2011 1st semester

MIMO Communications Systems

#2: Fundamentals of Wireless Communication

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Schedule (1^{st} half)

	Date	Text	Contents
#1	Apr. 12	A-1, B-1	Introduction
#2	Apr. 19	B-5, B-6	Fundamentals of wireless commun.
#3	Apr. 26	B-12	OFDM for wireless broadband
	May 3		No class
#4	May 10	B-7	Array signal processing
#5	Nov. 17	A-3, B-10	MIMO channel capacity
#6	Nov. 24	B-2, 3	Spatial channel model
	May 28		No class

Agenda

- Aim of today

Derive throughput performance of basic SISO system

- Contents

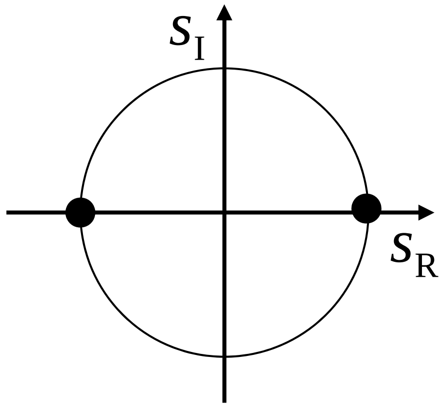
  - Narrow band system
    - Channel capacity
    - Path loss & multi-path fading
    - QAM signaling & throughput performance

Warming Up

Question

Given a transmission system with BPSK modulation whose transmission rate is 1 bit per T seconds. Calculate spectrum efficiency C [bit/s/Hz] of this system.

Constellation



Spectrum efficiency

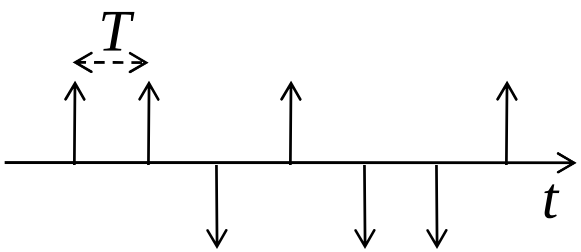
Nyquist filter

$$G(f)=\begin{cases}1, & 0\le |fT|\le 1/2\\ 0, & \text{otherwise}\end{cases}$$

Bandwidth

$$B = 1/T$$

Time series



Spectrum efficiency

?

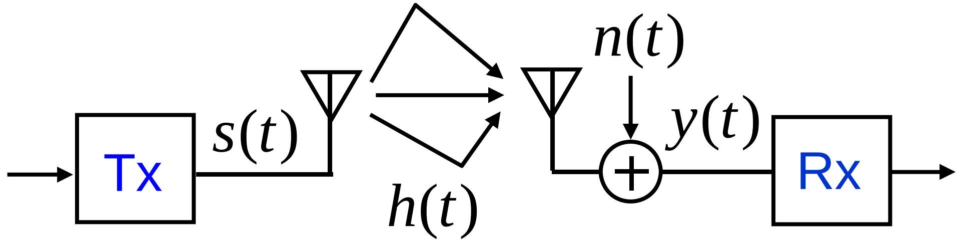
Narrow Band System

Received signal model

Received signal — Additive noise

$$y(t) = h(t)s(t) + n(t)$$

Channel response — Transmit signal



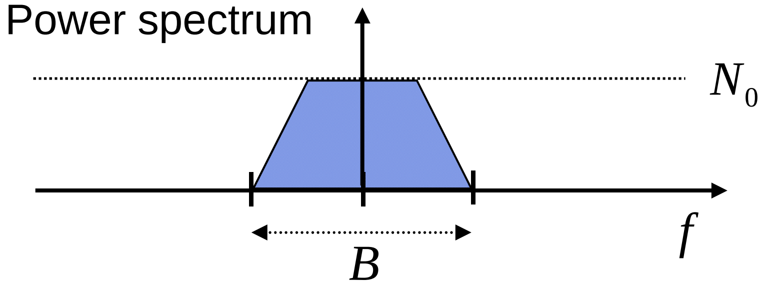
Propagation channel

$$h(t) \approx \text{const}$$

over unit signal packet

Transmit power

$$P = \mathrm{E}[|s(t)|^2]$$



Noise power

$$\sigma^2 = \mathrm{E}[|n(t)|^2] = N_0 B$$

Channel Capacity

Channel capacity of real number system

$$C_{\mathrm{R}} = \frac{1}{T} \max_{\mathrm{E}[s^2] \le P} I(S;Y) = \frac{1}{2T} \log_2\left(1 + \frac{P|h|^2}{\sigma^2}\right) = \frac{B}{2} \log_2\left(1 + \frac{P|h|^2}{\sigma^2}\right)$$

Mutual information

$$I(S;Y) = H(Y) - H(Y \mid S) = H(Y) - H(S + N \mid S) = H(Y) - H(N)$$

Entropy of Gaussian signal

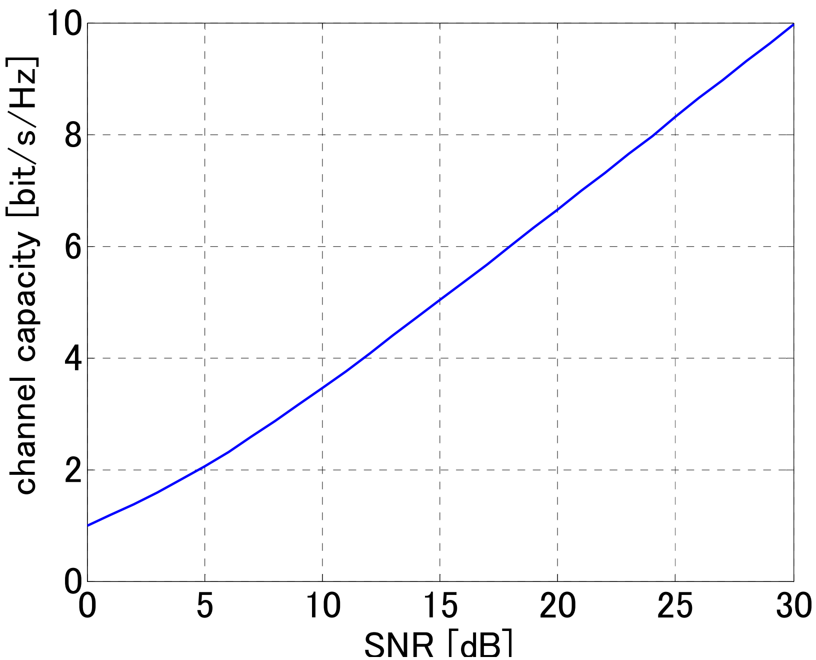
$$H(N) = \frac{1}{2}\log_2 2\pi e \sigma^2 \qquad H(Y) \le \frac{1}{2}\log_2 2\pi e(|h|^2 P + \sigma^2)$$

Channel capacity of complex number system

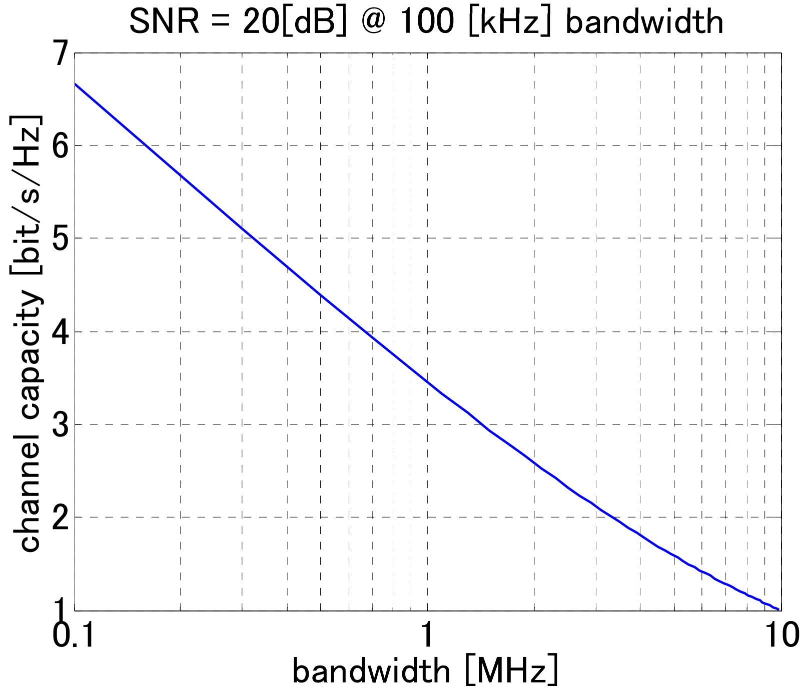
$$C_{\mathrm{C}} = 2 \times \frac{B}{2} \log_2\left(1 + \frac{P/2\,|h|^2}{\sigma^2/2}\right) = B \log_2\left(1 + \frac{P|h|^2}{\sigma^2}\right) \ \text{[bits/s]}$$

Achievable Data Rate of SISO

Power dependency

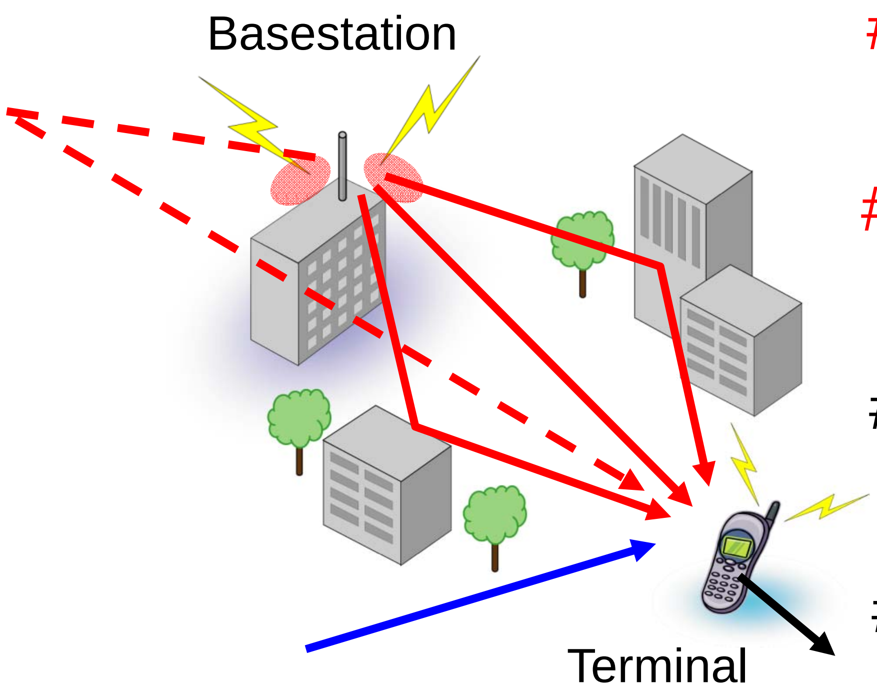


Bandwidth dependency



Wireless Communication Channel

Wireless is vulnerable!



#1: Path loss due to distance

⟶ Lower average SNR

#2: Multi-path fading

⟶ Deep SNR dip

#3: Time dispersive fading

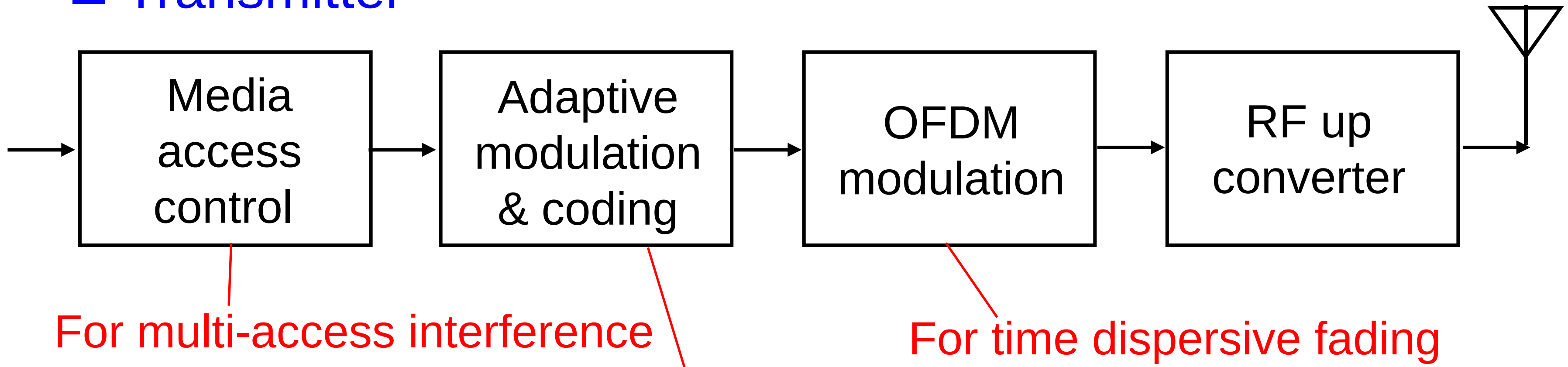
⟶ Inter symbol interference

#4: Multi-access interference

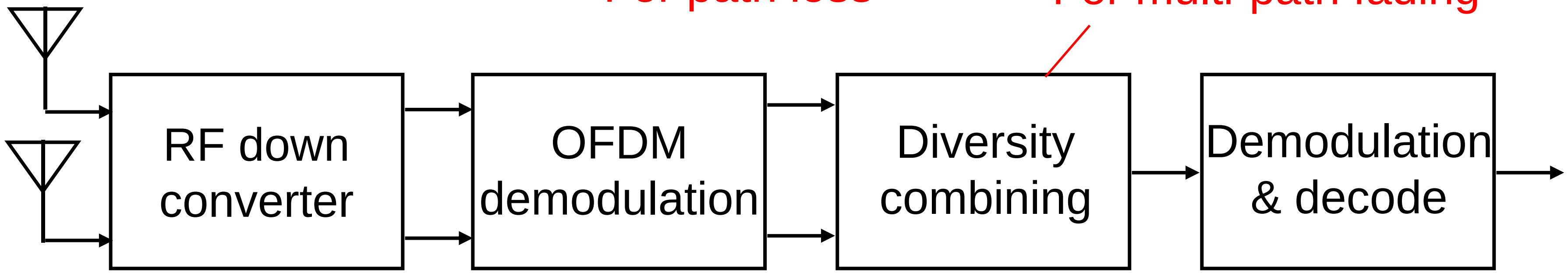
⟶ Co-channel interference

Modern Wireless Transceiver

- Transmitter



- Receiver

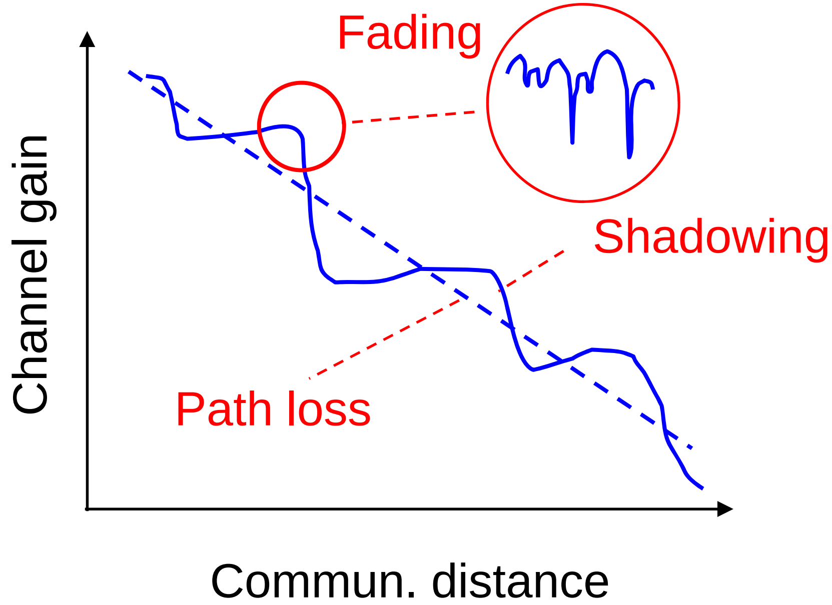


Issue #1: Path loss

- Channel gain decreases in accordance with distance between Tx & Rx
- It results in lower SNR, higher BER, and lower throughput

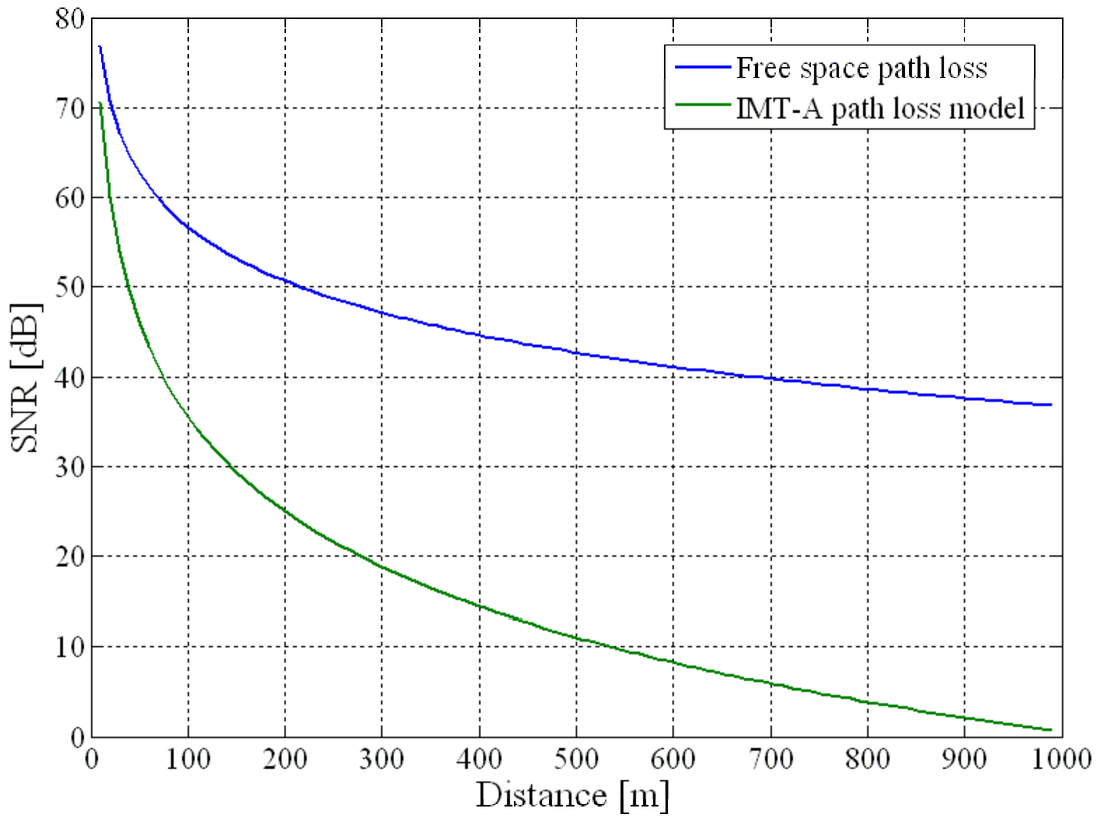
Free space path loss

$$G_{\mathrm{pl}} = 10\log_{10}\left(\frac{\lambda}{4\pi d}\right)^2$$
$$= -20\log_{10} d - 20\log_{10} f + 28$$



Receive SNR

Transmit power = 40dBm, Noise power = -100dBm, Frequency = 3.5GHz



QAM Modulation

QAM modulation

- Message

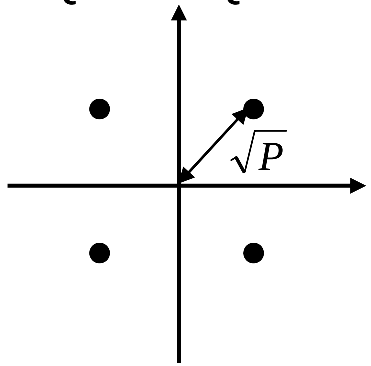
$$m_{\mathrm{R}} = \{0,1,\cdots,\sqrt{M_{\mathrm{ary}}}-1\} \quad m_{\mathrm{I}} = \{0,1,\cdots,\sqrt{M_{\mathrm{ary}}}-1\}$$

- Symbol

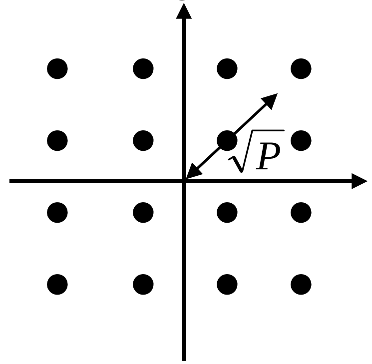
$$s = s_{\mathrm{R}} + js_{\mathrm{I}}$$
$$= (2m_{\mathrm{R}} - \sqrt{M_{\mathrm{ary}}} + 1) + j(2m_{\mathrm{I}} - \sqrt{M_{\mathrm{ary}}} + 1)$$

M_{ary} QAM $\longrightarrow$ $r = \log_2 M_{\mathrm{ary}}$ [bits/symbol]

4QAM = QPSK



16QAM



Power normalization

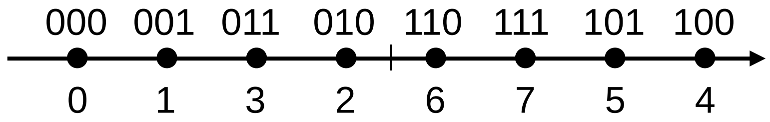
$$P = 2\left(\frac{2}{\sqrt{M_{\mathrm{ary}}}}\sum_{i=1}^{\sqrt{M_{\mathrm{ary}}}/2}(2i-1)^2\right) = \frac{2(M_{\mathrm{ary}}-1)}{3} \longrightarrow s = s/\sqrt{P}$$

Gray coding

Adjacent symbols differ by one binary digit

$$s_{\mathrm{R}} = \mathrm{gray}_{\sqrt{M_{\mathrm{ary}}}}(m_{\mathrm{R}})$$

Ex. $\mathrm{gray}_8(m_{\mathrm{R}})$



BER on QPSK Signaling

Channel estimation

$$\hat{h} = \int \frac{y_{\mathrm{T}}(t)}{s_{\mathrm{T}}(t)} \mathrm{d}t$$

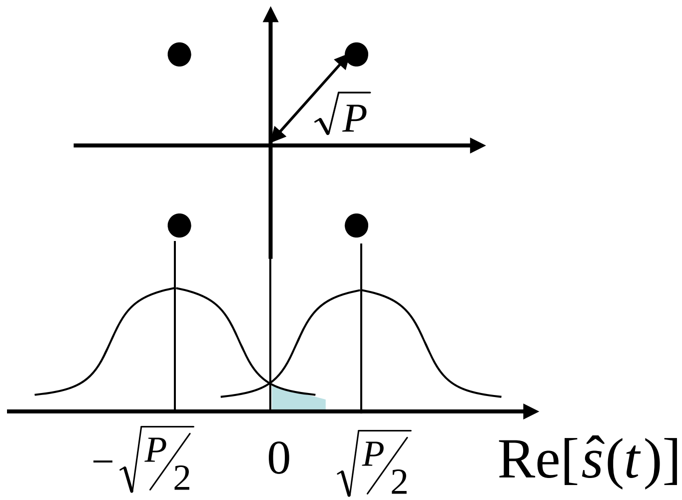
Coherent detection

$$\hat{s}(t) = \frac{y(t)}{\hat{h}} = s(t) + \frac{n(t)}{\hat{h}}$$

Bit error rate (BER)

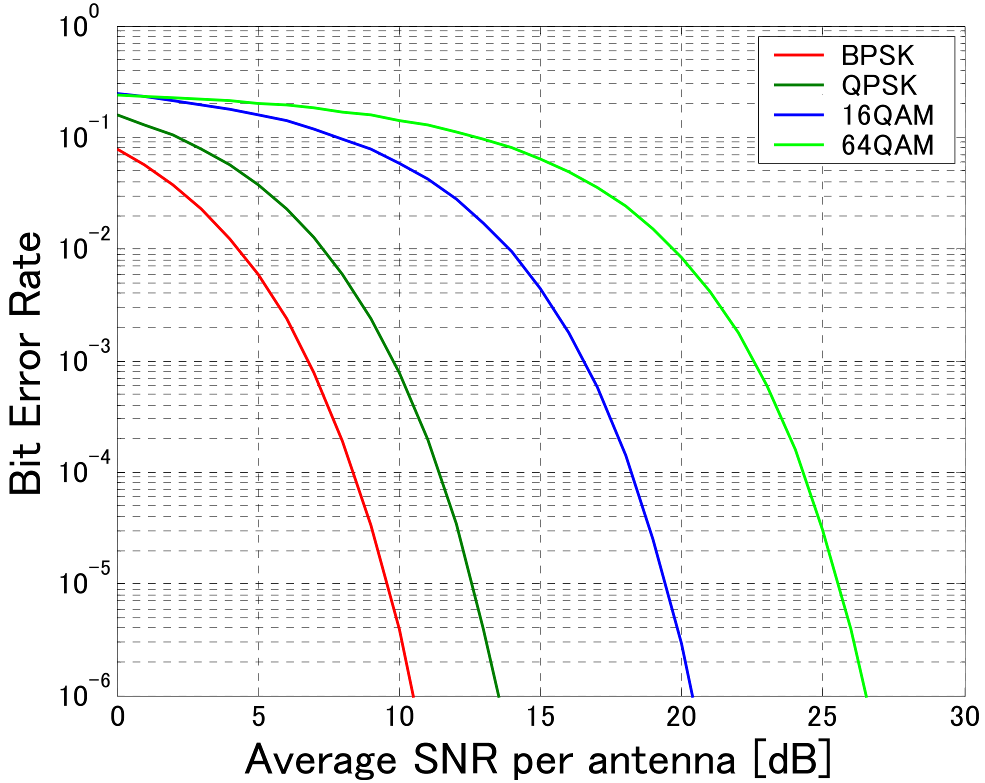
$$P_{\mathrm{eb}}(\gamma) = \frac{1}{2} \mathrm{erfc}\left(\sqrt{\frac{\gamma}{2}}\right) \qquad \gamma = \frac{P\left|\hat{h}\right|^2}{\sigma^2}$$

$$\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp\left(-z^2\right) \mathrm{d}z$$



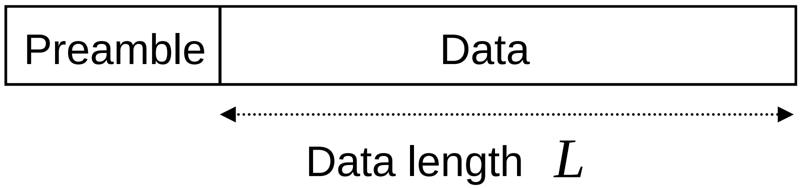
BER Performance

QAM signaling



Throughput Performance

Frame structure



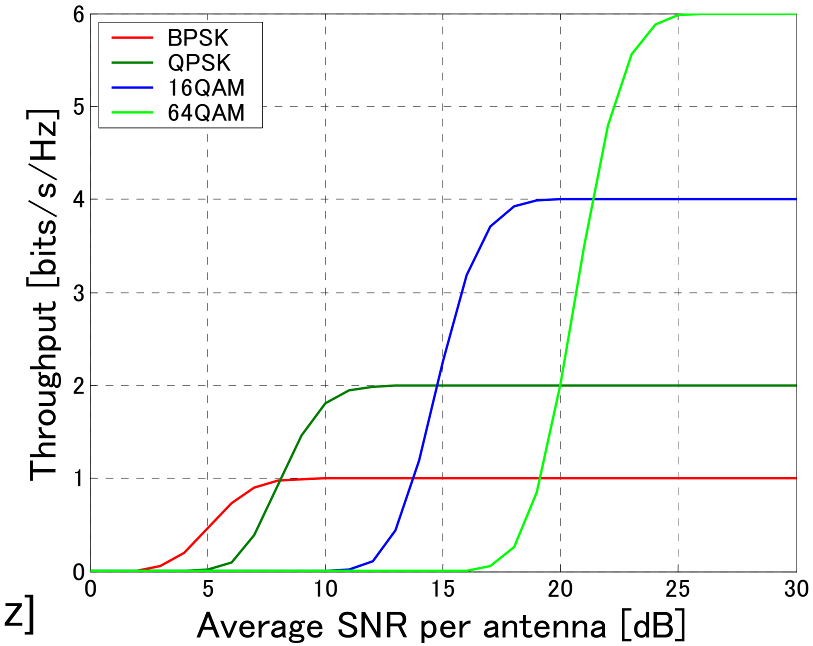
Packet error rate

$$P_{\mathrm{ep}}(\gamma) = 1 - \left(1 - P_{\mathrm{eb}}(\gamma)\right)^{L}$$

Throughput

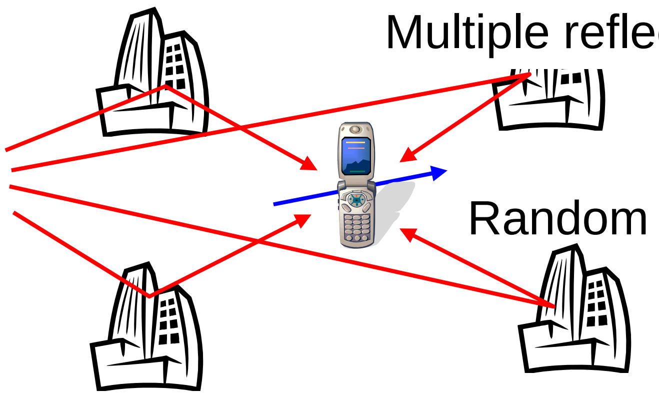
$$TP(\gamma) = \log_2 M_{\mathrm{ary}} \left(1 - P_{\mathrm{eb}}(\gamma)\right)^{L} \quad \text{[bits/s/Hz]}$$

Throughput performance

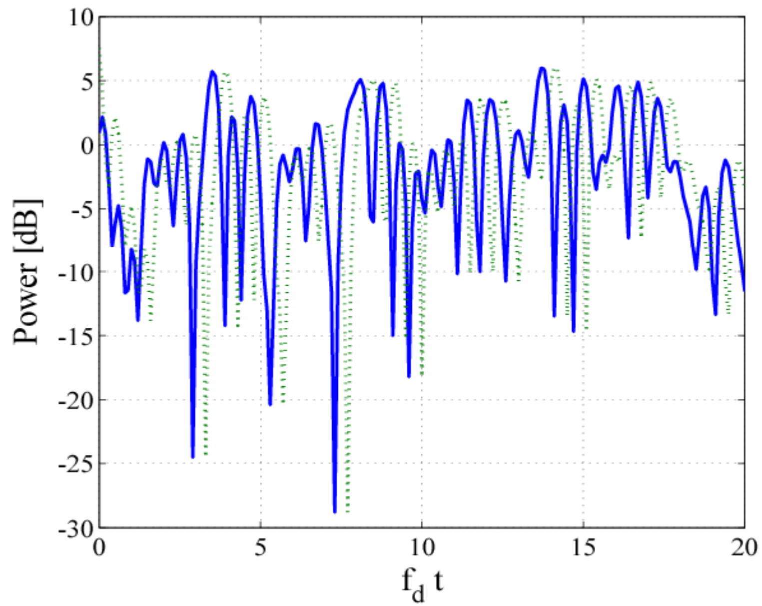


Issue #2: Multi-path fading

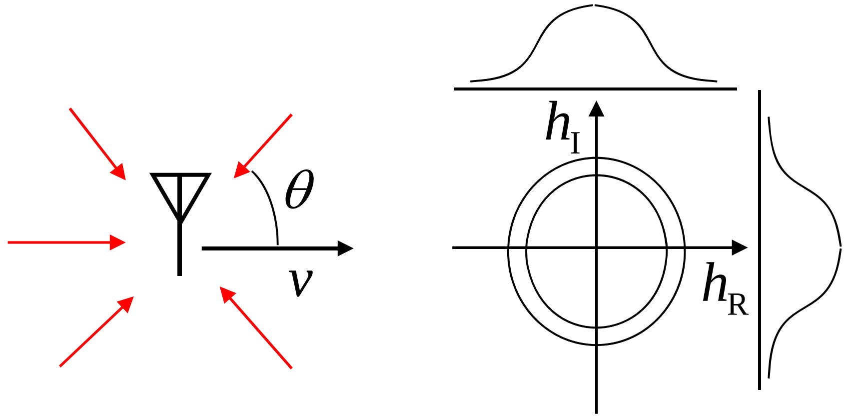
- Instantaneous SNR variation due to multi-path fading
- Deep SNR dip results in serious BER performance



Received power variation



Multi-path Rayleigh Fading



Multi-path channel

$$h(t) = \sum_l \beta_l \exp\left(j2\pi \frac{vt}{\lambda} \cos\theta \right)$$

Central limit theorem

$$f(h_R) = f(h_I) = \frac{1}{\sqrt{2\pi\alpha}} \exp\left(-\frac{x^2}{2\alpha^2} \right)$$

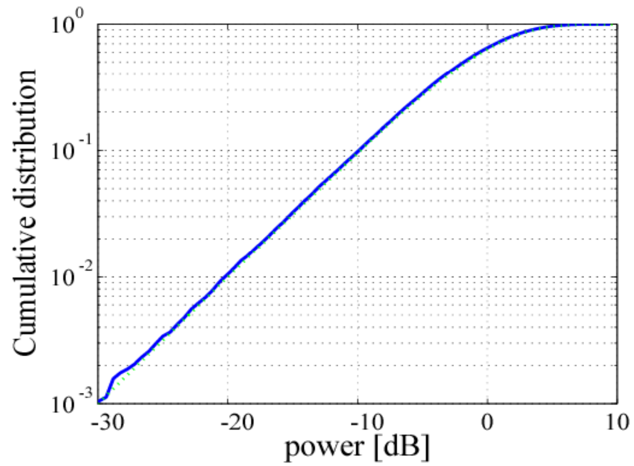
$$\alpha^2 = E[|h_R|^2] = E[|h_I|^2]$$

PDF of channel power or SNR

$$f(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}} \right)$$

$$\gamma = |h|^2 \quad \text{or} \quad \gamma = \frac{P|h|^2}{\sigma^2}$$

Cumulative distribution



BER Performance in Fading Environment

Instantaneous BER

$$P_{\mathrm{eb}}(\gamma) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{\gamma}{2}}\right) \qquad \gamma = \frac{P|h(t)|^2}{\sigma^2}$$

PDF of SNR

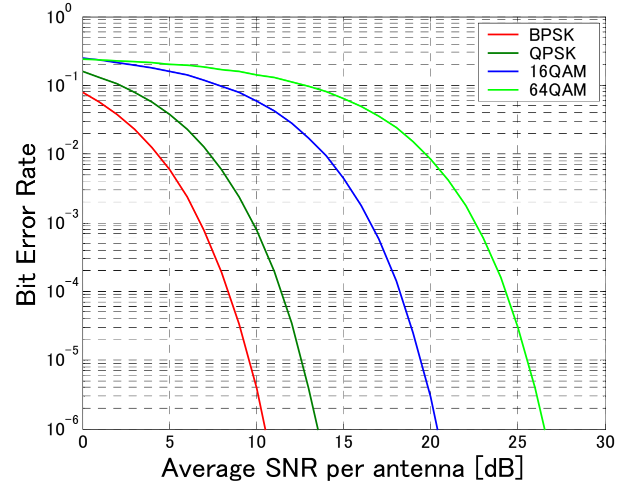
$$f(\gamma) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma}{\bar{\gamma}}\right) \qquad \bar{\gamma} = \mathrm{E}\left[\frac{P|h(t)|^2}{\sigma^2}\right]$$

Average BER & TP

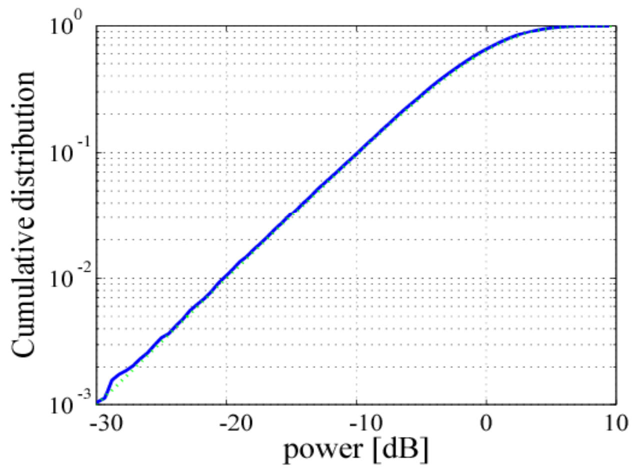
$$\overline{P}_{\mathrm{eb}}(\bar{\gamma}) = \int f(\gamma)P_{\mathrm{eb}}(\gamma)\mathrm{d}\gamma = \frac{1}{2}\left(1-\sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}}\right)$$

$$\overline{TP}(\bar{\gamma}) = \int f(\gamma)TP(\gamma)\mathrm{d}\gamma$$

BER performance

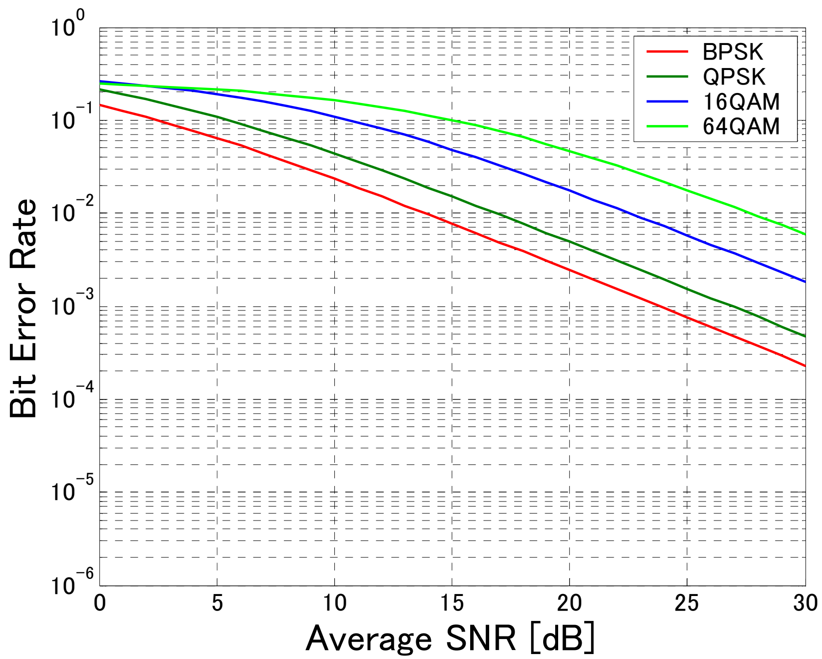


Cumulative distribution

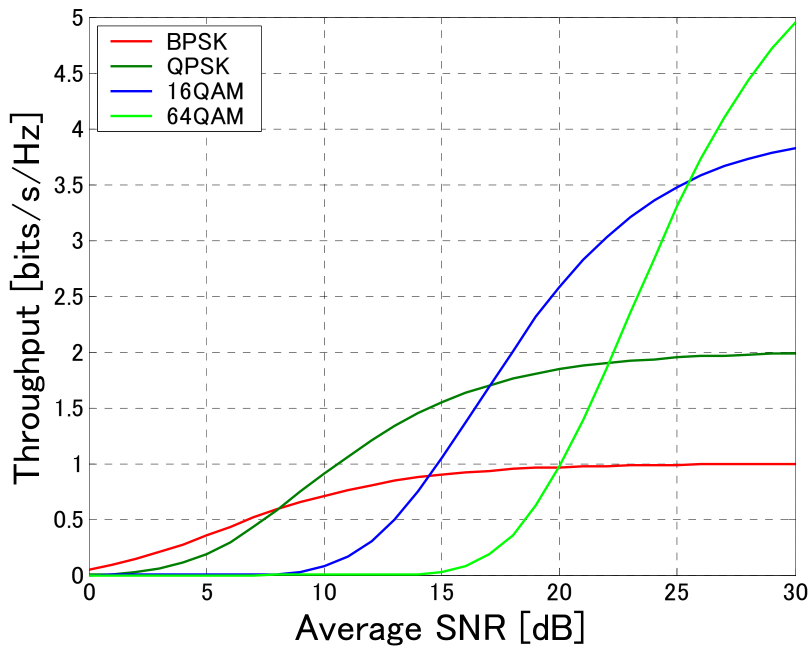


BER & Throughput Performance

Average BER performance



Average TP performance



Summary

- In narrow system
  - Channel capacity is achieved by Gaussian signaling
  - Path loss & multi-path fading are major issues in wireless communication channel
  - QAM for variable rate modulation with limited constellation
  - Derive throughput performance of QAM signal via BER calculation
  - Throughput performance degrades severely due to multi-path fading

For wideband system

OFDM for wireless broadband