Mechanical Vibration I (8)

Department of Mechanical and Control Engineering

Hiroshi Yamaura

System Identification and Harmonic Excitation Response

Identification of system parameter (1)



Fig.5 Free vibration of an actual vibration system to be modeled

Identification of system parameter (2)

Natural period

Logarithmic damping ratio

$$\delta = \frac{1}{N} \sum_{i=1}^{N} \ln\left(\frac{A_i}{A_{i+1}}\right)$$

Damping ratio



Open Course Ware, 2009, Tokyo Institute of Technology Copyright by Hiroshi Yamaura

 T_n

Identification of system parameter (3)

Case 1) *m* can be measured

$$k = m\omega_n^2 \quad c = 2\sqrt{mk}\zeta$$

Case 2) k can be measured

$$m = \frac{k}{\omega_n^2} \quad c = 2\sqrt{mk}\zeta$$

Harmonic exciting force and response (1)

Undamped one degree-of-freedom system

$$m\ddot{x} + kx = f$$

Exciting force

Response

$$f_c = F \cos \omega t$$

 $x(t) = A\cos(\omega t + \phi)$

$$A = \frac{F}{k - m\omega^2} \qquad \phi = \begin{cases} 0 & 0 \le \omega < \omega_n \\ -\frac{\pi}{2} & \omega = \omega_n \\ -\pi & \omega > \omega_n \end{cases}$$

Harmonic exciting force and response (2)

Damped one degree-of-freedom system

$$m\ddot{x} + c\dot{x} + kx = f$$

Exciting force

Response

 $f_c = F \cos \omega t$ $x(t) = A \cos(\omega t - \phi)$

$$A = \frac{F}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad \phi = \tan^{-1}\left(\frac{-c\omega}{k - m\omega^2}\right)$$