Probabilistic Concepts in Engineering Design

Class 5

Nov. 30th 2009

(2) Interval estimation of the mean

How good is the estimator X?

So far, we have discussed the point estimator of the mean and variance, however, do not convey information on the degree of accuracy of these estimators.

2

Estimating Parameters from Observed Data

Confidence intervals

➡ to supplement the point estimate

The method of estimation is known as "Interval estimation"

To estimate μ, we use the sample mean x
The accuracy of this estimate is concern

3

The expected value of sample mean $\overline{X}$ is equal to the population mean

Since X is a random variable, it also has a variance

4

Confidence interval with known variance

In some cases, σ may be assumed to be known from previous experience.

For large sample size, the sample mean $\overline{X}$ can be described with the normal distribution

By a simple transformation,

5

In general,
We denote (1ーα) as the specified confidence level, and as the values of the standard normal variate with cumulative probability levels $\alpha/2$ and（1ー$\alpha/2$）, respectively,
We may write

Rearrangement and substitution of the observed sample mean

6

More properly,
There is a confidence of (1−α) that the estimated interval contains the unknown

Thus, such an interval is called the 1−α confidence interval for the mean

7

One sided confidence limit for the mean

Upper and lower limits that bound the value of the population mean μ

Material strength, capacity of highway, flood channel,

For such purposes, the (1−α) lower confidence limit

8

Chap.3
Reliability and Reliability-Based Design

1. Introduction

The problems of reliability of engineering systems may be cast essentially

As problems of *"Supply versus Demand"*

9

The determination of the (***supply***) capacity of an engineering system

⇨ To meet certain (***demand***) requirement

In the consideration of safety of structure,

We are concerned with insuring that the strength of the structure (*supply*) is sufficient to withstand the life time maximum applied load (*demand*)

10

Flood control system

The adequacy of the reservoir capacity (**supply**) to control the largest flood (**demand**) that may occur over the life of system

11

Traditionally, the reliability of engineering systems is achieved through
"**the use of factors or margins of safety**"
and
"**adopting conservative assumptions**"
in the process of design

that is,
by ascertaining that a "worst", or minimum, supply condition will remain adequate by some margin under a "worst" or maximum demand requirement

12

Minimum supply and maximum demand?

Adequacy or inadequacy of the applied margin?

13

2. Basic problems

The available supply and required demand may be modeled as random variables.

X=the supply capacity
Y=the demand requirement

The objective of reliability analysis is to insure the event (X>Y) throughout the useful life

14

This assurance is possible only in terms of probability

$P(X>Y)$

Assume, the necessary probability distributions of X and Y are available, that is

$$F_X(x)\,or\;f_X(x)\,and\;F_Y(y)\,or\;f_Y(y)$$

15

The required probabilities may be formulated as follows,

$$P_F = P(X < Y) = \sum_{all} P(X < Y \mid Y = y)P(Y = y)$$

If the supply and demand, X and Y are statically independent,

$$P(X < Y \mid Y = y) = P(X < y)$$

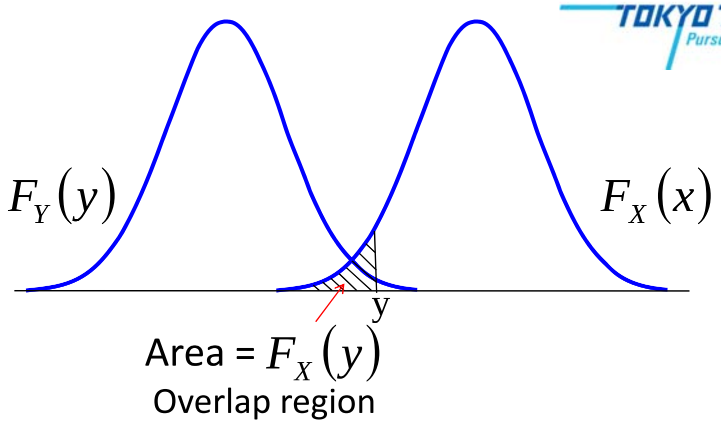
16

For continuous X and Y,

$$P_F = \int_0^{\infty} F_X(y) f_Y(y) dy$$

This is the convolution with respect to **"y"**

17



If **Y**=**y**, the conditional probability of failure would be $F_X(y)$

18

But, since **Y=y**,
more precisely $(y < Y < y + dy)$ is associated with probability $f_Y(y)dy$

Integration over all values of **Y** yields

$$P_F = \int_0^\infty F_X(y) f_Y(y) dy$$

Alternatively, the reliability may be formulated also by convolution with respect to x yielding

$$P_F = \int_0^\infty [1 - F_Y(x)] f_X(x) dx$$

19

The corresponding probability of nonfailure, therefore, is

$$P_S = 1 - p_F$$

END

20