2011 1st semester
MIMO Communication Systems

#6: Double Directional Spatial Channel Model

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May 24, 2011

Schedule (1^{st} half)

	Date	Text	Contents
#1	Apr. 12	A-1, B-1	Introduction
#2	Apr. 19	B-5, B-6	Fundamentals of wireless commun.
#3	Apr. 26	B-12	OFDM for wireless broadband
	May 3		No class
#4	May 10	B-7	Array signal processing
#5	Nov. 17	A-3, B-10	MIMO channel capacity
#6	Nov. 24	B-2, 3	Spatial channel model
	May 28		No class

Schedule (2nd half)

	Date	Text	Contents
#7	May 31	A-5	MIMO receiver
#8	June 7	A-3, 4	MIMO transmitter
#9	June 14	B-9	Adaptive commun. system
#10	June 21	A-6, B-14	Multi-user MIMO
#11	June 28	B-15, 16	Distributed MIMO networks
#12	July 5		Standardization of MIMO
	July 12		Final Examination

Agenda

■ Aim of today

Describe spatial channel model
& evaluate its effect on MIMO channel capacity

■ Contents

- Path loss model
- IID Ricean fading channel
- Correlated fading channel
- Antenna model

Warming Up

- Question

Fill out the following table with appropriate channel parameters

Correlation		Profile
Time	⟷	?
Space (antenna)	⟷	?
Frequency	⟷	?

- Auto correlation & power profile (spectrum)

Time variant channel $h(t)$ & its Fourier transform $\tilde{h}(f)$

Auto-correlation

$$R_h(\tau) = \mathrm{E}\left[h^*(t+\tau)h(t)\right]$$

⇕

$$R_h(\tau) = \int_{-\infty}^{\infty} P_h(f) e^{j2\pi f\tau} \mathrm{d}f$$

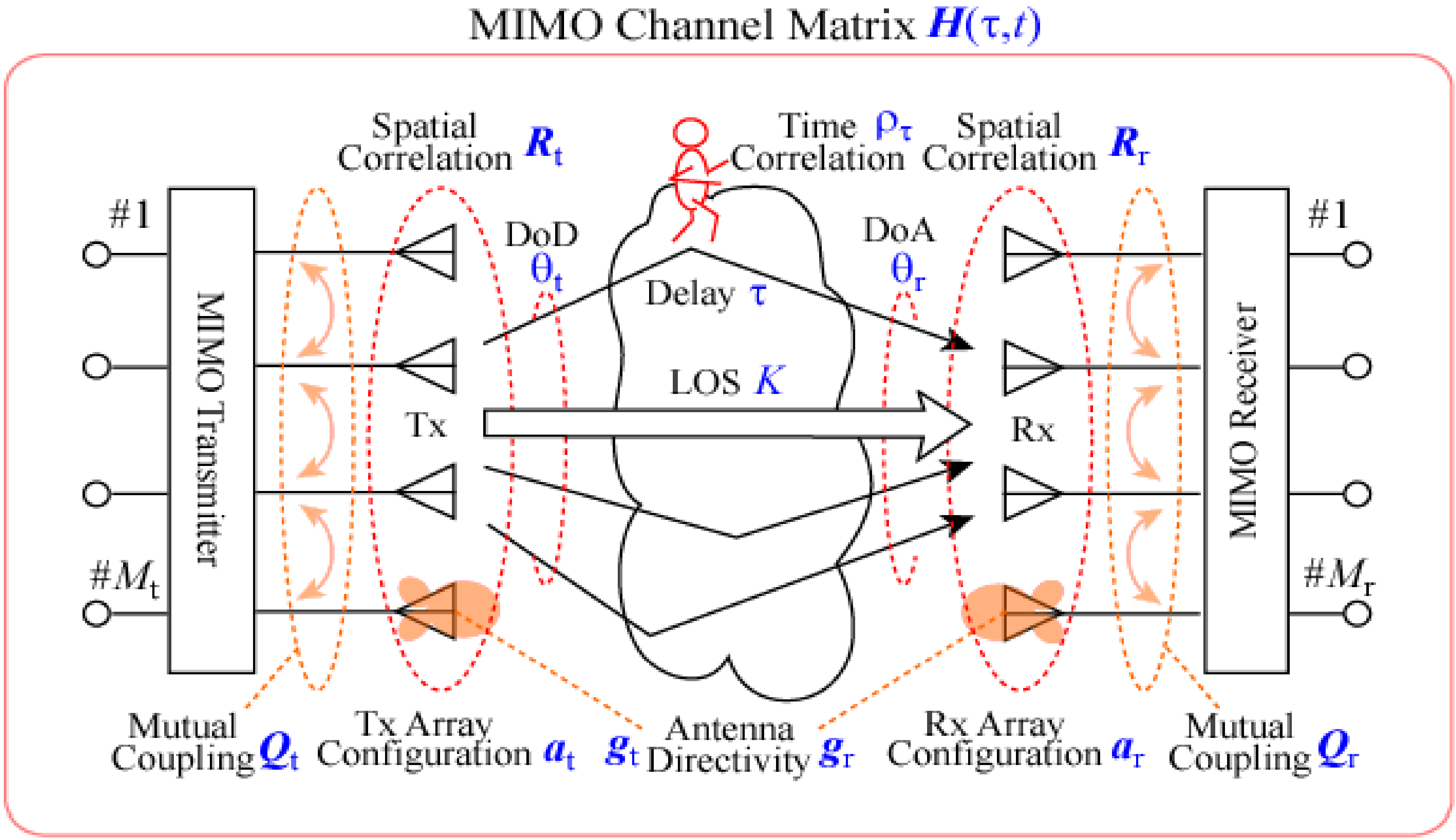
Power profile (spectrum)

$$P_h(f) = \mathrm{E}\left[\tilde{h}^*(f)\tilde{h}(f)\right]$$

⇕

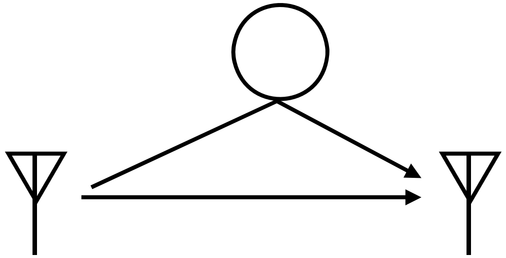
$$P_h(f) = \int_{-\infty}^{\infty} R_h(\tau) e^{-j2\pi f\tau} \mathrm{d}\tau$$

MIMO Propagation Channel



Channel Model

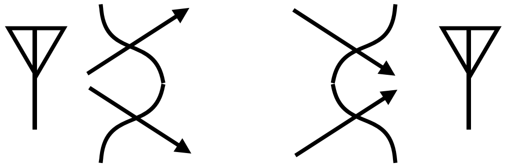
Deterministic model



- Channel is generated by solving electromagnetic field in the given geometry of scatterers (ex. FEM, PO)

  For fixed environment with small scatterers

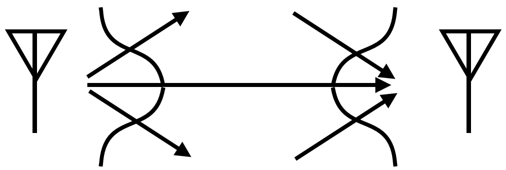
Stochastic model



- Channel is generated by stochastic process
- PDF of that process is derived empirically (ex. Rayleigh fading, Delay profile)

  For mobile environment with large scatterers

Hybrid model

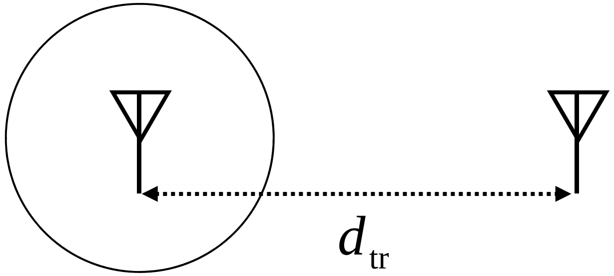


- Ex. path loss from deterministic model
- Ex. fading from stochastic model

Path Loss Model

Free space path loss

$$P_r = A_r \frac{G_t P_t}{4\pi d_{tr}^{\ 2}} = \left(\frac{\lambda}{4\pi d_{tr}}\right)^2 G_r G_t P_t$$



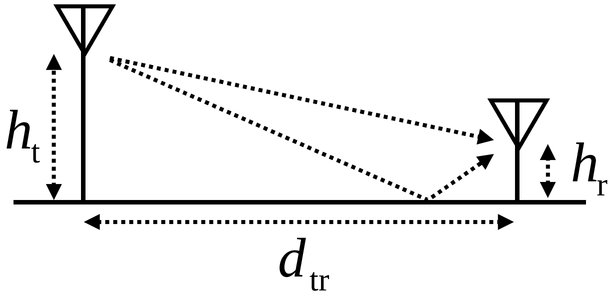
$$A_r = G_r \frac{\lambda^2}{4\pi}$$ Antenna effective aperture

$$\zeta_{db} = 10\log_{10}\left(\frac{\lambda}{4\pi d_{tr}}\right)^2 = -20\log_{10} d_{tr} - 20\log_{10} f + 28$$

Two-path model

When $d_{tr} >> \frac{4h_t h_r}{\lambda}$ Breakpoint

$$P_r = \frac{h_t^2 h_r^2}{d_{tr}^4} G_r G_t P_t$$



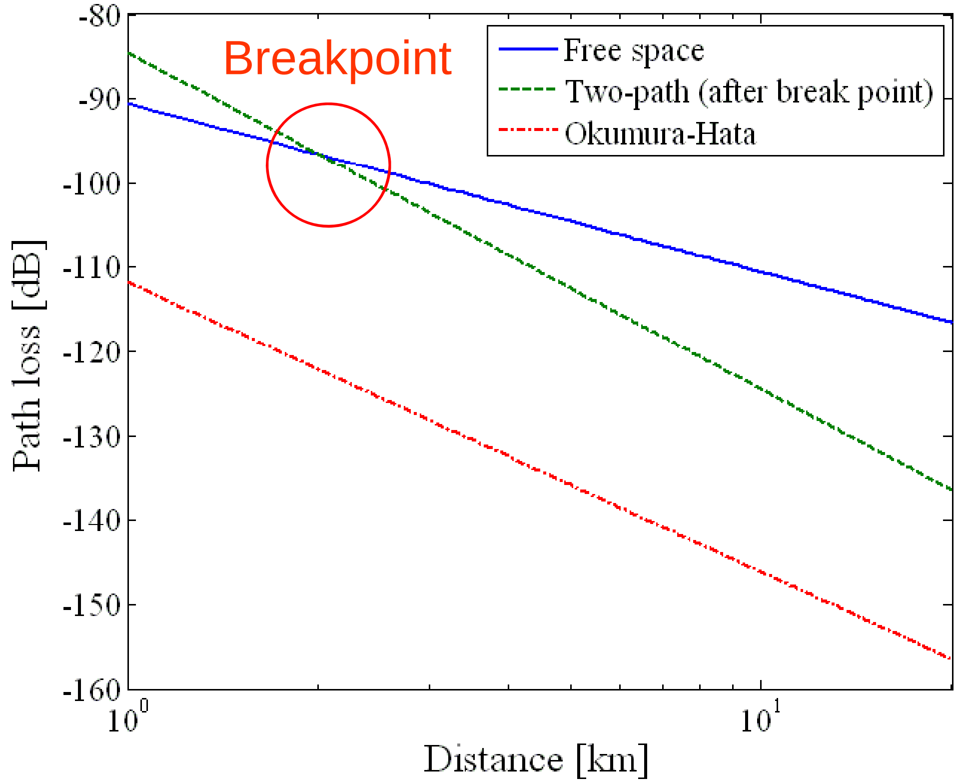
$$\zeta_{dB} = -40\log_{10} d_{tr} + 20\log_{10} h_t + 20\log_{10} h_r$$

Path Loss Model

Okumura-Hata model

$$\zeta_{\mathrm{db}} = -69.5 - 22.16\log_{10} f + 13.82\log_{10} h_{\mathrm{t}} + a(h_{\mathrm{r}}) - (44.9 - 6.55\log_{10} h_{\mathrm{t}})\log_{10} d_{\mathrm{tr}}$$

$$a(h_{\mathrm{r}}) = (1.1\log_{10} f - 0.7)h_{\mathrm{r}} - (1.56\log_{10} f - 0.8)$$



IID Rayleigh Channel

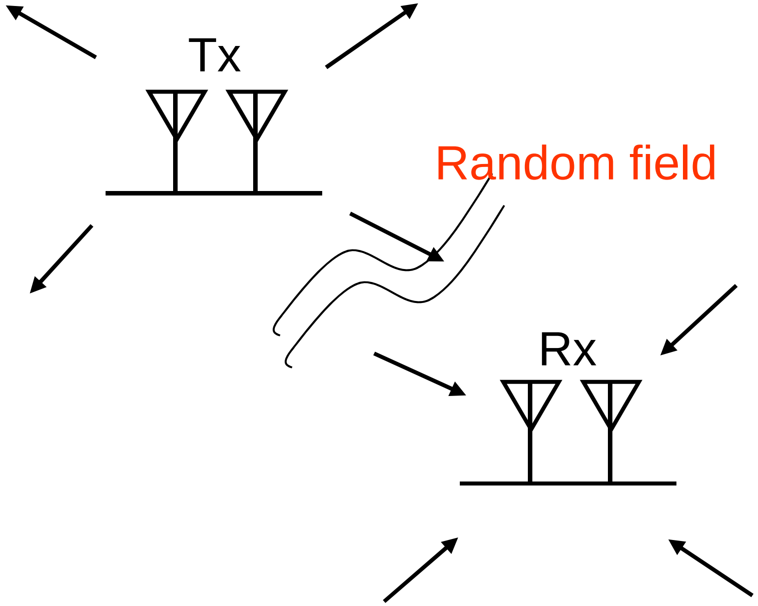
IID Rayleigh channel matrix

$$\mathbf{H}(t) = \sqrt{\zeta}\,\mathbf{H}_{\text{iid}}(t)$$

Joint PDF of $\gamma_{ij} = \left|\mathbf{H}_{ij}\right|^2$

$$f(\gamma_{ij}) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma_{ij}}{\bar{\gamma}}\right) \qquad \bar{\gamma} = \zeta$$

$$f(\gamma_{ij}, \gamma_{kl}) = f(\gamma_{ij}) f(\gamma_{kl})$$



Receive spatial correlation

$$\mathbf{R}_{\text{r}} = \frac{1}{M_{\text{t}}}\,\text{E}\left[\mathbf{H}(t)\mathbf{H}^H(t)\right] = \zeta\mathbf{I}_{M_{\text{r}}} \longrightarrow \rho_{ij} = \frac{\mathbf{R}_{\text{r}ij}}{\sqrt{\mathbf{R}_{\text{r}ii}\mathbf{R}_{\text{r}jj}}} = 0 \qquad i \neq j$$

Transmit spatial correlation

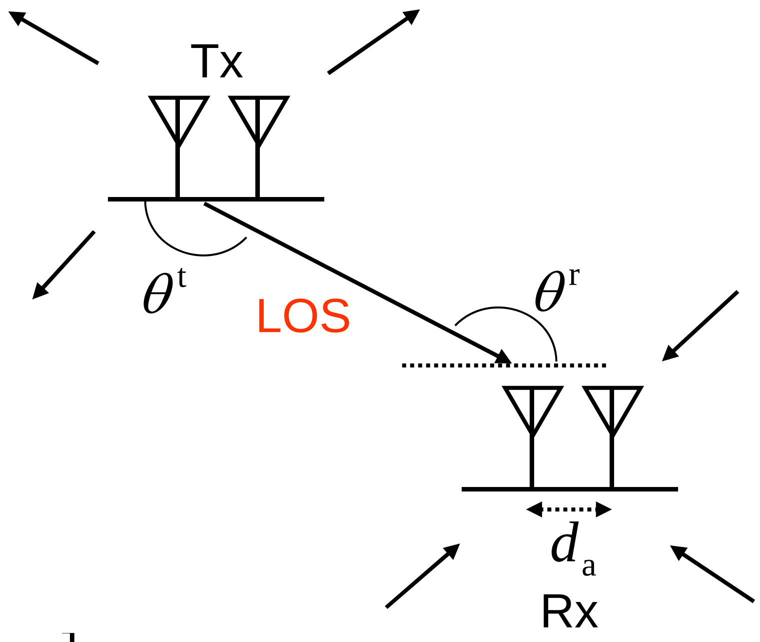
$$\mathbf{R}_{\text{t}} = \frac{1}{M_{\text{r}}}\,\text{E}\left[\left(\mathbf{H}^H(t)\mathbf{H}(t)\right)^*\right] = \zeta\mathbf{I}_{M_{\text{t}}} \longrightarrow \rho_{ij} = \frac{\mathbf{R}_{\text{t}ij}}{\sqrt{\mathbf{R}_{\text{t}ii}\mathbf{R}_{\text{t}jj}}} = 0 \qquad i \neq j$$

IID Ricean Channel

IID Ricean channel matrix

$$\mathbf{H} = \sqrt{\zeta}\left(\sqrt{\frac{1}{K+1}}\mathbf{H}_{\mathrm{iid}} + \sqrt{\frac{K}{K+1}}\mathbf{H}_{\mathrm{d}}\right)$$

Ricean *K* factor



LOS component

$$\mathbf{H}_{\mathrm{d}} = \mathbf{a}_{\mathrm{r}}(\theta^{\mathrm{r}})\mathbf{a}_{\mathrm{t}}^{\mathrm{T}}(\theta^{\mathrm{t}})$$

Array mode vector

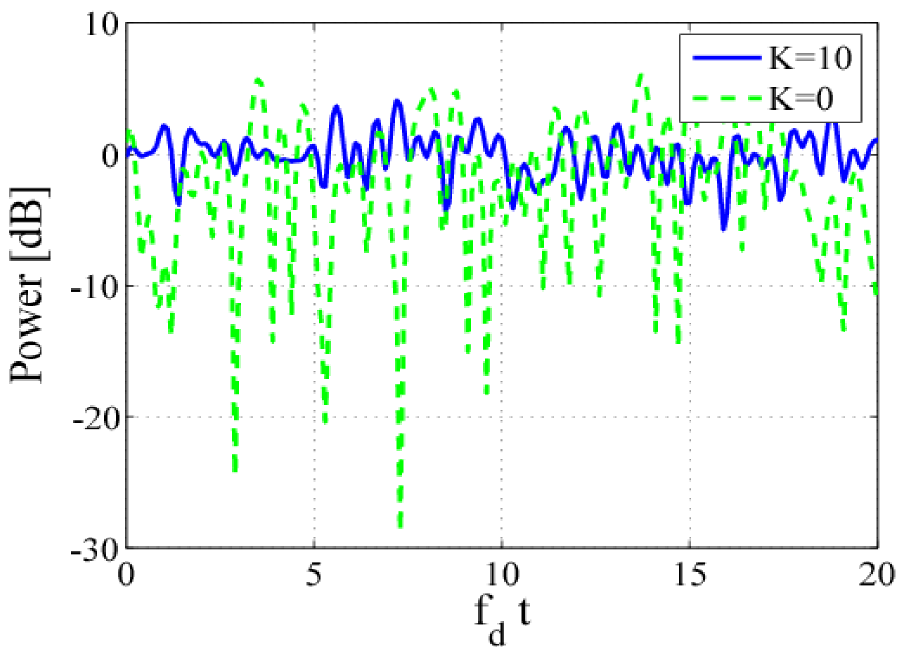
$$\mathbf{a}(\theta) = \left[1, e^{jkd_{\mathrm{a}}\cos\theta}, e^{jk2d_{\mathrm{a}}\cos\theta}, \cdots, e^{jk(M-1)d_{\mathrm{a}}\cos\theta}\right]$$

LOS component

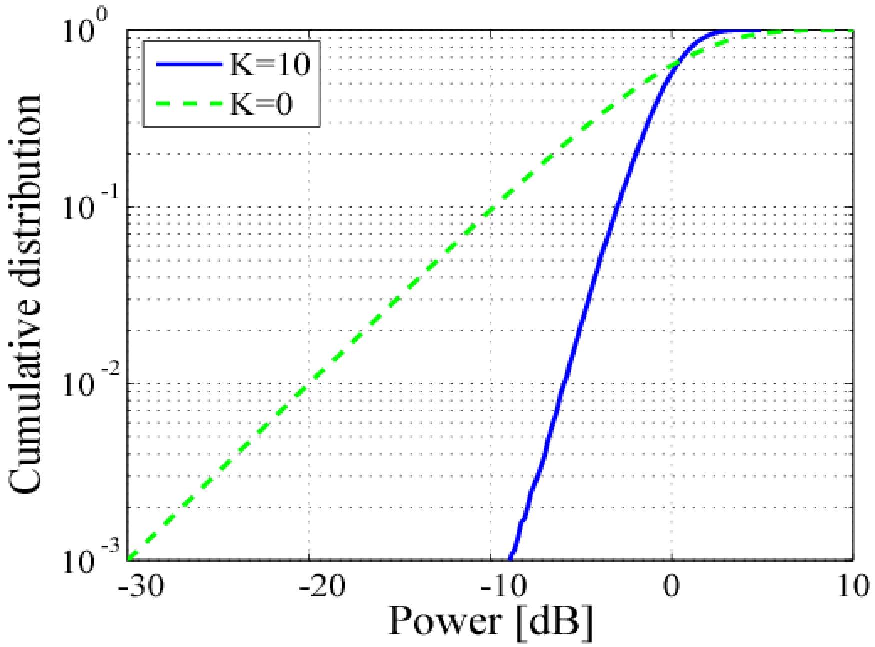
$$\mathbf{R}_{\mathrm{r}} = \zeta\left(\frac{1}{K+1}\mathbf{I}_{M_{\mathrm{r}}} + \frac{K}{K+1}\mathbf{a}_{\mathrm{r}}(\theta^{\mathrm{r}})\mathbf{a}_{\mathrm{r}}^{H}(\theta^{\mathrm{r}})\right) \longrightarrow \left|\rho_{\mathrm{r}}\right| = \left|\rho_{\mathrm{t}}\right| = \frac{K}{K+1}$$

CDF of Ricean Channel

Time variation

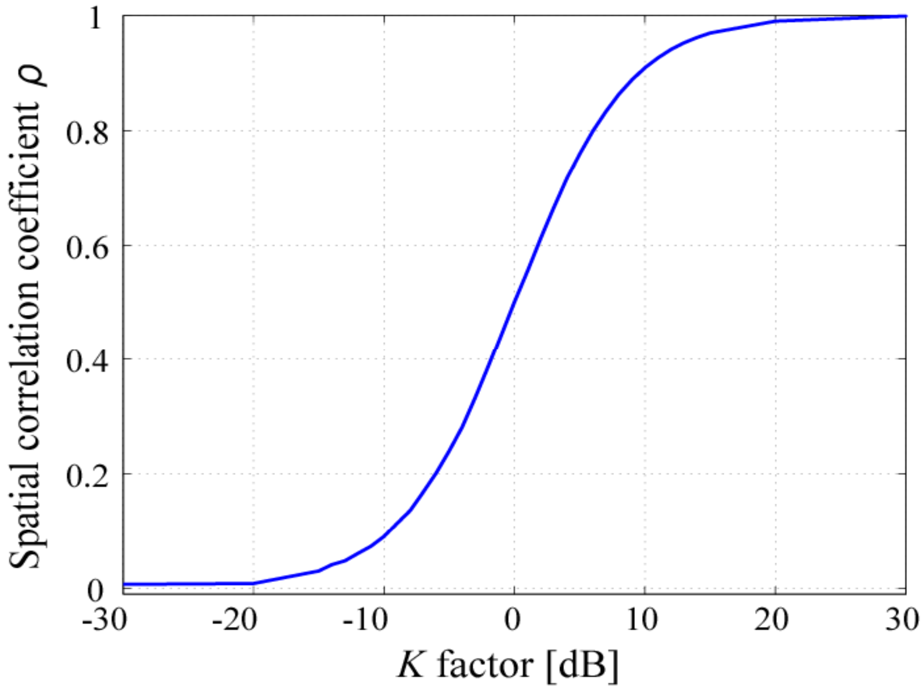


CDF of power

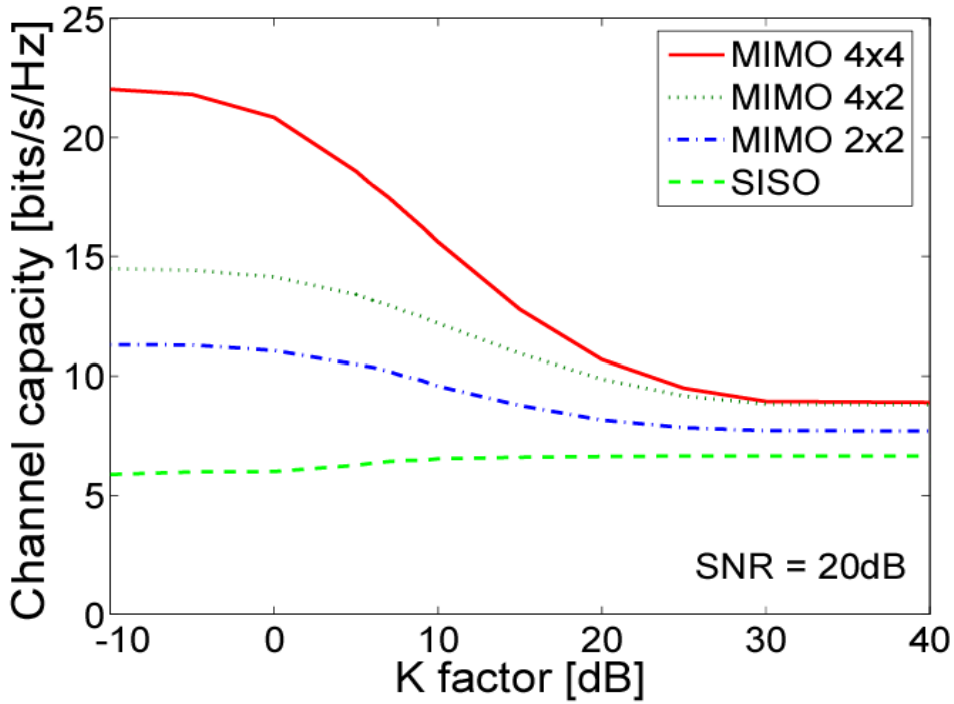


Capacity of Ricean Channel

Spatial correlation

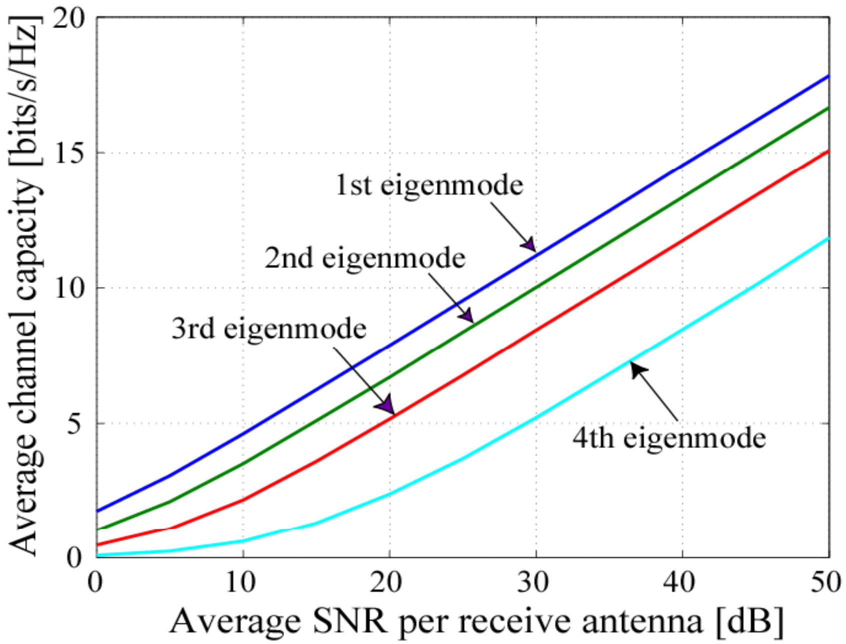


Channel capacity

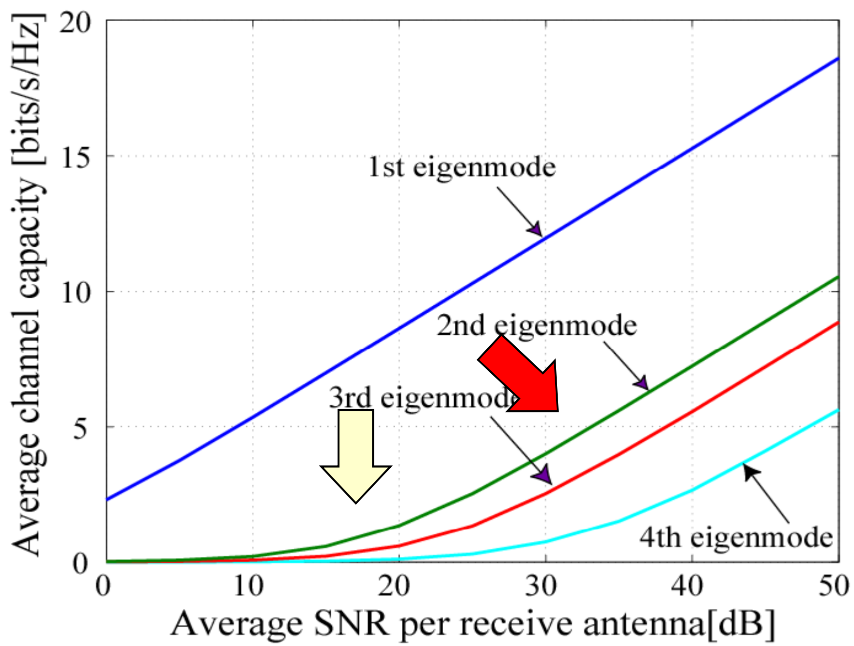


Capacity of Eigenmodes

K = 0



K = 100



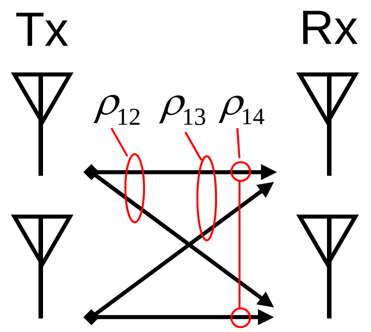
Correlated Rayleigh Channel

Correlated Rayleigh channel matrix

$$\mathbf{H} = \sqrt{\zeta}\,\mathrm{devec}\left(\sqrt{\overline{\mathbf{R}}}\mathbf{h}_{\mathrm{iid}}\right)$$

$$\mathbf{h}_{\mathrm{iid}} = \mathrm{vec}\left(\mathbf{H}_{\mathrm{iid}}\right)$$

Normalized correlation matrix



Channel correlation

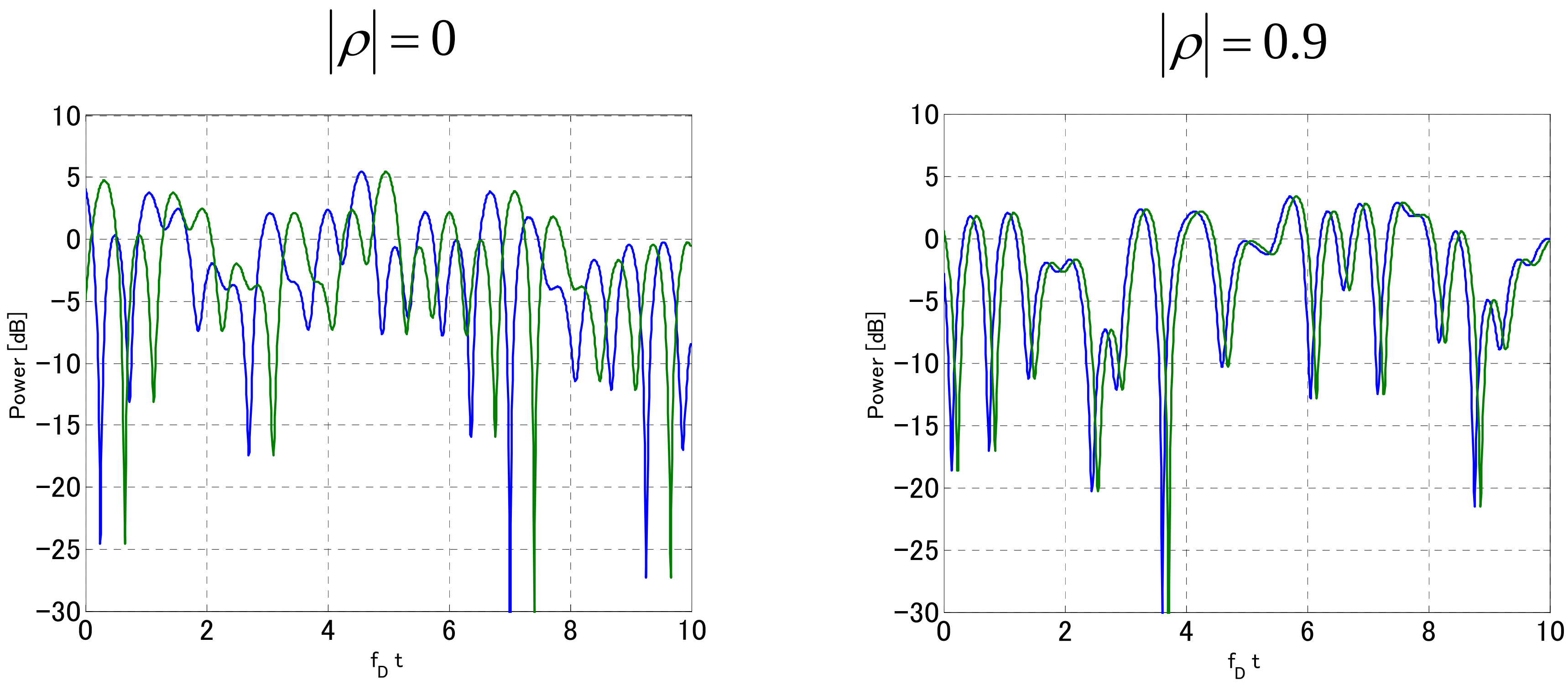
Correlation matrix

$$\mathbf{R} = \mathrm{E}[\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{H}]$$

$$= \zeta\sqrt{\overline{\mathbf{R}}}\mathbf{I}_{M_{\mathrm{r}}M_{\mathrm{t}}}\sqrt{\overline{\mathbf{R}}}^{H} = \zeta\overline{\mathbf{R}}$$

$$\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\\ \mathbf{c}\end{bmatrix} = \mathrm{vec}\begin{bmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\end{bmatrix}$$

Correlated Rayleigh Fading



Kronecker Model

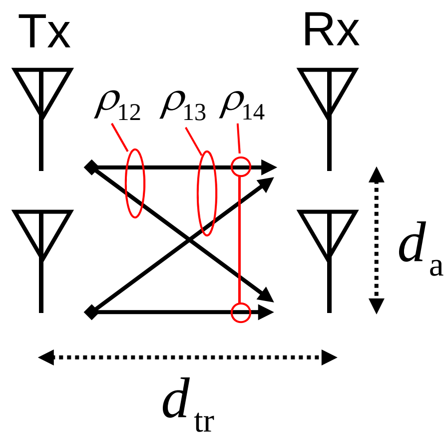
Correlation matrix

$$\mathbf{R} = \mathrm{E}[\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^H]$$

$$\cong \zeta \begin{bmatrix} \overline{\mathbf{R}}_\mathrm{r} & \rho_\mathrm{t} \overline{\mathbf{R}}_\mathrm{r} \\ \rho_\mathrm{t}^* \overline{\mathbf{R}}_\mathrm{r}^H & \overline{\mathbf{R}}_\mathrm{r} \end{bmatrix}$$

$$= \overline{\mathbf{R}}_\mathrm{t} \otimes \overline{\mathbf{R}}_\mathrm{r}$$

$$\mathrm{vec}(\mathbf{H}) = \begin{bmatrix} h_{11} \\ h_{21} \\ h_{12} \\ h_{22} \end{bmatrix}$$



Assumption $d_\mathrm{tr} >> d_\mathrm{a}$

Kronecker model

$$\mathbf{H} = \sqrt{\zeta}\,\mathrm{devec}\left(\sqrt{\overline{\mathbf{R}}}\mathbf{h}_\mathrm{iid}\right)$$

$$= \sqrt{\zeta}\,\mathrm{devec}\left(\sqrt{\overline{\mathbf{R}}_\mathrm{t} \otimes \overline{\mathbf{R}}_\mathrm{r}}\,\mathbf{h}_\mathrm{iid}\right)$$

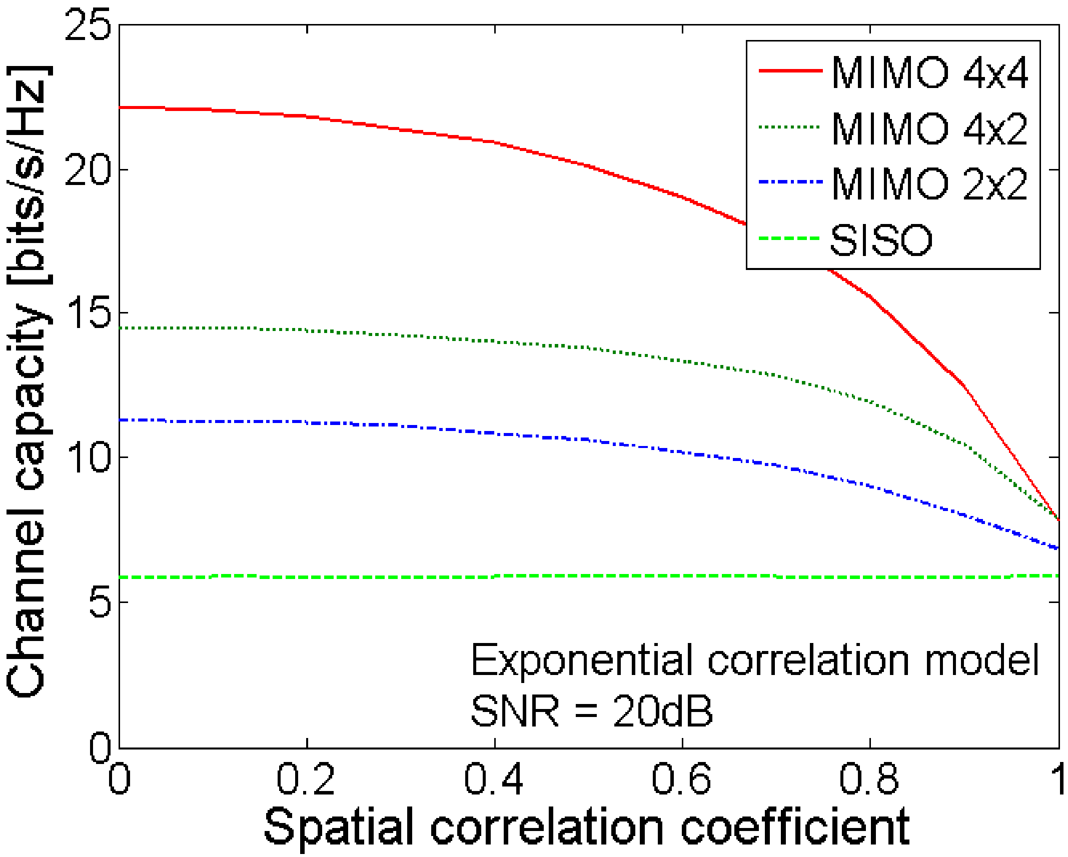
$$= \sqrt{\zeta}\sqrt{\overline{\mathbf{R}}_\mathrm{r}}\,\mathbf{H}_\mathrm{iid}\left(\sqrt{\overline{\mathbf{R}}_\mathrm{t}}\right)^T$$

$$(\mathbf{A} \otimes \mathbf{B})\mathrm{vec}(\mathbf{C}) = \mathrm{vec}\left(\mathbf{BCA}^T\right)$$

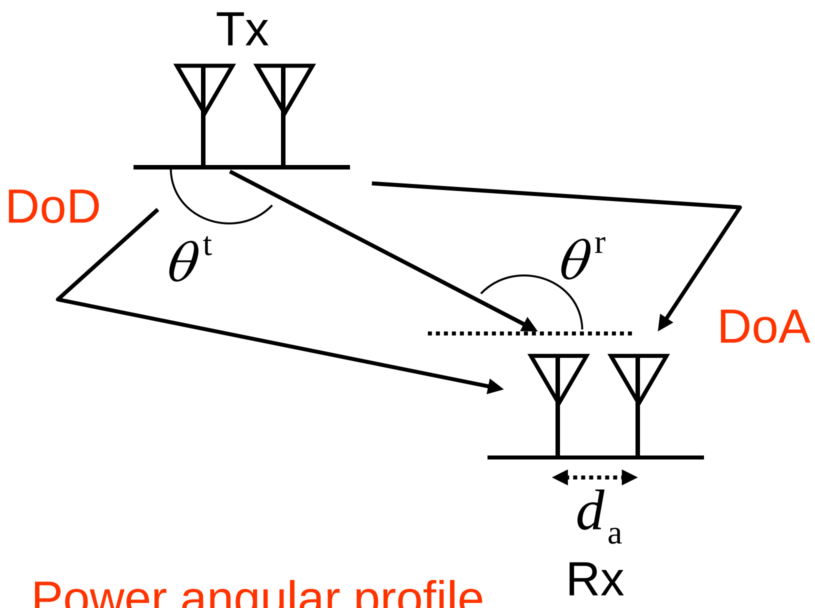
Capacity of Correlated Channel

Channel capacity in exponentially correlated channel

$$\overline{\mathbf{R}}_{\mathrm{r}} = \mathrm{teopliz}\left[1, \rho, \rho^2, \cdots, \rho^{M_{\mathrm{r}}-1}\right] \quad \overline{\mathbf{R}}_{\mathrm{t}} = \mathrm{teopliz}\left[1, \rho, \rho^2, \cdots, \rho^{M_{\mathrm{t}}-1}\right]$$



Double Directional Channel



Multi-path model

$$\mathbf{H} = \sqrt{\zeta}\sum_{l} \beta(\theta_l^{\mathrm{r}}, \theta_l^{\mathrm{t}})\mathbf{a}_{\mathrm{r}}(\theta_l^{\mathrm{r}})\mathbf{a}_{\mathrm{t}}^{T}(\theta_l^{\mathrm{t}})$$

Uncorrelated scattering

$$\mathrm{E}[\beta(\theta_i^{\mathrm{r}}, \theta_l^{\mathrm{t}})\beta^{*}(\theta_j^{\mathrm{r}}, \theta_l^{\mathrm{t}})] = 0 \qquad i \neq j$$

$$\mathrm{E}[\beta(\theta_l^{\mathrm{r}}, \theta_i^{\mathrm{t}})\beta^{*}(\theta_l^{\mathrm{r}}, \theta_j^{\mathrm{t}})] = 0 \qquad i \neq j$$

Power angular profile

Spatial correlation

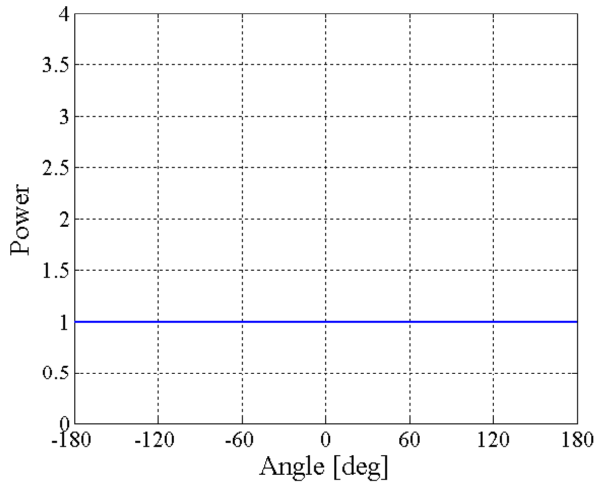
$$\mathbf{R}_{\mathrm{r}} = \zeta \int \mathrm{E}\left[\left|\beta(\theta^{\mathrm{r}})\right|^{2}\right]\mathbf{a}_{\mathrm{r}}(\theta^{\mathrm{r}})\mathbf{a}_{\mathrm{r}}^{H}(\theta^{\mathrm{r}})\mathrm{d}\theta^{\mathrm{r}} \longrightarrow \rho_{\mathrm{r}il} = \int P_{\mathrm{r}}\left(\theta^{\mathrm{r}}\right)e^{-jk(l-i)d_a\cos\theta^{\mathrm{r}}}\mathrm{d}\theta^{\mathrm{r}}$$

$$\mathbf{R}_{\mathrm{t}} = \zeta \int \mathrm{E}\left[\left|\beta(\theta^{\mathrm{t}})\right|^{2}\right]\mathbf{a}_{\mathrm{t}}(\theta^{\mathrm{t}})\mathbf{a}_{\mathrm{t}}^{H}(\theta^{\mathrm{t}})\mathrm{d}\theta^{\mathrm{t}} \longrightarrow \rho_{\mathrm{t}il} = \int P_{\mathrm{t}}\left(\theta^{\mathrm{t}}\right)e^{-jk(l-i)d_a\cos\theta^{\mathrm{t}}}\mathrm{d}\theta^{\mathrm{t}}$$

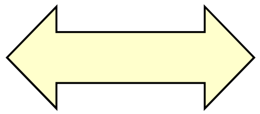
Angular Profile & Spatial Correlation

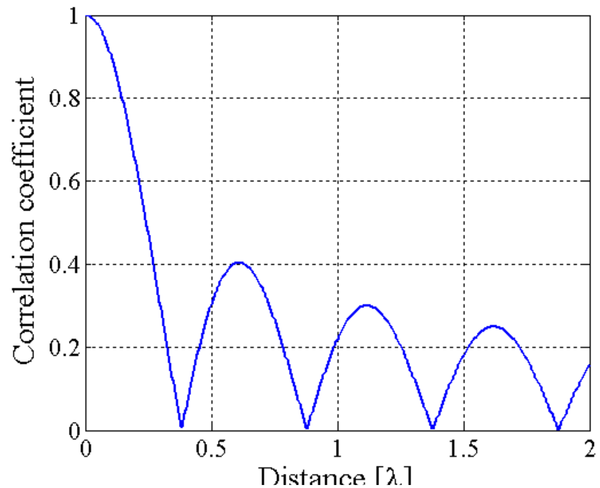
Angular profile

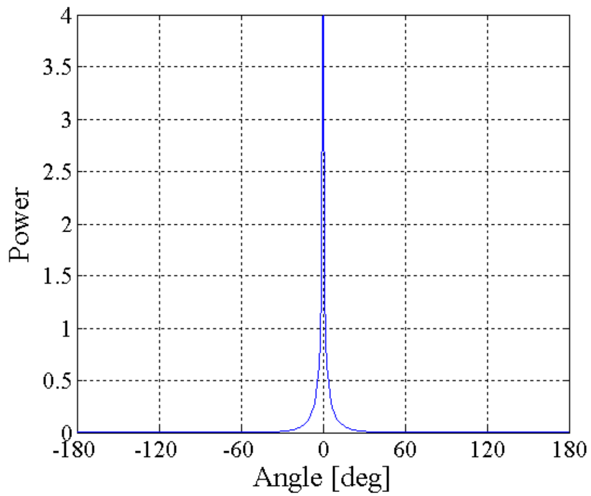
Spatial correlation



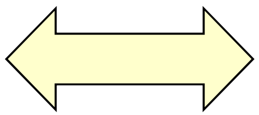
Uniform

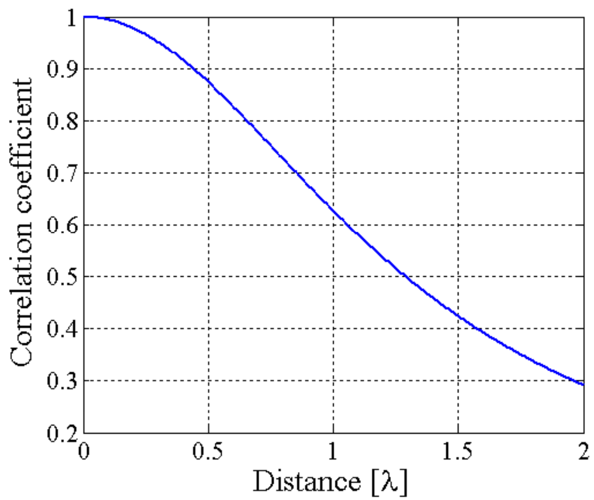






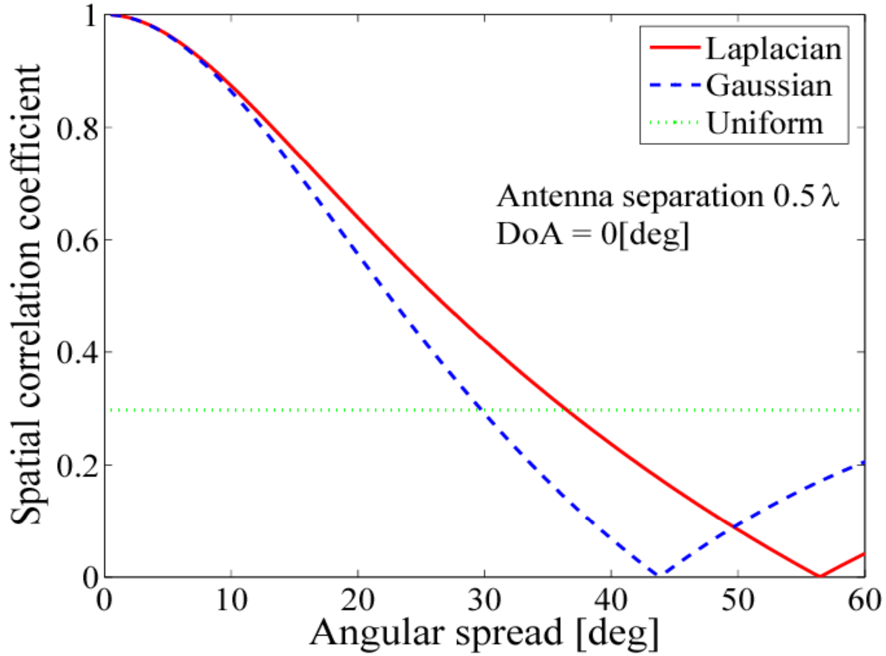
Laplacian



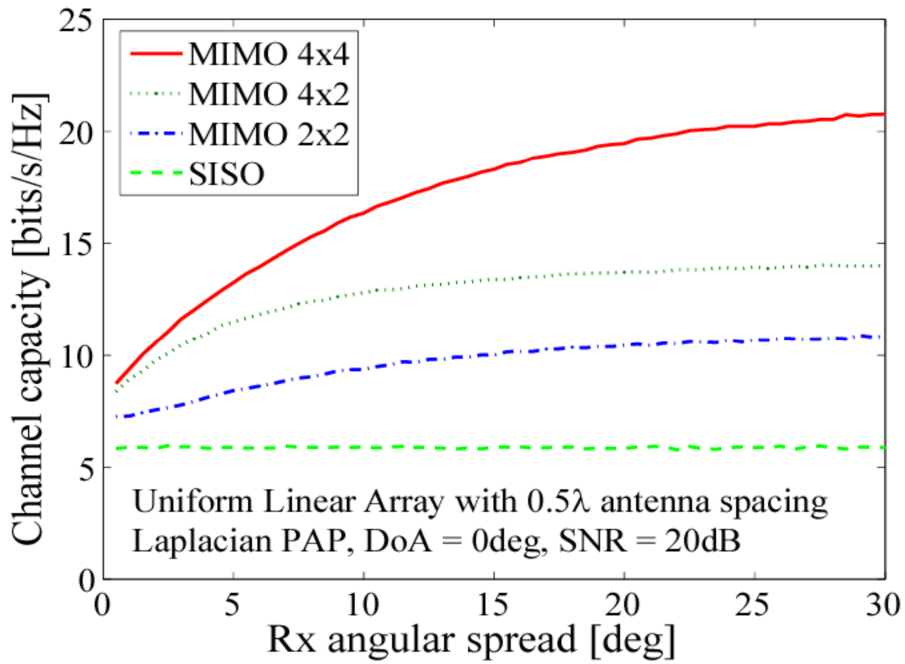


Angular Spread & Capacity

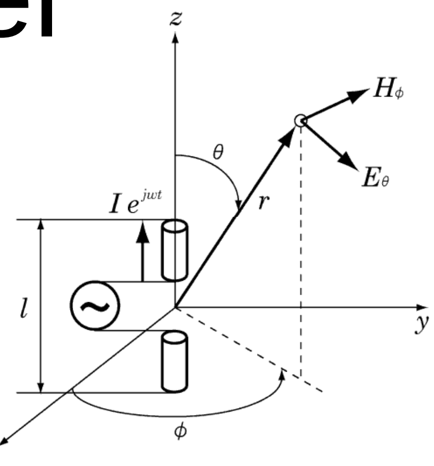
Spatial correlation



Channel capacity



Antenna Model



Scalar function channel matrix

$$\mathbf{H}=\sqrt{\zeta}\sum_{l=1}^{L}\beta\left(\theta_l^{\mathrm{r}},\theta_l^{\mathrm{t}}\right)\mathbf{a}_{\mathrm{r}}\left(\theta_l^{\mathrm{r}}\right)\mathbf{a}_{\mathrm{t}}^{T}\left(\theta_l^{\mathrm{t}}\right)$$

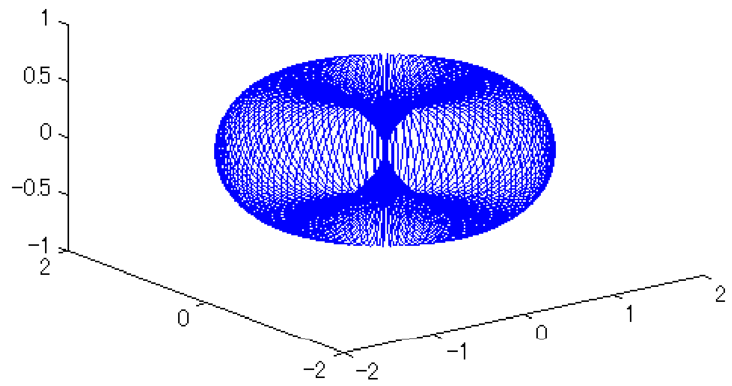
Vector function channel matrix

Polarization model

$$\mathbf{H}=\sqrt{\zeta}\sum_{l=1}^{L}\left[\tilde{\mathbf{a}}_{\mathrm{r}}^{\theta}\left(\psi_l^{\mathrm{r}}\right)\quad \tilde{\mathbf{a}}_{\mathrm{r}}^{\phi}\left(\psi_l^{\mathrm{r}}\right)\right]\begin{bmatrix}\beta^{\theta\theta}\left(\psi_l^{\mathrm{r}},\psi_l^{\mathrm{t}}\right) & \beta^{\theta\phi}\left(\psi_l^{\mathrm{r}},\psi_l^{\mathrm{t}}\right)\\ \beta^{\phi\theta}\left(\psi_l^{\mathrm{r}},\psi_l^{\mathrm{t}}\right) & \beta^{\phi\phi}\left(\psi_l^{\mathrm{r}},\psi_l^{\mathrm{t}}\right)\end{bmatrix}\left[\tilde{\mathbf{a}}_{\mathrm{t}}^{\theta}\left(\psi_l^{\mathrm{t}}\right)\quad \tilde{\mathbf{a}}_{\mathrm{t}}^{\phi}\left(\psi_l^{\mathrm{t}}\right)\right]^{T}$$

$$\psi_l^{\mathrm{r}}=\left(\theta_l^{\mathrm{r}},\phi_l^{\mathrm{r}}\right)\qquad \psi_l^{\mathrm{t}}=\left(\theta_l^{\mathrm{t}},\phi_l^{\mathrm{t}}\right)$$

Directivity of dipole antenna



Antenna directivity

$$\vec{\mathbf{g}}_{\mathrm{r}}\left(\theta^{\mathrm{r}},\phi^{\mathrm{r}}\right)=\mathbf{g}_{\mathrm{r}}^{\theta}\left(\theta^{\mathrm{r}},\phi^{\mathrm{r}}\right)\vec{\theta}+\mathbf{g}_{\mathrm{r}}^{\phi}\left(\theta^{\mathrm{r}},\phi^{\mathrm{r}}\right)\vec{\phi}$$

Array manifold with element directivity

$$[\tilde{\mathbf{a}}_{\mathrm{r}}^{\theta}\left(\psi^{\mathrm{r}}\right)\quad \tilde{\mathbf{a}}_{\mathrm{r}}^{\phi}\left(\psi^{\mathrm{r}}\right)]=[\mathbf{g}_{\mathrm{r}}^{\theta}\left(\psi^{\mathrm{r}}\right)\quad \mathbf{g}_{\mathrm{r}}^{\phi}\left(\psi^{\mathrm{r}}\right)]\circ\mathbf{a}_{\mathrm{r}}\left(\psi^{\mathrm{r}}\right)$$

Mutual Coupling

Circuit equation

$$\tilde{\mathbf{s}} = \mathbf{Z}_{\mathrm{A}}(\mathbf{Z}_{\mathrm{A}} + \mathbf{Z}_{\mathrm{L}})^{-1}\mathbf{s}$$

$$\mathbf{y} = \mathbf{Z}_{\mathrm{L}}(\mathbf{Z}_{\mathrm{A}} + \mathbf{Z}_{\mathrm{L}})^{-1}\tilde{\mathbf{y}}$$

$$\mathbf{Z}_{\mathrm{L}} = \mathrm{diag}\left[Z_L, \cdots, Z_L\right]$$

Mutual coupling matrix

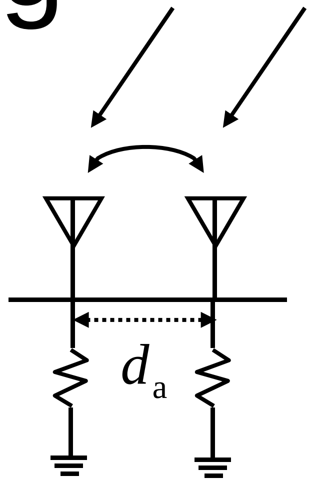
$$\mathbf{Q}_{\mathrm{r}} = \mathbf{Z}_{\mathrm{A}}(\mathbf{Z}_{\mathrm{A}} + \mathbf{Z}_{\mathrm{L}})^{-1}$$

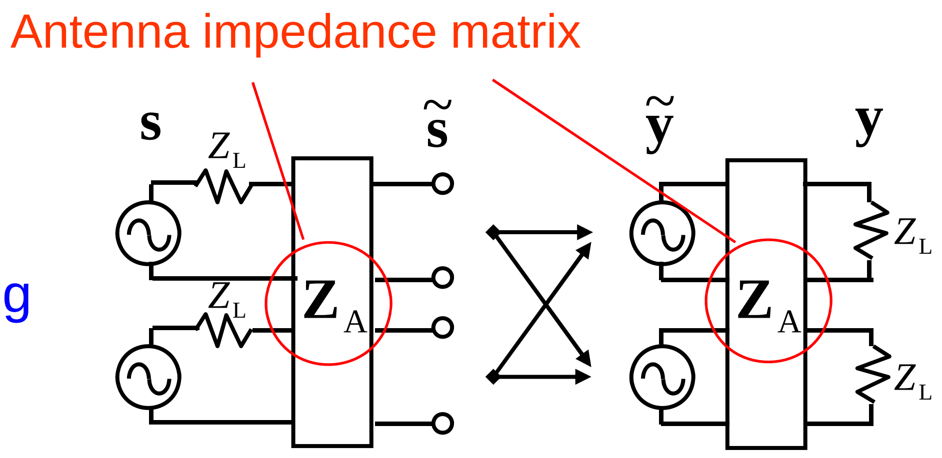
$$\mathbf{Q}_{\mathrm{t}} = \mathbf{Z}_{\mathrm{L}}(\mathbf{Z}_{\mathrm{A}} + \mathbf{Z}_{\mathrm{L}})^{-1}$$

Array manifold with mutual coupling

$$\tilde{\mathbf{a}}_{\mathrm{r}}(\theta) = \mathbf{Q}_{\mathrm{r}}\mathbf{a}_{\mathrm{r}}(\theta)$$

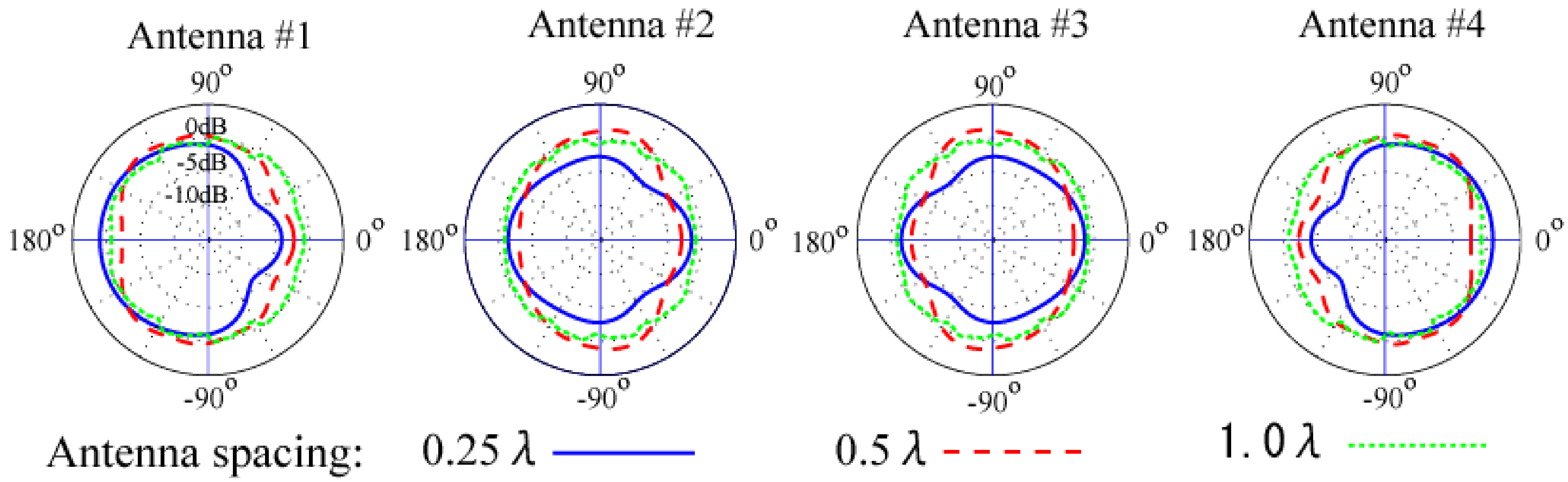
$$\tilde{\mathbf{a}}_{\mathrm{t}}(\theta) = \mathbf{Q}_{\mathrm{t}}\mathbf{a}_{\mathrm{t}}(\theta)$$





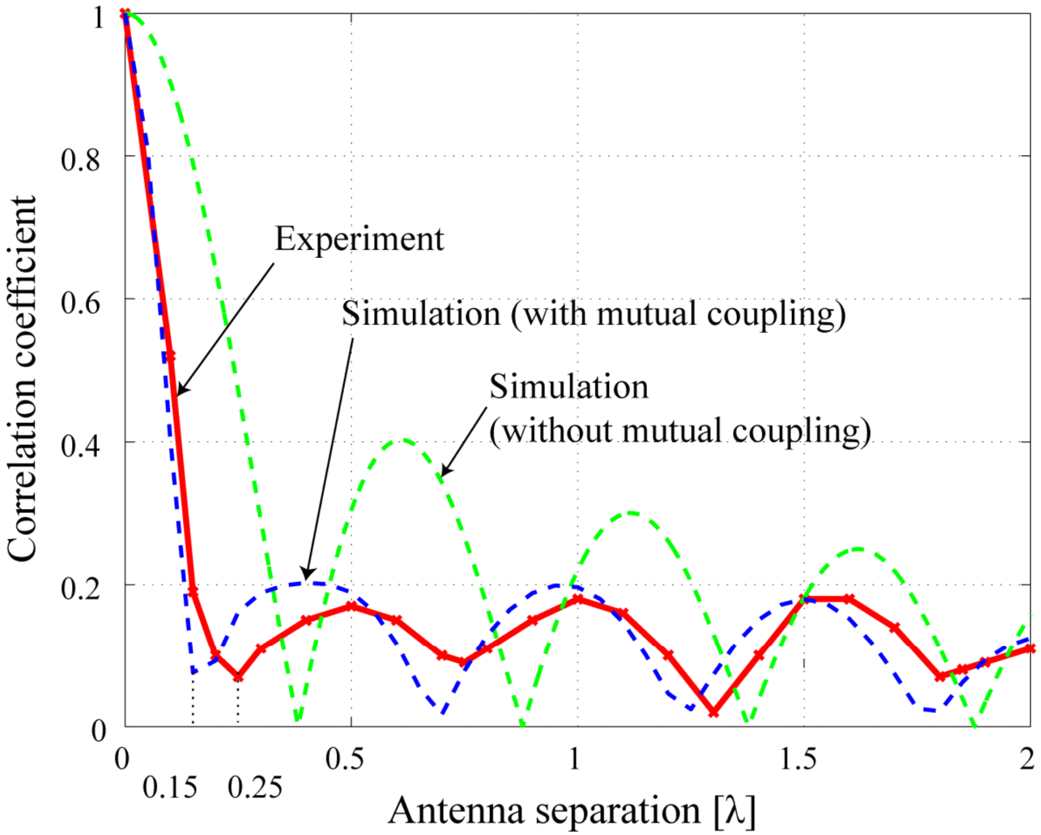
Effect of Mutual Coupling

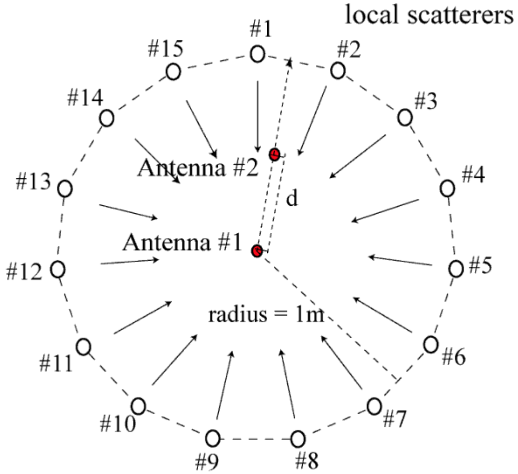
Directivity of each antenna element

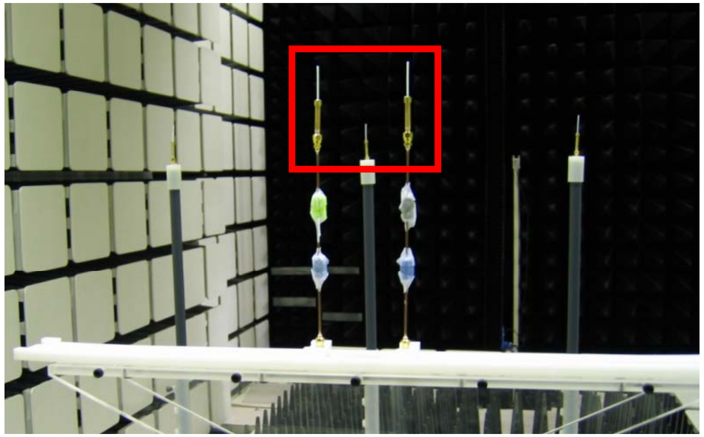


Effect of Mutual Coupling

Spatial correlation







Summary

- Spatial channel model for MIMO system
  - Deterministic model of path loss
  - Stochastic model of correlated fading channels via angular profiles
  - Stochastic parameters are given by empirical studies

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Design and assessment of MIMO system & MIMO antenna

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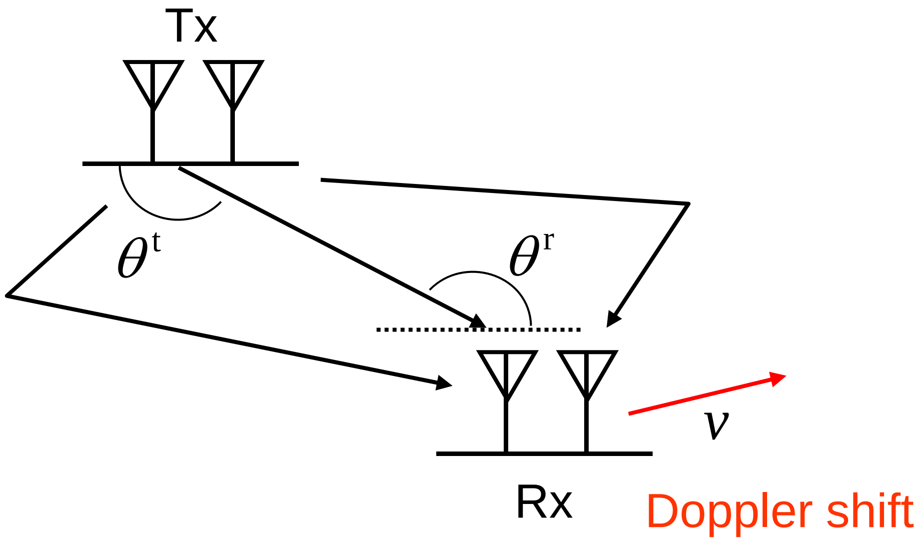
How about with practical transmission schemes?

Linear & non-linear MIMO receivers

Time Varing Channel



Double directional channel

$$\mathbf{H} = \sqrt{\zeta}\sum_{l} \beta(\theta_l^{\mathrm{r}}, \theta_l^{\mathrm{t}})\mathbf{a}_{\mathrm{r}}(\theta_l^{\mathrm{r}})\mathbf{a}_{\mathrm{t}}^{T}(\theta_{\mathrm{l}}^{\mathrm{t}})$$

Doppler shift

$$\beta(\theta_l^{\mathrm{r}}, \theta_l^{\mathrm{t}}, t) = \beta(\theta_l^{\mathrm{r}}, \theta_l^{\mathrm{t}})e^{jkvt\cos\theta_l^{\mathrm{r}}}$$

Uncorrelated scattering

$$\mathrm{E}[e^{jkvt\cos\theta_i^{\mathrm{r}}} e^{-jkvt\cos\theta_j^{\mathrm{r}}}] = 0 \qquad i \neq j$$

Time correlation

Doppler profile

$$\mathrm{E}[h(t)h^*(t+\Delta)] = \zeta\sum_{l} \mathrm{E}\left[\left|\beta\left(\theta_l^{\mathrm{r}}\right)\right|^2\right]e^{jkv\Delta\cos\theta_l^{\mathrm{r}}}$$

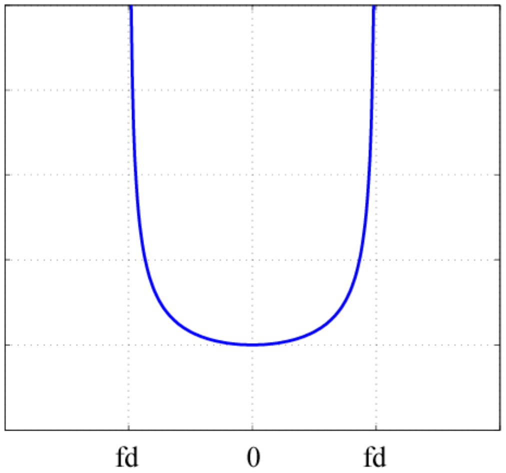
$$\rho(\Delta) = \int \mathrm{E}\left[\left|\beta\left(\theta^{\mathrm{r}}\right)\right|^2\right]e^{jkv\Delta\cos\theta^{\mathrm{r}}}\mathrm{d}\theta^{\mathrm{r}}$$

$$= \int P(f)e^{j2\pi f\Delta}\mathrm{d}f \longleftarrow \quad f = \frac{v\cos\theta^{\mathrm{r}}}{\lambda} \quad \longrightarrow \quad P(f) = \frac{\left|\beta\left(\mathrm{acos}\left(\frac{\lambda f}{v}\right)\right)\right|^2}{-\sqrt{\left(\frac{v}{\lambda}\right)^2 - f^2}}$$

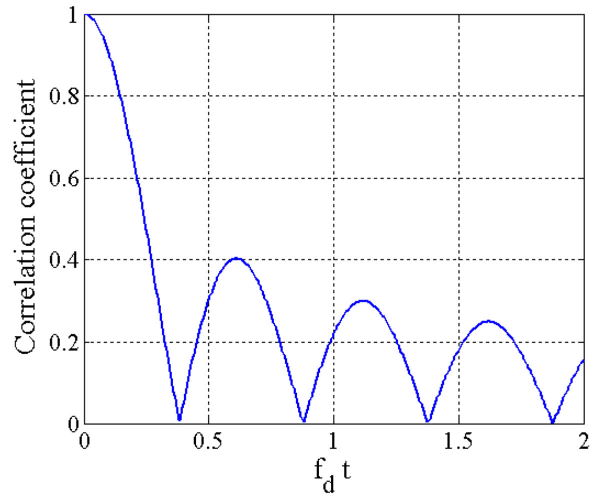
Doppler Profile & Time Correlation

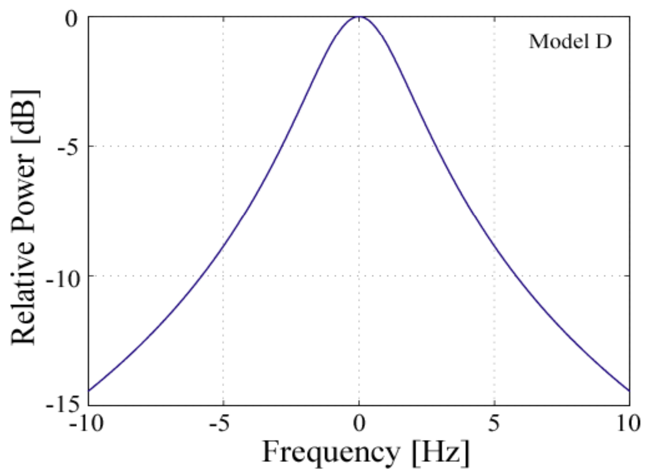
Doppler profile

Time correlation

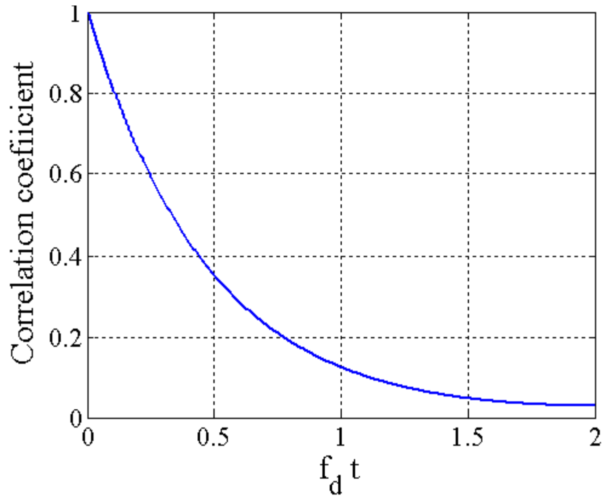


Jake's





Bell shape



Wide Band Channel

Time domain expression

$$\mathbf{y}(t) = \int \mathbf{H}(\tau)\mathbf{s}(t-\tau)\mathrm{d}\tau + \mathbf{n}(t)$$

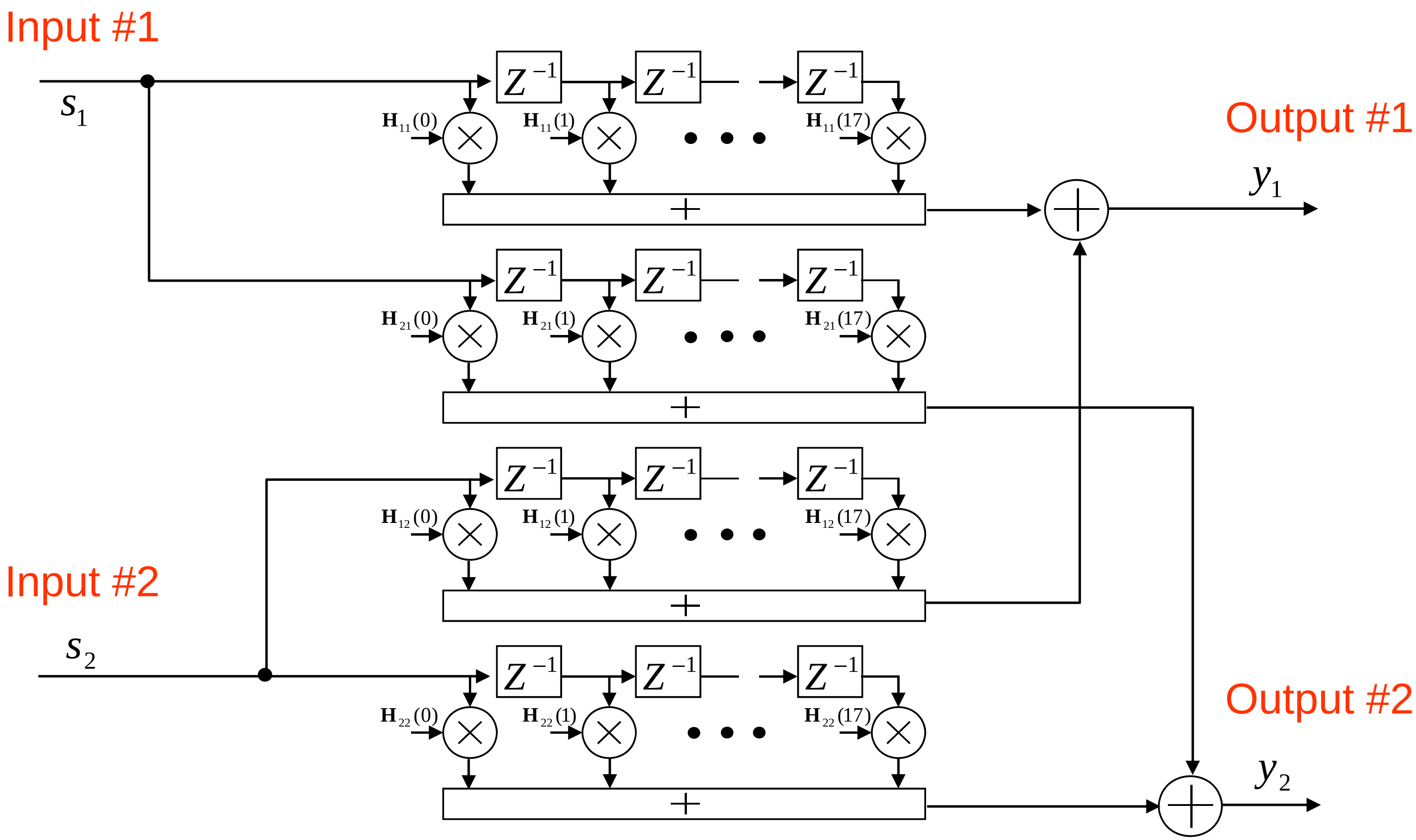
$$\mathbf{y}(t) = \sum_n \mathbf{H}(n\Delta\tau)\mathbf{s}(t-n\Delta\tau) + \mathbf{n}(t)$$

Frequency domain expression

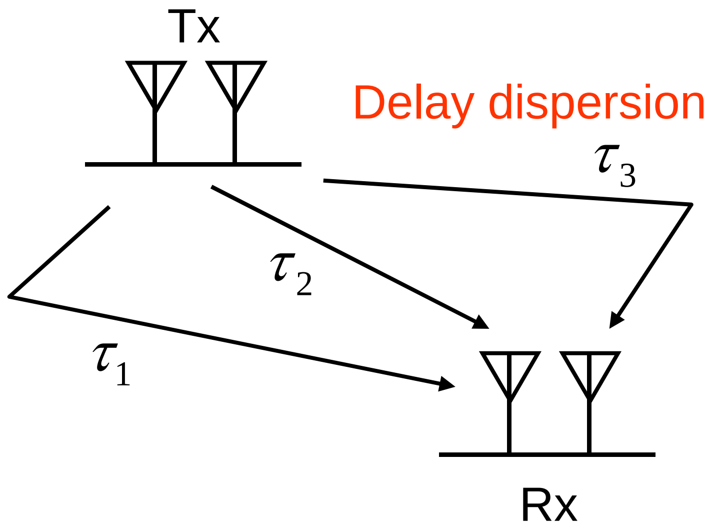
$$\tilde{\mathbf{y}}(f) = \tilde{\mathbf{H}}(f)\tilde{\mathbf{s}}(f) + \tilde{\mathbf{n}}(f)$$

$$\tilde{\mathbf{y}}(k\Delta f) = \tilde{\mathbf{H}}(k\Delta f)\tilde{\mathbf{s}}(k\Delta f) + \tilde{\mathbf{n}}(k\Delta f)$$

Tapped Delay Line (TDL)



Delay Profile & Frequency Correlation



Frequency domain signal

$$\tilde{y}(f) = \tilde{h}(f)\tilde{s}(f) + \tilde{n}(f)$$

Frequency response

$$\tilde{h}(f) = \sqrt{\zeta_\tau} \int \sum_{l_\tau=1}^{L_\tau} \beta\left(\theta_{l_\tau}, \tau\right) e^{-j2\pi f\tau} d\tau$$

Uncorrelated scattering

$$\mathrm{E}[\beta(\tau_i)\beta^*(\tau_j)] = 0 \quad i \neq j$$

Delay profile

Frequency correlation

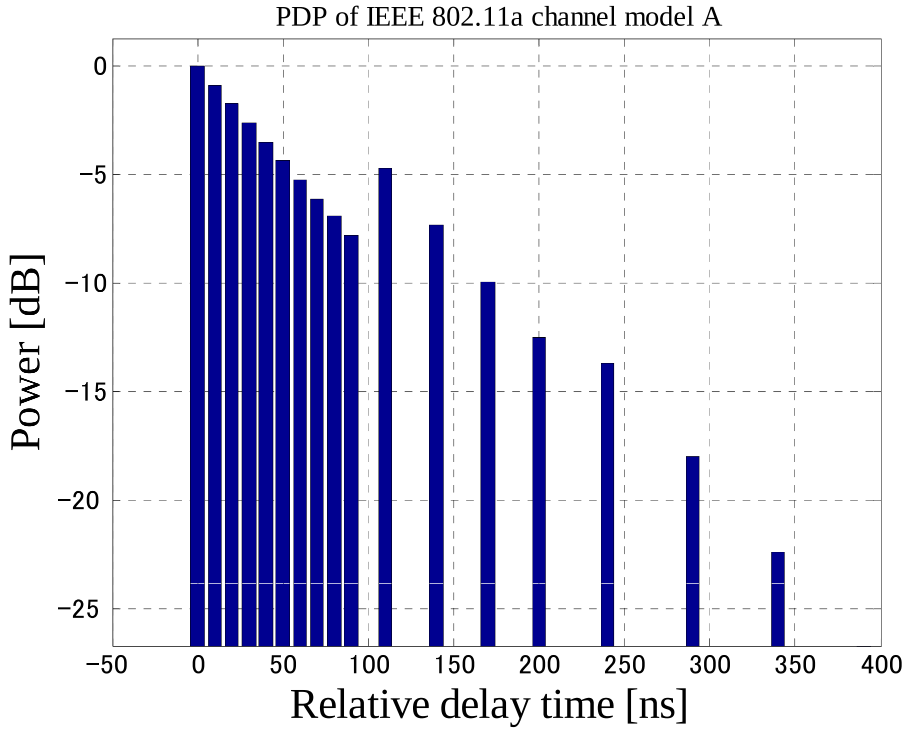
$$\mathrm{E}[\tilde{h}(f)\tilde{h}^*(f + \Delta f)] = \zeta \int \mathrm{E}\left[\left|\beta(\tau)\right|^2\right] e^{-j2\pi\Delta f\tau} \mathrm{d}\tau$$

$$\rho(\Delta f) = \int P(\tau) e^{-j2\pi\Delta f\tau} \mathrm{d}\tau$$

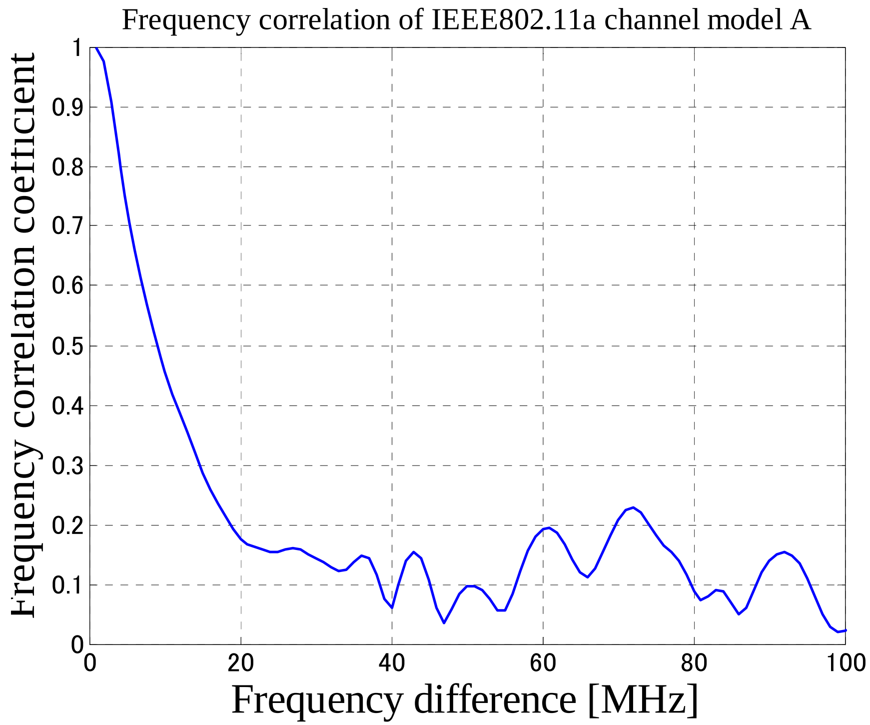
Delay Profile & Frequency Correlation

IEEE802.11a channel model A

Power delay profile



Frequency correlation



IEEE802.11n Spatial Channel Model