Mechanical Vibration Ⅰ (14)

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Response for an Arbitrary Exciting Force (3)

Impulse response function (1)

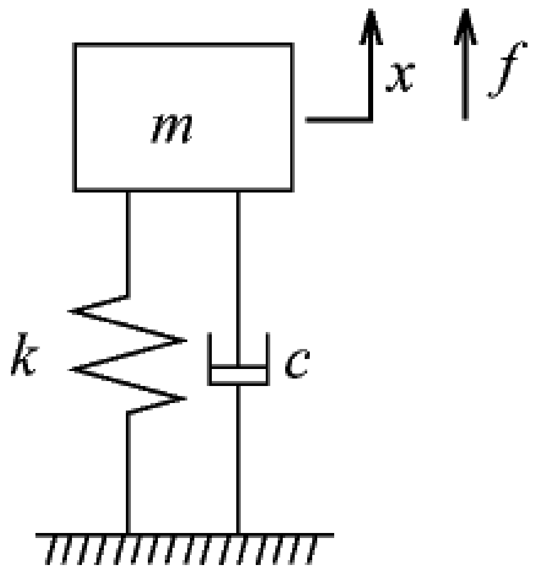

Fig.1 Damped one degree-of-freedom vibrationsystem with force excitation

Equation of motion

$$m\ddot{x} + c\dot{x} + kx = f$$

Unit impulse exciting force

$$f(t) = \delta(t)$$

Dirac's delta function

Impulse response function (2)

Table 1 Change of the states of the system with the unit impulse

Time	Momentum	Velocity	Displacement
$t=0$	$m\dot{x}=0$	$\dot{x}=0$	$x=0$
$t=\epsilon$	$\mathrm{m}\,\dot{x}=1$	$\dot{x}=1/m$	$x=0$

$$x(t)=e^{-\zeta\omega t}\frac{1}{m\omega_d}\sin\omega_d t\equiv h(t)$$

$h(t)$: Impulse response function

Transient response (2-1)

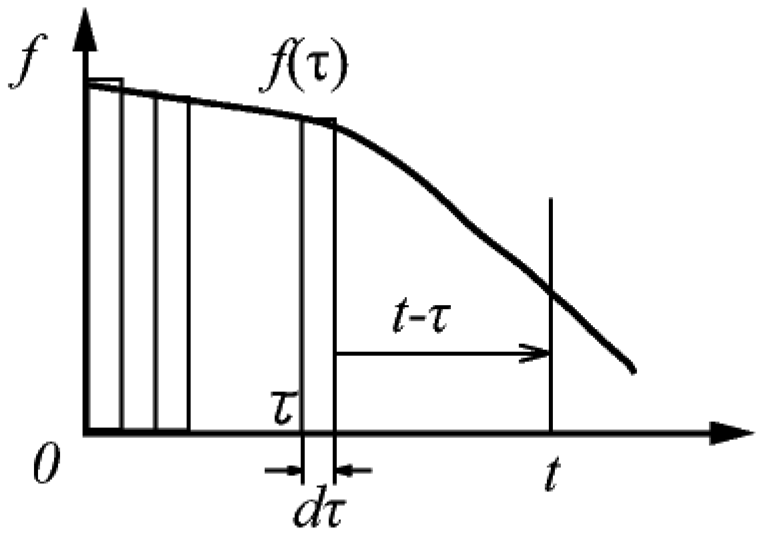

Fig.3 Decomposition of the exciting force into impulses

Transient response

$$x(t) = \int_0^t h(t-\tau) f(\tau) d\tau$$

Transient response (2-2)

Example

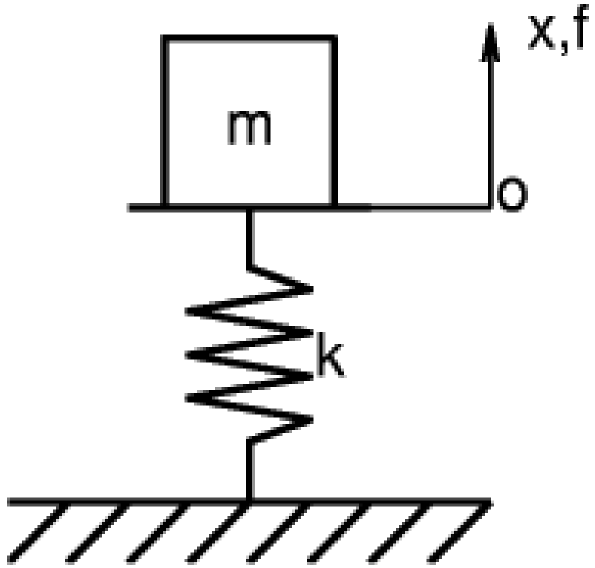

Fig.4 Undamped one degree-of-freedom vibration system

Impulse response function

$$h(t) = \frac{1}{m\omega_n} \sin \omega_n t$$

Transient response (2-3)

Example

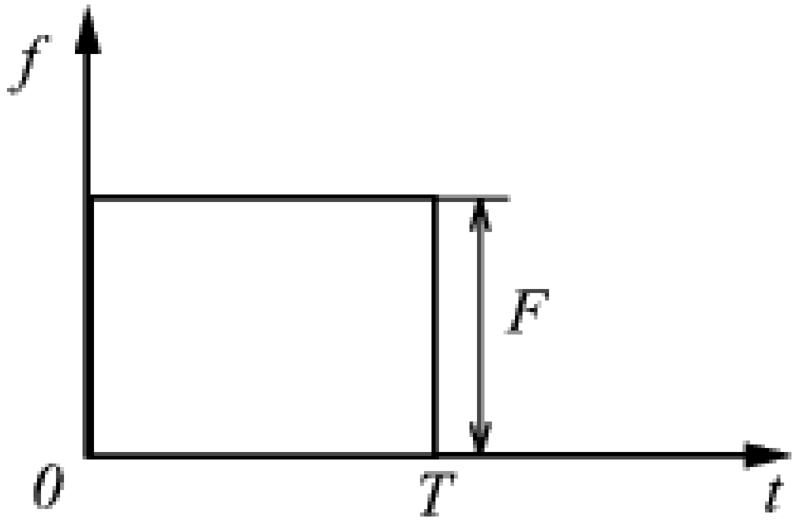

Fig.5 Exciting force

Exciting force

$$f(t) = \begin{cases} 0 & (t < 0) \\ F & (0 \le t \le T) \\ 0 & (t > T) \end{cases}$$

Transient response (2-4)

Example

$$x(t) = \begin{cases} 0 & (t < 0) \\ \int_0^t Fh(t-\tau)dt & (0 \le t \le T) \\ \int_0^T Fh(t-\tau)d\tau & (t > T) \end{cases}$$