Probabilistic Concepts in Engineering Design

Class 9

Jan. 25th 2010

Homework

- **Calculate "the circular constant π" by applying Monte Carlo Simulation**

Chap.5 Monte Carlo Simulation

1. Introduction

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Simulation is the process of replicating real world based on a set of assumptions and conceived models of reality.

For problems involving random variables with known probability distributions, Monte Carlo simulation is required.

A sample from Monte Carlo simulations is similar to a sample from experimental observations may be treated statistically.

Monte Carlo method should be used only as a last resort; that is, when and if analytical solution methods are not available or are ineffective.

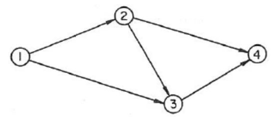


Figure 5.1 A construction network.

Table 5.1 Activity Durations

Activity	Duration (days)	Probability
1–2	2	$\frac{1}{3}$
	3	$\frac{1}{3}$
	4	$\frac{1}{3}$
1–3	4	$\frac{1}{2}$
	5	$\frac{1}{2}$
2–3	1	$\frac{2}{3}$
	2	$\frac{1}{3}$
2–4	2	$\frac{1}{2}$
	3	$\frac{1}{2}$
3–4	2	1

Table 5.2 Simulation of Project Duration

Sample #	1–2	1–3	2–3	2–4	3–4	1–2–4	1–3–4	1–2–3–4	Is Project Duration >6 Days?
1	2	5	1	3	2	5	7	5	Yes
2	2	4	1	3	2	5	6	5	No
3	4	4	2	2	2	6	6	8	Yes
4	4	5	1	2	2	6	7	7	Yes
5	3	5	1	3	2	6	7	6	Yes
6	2	5	2	2	2	4	7	6	Yes
7	3	5	1	3	2	6	7	6	Yes
8	3	5	2	2	2	5	7	7	Yes
9	3	4	1	2	2	5	6	6	No
10	3	5	1	2	2	5	7	6	Yes
11	4	5	2	2	2	6	7	8	Yes
12	4	5	1	3	2	7	7	7	Yes
13	3	5	2	2	2	5	7	7	Yes
14	3	5	1	3	2	6	7	6	Yes
15	2	5	1	2	2	4	7	5	Yes

Table 5.3 Estimated Completion Probabilities

Repetition No.	Estimated Probability of Completion	
	Sample Size 15	Sample Size 30
1	0.13	0.23
2	0.33	0.30
3	0.40	0.30
4	0.27	0.30
5	0.20	0.27
6	0.33	0.33
7	0.33	0.30
8	0.27	0.34
9	0.13	0.20
10	0.44	0.35
11	0.33	0.30
12	0.27	0.17
13	0.20	0.27
14	0.07	0.10
15	0.20	0.33
16	0.20	0.23
17	0.33	0.37
18	0.27	0.23
Mean	0.26	0.27
Standard Deviation	0.10	0.07
Range	0.07–0.44	0.10–0.37

Referred from
Probability Concepts in Engineering Planning and Design
Volume 1 and Volume 2, A.H. Ang and W.H. Tang

2. Generation of Random Numbers

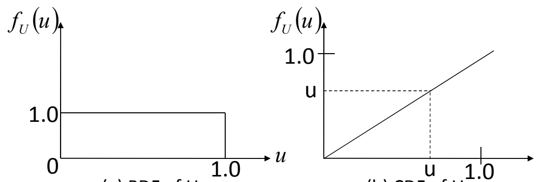
A key task in the application of Monte Carlo simulation is the generation of the appropriate values of the random variables, in accordance with the respective prescribed probability distributions.

- Tossing coin for random variables with two equally likely values
- Rolling a 6-faces dice for random variables with 6 equally likely possible values

The automatic generation of the requisite random numbers with specified distributions will be necessary.

This can be accomplished systematically for each variable by first generating a uniformly distributed random number between 0 and 1.0, and through appropriate transformations obtaining the corresponding random number with the specified probability distribution.

Suppose a random variable X with CDF Fx(x).
Then, at a given cumulative probability Fx(x)=u,
The value X is $x = F_X^{-1}(u)$ ------- Eq.(1)



(a) PDF of U Fig.5.2 (b) CDF of U

Now suppose that u is a value of the standard uniform variate, U, with a uniform PDF between 0 and 1.0; then, as shown in fig. 5.2(b)

$F_U(u) = u$ ------- Eq.(2)

That is, the cumulative probability of $\underline{U \le u}$ is equal to u

Therefore, if u is a value of U, the corresponding value of the variate X obtained through Eq.(1) will have a cumulative probability,

$$
\begin{aligned}
P(X \le x) &= P\left[F_X^{-1}(U) \le x\right] \\
&= P\left[U \le F_X(x)\right] \\
&= F_U\left[F_X(x)\right] = F_X(x)
\end{aligned}
$$

Which means that if ($u_1, u_2, \ldots, u_n$) is a set of values from U, the corresponding set of values obtained through Eq.(1), that is

$$x_i = F_X^{-1}(u_i); \quad i = 1,2,\ldots,n \qquad \text{------- Eq.(3)}$$

will have the desired CDF Fx(x). the relationship between u and x may be seen graphically in Fig. 3.

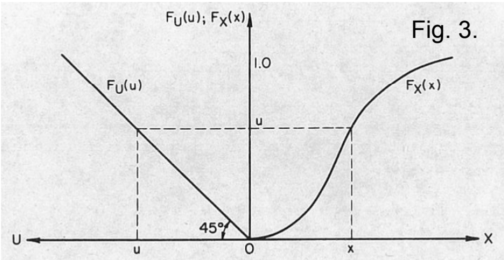


Fig. 3.

Random numbers with a prescribed distribution may be generated through Eq.(3) once the standard uniformly distributed random numbers have been obtained.
The generation of random numbers through Eq.(3) is known as **the inverse transform method**

Example 1

Consider the exponential distribution with the CDF

$$F_X(x) = 1 - e^{-\lambda x}; \qquad x \geq 0$$

The inverse function is

$$x = F_X^{-1}(u) = -\frac{1}{\lambda}\ln(1-u)$$

Therefore, in this case, once the standard uniformly distributed random numbers $u_i, \ i = 1,2,\ldots$ are generated, we obtain the corresponding exponentially distributed random numbers, according to Eq.(3), as

$$x_i = -\frac{1}{\lambda}\ln(1-u_i)$$

Since (1-u_i) is also uniformly distributed, the required random numbers may also generated by $x_i = -\frac{1}{\lambda}\ln u_i \quad i = 1,2\ldots$

Example 2

The CDF of the Type I asymptotic distribution of larges value is

where β is the most probable value of X, and α is the shape parameter.

At a given probability value Fx(x)= u,
we have

$$x = \beta - \frac{1}{\alpha}\ln\left(\ln\frac{1}{u}\right)$$

Therefore, random numbers with the type I asymptotic distribution can be generated from the corresponding uniformly distributed random numbers using the above relation