The Three Types of Asymptotic Extremal Distributions

Type	Tail Characteristi-cs of Initial Variate	Extreme	Cumulative Distribution Function	Mean Value	Standard Deviation	Standard Extremal Variate, S
I	Exponential	Largest	$\exp\left[-e^{-\alpha_n(x_n-u_n)}\right]$	$u_n+\frac{0.577}{\alpha_n}$	$\frac{\pi}{\sqrt{6}\alpha_n}$	$\alpha_n(X_n-u_n)$
		Smallest	$\exp\left[-e^{-\alpha_1(x_1-u_1)}\right]$	$u_1+\frac{0.577}{\alpha_1}$	$\frac{\pi}{\sqrt{6}\alpha_1}$	$-\alpha_1(X_1-u_1)$
II	Polynomial	Largest	$\exp\left[-\left(\frac{v_n}{y_n}\right)^k\right]$	$v_n\Gamma\left(1-\frac{1}{k}\right)$	$v_n\left[\Gamma\left(1-\frac{1}{k}\right)-\Gamma^2\left(1-\frac{1}{k}\right)\right]^{1/2}$	$k\ln\frac{Y_n}{v_n}$
		Smallest	$\exp\left[-\left(\frac{v_1}{y_1}\right)^k\right]$	$v_1\Gamma\left(1-\frac{1}{k}\right)$	$v_1\left[\Gamma\left(1-\frac{1}{k}\right)-\Gamma^2\left(1-\frac{1}{k}\right)\right]^{1/2}$	$k\ln\frac{Y_1}{v_1}$
III	Bounded (in direction of extreme)	Largest	$\exp\left[-\left(\frac{\omega-z_n}{\omega-w_n}\right)^k\right]$	$\omega-(\omega-w_n)\Gamma\left(1+\frac{1}{k}\right)$	$(\omega-w_n)\left[\Gamma\left(1+\frac{2}{k}\right)-\Gamma^2\left(1+\frac{1}{k}\right)\right]^{1/2}$	$-k\ln\left(\frac{\omega-z_n}{w_1-\varepsilon}\right)$
		Smallest	$\exp\left[-\left(\frac{z_1-\varepsilon}{w_1-\varepsilon}\right)^k\right]$	$\varepsilon+(w_1-\varepsilon)\Gamma\left(1+\frac{1}{k}\right)$	$(w_1-\varepsilon)\left[\Gamma\left(1+\frac{2}{k}\right)-\Gamma^2\left(1+\frac{1}{k}\right)\right]^{1/2}$	$-k\ln\left(\frac{z_1-\varepsilon}{w_1-\varepsilon}\right)$