Mechanical Vibration Ⅰ (13)

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Response for an Arbitrary Exciting Force (2)

Transient response (1-1)

Define the frequency response function $H(i\ \omega)$

$$H(i\omega) = \frac{X(i\omega)}{F}$$

Assume a complex exciting force

$$f(t) = F\exp(i\omega t)$$

Response against the complex exciting force

$$x(t) = FH(i\omega)\exp(i\omega t)$$

Transient response (1-2)

Inverse Fourier transformation of the exciting force

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\omega) \exp(i\omega t) d\omega$$

Response against the exciting force

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\omega) H(i\omega) \exp(i\omega t) d\omega$$

Fourier transformation of the response

$$X(i\omega) = F(i\omega) H(i\omega)$$

Transient response (1-3)

Necessary condition of the Laplace transformation is not strict compared to that of the Fourier transformation.

$$x(t) = \frac{1}{2\pi} \int_{\sigma - i\infty}^{\sigma + i\infty} F(s) H(s) \exp(st) ds$$

where

$$F(s) = \int_0^\infty f(t) \exp(-st) dt$$

$$H(s) = \int_0^\infty h(t) \exp(-st) dt = H(i\omega) \big|_{i\omega = s}$$

Transient response (1-4)

Table 2 Laplace transformation

Time function	Laplace transformation
$\delta(t)$	1
$u(t)$ (Unit function)	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$f(t-T)$	$F(s)e^{-sT}$
$\frac{d}{dt}f(t)$	$sF(s)-f(0)$
$\frac{d^2}{dt^2}f(t)$	$s^2F(s)-sf(0)-\frac{d}{dt}f(0)$

Transient response (1-5)

Example 1　Unit impulse response function

$$m\ddot{x} + c\dot{x} + kx = f \quad \text{and} \quad f(t) = \delta(t)$$

$$\Rightarrow \quad ms^2 X(s) + csX(s) + kX(s) = 1$$

$$\Rightarrow \quad X(s) = \frac{1}{ms^2 + cs + k} \qquad \text{Transfer function}$$

$$= \frac{1}{m}\frac{i}{2\omega_d}\left(\frac{-1}{s-\lambda_1} + \frac{1}{s-\lambda_2}\right)$$

where $\lambda_{1,2} = -\zeta\omega_n \pm i\omega_d$

Transient response (1-6)

Example 1　Unit impulse response function

$$x(t) = \frac{1}{m}\frac{i}{2\omega_d}\left(-e^{\lambda_1 t} + e^{\lambda_2 t}\right)$$

$$= e^{-\zeta\omega_n t}\frac{1}{m\omega_d}\sin\omega_d t \equiv h(t)$$

Transient response (1-7)

Example 2

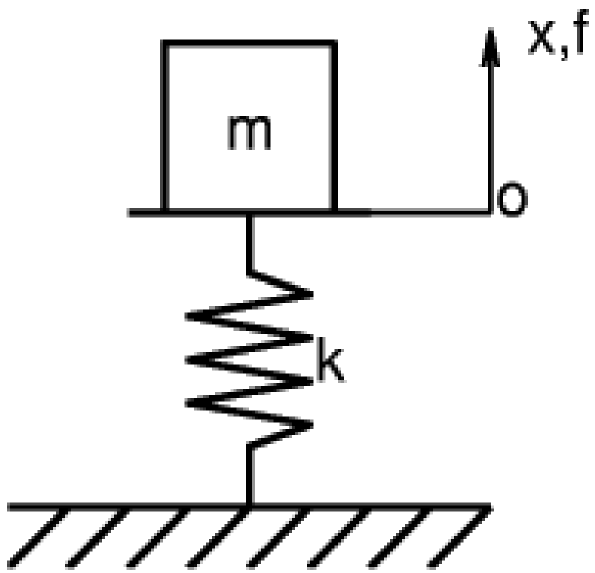

Fig.4 Undamped one degree-of-freedom vibration system

Transfer function

$$H(s) = \frac{1}{m}\frac{1}{s^2 + \omega_n^2}$$

Transient response (1-8)

Example 2

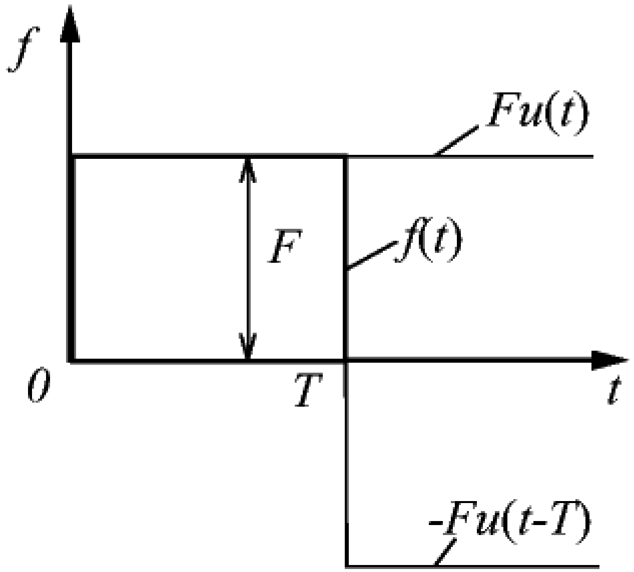

Fig.6 Decomposition of the exciting force into unit step functions

Exciting force

$$f(t) = \begin{cases} 0 & \\ Fu(t) & (0 \le t \le T) \\ Fu(t) - Fu(t-T) & (t > T) \end{cases}$$

Transient response (1-9)

Example 2

$$x(t) = \begin{cases} 0 & 0 \\ L^{-1}\left[H(s)\dfrac{F}{s} \right] & (0 \le t \le T) \\ L^{-1}\left[H(s)\dfrac{F}{s}\left(1 - e^{-sT}\right) \right] & (t > T) \end{cases}$$

$L^{-1}[\ ]$: Inverse Laplace transformation