Probabilistic Concepts in Engineering Design

Class 4

Nov. 16th 2009

The Poisson Process and Poisson Distribution

Many physical problems, possible occurrences of events at any point and/or space

The event can occur at any instant, it may occur more than once at a given time or space interval

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- **Poisson Sequence**
- **Poisson Process**

The Poisson Process and Poisson Distribution

Assumptions

1. An event can ocuur at random
2. independent
3. small interval ⊿t, ν ⊿t
   where ν is the mean rate of occurrence of event

Similarities
Differences
→ **Between** { Bernoulli sequence / Poisson process }

Interval : Shorter interval

The Exponential Distribution

- **Related to the Poisson process**
- **If events occur according to a Poisson process**
- **Time till the first occurrence of the event has an exponential distribution**
- **Recurrence time or return period**

Estimating Parameters from Observed Data

The determination the parameters, such as the mean value μ and variance σ 2, and the choice of specific distributions are of interest.

Inherent variability and estimation error

Uncertainty arises from inherent randomness of the natural phenomenon, the inaccuracies in the estimation of the parameters in the choice of distributions

1. Classical approach to Estimation of Parameters

- •Point estimation
- •Interval estimation

Random sampling and Point estimation

- •method of moment
- •method of maximum likelihood

Sample of size n from the population X

X1, X2, X3, -----Xn

The method of moments

The mean and variance are the weighted averages of

X and (X－μ)²

The method of maximum likelihood

A procedure for deriving the point estimator of the parameter directly

Consider a random variable X with density function f(x; θ), in which θ is the parameter, such as the mean λ in the exponential distribution

What is the most likelihood of the sample value of θ that produces the set of the set of observations, X1. X2, X3,…..Xn,?

The method of maximum likelihood

In other words, among the possible value of θ , what is the value that will maximize the likelihood of obtaining the set of observations?

Likelihood function of observing set X1, X2, X3, ……Xn

$$L(X_1, X_2, X_3, Xn; \theta)$$
$$= f(X_1;\theta) f(X_2;\theta) f(X_3;\theta) \ldots f(X_n;\theta)$$

The maximum likelihood estimator may be obtained by differentiating L

Interval estimation of the mean

So far, we have discussed the point estimator of the mean and variance, however, do not convey information on the degree of accuracy of these estimators.

Confidence intervals:
to supplement the point estimate Interval estimation

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