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Probabilistic Concepts in Engineering Design

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1. Introduction

Reliability-Based Civil Engineering Concept

- **Low probability, high consequence events**
- **Risk**
- **Structural reliability**
- **Risk and risk perception**

1. Role of Probability in Engineering

1.1 Introduction

- **Decisions are required**
- **Irrespective of the state of completeness and quality of information**
- **Under condition of uncertainty**
- **Many problems in engineering involve natural process and phenomena are naturally indeterminate and random**

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1.2 Uncertainty in Real-World Information

1.2.1 Uncertainty associated with randomness

- **Many phenomena or processes contain randomness, that is the actual outcomes are unpredictable**
- **Such Phenomena are characterized by experimental observations**
  - **Portrayed graphically tin the form of histogram**
  - **Frequency diagram**
- **Frequency diagram → Probability density function**

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1.2 Uncertainty in Real-World Information

1.2.2 Uncertainty associated with imperfect modeling and estimation

First

Especially when data are limited

⬇

Estimated values of a given variable(such as the mean) based on observed data will not be error-free,.

Second

The mathematical or simulation models are imperfect representations of reality.

→ **Prediction and/or calculations made on the basis of there models may be inadequate.**

1.3 Design and Decision Making under Uncertainty

1.4 Controls and Standards

In order to Assure
- Some minimum level of Quality
- Performance of engineering products

Necessary

Inspections and Standards of acceptance

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2. Analytical Models of Random Phenomena

2.1 Random Variables

Random Phenomena

→ Numerical Terms

Through

Values of Function

- Outcomes

or

- Events

Should be Identified

Such a function is a random variable which is usually denoted with a capital letter.

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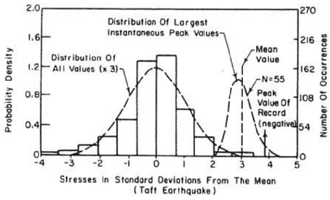
2.2 Probability Distribution of a Random Variable



<u>If X = A Random Variable,</u>

Its probability distribution can always be described by its Cumulative Distribution Function (CDF)

Cumulative Distribution Function (CDF) **Fx(x)**

Probability Density Function (PDF) **fx(x)**

$$f_X(x) \quad = \quad dF_X(x)/dx$$

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2.3 Main Descriptions of a Random Variable

(1) Mean or expected value(a central value)
Some central value, such as mean value
Weighted average-----mean value or expected value

<u>The Mode is the most probable value of random variable</u>

Median

The value of random valuable, above it or below it are equally probable

In general,

- **Mean**
- **Median**
- **Mode of random variable**

} **Different**

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2.3 Main Descriptions of a Random Variable

(2) Variance and standard deviation

Measure of dispersion or **Variability**

Should be no significance

A deviation is above or below the central value

The weighted average of squared deviations

or

Mathematical expectation of second central moment

2.3 Main Descriptions of a Random Variable

Dimensionally,

A more convenient measure of dispersion

⇒ **The square root of the variance**

or

The standard deviation σ

It is hardly to say on the bases of the variance or standard deviation

⇕

whether the dispersion is large or small

The measure of dispersion relative to the central value is more convenient

➡ **Coefficient of Variation(COV)**

2.3 Main Descriptions of a Random Variable

(3) Measure of Skewness

The symmetry or lack of symmetry variability

Third central Moment

2.4 Useful Probability Distributions

(1) The Normal Distribution

The best-known and most widely used probability distribution

- •Normal distribution
- •Gaussian distribution

The standard normal distribution

Gaussian Distribution with parameters

$\mu=0$ $\sigma=1.0$

N(0,1)

Because of its wide use, special notation $\Phi(s)$ is commonly used

Standard normal variant S

That is $\Phi(s) = F_S(s)$, where S has N(0,1)

2.4 Useful Probability Distributions

(2) The logarithmic normal distribution

If lnX(the natural logarithm of X) is normal,

A random variable has a logarithmic normal probability distribution

2.4 Useful Probability Distributions

(3) Bernoulli sequence and the binominal distribution

- **In engineering design and planning**
- **The potential occurrence or recurrence of event**
- **In a sequence of repeated trials**

For example,

- **Annual Max.rain, Annual river flow, Earthquake**
- **The operational conditions of equipment may or may not malfunction over duration of the project**

2.4 Useful Probability Distributions

(3) Bernoulli sequence and the binominal distribution

- **The probability is only two only two possible outcomes**
- **Occurrence or Non occurrence of an event**

These problems may be modeled by "<u>Bernoulli Sequence</u>" which is based on the following assumptions

1. **<u>Each trial has only two possible outcomes</u>**
   - The occurrence or non-occurrence
2. **<u>The probability of occurrence of the event in each trial is constant</u>**
3. **<u>The trials are statically independent</u>**

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2.4 Useful Probability Distributions

(4) Binominal Distribution

p : The probability of occurrence of an event in each trial

1-p : The probability of non-occurrence

Probability of exactly x occurrence among n trials in Bernoulli sequence is given by the binominal PMF as follows

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2.4 Useful Probability Distributions

(5) The geometric distribution

In a Bernoulli sequence, the number of trials until a specified event occurs for the first time is governed by the geometric distribution.

If the first occurrence of event is realized on the t-th trial, then there must be no occurrence of this event in any of the prior (t-1) trials. Therefore, if T is the appropriate random variable,

$P(T=t) = p\, q^{t-1}$ $t=1,2,\ldots$

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2.4 Useful Probability Distributions

(6) The return period

In a time (or space) problem, that can be modeled as a Bernoulli sequence, the number of time (or space) intervals until the first occurrence of an event is called the first occurrence time.

If the individual trials (or intervals) in the sequence are statically independent, the first occurrence time must also be the time between any two consecutive occurrences of the same event; that is, the recurrence time is equal to the first occurrence time.

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2.4 Useful Probability Distributions

(6) The return period

The recurrence time, therefore, in a Bernoulli sequence also has a geometric distribution; the mean recurrence time, which is popularly known in engineering as the average return period is

End