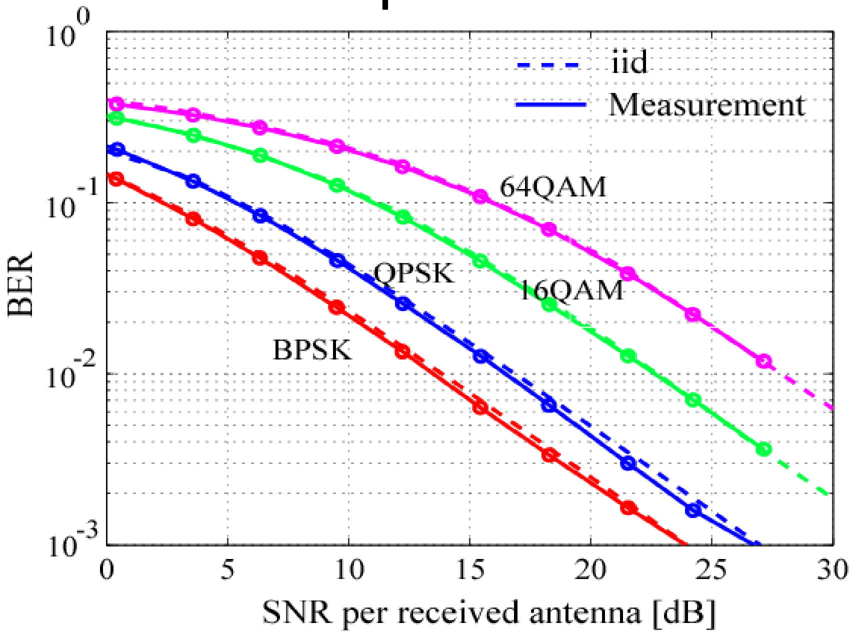
Spatial correlation



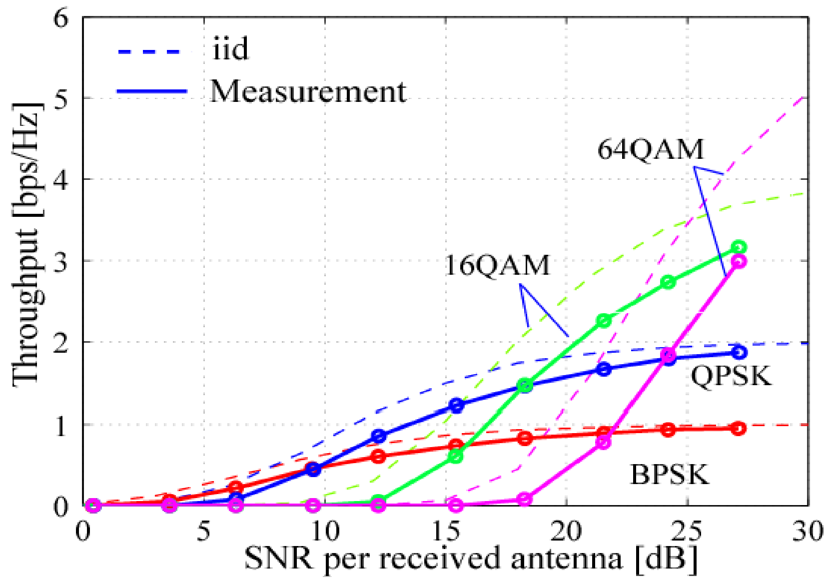
SISO Transmission Performance

- BER performance agrees with that in Rayleigh environment
- Degradation of throughput performance is due to imperfection of frequency characteristics in RF transceiver

BER performance



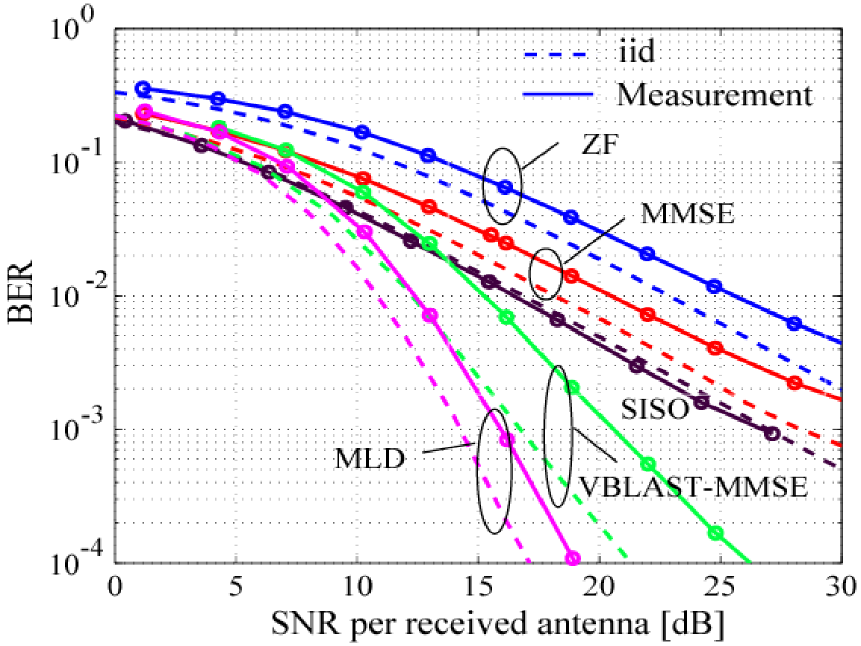
Throughput performance



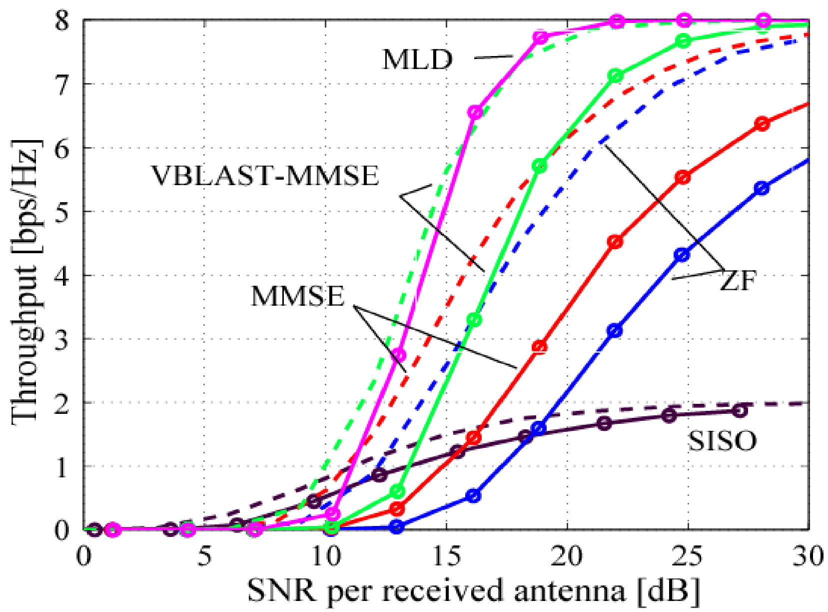
MIMO Transmission Performance

- Performance degradation of BER is due to spatial correlation
- RF impairment & spatial correlation degrades throughput performance

BER performance



Throughput performance



Summary

- MIMO receiver architecture
  - Transmit & receive diversity on frame synchronization
  - Orthogonal preamble sequences for MISO channel estimation
  - MIMO detection algorithm to resolve the transmit signal
- MIMO detection algorithm
  - Multiplexing gain to improve throughput performance
  - Diversity gain to improve BER performance
  - Tradeoff between complexity & performance (diversity order)
    Linear schemes (ZF, MMSE)
    Non-linear schemes (VBLAST, MLD, QRM-MLD)

Further improvement by transmit MIMO schems