Probabilistic Concepts in Engineering Design

Class 6

Dec. 21st 2009

The measure of safety or reliability

Function of the relative positions of $f_X(x)$ and $f_Y(y)$

As well as of the degree of dispersions

The quantitative evaluation of the true pf often poses major problems :
for example, the determination of the correct forms of $f_X(x)$ and $f_Y(y)$

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We assume that X and Y are statistically independent.
In general, these variables may be correlated;
that is,

$$P(Y < X = x) \neq P(Y < x)$$

and

$$P(X < Y = y) \neq P(X < y)$$

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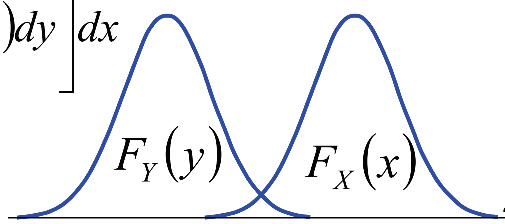
In such cases, the probability of failure may be expressed in terms of the joint PDF as follows

$$P_F = \int_0^\infty \left[\int_0^y f_{X,Y}(x,y)dx \right] dy$$

The corresponding reliability is

$$P_S = \int_0^\infty \left[\int_0^x f_{X,Y}(x,y)dy \right] dx$$



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Margin of safety

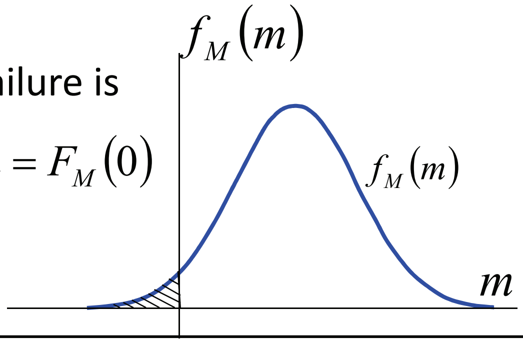
Safety margin, M

$$M = X - Y$$

As X and Y are random variables, M is also random variable with corresponding **PDF**

In this case, failure is clear
The event M<0,
The probability of failure is

$$P_F = \int_{-\infty}^{0} f_M(m)dm = F_M(0)$$



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Example 1

Consider a structure whose **R** is normal random variable $N(\mu_R, \sigma_R)$

Similarly, the load **Q** is also normal $N(\mu_Q, \sigma_Q)$

The probability distribution of the safety margin **M=R-Q** is also normal $N(\mu_M, \sigma_M)$

In which

$$\mu_M = \mu_R - \mu_Q$$

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And for statistically independent **R** and **Q**

$$\sigma_M^2 = \sigma_R^2 - \sigma_Q^2$$

Furthermore, is N(0,1)

$$P_F = F_M(0) = \Phi\left(-\frac{\mu_M}{\sigma_M}\right) = 1 - \Phi\left(\frac{\mu_M}{\sigma_M}\right)$$

$$P_S = 1 - P_F = \Phi\left(\frac{\mu_M}{\sigma_M}\right)$$

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The Factor of Safety

Familiar term in engineering is the factor of safety

$$\Theta = \frac{X}{Y}$$

If the supply **X** and demand **Y** are random variables, the safety factor Θ will be a random variable.

In this case, failure would be the event $\Theta < 1$

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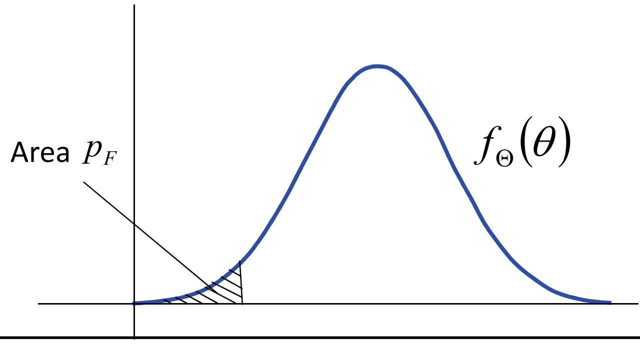
The corresponding probability of failure is

$$p_F = \int_0^1 f_\Theta(\theta) d\theta = F_\Theta(1,0)$$

which is the area under $f_\Theta(\theta)$ between 0 and 1



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Second-Moment Formulation

The calculation of the probability of safety, requires

$$f_x(x) \;\; and \;\; f_Y(y) \;\; or \;\; f_{X,Y}(x,y)$$

In practice, this information is often unavailable or difficult to obtain

The available information or data may be sufficient only to evaluate the first moment and second moment. The mean values and variances of the random variables.

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With the second moment approach, the reliability may be measured entirely with a function of the first and second moments of the design variables, namely safety index β.

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The safety margin M=X-Y
The safe state may be defined as M>0

The boundary separating the safe and failure states is the limit state defined by the equation M=0

Introduce the reduced variables

$$X' = \frac{X - \mu_X}{\sigma_X}$$

$$Y' = \frac{Y - \mu_Y}{\sigma_Y}$$

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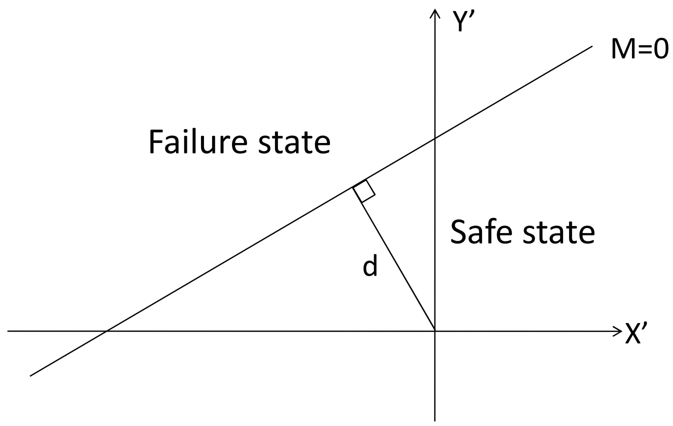
In the space of these reduced variants, the safe state and failure state may be represented



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Also, in terms of the reduced variants, the limit state equation M=0 becomes

$$\sigma_x X' - \sigma_Y X' + \mu_X - \mu_Y = 0$$

which is a straight line in the figure.
The distance from the linear failure line to the origin “0” is a measure of reliability; this distance “d” is given as

$$d = \frac{\mu_X - \mu_Y}{\sqrt{\sigma_X^2 + \sigma_Y^2}}$$

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For normal X and Y, this distance d is also safety index β and the reliability is

$$p_s = \Phi(d)$$

END Dec 21st