Filtering: Signal Conditioning and Processing

Agenda

- Review of Filter & Signal Processing
- Linear & Non-linear Signal Processing
- Filter Design & Synthesis
- Gaussian Filter
- Nyquist Filter
- Partial Response Filter

Review of Filter & Signal Processing

1) Filter = Hardware and/or Algorithm

2) Stochastic vs. Deterministic

- Deterministic:

How to realize a filter circuit which has a desired frequency characteristics

- Linear Signal Processing
  - Noise & Interference Suppression
  - Inter-Symbol Interference Problem

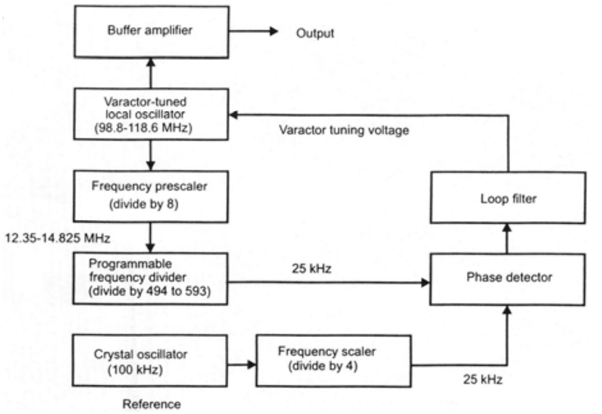
↓

(Negative) Remove → Nyquist Filter (1920's)
Nyquist Criteria
(Positive) Utilize → Partial Response Filter (1960's)
Spectrum Shaping

- Non-Linear Signal Processing
  - Envelope Detection (Diode + LPF) : No phase Information
  - PLL (Phase Comparator + LPF + VCO) : Frequency Synthesizer
  - Pre-emphasis in FM System

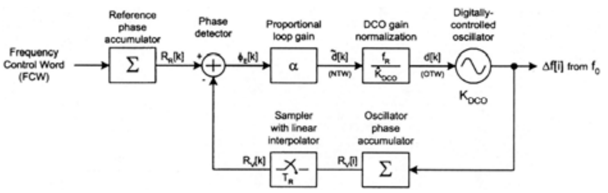
PLL (Phase Lock Loop) Principle

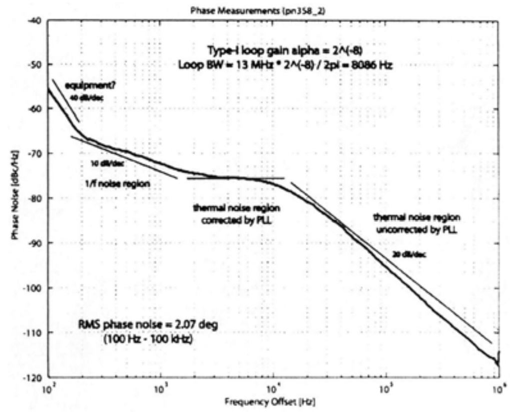
- Reference Frequency by Stable Crystal Oscillator
- Pre-scaler
- VCO (Voltage Controlled Oscillator)



Principle of ADPLL (All- Digital PLL)

- Digital Loop Filter
- Digital Controlled Oscillator
- TDC (Time-to-Digital Converter)





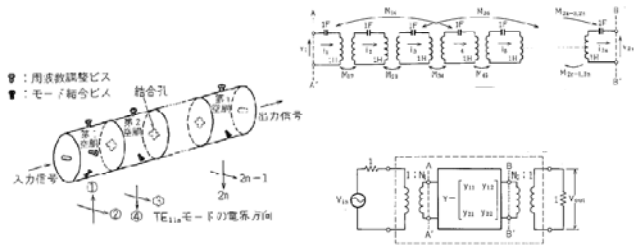
History of Filter Design

Design Theory: Butterworth (1930's)
Chebyshev (1950's),
Elliptic (1960's)

Hardware: LCR, Active, Digital, Ceramic,
SAW, SC, Waveguide

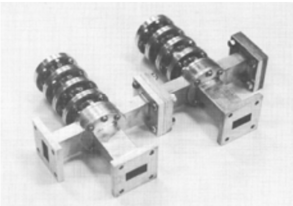
Dual-mode Filter

In-Line（Longitudinal) Type

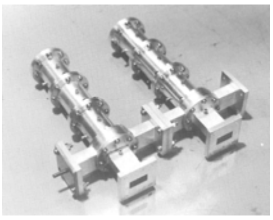


(a) Filter Structure (b) Equivalent Circuit

Multiplexer/Demultiplexer

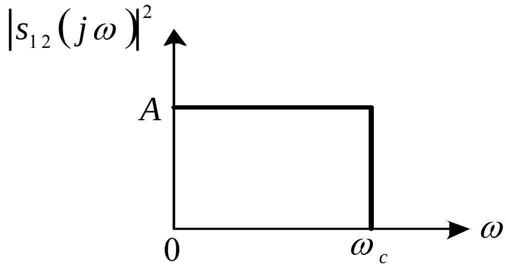


(a) Multiplexer (TE$_{111}$)

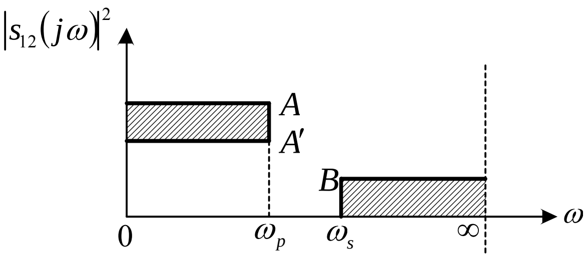


(b) Demultiplexer (TE$_{113}$)

Design Procedure



Ideal low-pass prototype response



Low-pass prototype specification

	Pass-band	Stop-band
Butterworth	Flat	Flat
Chebyshev	Equal-Ripple	Flat
Inv. Chebyshev	Flat	Equal-Ripple
Elliptic	Equal-Ripple	Equal-Ripple

- Maximally Flat (Butterworth)

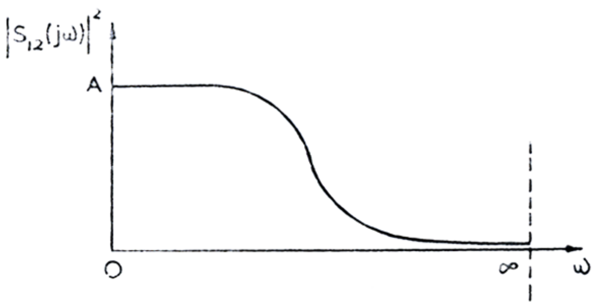
$$|S_{12}(j\omega)|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

- Equal Ripple (Chebyshev)

$$|S_{12}(j\omega)|^2 = \frac{1}{1+\varepsilon^2 T_n^{\,2}(\omega)}$$

$T_n(\omega)$: n-th order Chebyshev Polynomial

ε : Ripple Level

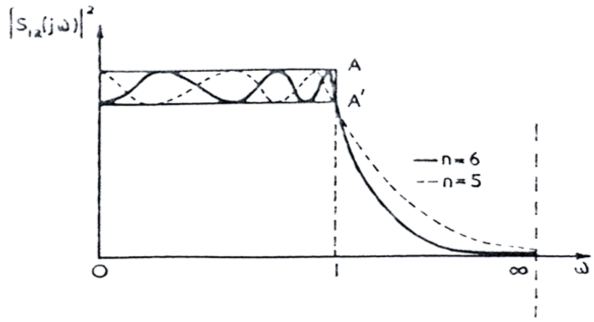


Maximally flat response

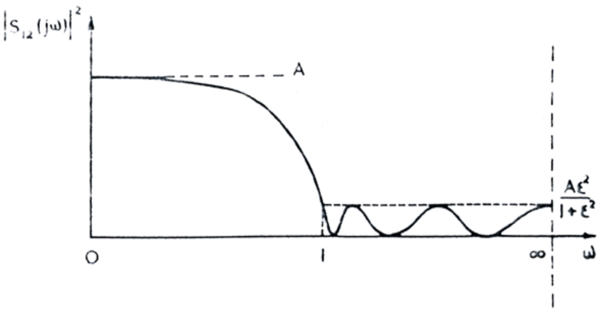
Chebyshev Polynomial

$$T_n(x) = \cos\left(n\cos^{-1}(x)\right)$$
$$T_0(x) = 1$$
$$T_1(x) = x$$
$$T_2(x) = 2x^2 - 1$$
$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$$|x| \le 1 \quad \rightarrow \quad |T_n(x)| \le 1$$
$$|x| \ge 1 \quad \rightarrow \quad |T_n(x)| \ge 1$$



Chebyshev response



Inverse Chebyshev response

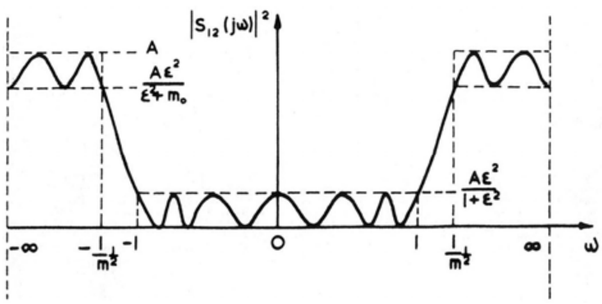
Elliptic Filter

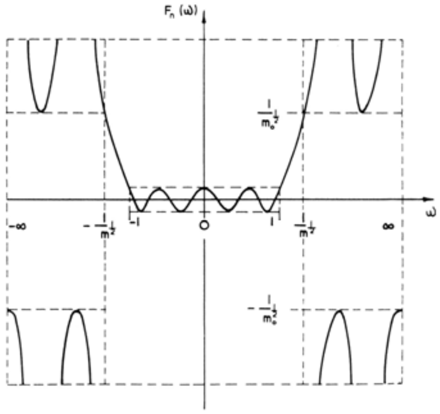
- Sharp Transition
- Equal-Ripple Characteristics both in PB and SB
- Elliptic function is used for the design of Filter Transfer Function

Equal Ripple Rational Function

$$|x| \le 1 \quad |F(x)| \le 1$$
$$|x| \ge k \quad |F(x)| \ge K$$

$$F = \mathrm{ch}(nu, K)$$
$$x = \mathrm{ch}(u, k)$$
$$\mathrm{ch}(u, k): \text{Jacobi's Elliptic Function}$$





Design Example

- LA=50dB, LR=20dB, ωs/ωp=2 :

  Elliptic Filter n=5

  Chebyshev Filter n=7

  Maximally Flat Filter n=12

Filter Synthesis

$$|S_{21}(j\omega)|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

$$|S_{11}(j\omega)|^2 = 1-|S_{21}(j\omega)|^2 = \frac{\varepsilon^2\omega^{2n}}{1+\varepsilon^2\omega^{2n}} = S_{11}(s)\,S_{11}(-s)$$

[Factorization Technique]

$s = j\omega$

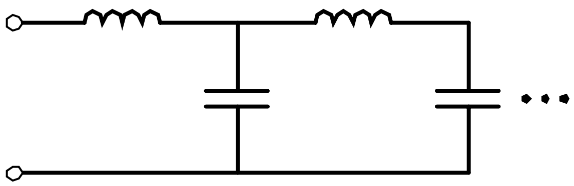
All poles of $S_{11}(x)$ exist in left half plane

$$Z_{in}(s)/Z_o = \frac{1+S_{11}(s)}{1-S_{11}(s)}$$

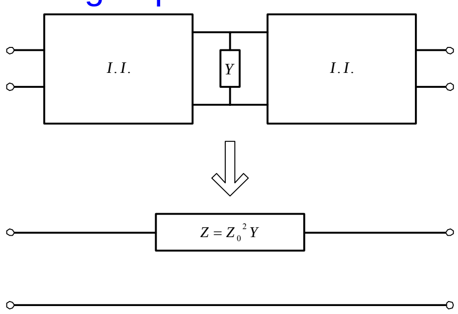
Continued Faction Technique

e.g. $$\frac{3s^4+5s^2+1}{2s^3+s} = \frac{3}{2}s + \cfrac{1}{\frac{4}{9}s + \cfrac{1}{\frac{49}{6}s + \cfrac{1}{\frac{3}{7}s}}}$$

LC-Ladder Circuit with *n*-elements

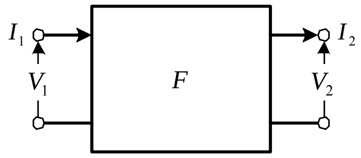


Parallel to Serial Transform by using Impedance Inverters

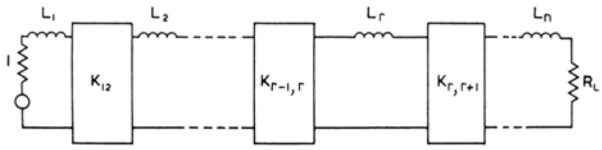


Impedance Inverters

- $\lambda/4$ Transmission Lines (Passive)
- Operational Amplifiers (Active)



$$F = \begin{bmatrix} 0 & jZ_0 \\ j\frac{1}{Z_0} & 0 \end{bmatrix}$$

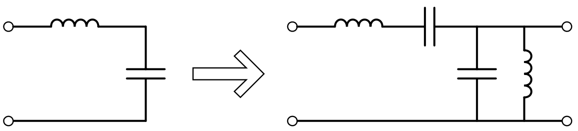


Frequency Transformation

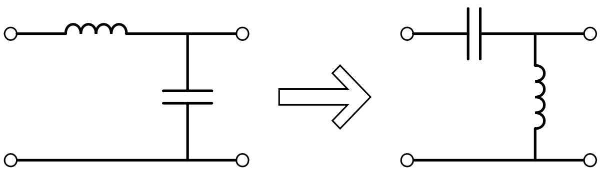
$$\underset{\text{LPF}}{S} \rightarrow \underset{\text{BPF}}{\left(\frac{S}{\omega_0} + \frac{\omega_0}{S}\right) \Big/ \Delta\omega}$$

ω_0 : center frequency

$\Delta\omega$: band width



$$\underset{\text{LPF}}{S} \rightarrow \underset{\text{HPF}}{\frac{1}{S}}$$



- Stochastics:

  How to select signal and noise

  Estimation and Prediction Theory

  – **Gauss** (1795):

  Least Square Mean Concept →
  Astronomy (Prediction of Satellite Orbit),
  → Gauss Distribution

– Wiener and Kolmogorov (1940's):

Linear Prediction for Stationary Stochastic Process using 2-nd order statistics (Correlation Matrix)

Generalized Harmonic Analysis
(Stochastic Theory + Fourier Analysis)

Wiener-Hopf Integral Equation
(Semi-infinite Singular Boundary Value Problem)
Communication + Control ⇒ Cybernetics

Wiener Filter based on Correlation Function

$$x(t) = s(t) + n(t)$$
$$y(t) = \int_0^{\infty} x(t-\tau)\, h(\tau)\, d\tau$$
$$\text{Min } E\left[\left| y(t) - s(t) \right|^2 \right]$$

$\rightarrow$ Wiener-Hopf Equation for $h(\tau)$

$$\int_0^{\infty} \left[R_{ss}(\tau - \tau') + R_{nn}(\tau - \tau') \right] h(\tau')\, d\tau' = R_{ss}(\tau)$$

$R_{ss} = Signal - Auto - Correlation$

$R_{nn} = Noise - Auto - Correlation$

– Kalman (1960's):

Non-stationary Process Prediction by using Kalman algorithm

State Space Approach, Linear System Theory, Control Theory, Controlability, Observability, Optimum Regulator, Optimum Filter, Stability, etc.

– Godard (1974):

Learning Theory, Adaptive Equalizer for Wired Transmission
Unknown state variables = Transmission Characteristics

– RLS (Recursive LSM) (1990'):

$\rightarrow$ Inter-symbol Interefence Canceller, Multi-user Detection for Wireless Communication

Frequency Characterstics and Impulse Response

- Transfer Function of Linear Filter:

[Linearity + Time-Invariance]

$\rightarrow$ Impulse response function $h(t)$ is enough for system description.

Output signal $y(t)$ is given by a convolution of Input signal $x(t)$ and Impulse response function $h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

Linear System

- Linear Time-Invariant : Impulse Function (filter)
- Linear Periodic-variant : Multi-rate System (mixer)
- Application : Band aggregation, Rate Conversion, Frequency Conversion

→ Exponential time function

$\exp(at) =$ eigen-function

→ Fourier Analysis

$$Y(f) = X(f)H(f)$$

→ Transfer Function *H*(*f*)

$$H(f) = \int_{-\infty}^{\infty} h(t)\exp(-j2\pi ft)\,dt$$

$\lvert H(f)\rvert$: Amplitude Characteristics
$\angle H(f)$: Phase Characteristics
$-\partial\angle H(f)/\partial f$: Delay - time Charactersticss

- Ideal Filter and Physical Realizability: Causality
  - Ideal Low Pass Filter: Flat Amplitude, Sharp Cutoff, Linear Phase

$$H(f) = A \cdot \mathrm{rect}\left(\frac{f}{2W}\right)\exp(-j2\pi f\tau)$$

where

$$\mathrm{rect}(x) = \begin{cases} 1 & \text{for } \lvert x\rvert \le \frac{1}{2} \\ 0 & \text{for } \lvert x\rvert > \frac{1}{2} \end{cases}$$

W: Bandwidth τ : delay time

– Impulse Response : **sinc** function, equal-distance zero-crossing

$$h(t) = 2AW\frac{\sin[2\pi W(t-\tau)]}{2\pi W(t-\tau)}$$

⇒ Non-causal !

– **Uncertainty Principle**: $\triangle f \cdot \triangle t \ge 1/4\pi$

Impulse function $(\triangle t \to 0)$ has flat spectrum $(\triangle f \to \infty)$ Sinusoidal function $(\triangle f \to 0)$ is widely spread $(\triangle t \to \infty)$

(cf. In Quantum Physics, $\triangle E \cdot \triangle t \ge h/4\pi$, *E*: Energy, *h*: Planck constant)

Gaussian function is optimum with respect to the product of time spread and frequency spread; $\triangle t \cdot \triangle f$.

– Finiteness of system:

→ Transfer function is a Rational function of *f*

– Causality ⇔ $h(t) = 0$ for $t < 0$

⇔ Wiener-Palay Condition

$$\int_{-\infty}^{\infty} \frac{\lvert H(f)\rvert^2}{1 + f^2}\,df < \infty$$

→ Real part, $R(f)$

← Hilbert Transform → Imaginary Part, $X(f)$

$$R(f) = -\frac{2}{\pi}\int_0^{\infty} \frac{u}{u^2 - f^2} X(u)\,du + R(\infty)$$

$$X(f) = \frac{2}{\pi}\int_0^{\infty} \frac{f}{u^2 - f^2} R(u)\,du$$

– For Minimum Phase System:

Amplitude Characteristics $|H(f)|$ determines Phase Characteristics $\angle H(f)$

But when delayed waves are larger than the first arriving wave in the multi-path environment, it becomes Non-minimum Phase.

Gaussian Filter

- Transfer Function: $H(f) = \exp(-(f/f_0)^2)$
- Impulse Response: $h(t) = f_0\sqrt{\pi}\exp(-(\pi f_0 t)^2)$
- Step Response: $s(t) = 1 - \frac{1}{2}\mathrm{erfc}(\pi f_0 t)$
  where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-u^2)\,du$: complementary error function
- Mono pulse (T) response:
  $g(t) = s(t) - s(t-T)$
  $= \frac{1}{2}[\mathrm{erfc}(\pi f_0 t(\frac{t}{T} - 1)) - \mathrm{erfc}(\pi f_0 t(\frac{t}{T}))]$

- For Random pulse sequence $\{a_n\}$,

  $g_r(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT)$

- Eye pattern is determined by $f_0 T$
  $f_0 T \rightarrow$ large, Good eye pattern
- Bessel Filter of 5-th order ≈ Gaussian Filter (Maximally Flat in delay characteristics)

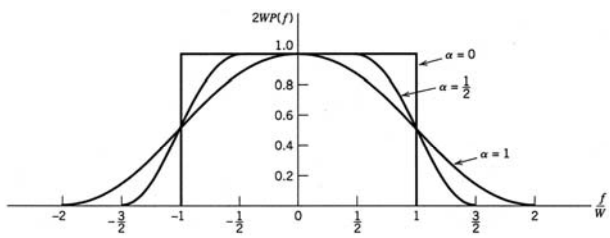
Nyquist Filter

- No interference condition at sampling time
- Roll-off Filter

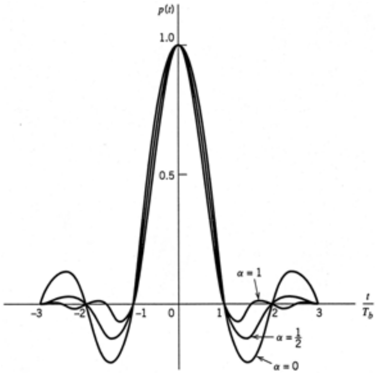
$$R(f) = \begin{cases} 1 & \text{for } 0 \le |fT| \le \frac{1-\alpha}{2} \\ \frac{1}{2}\left[1 - \sin\left(\frac{\pi}{2\alpha}(2fT - 1)\right)\right] & \text{for } \frac{1-\alpha}{2} \le |fT| \le \frac{1+\alpha}{2} \\ 0 & \text{for } \frac{1+\alpha}{2} \le |fT| \end{cases}$$

- Roll-off Response

$$r(t) = \frac{\sin(\pi t/T)}{\pi t/T}\,\frac{\cos(\alpha t/T)}{\alpha t/T} \qquad \alpha : \text{roll-off factor} (0 \le \alpha \le 1)$$



Frequency response



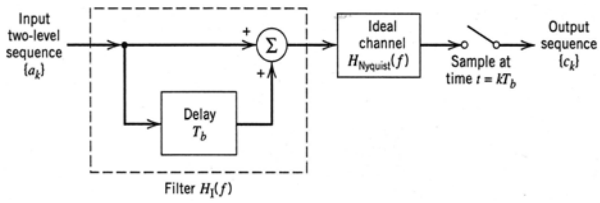
Time response

Partial Response Filter

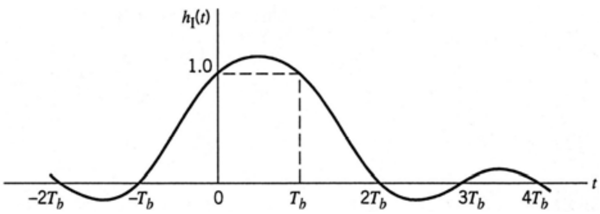
Controlled Inter-symbol Interference

- Class of Partial Response Filter
  Partial Response Filter: Binary sequence → Multi-valued sequence → Spectrum Shaping
  Partial Response Filter:
  FIR Filter with Integer coefficient
- Similar concept: Thomlinson-Harashima Precoding in Dirty-paper Coding

Class	c_0, c_1, c_2, c_3, c_4	$H(f), \lvert f \rvert \le \frac{1}{2T}$	Impulse response
1	1, 1	$2T \cos \pi f T$	$\frac{4}{\pi} \frac{\cos(\pi T)}{1-4(t/T)^2}$
	1, −1	$-2T \sin \pi f T$	$\frac{8t/T}{\pi} \frac{\cos(\pi T)}{4(t/T)^2-1}$
2	1, 2, 1	$4T \cos^2 \pi f T$	$\frac{2}{\pi t/T} \frac{\sin(\pi T)}{1-(t/T)^2}$
3	2, 1, −1	$T(1+\cos 2\pi f T + 3j \sin 2\pi f T)$	$\frac{3t/T-1}{\pi t/T} \frac{\sin(\pi T)}{(t/T)^2-1}$
4	1, 0, −1	$j2T \sin \pi f T$	$\frac{2}{\pi} \frac{\sin(\pi T)}{(t/T)^2-1}$
5	1, 0, 2, 0, 1	$-4T \sin^2 2\pi f T$	$\frac{8}{\pi t/T} \frac{\sin(\pi t/T)}{(t/T)^2-4}$



Duo-binary signaling scheme



Impulse response of the duo-binary conversion filter

- Error Propagation and Pre-coder
  Full response system : No error propagation
  Partial response system: Error propagation
  Pre-coder is necessary for prevention of error propagation

Pre-coder in TX

Source information $\{a_n\} \rightarrow$

Pre-coded information $\{s_n\}$

Digital calculation (Logical calculation)

$$s_n = \sum_{i=1}^{k} c_i \cdot s_{n-i} + a_n \quad (\bmod 2)$$

Pre-coded information $\{s_n\} \rightarrow$

Partial response information $\{g_n\}$

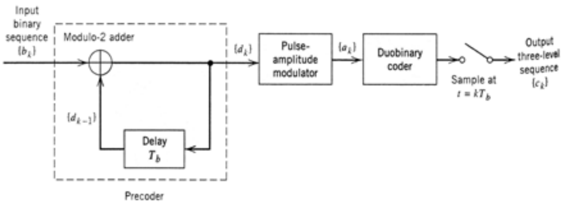
Analog calculation (Physical calculation)

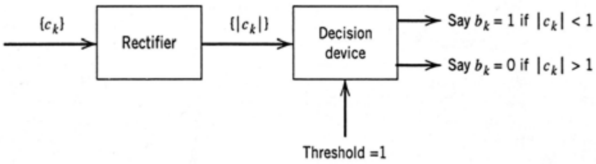
$$g_n = \sum_{i=0}^{k} c_i \cdot s_{n-i}$$

Decoding in RX

$$a_n = g_n \quad (\bmod 2)$$

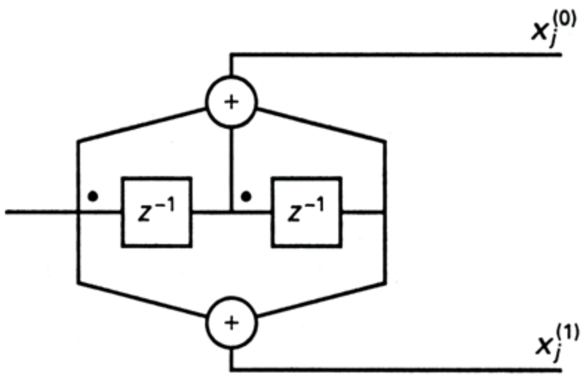
Apparently, error propagation is eliminated.



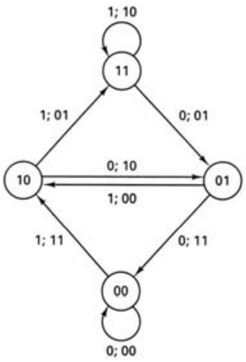


PR-VA (Partial Response & Viterbi Algorithm) is a most powerful recording method in magnetic recording.

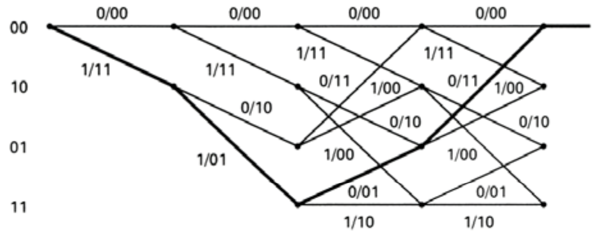
Convolutional Code also utilizes a partial response in the codeword



$R = \frac{1}{2}$, nonsystematic, $m = 2$ encoder



State transition diagram for $R = \frac{1}{2}$, $m = 2$ code



Trellis for four levels, $R = \frac{1}{2}$, $v = 2$ code.

Heavy line denotes route for message $\mathbf{u} = (11000\ldots)$.