# Filtering: Signal Conditioning and Processing

## Agenda

- Review of Filter & Signal Processing
- Linear & Non-linear Signal Processing
- Filter Design & Synthesis
- Gaussian Filter
- Nyquist Filter
- Partial Response Filter



## Review of Filter & Signal Processing

1) Filter = Hardware and/or Algorithm

2) Stochastic vs. Deterministic



- Deterministic:

How to realize a filter circuit which has a desired frequency characteristics

- Linear Signal Processing
  - Noise & Interference Suppression
  - Inter-Symbol Interference Problem

↓

(Negative) Remove → Nyquist Filter (1920's)
Nyquist Criteria
(Positive) Utilize → Partial Response Filter (1960's)
Spectrum Shaping

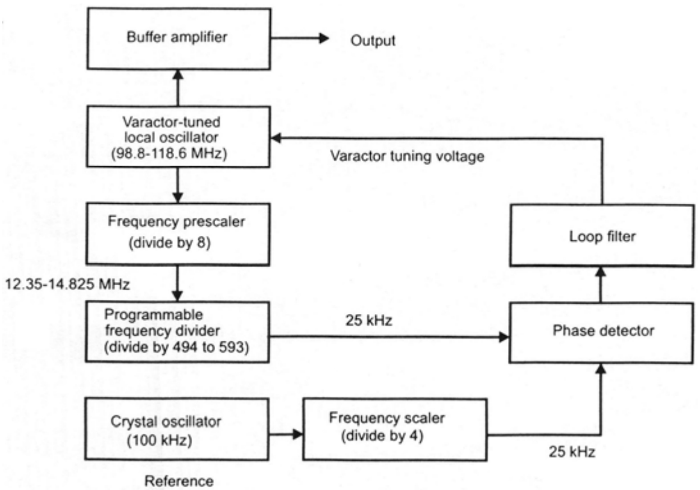
– Non-Linear Signal Processing
- Envelope Detection (Diode + LPF) : No phase Information
- PLL (Phase Comparator + LPF + VCO) : Frequency Synthesizer
- Pre-emphasis in FM System



# PLL (Phase Lock Loop) Principle

- Reference Frequency by Stable Crystal Oscillator
- Pre-scaler
- VCO ( Voltage Controlled Oscillator )

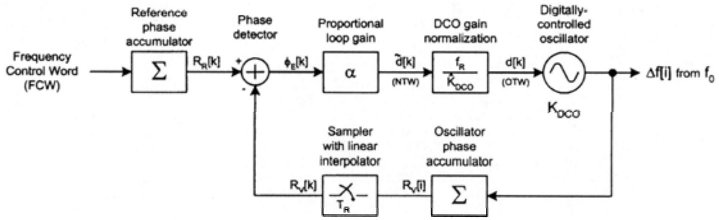




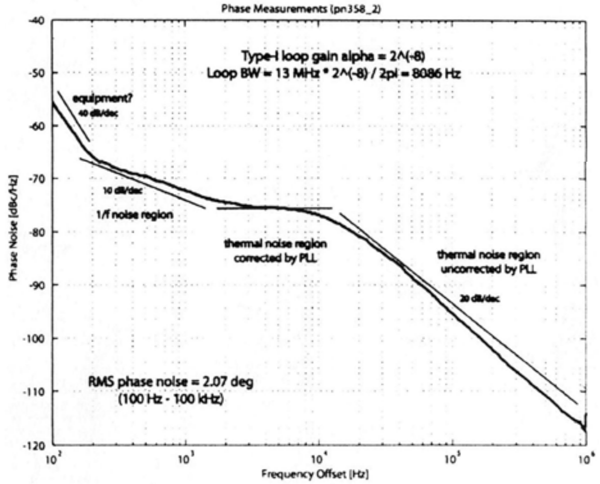



# Principle of ADPLL ( All- Digital PLL)

- Digital Loop Filter
- Digital Controlled Oscillator
- TDC ( Time-to-Digital Converter )








## History of Filter Design

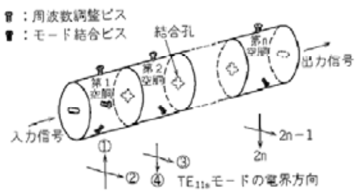
Design Theory: Butterworth (1930's)
Chebyshev (1950's),
Elliptic (1960's)

Hardware: LCR, Active, Digital, Ceramic,
SAW, SC, Waveguide

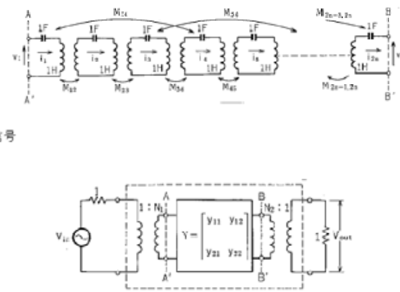


## Dual-mode Filter

In-Line（Longitudinal) Type

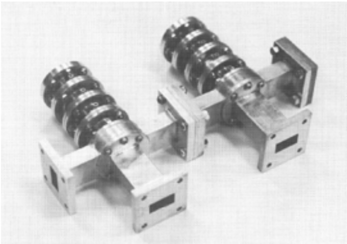



(a) Filter Structure

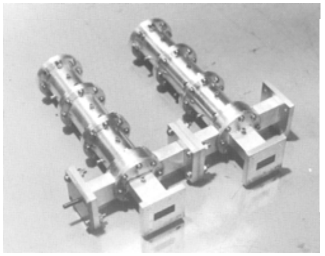


(b) Equivalent Circuit

# Multiplexer/Demultiplexer



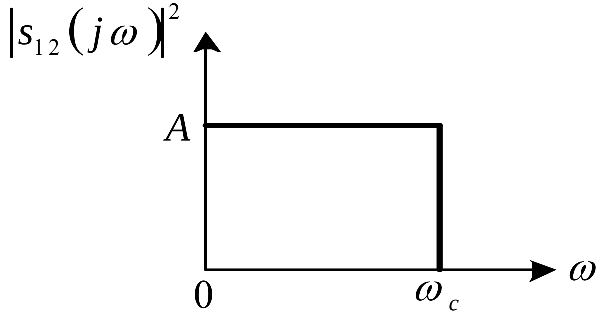
(a) Multiplexer（$TE_{111}$）



(b) Demultiplexer（$TE_{113}$）

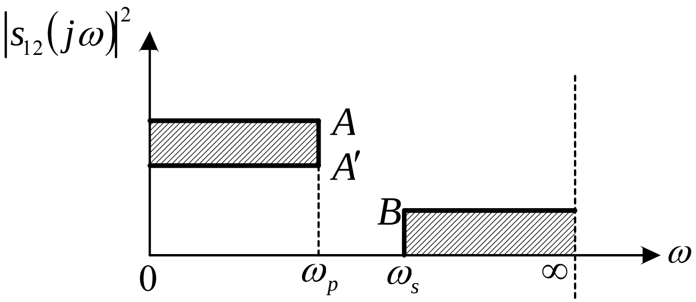


## Design Procedure




Ideal low-pass prototype response






Low-pass prototype specification



| | Pass-band | Stop-band |
|---|---|---|
| Butterworth | Flat | Flat |
| Chebyshev | Equal-Ripple | Flat |
| Inv. Chebyshev | Flat | Equal-Ripple |
| Elliptic | Equal-Ripple | Equal-Ripple |

- Maximally Flat (Butterworth)

$$|S_{12}(j\omega)|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

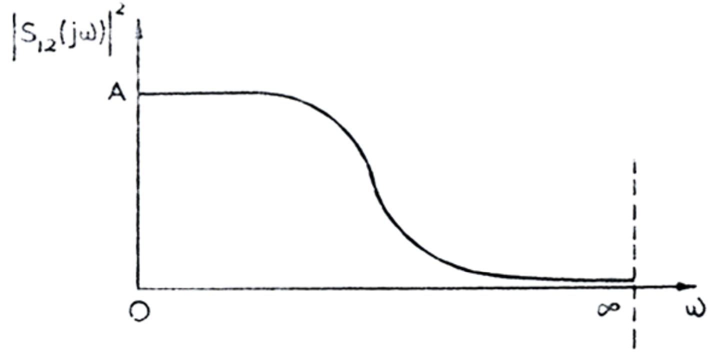
- Equal Ripple (Chebyshev)

$$|S_{12}(j\omega)|^2 = \frac{1}{1+\varepsilon^2 T_n{}^2(\omega)}$$

$T_n(\omega)$: $n$ - th order Chebyshev Polynomial

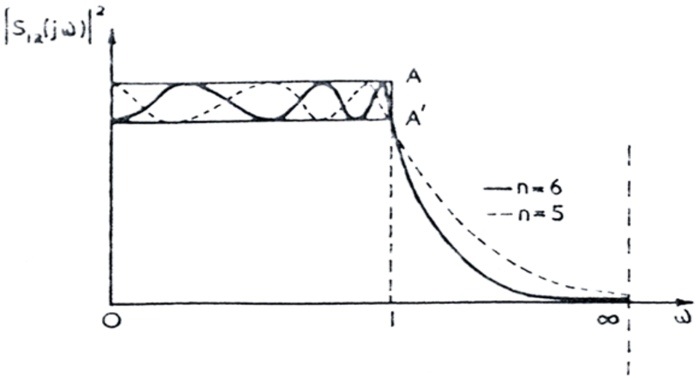
$\varepsilon$ : Ripple Level





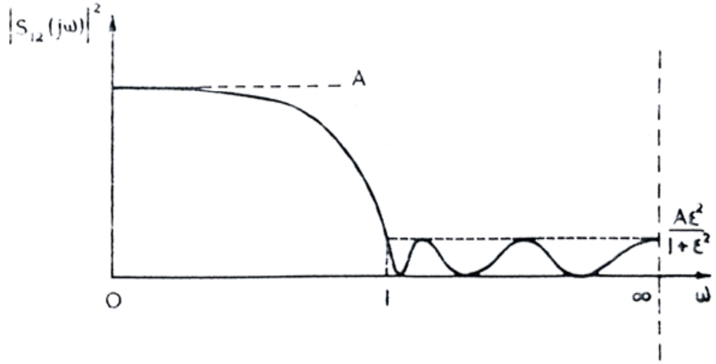

Maximally flat response






Chebyshev response



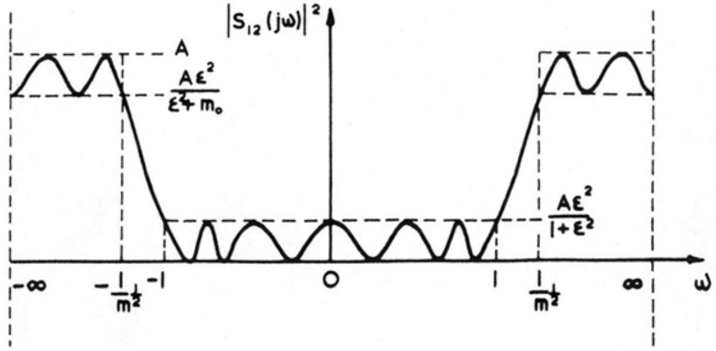



Inverse Chebyshev response

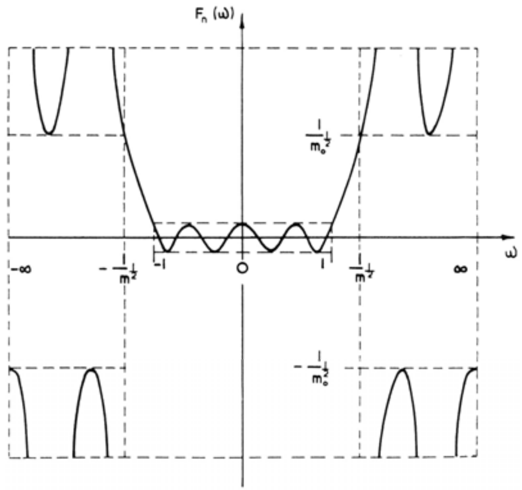
## Elliptic Filter

- Sharp Transition
- Equal-Ripple Characteristics both in PB and SB
- Elliptic function is used for the design of Filter Transfer Function










## Design Example

- LA=50dB, LR=20dB, ωs/ωp=2 :

  Elliptic Filter n=5

  Chebyshev Filter n=7

  Maximally Flat Filter n=12

# Filter Synthesis

$$\left|S_{21}(j\omega)\right|^2 = \frac{1}{1+\varepsilon^2\omega^{2n}}$$

$$\left|S_{11}(j\omega)\right|^2 = 1-\left|S_{21}(j\omega)\right|^2 = \frac{\varepsilon^2\omega^{2n}}{1+\varepsilon^2\omega^{2n}} = S_{11}(s)\,S_{11}(-s)$$

[ Factorization Technique]

$$s = j\omega$$



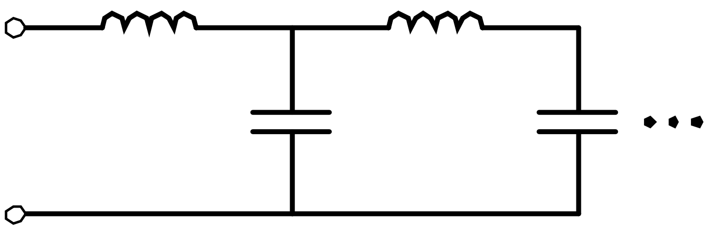
$$Z_{\text{in}}(s) = \frac{1-S_{11}(s)}{1+S_{11}(s)}$$

Continued Faction Technique

e.g. $$\frac{3s^4+5s^2+1}{2s^3+s} = \frac{3}{2}s + \cfrac{1}{\frac{4}{9}s + \cfrac{1}{\frac{49}{6}s + \cfrac{1}{\frac{3}{7}s}}}$$

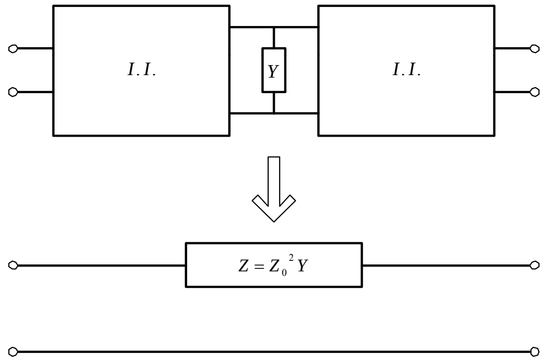


# LC-Ladder Circuit with *n*-elements



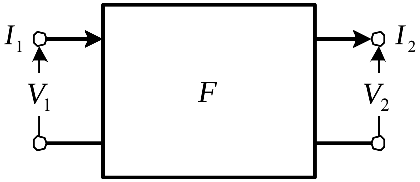


# Parallel to Serial Transform by using Impedance Inverters



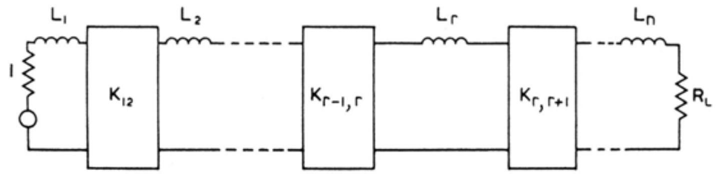
# Impedance Inverters

- $\lambda/4$ Transmission Lines (Passive)
- Operational Amplifiers (Active)




$$F = \begin{bmatrix} 0 & jZ_0 \\ j\frac{1}{Z_0} & 0 \end{bmatrix}$$



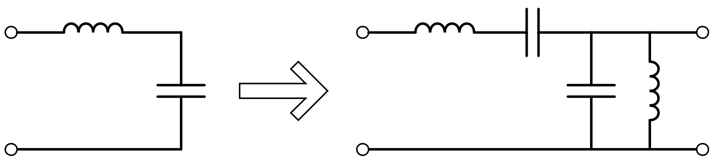





# Frequency Transformation

$$S \rightarrow \frac{S}{\omega_0} + \frac{\omega_0}{S}$$

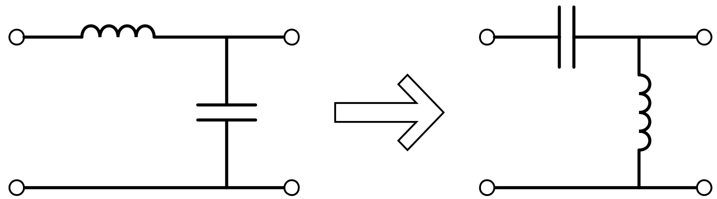
LPF BPF





$$S \rightarrow \frac{1}{S}$$

LPF HPF



- Stochastics:

How to select signal and noise

*Estimation and Prediction Theory*

– **Gauss** (1795):
Least Square Mean Concept →
Astronomy (Prediction of Satellite Orbit),
→ Gauss Distribution



– Wiener and Kolmogorov (1940's):

Linear Prediction for Stationary Stochastic Process using 2-nd order stastitics (Correlation Matrix)

Generalized Harmonic Analysis
(Stochastic Theory + Fourier Analysis)

Wiener-Hopf Integral Equation
(Semi-infinite Singular Boundary Value Problem)
Communication + Control ⇒ Cybernetics



## Wiener Filter based on Correlation Function

$$x(t) = s(t) + n(t)$$

$$y(t) = \int_0^\infty x(t-\tau)\, h(\tau)\, d\tau$$

$$\text{Min } E\left[\left|y(t) - s(t)\right|^2\right]$$

→ Wiener-Hopf Equation for $h(\tau)$

$$\int_0^\infty \left[R_{ss}(\tau-\tau') + R_{nn}(\tau-\tau')\right] h(\tau')\, d\tau' = R_{ss}(\tau)$$

$R_{ss} = Signal - Auto - Correlation$

$R_{nn} = Noise - Auto - Correlation$



– Kalman (1960's):

Non-stationary Process Prediction by using Kalman algorithm

State Space Approach, Linear System Theory,
Control Theory, Controlability, Observability,
Optimum Regulator, Optimum Filter, Stability, etc.

– Godard (1974):

Learning Theory, Adaptive Equalizer for Wired Transmission
Unknown state variables = Transmission Characteristics

– RLS (Recursive LSM) (1990'):

→ Inter-symbol Interefence Canceller, Multi-user Detection for Wireless Communication



# Frequency Characterstics and Impulse Response

- Transfer Function of Linear Filter:

[Linearity + Time-Invariance]

→ Impulse response function $h(t)$ is enough for system description.



Output signal $y(t)$ is given by a convolution of Input signal $x(t)$ and Impulse response function $h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau$$



# Linear System

- Linear Time-Invariant : Impulse Function
- Linear Periodic-variant : Multi-rate System
- Application : Band aggregation, Rate Transform

→ Exponential time function

$\exp(at)$ = eigen-function

→ Fourier Analysis

$$Y(f) = X(f)H(f)$$



→ Transfer Function $H(f)$

$$H(f) = \int_{-\infty}^{\infty} h(t)\exp(-j2\pi ft)\,dt$$

| | |
|---|---|
| $\lvert H(f)\rvert$ | : Amplitude Characteristics |
| $\angle H(f)$ | : Phase Characteristics |
| $-\partial\angle H(f)/\partial f$ | : Delay - time Characterstics |



- Ideal Filter and Physical Realizability: Causality
  - Ideal Low Pass Filter: Flat Amplitude, Sharp Cutoff, Linear Phase

$$H(f) = A\cdot \mathrm{rect}\left(\frac{f}{2W}\right)\exp(-j2\pi f\tau)$$

where

$$\mathrm{rect}(x) = \begin{cases} 1 & \text{for} \quad |x| \le \frac{1}{2} \\ 0 & \text{for} \quad |x| > \frac{1}{2} \end{cases}$$

$W$: Bandwidth $\tau$ : delay time



– Impulse Response : **sinc** function, equal-distance zero-crossing

$$h(t) = 2AW\,\frac{\sin[2\pi W(t-\tau)]}{2\pi W(t-\tau)}$$

⇒ Non-causal !

– **Uncertainty Principle**: $\triangle f \cdot \triangle t \geq 1/4\pi$

Impulse function $(\triangle t \to 0)$ has flat spectrum $(\triangle f \to \infty)$ Sinusoidal function $(\triangle f \to 0)$ is widely spread $(\triangle t \to \infty)$
(cf. In Quantum Physics, $\triangle E \cdot \triangle t \geq h/4\pi$, $E$: Energy, $h$: Planck constant)
Gaussian function is optimum with respect to the product of time spread and frequency spread; $\triangle t \cdot \triangle f$.



– Finiteness of system:
→ Transfer function is a Rational function of $f$

– Causality ⇔ $h(t) = 0$ for $t < 0$
⇔ Wiener-Palay Condition

$$\int_{-\infty}^{\infty} \frac{|H(f)|^2}{1 + f^2} df < \infty$$



→ Real part, $R(f)$
← Hilbert Transform → Imaginary Part, $X(f)$

$$R(f) = -\frac{2}{\pi}\int_0^{\infty} \frac{u}{u^2 - f^2} X(u)\, du + R(\infty)$$

$$X(f) = \frac{2}{\pi}\int_0^{\infty} \frac{f}{u^2 - f^2} R(u)\, du$$



– For Minimum Phase System:

Amplitude Characteristics $|H(f)|$ determines Phase Characteristics $\angle H(f)$
But when delayed waves are larger than the first arriving wave in the multi-path environment, it becomes Non-minimum Phase.

# Gaussian Filter

- Transfer Function: $H(f) = \exp(-(f/f_0)^2)$
- Impulse Response: $h(t) = f_0\sqrt{\pi}\exp(-(\pi f_0 t)^2)$
- Step Response: $s(t) = 1 - \frac{1}{2}\mathrm{erfc}(\pi f_0 t)$
  where $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-u^2)\,du$ :
  complementary error function
- Mono pulse ($T$) response:

$$g(t) = s(t) - s(t-T)$$
$$= \frac{1}{2}[\mathrm{erfc}(\pi f_0 t(\tfrac{t}{T}-1)) - \mathrm{erfc}(\pi f_0 t(\tfrac{t}{T}))]$$



- For Random pulse sequence $\{a_n\}$,

$$g_r(t) = \sum_{n=-\infty}^{\infty} a_n g(t-nT)$$

- Eye pattern is determined by $f_0T$
  $f_0T \to$ large, Good eye pattern
- Bessel Filter of 5-th order ≈Gaussian Filter
  (Maximally Flat in delay characteristics)



# Nyquist Filter

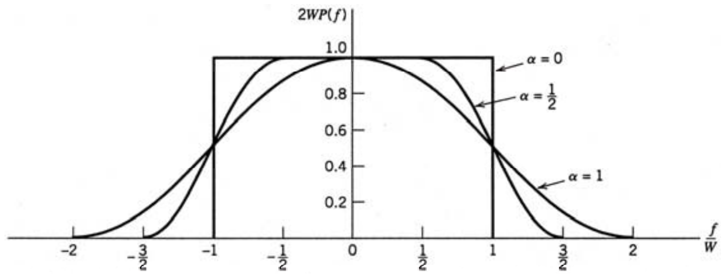
- No interference condition at sampling time
- Roll-off Filter

$$R(f) = \begin{cases} 1 & \text{for} \quad 0 \le |fT| \le \frac{1-\alpha}{2} \\ \frac{1}{2}\left[1 - \sin\left(\frac{\pi}{2\alpha}(2fT-1)\right)\right] & \text{for} \quad \frac{1-\alpha}{2} \le |fT| \le \frac{1+\alpha}{2} \\ 0 & \text{for} \quad \frac{1+\alpha}{2} \le |fT| \end{cases}$$

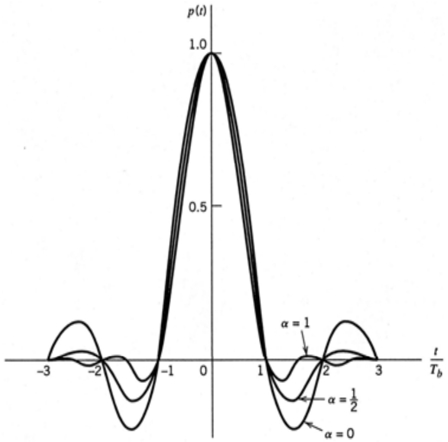
- Roll-off Response

$$r(t) = \frac{\sin(\pi t/T)}{\pi t/T}\frac{\cos(\alpha t/T)}{\alpha t/T} \qquad \alpha: \text{roll-off factor}(0 \le \alpha \le 1)$$






Frequency response




Time response



# Partial Response Filter

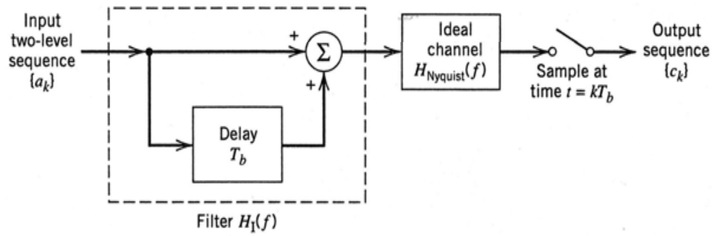
## Controlled Interference

- Class of Partial Response Filter
  Partial Response Filter: Binary sequence → Multi-valued sequence → Spectrum Shaping
  Partial Response Filter:
  FIR Filter with Integer coefficient
- Similar concept: Thomlinson-Harashima Precoding in Dirty-paper Coding

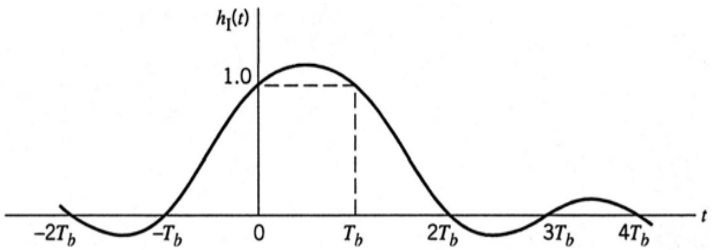


| Class | $c_0, c_1, c_2, c_3, c_4$ | $H(f), \lvert f \rvert \le \frac{1}{2T}$ | Impulse response |
|---|---|---|---|
| 1 | 1, 1 | $2T\cos \pi f T$ | $\frac{4}{\pi}\frac{\cos(\pi t T)}{1-4(t/T)^2}$ |
| | 1, −1 | $-2T\sin \pi f T$ | $\frac{8t/T}{\pi}\frac{\cos(\pi t T)}{4(t/T)^2-1}$ |
| 2 | 1, 2, 1 | $4T\cos^2 \pi f T$ | $\frac{2}{\pi t/T}\frac{\sin(\pi t T)}{1-(t/T)^2}$ |
| 3 | 2, 1, −1 | $T(1+\cos 2\pi f T + 3j\sin 2\pi f T)$ | $\frac{3t/T-1}{\pi t/T}\frac{\sin(\pi t T)}{(t/T)^2-1}$ |
| 4 | 1, 0, −1 | $j2T\sin \pi f T$ | $\frac{2}{\pi}\frac{\sin(\pi t T)}{(t/T)^2-1}$ |
| 5 | 1, 0, 2, 0, 1 | $-4T\sin^2 2\pi f T$ | $\frac{8}{\pi t/T}\frac{\sin(\pi t/T)}{(t/T)^2-4}$ |






Duo-binary signaling scheme




Impulse response of the duo-binary conversion filter



- Error Propagation and Pre-coder

  Full response system : No error propagation

  Partial response system: Error propagation

  Pre-coder is necessary for prevention of error propagation



**Pre-coder in TX**

Source information $\{a_n\} \rightarrow$

Pre-coded information $\{s_n\}$

Digital calculation (Logical calculation)

$$s_n = \sum_{i=0}^{k} c_i \cdot s_{n-i} \quad (\text{mod}\, 2)$$



Pre-coded information $\{s_n\} \rightarrow$

Partial response information $\{g_n\}$

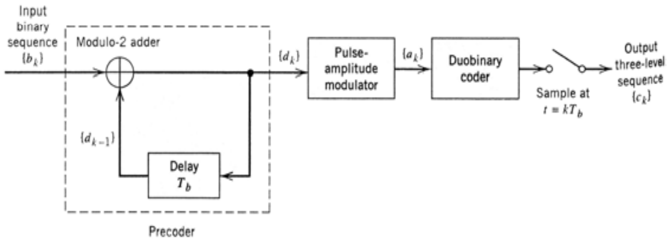
Analog calculation (Physical calculation)

$$g_n = \sum_{i=0}^{k} c_i \cdot s_{n-i}$$

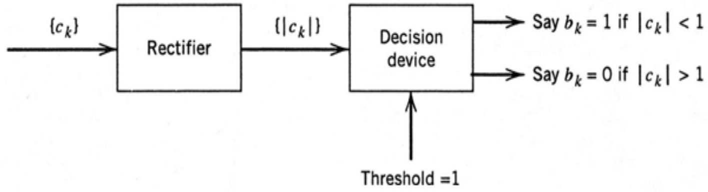
Decoding in RX

$$a_n = g_n \quad (\text{mod}\, 2)$$

Apparently, error propagation is eliminated.





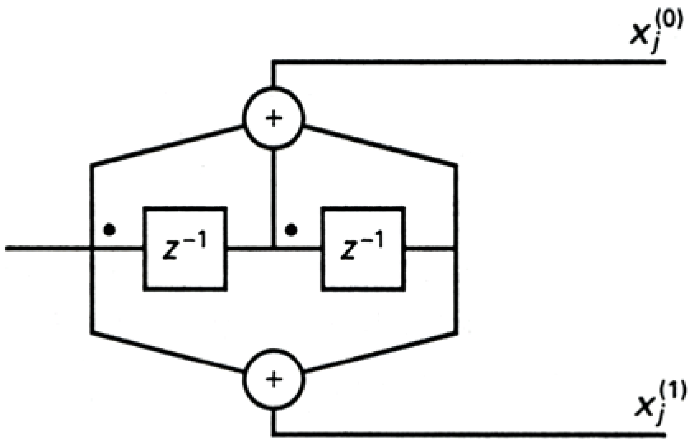





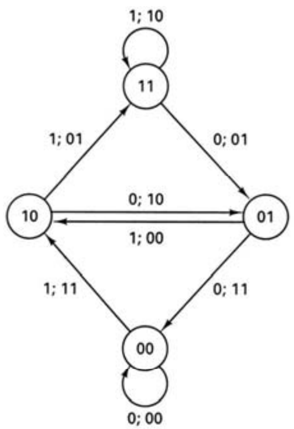

PR-VA (Partial Response & Viterbi Algorithm) is a most powerful recording method in magnetic recording.

Convolutional Code also utilizes a partial response in the codeword



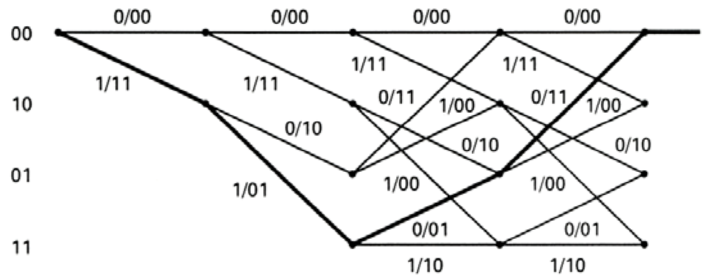



$R = \frac{1}{2}$, nonsystematic, $m = 2$ encoder




State transition diagram for $R = \frac{1}{2}, m = 2$ code






Trellis for four levels, $R = \frac{1}{2}, v = 2$ code.

Heavy line denotes route for message $\mathbf{u} = (11000\ldots)$.