Fading Theory

- In many circumstances, it is too complicated to describe all reflection, diffraction, and scattering processes that determine the different Multi-path Components.

  Rather, it is often preferable to describe the probability that a channel parameter attains a certain value.

Deterministic vs. Stochastic

- Deterministic case : "x=y" means 2=2.
- Stochastic case : "x=y" means "p(x)=p(y)".
- For example, $x = 1-x$ holds

  when x is a uniform distributed random variable in the interval [0,1]

  z: zero-mean Complex Gaussian Noise

  ∴ "$z=-z=z^*=-z^*$"

  Z:zero-mean Complex Gaussian Independent Vector

  ∴ "$\mathbf{Z}=U\mathbf{Z}$" where U: Unitary matrix

Stochastic Signal Processing

- Realized value of random number is known → e.g. MRC
- Pdf of random number is known→ e.g. Wiener Filtering
- Moments of random number is known → e.g. Decoupling Circuit Design

Stochastic Process in Wireless Communication

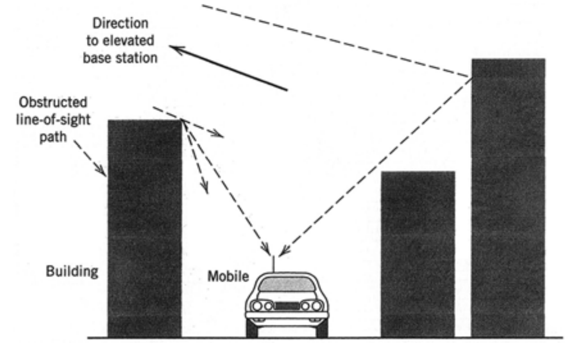
- Noise (white spectrum)
- Signal (modulated bandwidth)
- Channel (Doppler frequency)
- Noise > Signal > Channel

Contents

- Path Loss Formula
- Log-normal distribution
- Rayleigh/Rice distribution
- Envelope/Phase distribution
- Power Spectrum & Doppler effect
- Fading Coefficient
- MAP Estimation of Fading Channel in PHS

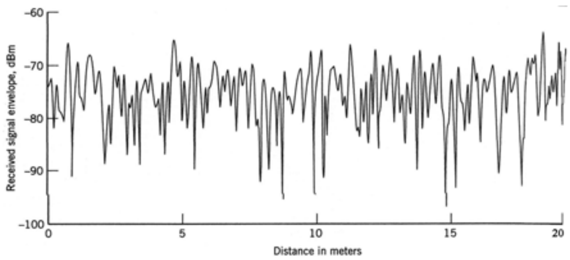
Mobile Communication Channel

In addition to Direct wave, there are many Reflection, Refraction and Diffraction waves.



- Received signal fluctuates dramatically

→ Fading (Long-range, Medium-range, Short-range)



Hierarchical stochastic structure

- Path loss : The large-scale mean itself depends on the "distance" between transmitter and receiver.
- Log-normal : Mean power, averaged over about 10 wavelengths, itself shows fluctuations due to "shadowing" by large objects.
- Rayleigh and Nakagami-Rice : On a very-short-distance scale, power fluctuates around a local mean value due to "interference" between different MPCs.

Path loss and Power Control

- For 3G Wireless Communication System, i.e. W-CDMA (Wideband Code Division Multiple Access) Power Control is used in order to alleviate "Near-Far Problem".

  Dynamic Range for Power Control is required more than 74dB.

Path Loss Formula

- **Land mobile electromagnetic wave propagation**
  Propagation characteristics are important in designing a cell size, a transmitter and a receiver.
  - **Long** distance variation (Okumura curve): The CCIR adopted the basic formula for the median path loss, based on Okumura's measurements.

$$L = 69.55 + 26.16\log(f) - 13.82\log(H_b) + [44.9 - 6.55\log(H_b)]\log(d) + a_x(H_m)$$

f : frequency in MHz

H_b : Base station antenna height in meter

d : Range in Km

H_m : Mobile station antenna height in meter

$a_x(H_m)$: Correction factor

– **Middle** distance variation (**Log-normal** distribution: Shadowing) Median over several ten or hundred wavelengths obeys a log-normal distribution.

$$E_r = T_1 \times T_2 \times T_3 \times \cdots \times E_s$$

E_r : Signal Strength at the receiver

E_s : Signal Strength at the transmitter

T_i : Transmission coefficent at the i-th obstacle

$$\therefore \quad \log E_r = \log T_1 + \log T_2 + \cdots + \log E_s$$

Central Limit Theorem

- The sum of statistically independent and identically distributed random variables with finite mean and variance approaches to a **Gaussian distribution** as the number of variables increases.
- **Gaussian distribution** is characterized only by mean and variance (2 parameters).
- An instantaneous complex amplitude of OFDM signal can be also approximated by Gaussian variable.

Shadowing effect

- Typical shadowing range is around 4-10dB
- 3GPP Channel model:

Suburban Macro	8dB	
Urban Macro	8dB	
Urban Micro	10dB(NLOS)	4dB(LOS)

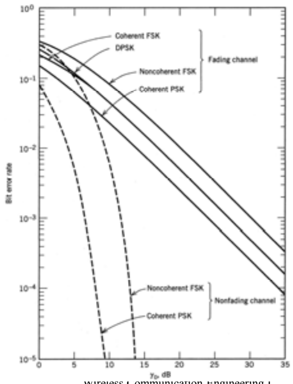
Rayleigh Fading

– **Short** distance variation (Rayleigh Fading) There are so many reflection and diffraction waves to generate a complicate standing wave pattern. The mobile station moves through there.

BER Performance in Rayleigh Fading Channel

- BER (Bit Error Rate) is proportional to an exponential function of SNR in non-fading channel (AWGN channel).
- BER is proportional to an inverse of SNR in fading channel.
- Because SNR in fading channel is a random variable of which PDF (probability density function) is an exponential function.

- Fading significantly deteriorates QoS (i.e. bit error rate).



BER in Rayleigh channel

- Instantaneous BER: $Pe(\gamma) \cong \exp(-\gamma)/2$
- Averaged BER: $\overline{Pe(\gamma)} = \int Pe(\gamma) \times P(\gamma) d\gamma = 1/\{2(\Gamma+1)\}$
- Pdf of SNR: $P(\gamma) = \exp(-\gamma/\Gamma)/\Gamma$
  where Γ : average SNR

Interference between Multi-path Components

- **Rayleigh Fading Model**

The n - th elementary arriving wave $e_n(t)$ at an angle of ϕ_n

$$e_n(t) = \mathrm{Re}[z_n(t)\exp(j2\pi[f_c + f_D\cos(\phi_n)]t)]$$

$\mathrm{Re}[\ \ ]$: Real part complex number

$z_n(t)$: Complex envelope $= R_n e^{j\theta_n}$

f_c : Carrier frequency

f_D : Maximum Doppler frequency shift $(= v/\lambda)$

v : Velocity of mobile station

λ : Wavelength $(= c/f_c)$

– Envelope and phase distribution

Received signal $e(t)$ is composed of N elementary waves.

$$e(t)=\sum_{n=1}^{N} e_n(t)$$
$$=\mathrm{Re}\left[\sum_{n=1}^{N} z_n(t)\exp(j2\pi f_c t)\right]$$
$$z(t)=\sum_{n=1}^{N} z_n(t)$$
$$=x(t)+jy(t)$$

$x(t)$: In-phase component $=R(t)\cos(\theta(t))$

$y(t)$: Quadrature component $=R(t)\sin(\theta(t))$

In the limit $(N\to\infty)$, $x(t)$ and $y(t)$ become an independent Gaussian random variable with zero mean.

Thus, a joint pdf (probability density function) of x and y

$$p(x,y)=\exp\left(-\frac{x^2+y^2}{2b_0}\right)\Big/2\pi b_0$$

where $2b_0$: average received power $=E\left[x^2+y^2\right]=E\left[R^2\right]$

A joint pdf of R and θ is

$$p(R,\theta)=\frac{R}{2\pi b_0}\exp\left(-\frac{R^2}{2b_0}\right)=p(R)\,p(\theta)$$

where R : envelope

θ : phase

Rayleigh Distribution

A pdf of envelope R is a **Rayleigh** distribution

$$p(R)=\frac{R}{b_0}\exp\left(-\frac{R^2}{2b_0}\right)$$

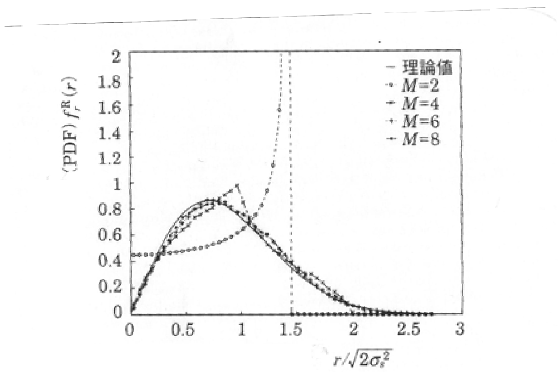
A pdf of phase θ is a **uniform** distribution

$$p(\theta)=1/2\pi$$

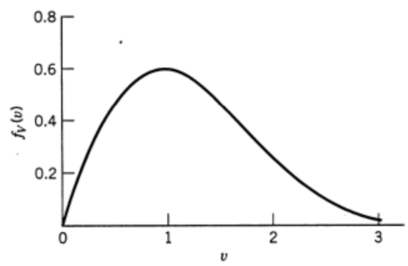
CNR (Carrier-to-noise radio), $\gamma=R^2/p_n$ is **exponential** distribution with noise power of p_n

$$p(\gamma)=\frac{1}{\Gamma}\exp\left(-\frac{\gamma}{\Gamma}\right)$$

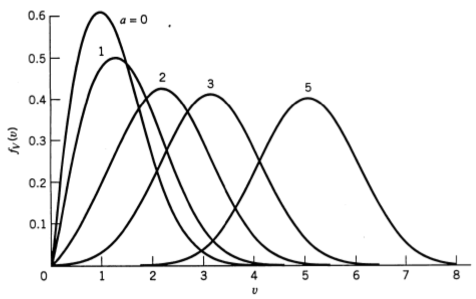
where Γ : Average CNR



Normalized Rayleigh Distribution



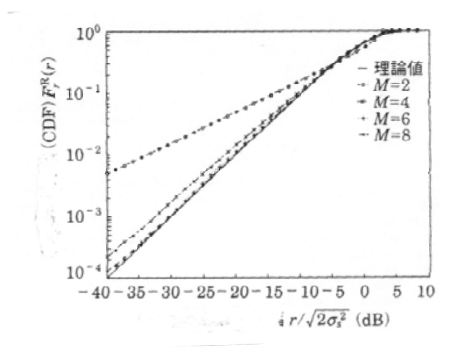
Normalized Nakagami/Rice Distribution



Rician Distribution

$$f_V(v) = v \exp\left(-\frac{v^2 + a^2}{2}\right) I_0(av)$$

$I_0(\ )$: 0-th Modified Bessel Function
a: Rice factor = LOS / NLOS



- **Power spectrum & Doppler effect**

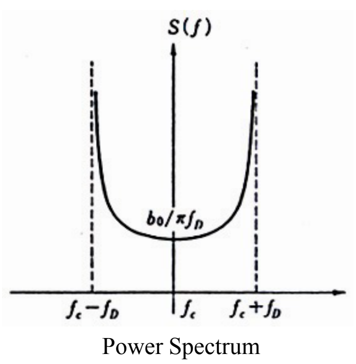
Elementary wave of arrival angle ϕ has a different frequency from f_c due to the Doppler effect.

$$f = f_c + f_D \cos\phi$$

Arriving angle is uniformly distributed so that received power $S(f)\,df$ in the range $[f,\ f+df]$ is

$$S(f)df = 2 \times \frac{b_0}{2\pi} df$$
$$= \frac{b_0}{\pi f_D \sqrt{1 - [(f - f_c)/f_D]^2}} df$$

(cf. $f_c = 1.5\text{GHz}, v = 50\text{km/h}, f_D = 135\text{Hz}$)



Power Spectrum

Time derivative of random variables

$$dx(t)/dt = dR(t)/dt \times \cos(\theta(t)) - R(t) \times \sin(\theta(t)) \times d\theta(t)/dt$$

$$dy(t)/dt = dR(t)/dt \times \sin(\theta(t)) + R(t) \times \cos(\theta(t)) \times d\theta(t)/dt$$

$$pdf\ (x, y, dx/dt, dy/dt) \rightarrow pdf\ (R, \theta, dR/dt, d\theta/dt)$$

Joint pdf

$$p(x,y,\dot{x},\dot{y})=\frac{1}{2\pi b_0}e^{-\frac{x^2+y^2}{2b_0}}\frac{1}{2\pi b_2}e^{-\frac{\dot{x}^2+\dot{y}^2}{2b_2}}$$

$$\downarrow$$

$$p(R,\theta,\dot{R},\dot{\theta})=\frac{R^2}{2\pi b_0}e^{-\frac{R^2}{2b_0}}\frac{1}{2\pi b_2}e^{-\frac{\dot{R}^2+R^2\dot{\theta}^2}{2b_2}}$$

$$p(R,\dot{R},\dot{\theta})=\frac{R^2}{b_0}e^{-\frac{R^2}{2b_0}}\frac{1}{2\pi b_2}e^{-\frac{\dot{R}^2+R^2\dot{\theta}^2}{2b_2}}$$

$$p(R,\dot{R})=\frac{R}{b_0}e^{-\frac{R^2}{2b_0}}\frac{1}{\sqrt{2\pi b_2}}e^{-\frac{\dot{R}^2}{2b_2}}=p(R)\cdot p(\dot{R})$$

$$p(R,\dot{\theta})=\frac{R^2}{b_0}e^{-\frac{R^2}{2b_0}}\frac{1}{\sqrt{2\pi b_2}}e^{-\frac{R^2\dot{\theta}^2}{2b_2}}$$

$$p(\theta)=\frac{1}{2}\sqrt{\frac{b_0}{b_2}}\frac{1}{\left(1+\frac{b_0}{b_2}\dot{\theta}^2\right)^{3/2}}$$

- **Level crossing number & Fade duration**

  They are important parameters for mobile communication quality.

  - **Level crossing number**

$\dot{R}$: time derivative of envelope R

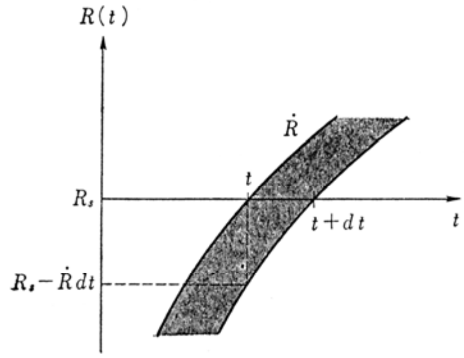
A joint pdf of R and $\dot{R}$, $p(R,\dot{R})$ is

$$p(R,\dot{R})=\frac{R}{b_0}\exp\left[-\frac{R^2}{2b_0}\right]\frac{1}{\sqrt{2\pi b_2}}\exp\left[-\frac{\dot{R}^2}{2b_2}\right]$$

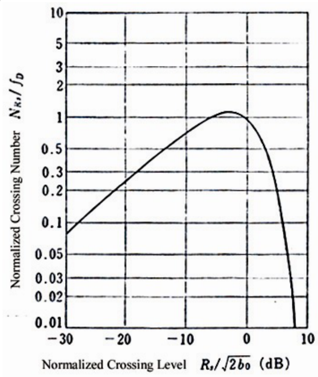
Level crossing number of envelope per unit time $N(R_s)$ at the level R_s

$$N(R_s)=\int_0^\infty \dot{R}\cdot P(R_s,\dot{R})d\dot{R}$$

where $b_2=E[\dot{R}^2]$



Therefore



$$N(R_s)=\sqrt{\frac{b_2}{\pi b_0}}\frac{R_s}{\sqrt{2b_0}}\exp\left[-\frac{R_s^{\,2}}{2b_0}\right]$$

$$=\sqrt{2\pi}\,f_D\frac{R_s}{\sqrt{2b_0}}\exp\left[-\frac{R_s^{\,2}}{2b_0}\right]$$

$$N\left(\sqrt{b_0}\right)_{\max}=f_D\sqrt{\pi/e}$$

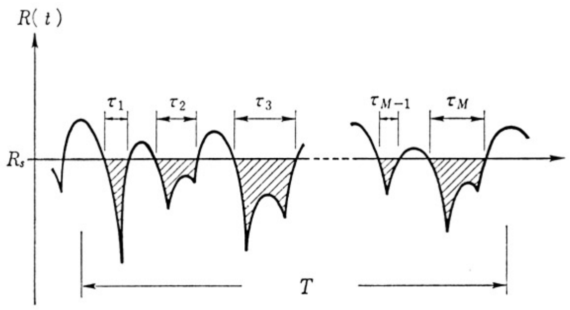
$$2\pi f_0=\sqrt{2b_2/b_0}$$

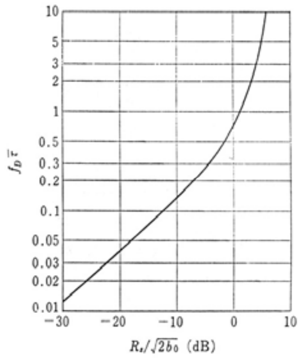
– Average fade duration time at the level R_s, $\bar{\tau}$

$$\bar{\tau}=\frac{\Pr[R(t)\le R_s]}{N(R_s)}$$

$$=\frac{\sqrt{2b_0}}{\sqrt{2\pi}\,f_D R_s}\left[\exp\left(\frac{R_s^{\,2}}{2b_0}\right)-1\right]$$

(cf. When $R_s/\sqrt{2b_0}=0.1$ (20dB down), $f_c=1.5\text{GHz}$, $v=50\text{km/h}$, $\bar{\tau}=2\text{ms}$)





- Random FM noise

$\theta(t)$ fluctuates randomly $\rightarrow$ FM noise

A pdf of $\dot{\theta}$, $p(\dot{\theta})$ is

$$p(\dot{\theta}) = \frac{1}{2}\sqrt{\frac{b_0}{b_2}}\left[1 + \frac{b_0}{b_2}\dot{\theta}^2\right]^{-3/2}$$

Random FM noise is independent on average received power.
This determines a lower bound of bit error rate.

- **Fading correlation**
  The correlation characteristics are necessary for the design of diversity system.
  - Time correlation

$$\rho(\tau) = \frac{E\left[z^*(t)z(t+\tau)\right]}{E\left[z(t)^* z(t)\right]} = J_0(2\pi f_D \tau)$$

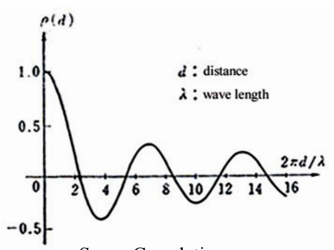
$J_0(\ )$: 0 - th order Bessel function of the first kind

- **Space correlation**

Space distance $d = v\tau$

$$\rho(d) = J_0(2\pi d/\lambda)$$

Around half wavelength spacing $(d \sim \lambda/2) \rightarrow$ no correlation



Space Correlation

MIMO Transmission and Antenna correlation

- Antenna correlation decreases MIMO channel capacity if average SNR at RX antenna is equal to each other.

– **Frequency correlation**
This is important parameter for Wide-band transmission.

$$\rho(\Omega)=\frac{1}{1+j2\pi\Omega(\delta\ell/c)}\exp\left(j2\pi\ell_0/c\right)$$

ℓ_0 : minimum path length
$\delta\ell$: deviation in path length

(cf. For $\delta\ell$ = 200m, coherent bandwidth is 400kHz)

A MAP Estimation of Rayleigh Fading Channel

– A Filter Theory of Complex Gaussian Process –
and Its Application to PHS SDMA

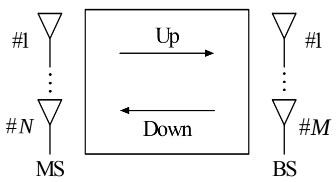
Contents

- Background & Motivation
- Complex Gaussian Stochastic Process
- Noisy Rayleigh Fading Channel
- MAP Estimation of Channel Transfer Coefficient
- Numerical Results
- Conclusion
- Future Work

Background & Motivation

- Recursive Simulation Method for Rayleigh Fading Channel.
  – How to write a computer program ?
- Fading Channel Coefficients should be estimated in SDMA PHS Systems

- Mobile Communication Channel with MIMO Systems
  – Time Variant Linear Reciprocal System



For $(N+M)$-port Circuit, a $(N+M)\,(N+M)$ scattering matrix S is defined;

$$S(f,t)=\begin{array}{cc} \begin{array}{cc} \leftarrow N\rightarrow & \leftarrow M\rightarrow \end{array} & \\ \begin{bmatrix} S_{MM} & S_{BM} \\ S_{MB} & S_{BB} \end{bmatrix} & \begin{array}{c} \updownarrow N \\ \updownarrow M \end{array} \end{array}$$

where

S_{BM} : $M \times N$ Transfer Matrix of Up-Link from MS to BS
S_{MB} : $N \times M$ Transfer Matrix of Down-Link from BS to MS

By the reciprocity,

$$S = S^t$$

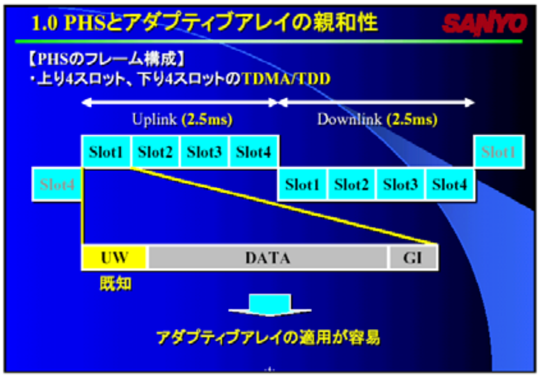
$$\therefore \; S_{MB}(f,t) = S_{BM}(f,t)^t$$

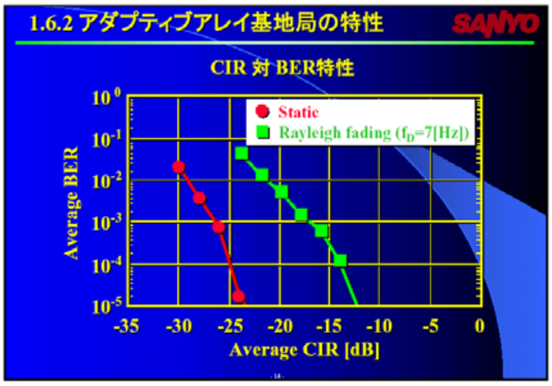
Thus, the Down-Link Transfer Characteristics can be determined by the Up-Link one.

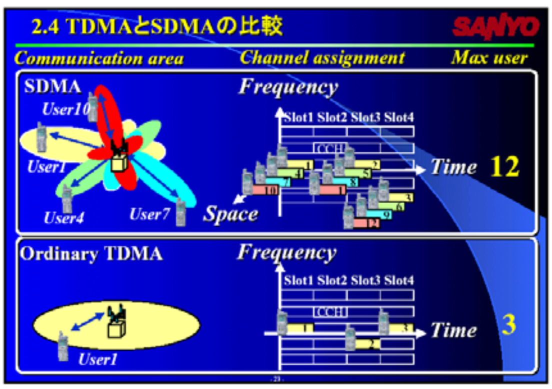
The above equality, however, holds only for the same frequency and time.

PHS system

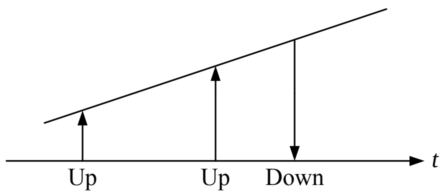
- TDD (Time Domain Duplex)
- TDMA (Time Domain Multiple Access)
- 4 Time Slot Segmentation
- Introduction of SDMA increases a channel capacity by 3 times or more.
- At the PHS base station, 4 antennas are installed.
- At most 4 data streams can be transmitted simultaneously by pre-coding at BS for down link.
- The idea is used in “i-Burst” system (IEEE802.20)







- Conventionally
  - Linear Extrapolation for Channel coefficient is used.



  - Noise Filtering is not taken into account.

Complex Gaussian Stochastic Process

1) Rayleigh (or Rice) Fading Coefficient : $X(t)$
2) Random White Gaussian Noise : $Y(t)$

↓

3) Rayleigh Fading Coefficient contaminated with Noise:

$$Z(t) = X(t) + Y(t)$$

Stationary Gaussian Process can be characterized only by Autocorrelation Function

$$R_{ZZ}(\tau) = \overline{Z(t)Z(t+\tau)} = R_{XX}(\tau) + R_{YY}(\tau)$$

where

$$R_{XX}(\tau) = A\,\mathrm{J}_0(2\pi f_D \tau)$$

$A = \overline{|X(t)|^2}$: Average Fading Level

J_0 : 0th Order Bessel Function of First Kind

f_D : Maximum Doppler Frequency $\left(= f_c \frac{v}{c}\right)$

f_c : Carrier Frequency

v : velocity of MS

c : velocity of Light

$$R_{YY}(\tau) = \begin{cases} N & (\tau = 0) \\ 0 & (\tau \neq 0) \end{cases}$$

$N = \overline{|Y(t)|^2}$: Average Noise Level

For MAP Estimation, Cross-correlation Function is also needed

$$R_{ZX}(\tau) = \overline{Z(t)X(t+\tau)} = \overline{(X(t)+Y(t))X(t+\tau)} = \overline{X(t)X(t+\tau)} = R_{XX}(\tau)$$

$\because$ $X(t)$ and $Y(t)$ are independent.

- MAP (LS) Estimation and Optimal Noise Reduction
  - Wiener-Hopf Equation

Optimal Linear Combination Estimator Vector : **b**

$$\begin{bmatrix} 1+\frac{N}{A} & J_0(2\pi f_D(t_1-t_0)) & \cdots & J_0(2\pi f_D(t_{n-1}-t_0)) \\ & & \ddots & \\ & & & 1+\frac{N}{A} \end{bmatrix} [\mathbf{b}] = \begin{bmatrix} J_0(2\pi f_D(t_n - t_0)) \\ \vdots \\ J_0(2\pi f_D(t_n - t_{n-1})) \end{bmatrix}$$

MAP Estimator for $X(t_n)$

$$X(t_n)_{\mathrm{MAP}} = \mathbf{b}^{\dagger}\,\mathbf{Z}$$

where

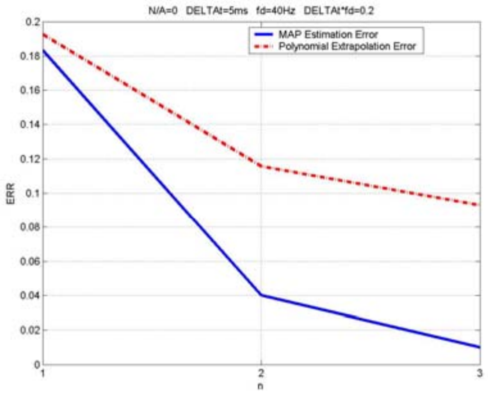
$\mathbf{Z} = (Z(t_0), \ldots, Z(t_{n-1}))$: Observed Noisy Data

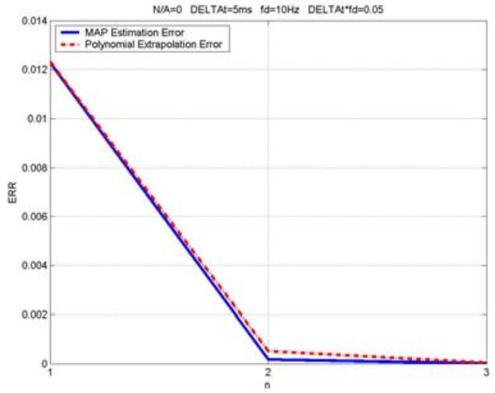
Numerical Results

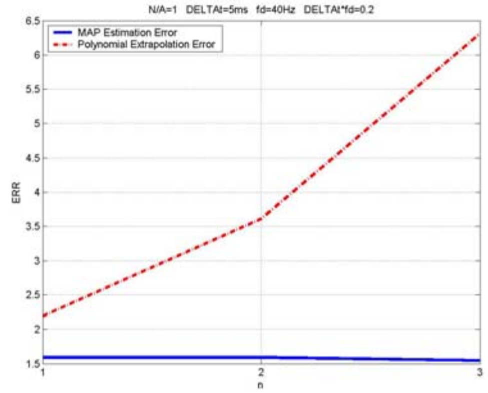
(1) Noise Level : $N/A = 0, 0.1, 1$

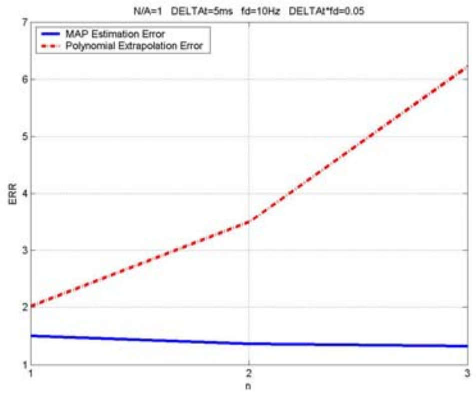
(2) Doppler Frequency: $f_D = 10,\ 40[\text{Hz}]$

(3) No. of Data : $n = 1, 2, 3$









Conclusion

- Estimation of Fading Coefficient is useful for TDMA/ TDD.
- Conventional Estimation is not satisfactory.
- Estimation Error can be greatly reduced by MAP Estimation.