2011 1st semester

MIMO Communication Systems

#3: OFDM of Wireless Broadband

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Schedule (1st half)

	Date	Text	Contents
#1	Apr. 12	A-1, B-1	Introduction
#2	Apr. 19	B-5, B-6	Fundamentals of wireless commun.
#3	Apr. 26	B-12	OFDM for wireless broadband
	May 3		No class
#4	May 10	B-7	Array signal processing
#5	Nov. 17	A-3, B-10	MIMO channel capacity
#6	Nov. 24	B-2, 3	Spatial channel model
	May 28		No class

Agenda

- Aim of today

  Derive throughput performance of
  OFDM based wide band system

- Contents

  - Wide band system
    - Frequency selective & time dispersive fading
    - Inter-Symbol Interference (ISI) and SINR degradation
    - Orthogonal Frequency Division Multiplexing (OFDM)

Warming Up

- Question

Describe *N*-point DFT in matrix form

- Discrete Fourier Transform (DFT)

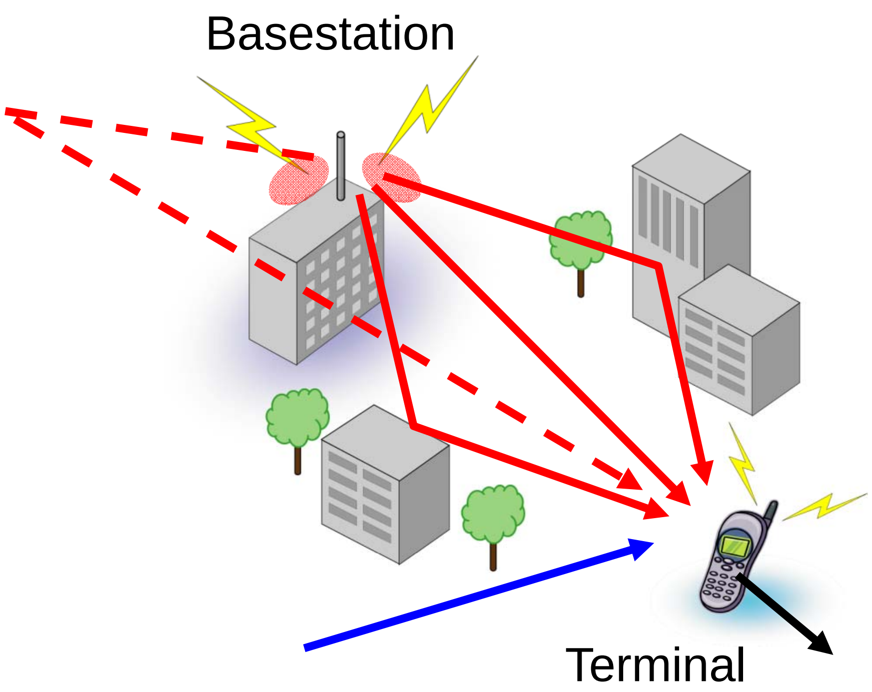
$$\tilde{x}(k\Delta f) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n\Delta t) \exp\left(-j2\pi k\Delta f n\Delta t\right)$$

$$= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) \exp\left(-j2\pi \frac{nk}{N}\right)$$

$$\Delta f = \frac{1}{N\Delta t}$$

Wireless Communication Channel

Wireless is vulnerable!



#1: Path loss due to distance

⟶ Lower average SNR

#2: Multi-path fading

⟶ Deep SNR dip

#3: Time dispersive fading

⟶ Inter symbol interference

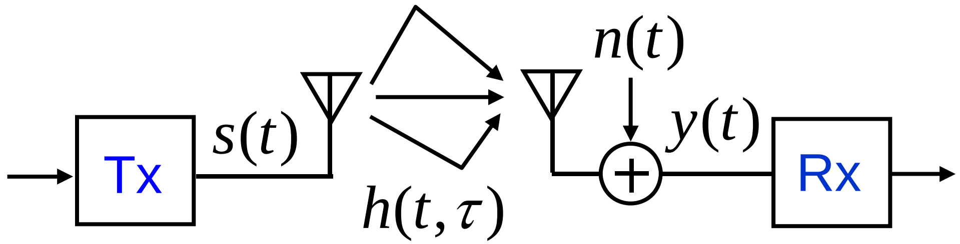
#4: Multi-access interference

⟶ Co-channel interference

Wide Band Signal Model

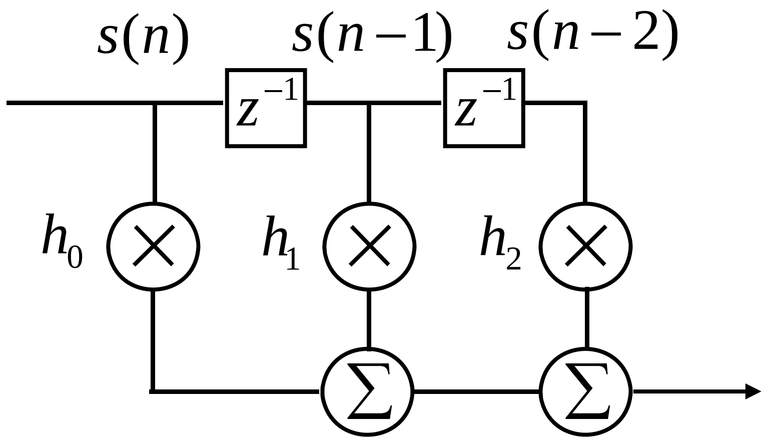
Received signal

$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}\tau + n(t)$$



Discrete representation

$$y(n\Delta t) = \sum_l h_l s(n\Delta t - l\Delta\tau) + n(n\Delta t)$$



Frequency domain

$$\tilde{y}(f) = \tilde{h}(f)\tilde{s}(f) + \tilde{n}(f)$$

Uncertainty theorem

$$\tilde{y}(k\Delta f) = \tilde{h}(k\Delta f)\tilde{s}(k\Delta f) + \tilde{n}(k\Delta f) \quad \Delta F = 1/\Delta t \quad \Delta f = 1/\Delta T$$

Frequency Selective Fading

Two-path model

$$y(t) = h_0 s(t) + h_1 s(t - \Delta\tau) + n(t)$$

Impulse response

$$h(\tau) = h_0 \delta(\tau) + h_1 \delta(\tau - \Delta\tau)$$

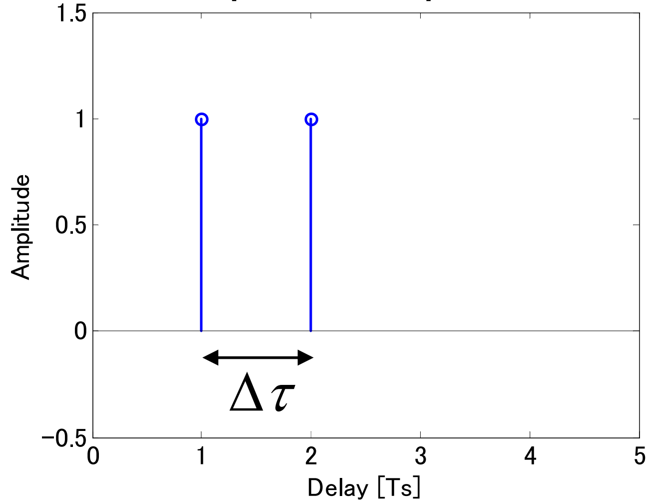
Frequency response

$$\tilde{h}(f) = h_0 + h_1 \exp(-j2\pi f \Delta\tau)$$

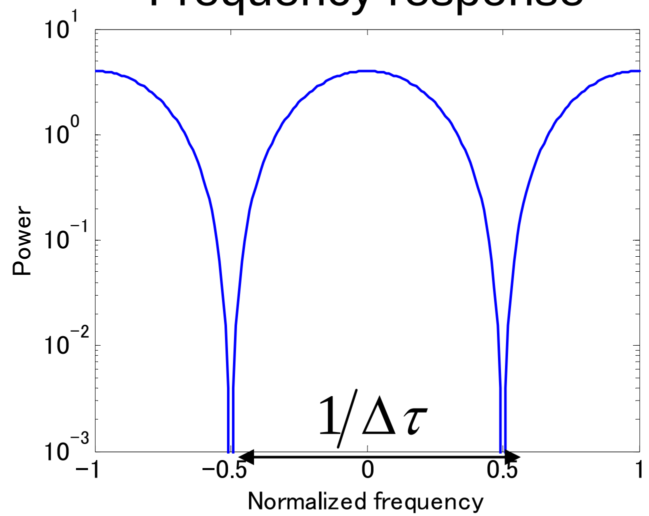
Condition for narrow band signal

Bandwidth $\Delta F << 1/\Delta\tau$

Impulse response



Frequency response



Frequency Spectrum

Received signal

$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}\tau + n(t)$$

Auto correlation

$$R_s(\tau) = \mathrm{E}\left[s^*(t)s(t+\tau)\right]$$

Power spectrum of transmit signal

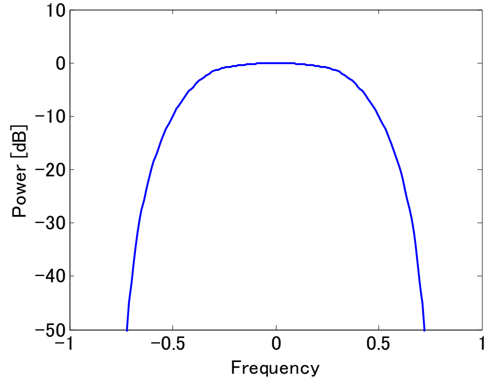
$$S_s(f) = \int R_s(\tau)\exp\left(-j2\pi f\tau\right)\mathrm{d}\tau$$

Power spectrum of receive signal

$$S_y(f) = \left|\tilde{h}(f)\right|^2 S_s(f)$$

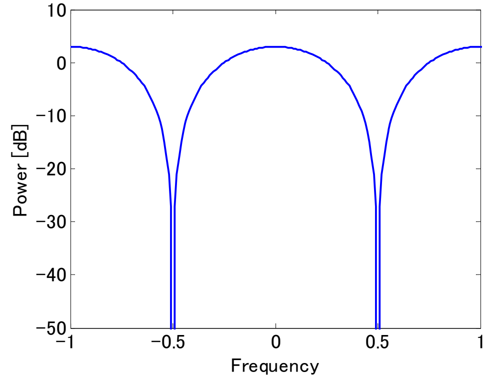
$$\tilde{h}(f) = \int h(t)\exp\left(-j2\pi ft\right)\mathrm{d}t$$

Transmit signal

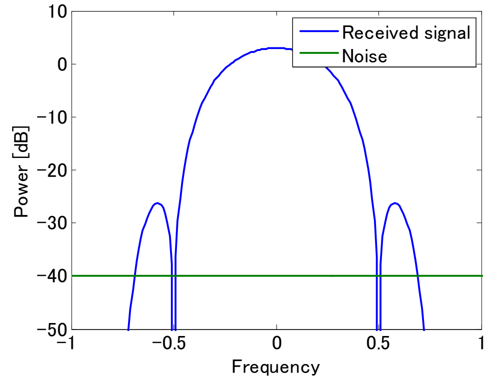


$$\Delta F > 1/\Delta\tau$$

Frequency response



Receive signal



Distortion in power spectrum

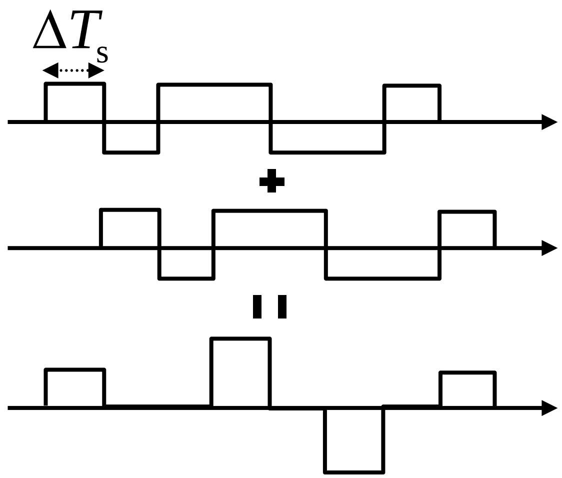
Time Dispersive Fading & Inter Symbol Interference

Two-path model

$$y(t) = \sum h(\Delta\tau)s(t-\Delta\tau)+n(t)$$

$$= h_0 s(t) + h_1 s(t-\Delta T_s) + n(t)$$

if $\Delta\tau = \Delta T_s$

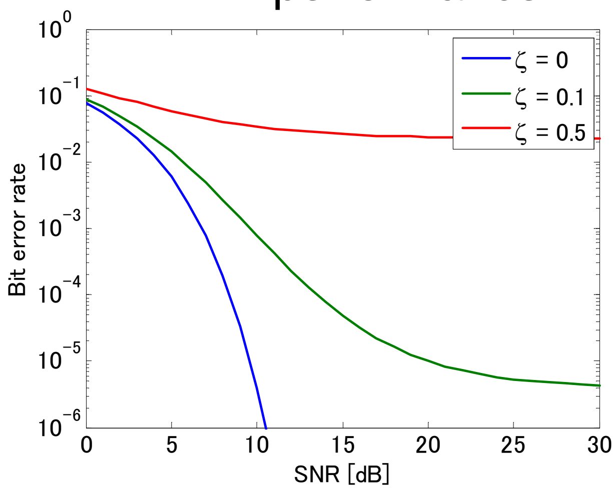


Signal to Interference & Noise Ratio（SINR）

$$\gamma_I = \frac{|h_0|^2 P}{|h_1|^2 P + \sigma^2} = \frac{|h_0|^2}{|h_1|^2 + \sigma^2 / P}$$

$$P_{eb} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma_I}\right)$$ for BPSK

BER performance



Channel Capacity

Two-path model

$$y(t) = h_0 s(t) + h_1 s(t - \Delta\tau) + n(t)$$

Transmit signal with pulse shaping

$$s(t) = \sum_i s_i g(t - iT_s)$$

Receive SINR

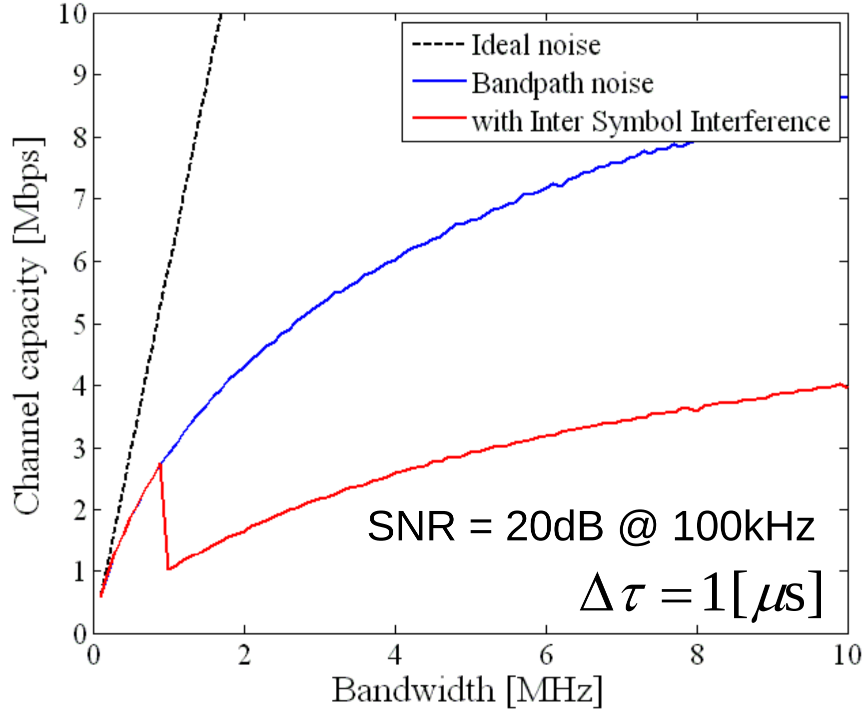
$$\gamma_{\mathrm{I}} = \frac{\left|h_0 g(0) + h_1 g(-\Delta\tau)\right|^2 P}{\sum_{i \neq 0} \left|h_1 g(iT_s - \Delta\tau)\right|^2 P + \sigma^2}$$

Channel capacity

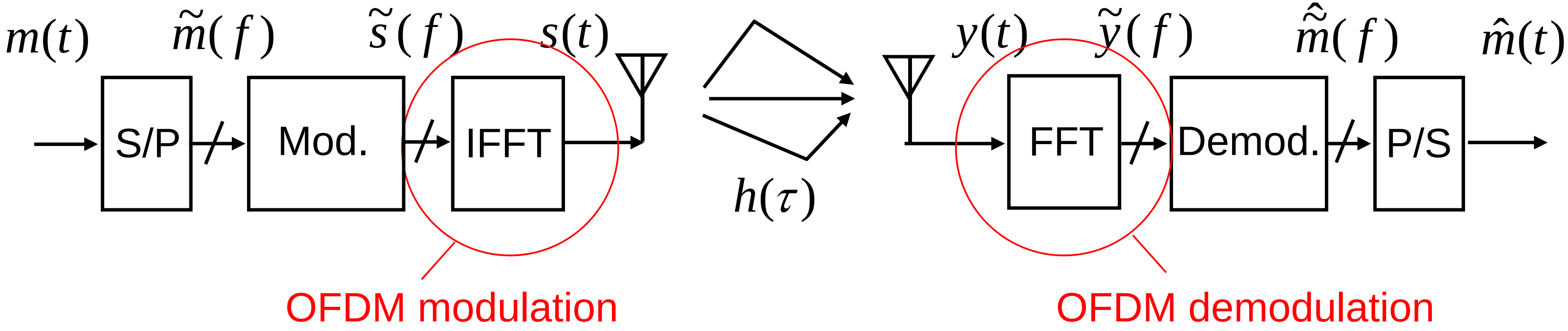
$$C = B \log_2\left(1 + \gamma_{\mathrm{I}}\right) \approx \frac{1}{T_s} \log_2\left(1 + \gamma_{\mathrm{I}}\right)$$

$$\sigma^2 = BN_0$$

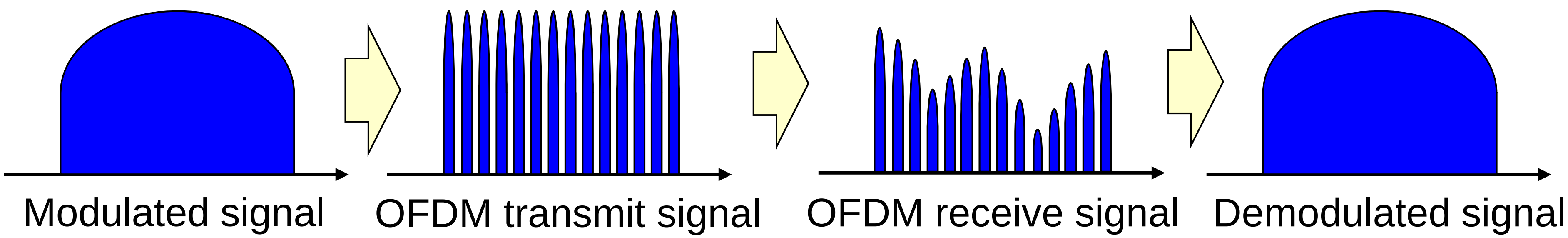
Average channel capacity



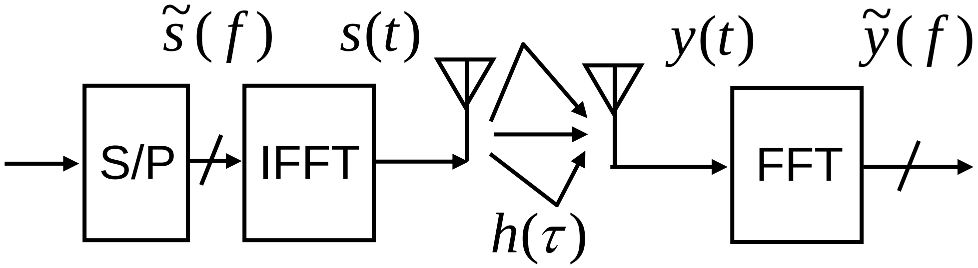
Orthogonal Frequency Division Multiplexing (OFDM)

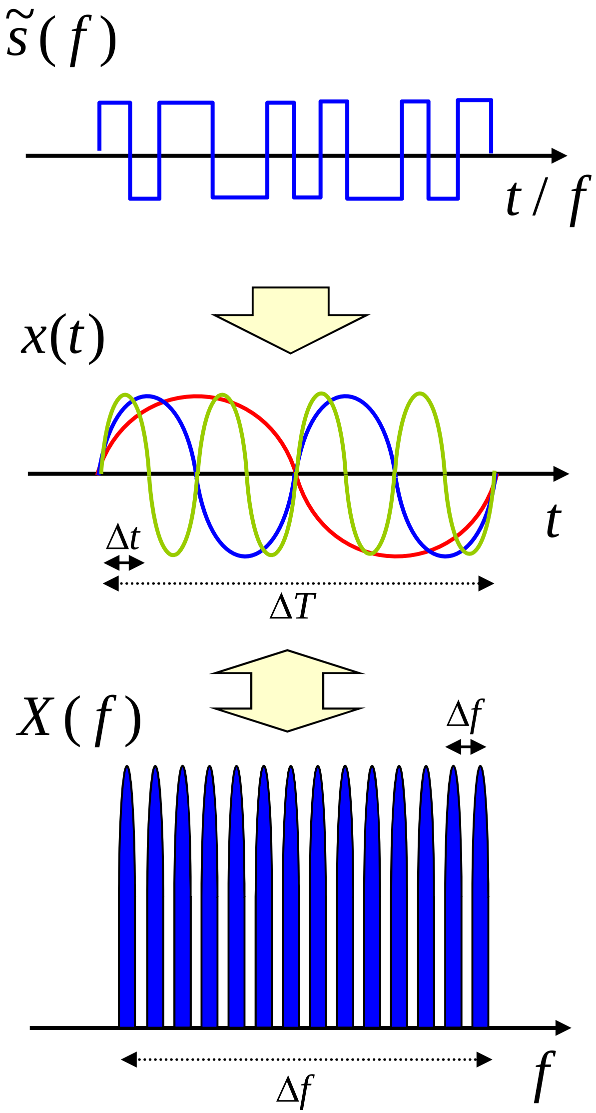


- Block transmission system using S/P & P/S converter
- Convert wide band signal to super position of narrow band signals satisfying $\Delta f << 1/\Delta\tau$



OFDM Modulation





OFDM modulation

Number of subcarriers

$$s(t) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} \tilde{s}(k\Delta f) \exp\left(j 2\pi k \Delta f t\right)$$

Subcarrier interval

$$0 \le t \le \Delta T = 1/\Delta f$$

OFDM parameters

Subcarrier interval

$$\Delta f << 1/\Delta\tau$$

Bandwidth

$$\Delta F = K\Delta f$$

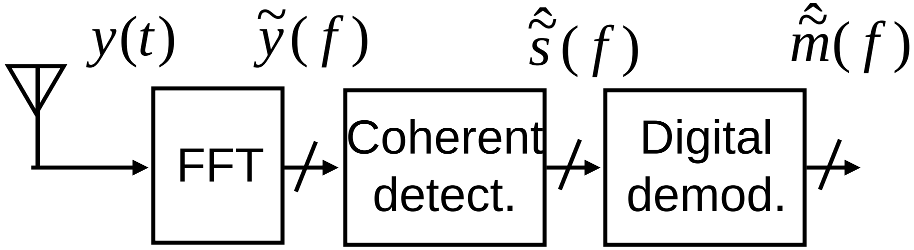
Sampling period

$$\Delta t \le 1/\Delta F$$

OFDM symbol period

$$\Delta T = 1/\Delta f$$

OFDM demodulation



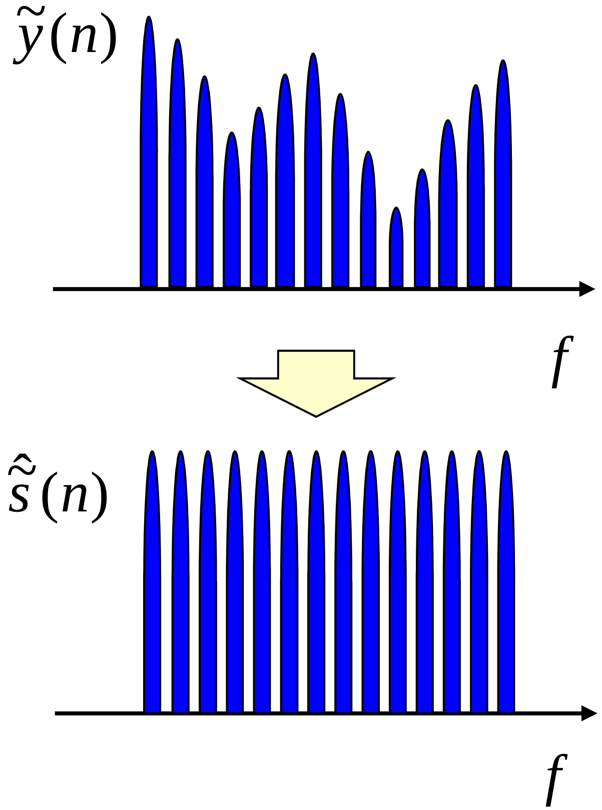
Receive signal

$$y(t) = \int h(\tau)s(t-\tau)\mathrm{d}t + n(t)$$

OFDM demodulation

$$\tilde{y}(k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} y(n)\exp\left(-j2\pi\frac{nk}{N}\right)$$

$$= \tilde{h}(k)\tilde{s}(k) + \tilde{n}(k)$$

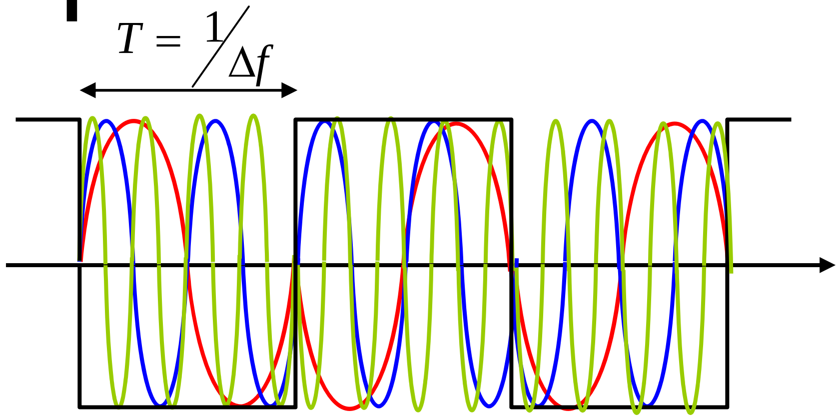


Frequency Domain Equalizer（FDE）

$$\hat{\tilde{s}}(k) = \tilde{y}(k) \big/ \tilde{h}(k)$$

Coherent detection
in frequency domain

Frequency Spectrum



OFDM signal

$$s(t) = \frac{1}{\sqrt{K}} \sum_{k=0}^{K-1} \tilde{s}(k) \exp(j2\pi k \Delta f t)$$

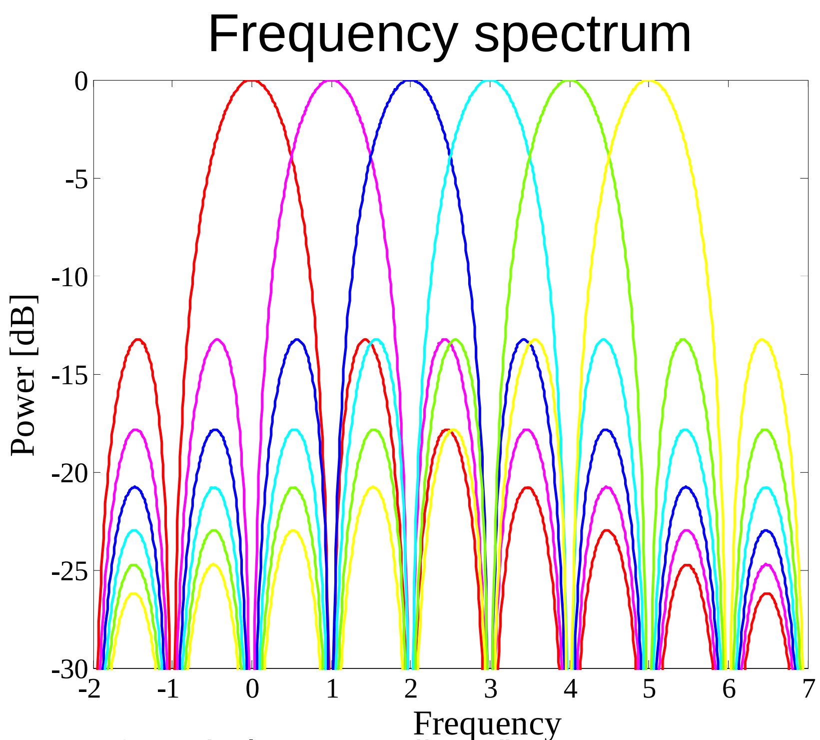
Auto correlation

$$R_s(\tau) = \mathrm{E}\left[s^*(t) s(t+\tau)\right] = \frac{1}{K} \sum_{k=0}^{K} \left(1 - \frac{|\tau|}{T}\right) \exp(j2\pi k \Delta f \tau)$$

Auto correlation function of rectangular pulse

Power spectrum

$$S_S(f) = \frac{1}{K} \sum_{n=0}^{K-1} \mathrm{sinc}^2\left(T(f - n\Delta f)\right)$$



Matrix representation

OFDM modulation

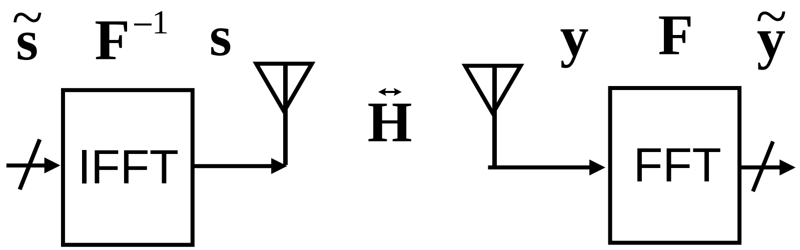
Transmit signal block

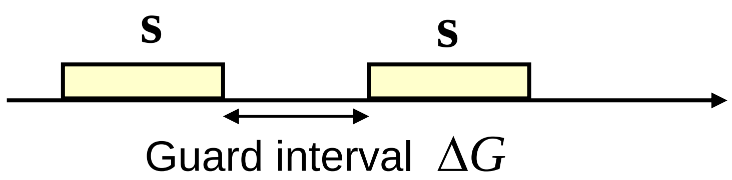
$$\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{s}_0 & \tilde{s}_1 & \cdots & \tilde{s}_{K-1} \end{bmatrix}^T$$

$$\mathbf{s} = \begin{bmatrix} s_0 & s_1 & \cdots & s_{N-1} \end{bmatrix}^T$$

$$\mathbf{s} = \mathbf{F}^{-1}\tilde{\mathbf{s}}$$

Inverse DFT





Convolution in matrix form

L-path model

$$\mathbf{h} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{L-1} & 0 & 0 \end{bmatrix}^T$$

Receive signal block

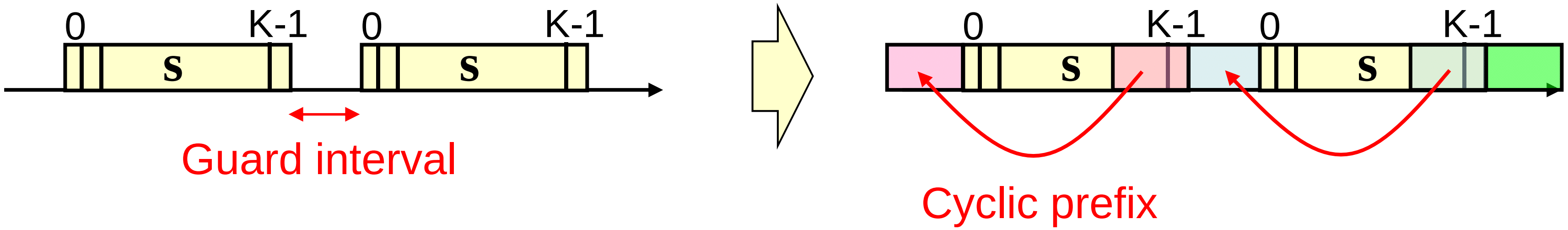
$$\mathbf{y} = \begin{bmatrix} y_0 & y_1 & \cdots & y_{N-1} \end{bmatrix}^T$$

If $\Delta G > \Delta\tau_{\max}$

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & \cdots & 0 & 0 \\ h_1 & h_o & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ \vdots \\ n_{N-2} \\ n_{N-1} \end{bmatrix}$$

Cyclic Prefix

Block transmission | **Cyclic prefix**



Matrix representation of received signal block

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & \cdots & 0 & 0 \\ h_1 & h_o & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix}$$

Cyclic shift matrix

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-2} \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_o & 0 & h_{L-1} & \cdots & h_1 \\ h_1 & h_o & \cdots & h_{L-1} & \vdots \\ \vdots & \vdots & \ddots & \vdots & h_{L-1} \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ \vdots \\ s_{N-2} \\ s_{N-1} \end{bmatrix}$$

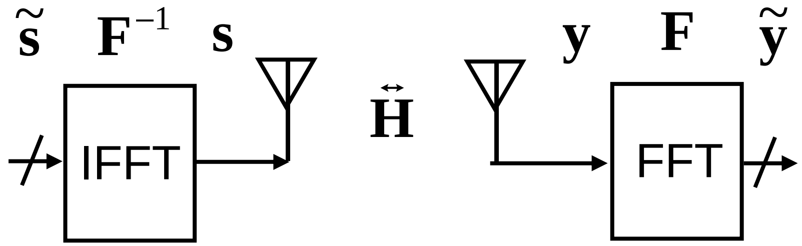
Psudo-periodical transmission

OFDM Transmission

Diagonalization of cyclic shift matrix

Cyclic shift matrix

$$\overleftrightarrow{\mathbf{H}} = \begin{bmatrix} h_o & 0 & h_{L-1} & \cdots & h_1 \\ h_1 & h_o & \cdots & h_{L-1} & \vdots \\ \vdots & \vdots & \ddots & \vdots & h_{L-1} \\ h_{L-1} & \cdots & h_1 & h_o & 0 \\ 0 & h_{L-1} & \cdots & h_1 & h_o \end{bmatrix}$$



OFDM transmission

$$\mathbf{y} = \overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{n}$$

$$\tilde{\mathbf{y}} = \mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{s} + \mathbf{n} = \mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{F}^{-1}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

$$= \mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{F}^{-1}\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

$$= \mathrm{diag}[\tilde{\mathbf{h}}]\tilde{\mathbf{s}} + \tilde{\mathbf{n}}$$

K-parallel transmission

Interesting feature of cyclic shift matrix

$$\mathbf{F}\overleftrightarrow{\mathbf{H}}\mathbf{F}^{-1} = \begin{bmatrix} \tilde{h}_0 & 0 & \cdots & 0 \\ 0 & \tilde{h}_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \tilde{h}_{K-1} \end{bmatrix} = \mathrm{diag}[\tilde{\mathbf{h}}]$$

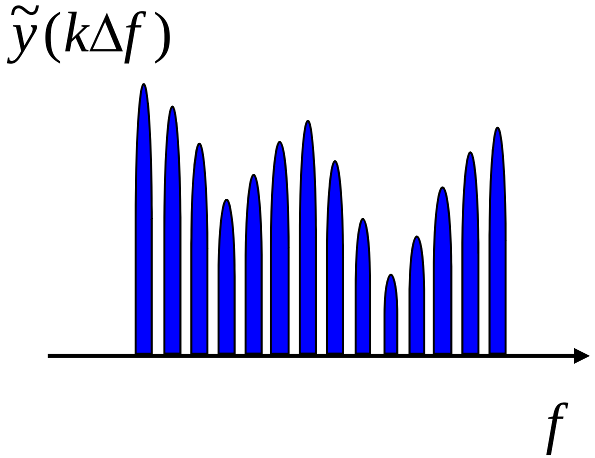
Frequency response

$$\tilde{\mathbf{h}} = \mathbf{F}\mathbf{h}$$

BER Performance

SNR per subcarrier

$$\gamma_k = \frac{P/K \left|\tilde{h}_k\right|^2}{\sigma^2/K} = \frac{P\left|\tilde{h}_k\right|^2}{\sigma^2} \quad \longleftrightarrow \quad \bar{\gamma}_k = \bar{\gamma}$$



PDF of SNR for each subcarrier

$$f(\gamma_k) = \frac{1}{\bar{\gamma}_k}\exp\left(-\frac{\gamma_k}{\bar{\gamma}_k}\right)$$

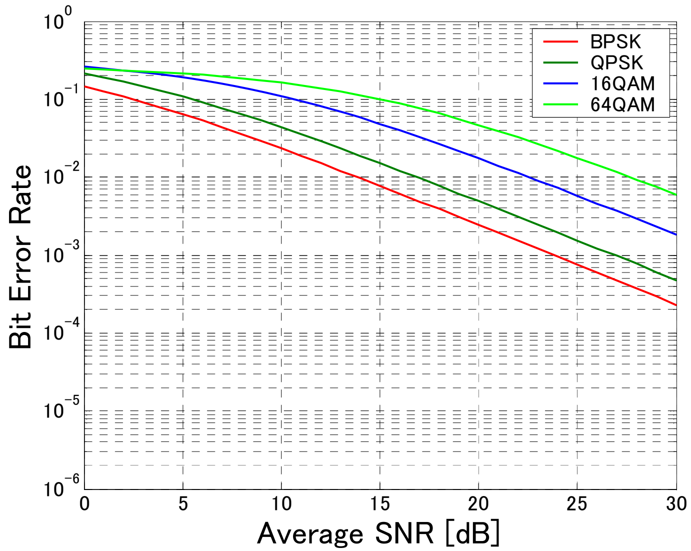
Average BER for each subcarrier

$$\bar{P}_{\mathrm{eb}}^{k}(\bar{\gamma}_k) = \int f(\gamma_k) P_{\mathrm{eb}}(\gamma_k)\mathrm{d}\gamma_k$$

Overall average BER

$$\bar{P}_{\mathrm{eb}}(\bar{\gamma}) = \frac{1}{K}\sum_{k=0}^{K-1}\bar{P}_{\mathrm{eb}}^{k}\left(\bar{\gamma}_k\right)$$

Average BER performance



Channel Capacity

OFDM signal model

$$\tilde{y}_k = \tilde{h}_k \tilde{s}_k + \tilde{n}_k, \quad k = 1, \cdots, K$$

SNR for each subcarrier

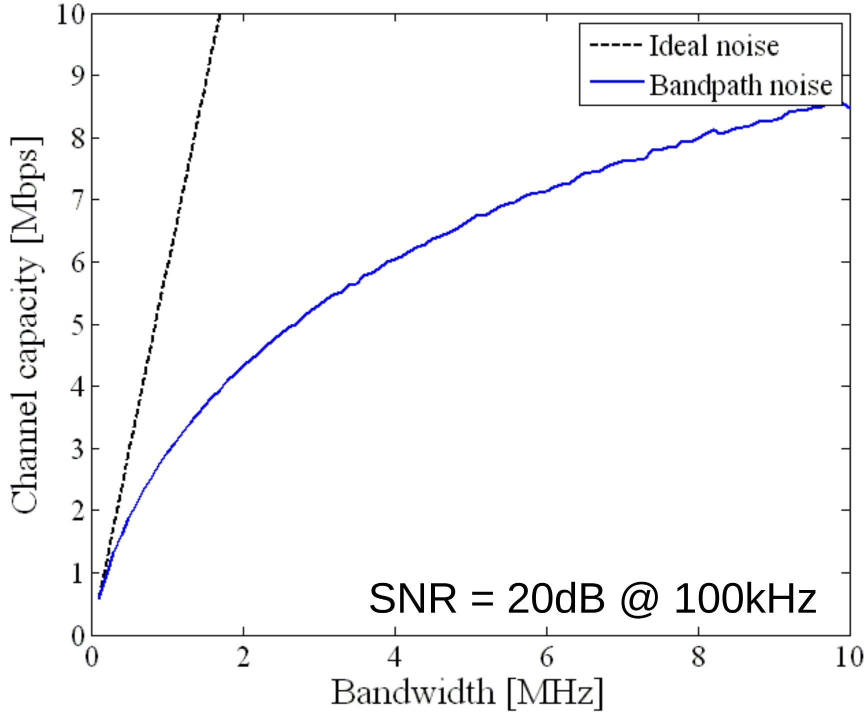
$$\gamma_k = \frac{P/K \, |h_k|^2}{\sigma^2 / K} = \frac{P |h_k|^2}{\sigma^2}$$

Channel capacity

$$C = \sum_{k=0}^{K-1} \Delta f \log_2 (1 + \gamma_k)$$

$$\sigma^2 = B N_0$$

Average channel capacity



Summary

- In wide band signal
  - Time dispersive fading causes inter-symbol interference
  - Frequency selective fading causes distortion in power spectrum
  - OFDM converts wide band signal to multiple narrow band signals
  - IFFT, FFT, and cyclic prefix creates parallel orthogonal channels
  - Problem of Rayleigh fading still remains even by using OFDM

↓

Some measure for Rayleigh fading

Array signal processing