Mechanical Vibration Ⅰ (7)

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Damped Free Vibration System

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Analysis of a damped one degree-of-freedom vibration system (1)

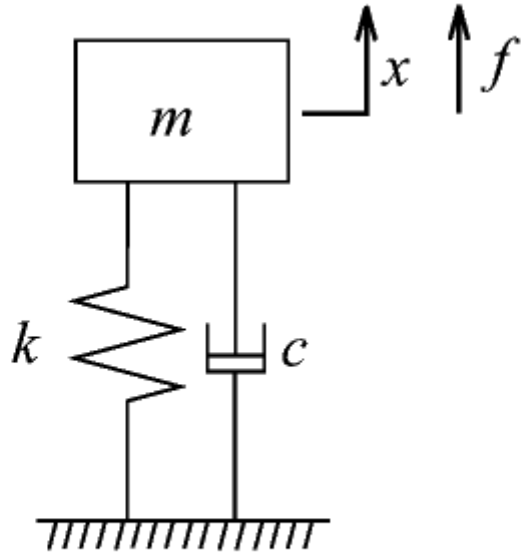

Fig.3 Damped one-degree-of-freedom vibration system

$$m\ddot{x} + c\dot{x} + kx = f$$

Analysis of a damped one degree-of-freedom vibration system (2)

Free Vibration $m\ddot{x} + c\dot{x} + kx = 0$

Free Vibration Response $x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

where

$$\lambda_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$\zeta = \frac{c}{2\sqrt{mk}}$$ Damping Ratio

Damping ratio and initial value response (1)

(a) Unstable $\zeta < 0$

$$x(t) = \frac{x_0\lambda_2 - v_0}{\lambda_2 - \lambda_1} e^{\lambda_1 t} + \frac{x_0\lambda_1 - v_0}{\lambda_1 - \lambda_2} e^{\lambda_2 t}$$

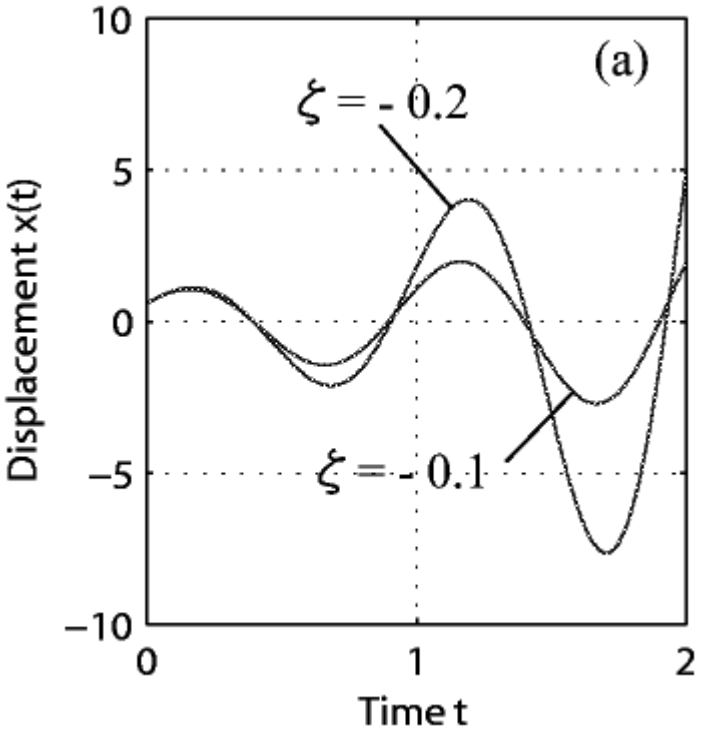

Fig.4 Initial value response
$(x_0 = 0.6, v_0 = 0.8\omega_n, \omega_n = 2\pi)$

Damping ratio and initial value response (2)

(b) Under damping $0 < \zeta < 1$

$$x(t) = A_0 e^{-\zeta\omega_n t}\cos(\omega_d t - \phi)$$

$$A_0 = \sqrt{x_0^2 + \left(\frac{x_0\zeta\omega_n + v_0}{\omega_d}\right)^2}$$

$$\phi = \tan^{-1}\left(\frac{x_0\zeta\omega_n + v_0}{x_0\omega_d}\right)$$

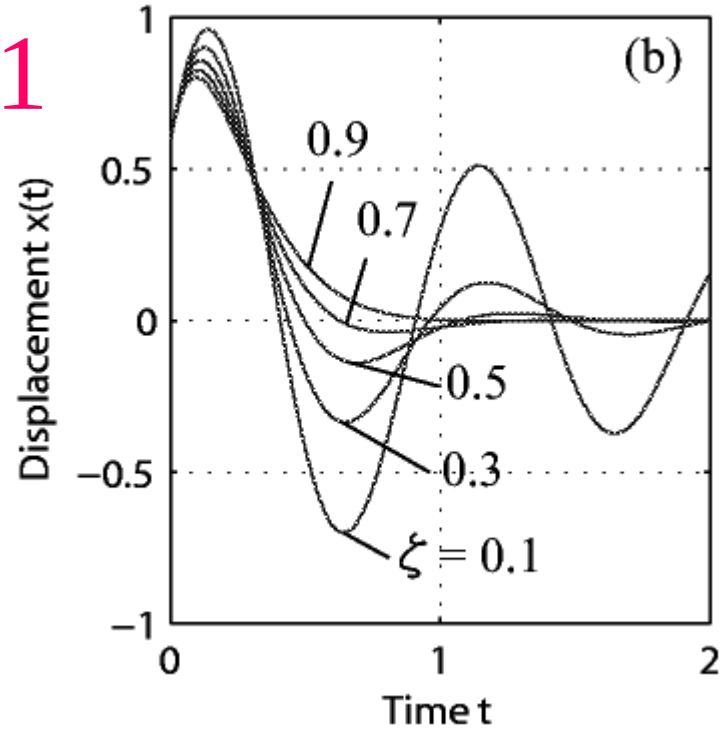

Fig.4 Initial value response
$(x_0 = 0.6, v_0 = 0.8\omega_n, \omega_n = 2\pi)$

Damping ratio and initial value response (3)

(c) Critical damping

$$\zeta = 1$$

$$x(t) = x_0 e^{\lambda_1 t} + (v_0 - x_0\lambda_1) t e^{\lambda_1 t}$$

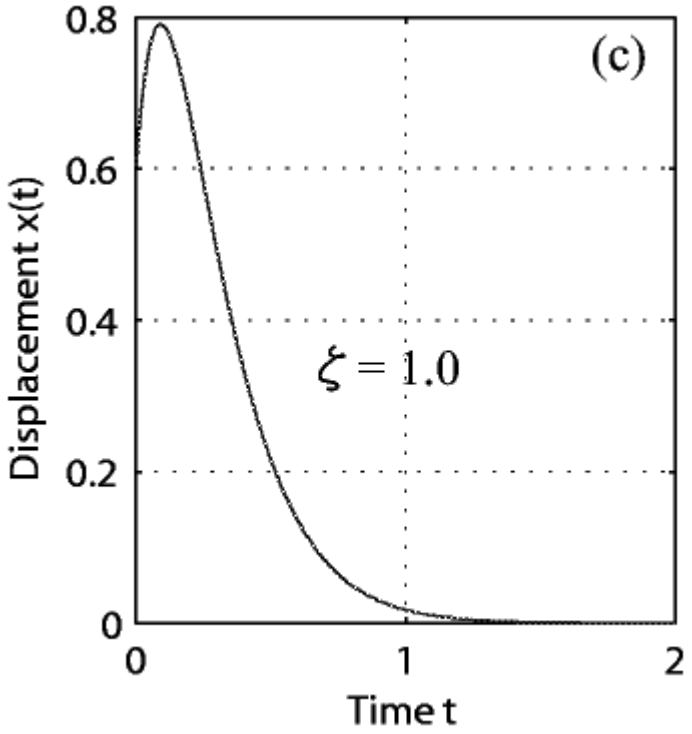

Fig.4 Initial value response
$(x_0 = 0.6, v_0 = 0.8\omega_n, \omega_n = 2\pi)$

Damping ratio and initial value response (4)

(d) Over damping

$$\zeta > 1$$

$$x(t) = \frac{x_0\lambda_2 - v_0}{\lambda_2 - \lambda_1} e^{\lambda_1 t} + \frac{x_0\lambda_1 - v_0}{\lambda_1 - \lambda_2} e^{\lambda_2 t}$$

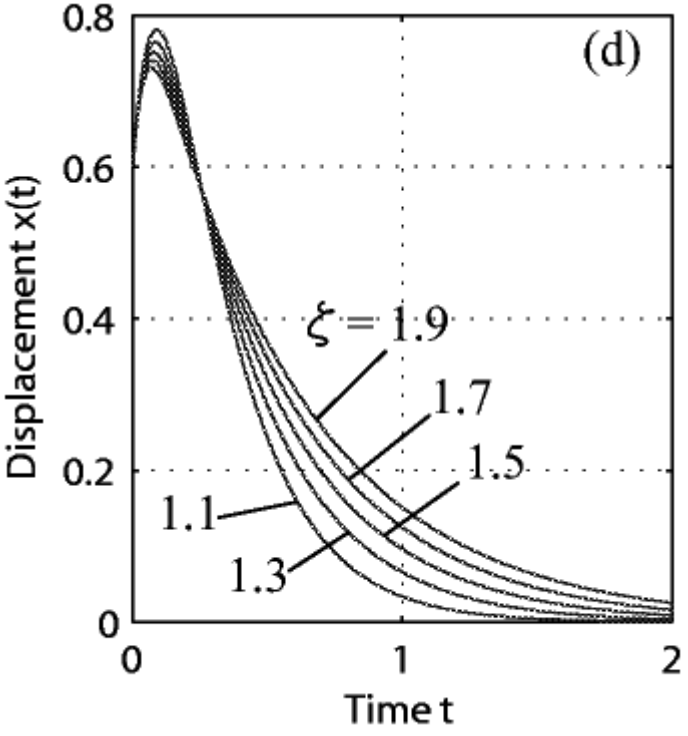

Fig.4 Initial value response
$(x_0 = 0.6, v_0 = 0.8\omega_n, \omega_n = 2\pi)$