# Mechanical Vibration Ⅰ (3)

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# Undamped One-degree-of-freedom Vibration System

Natural frequency

Free vibration

# Natural angular frequency

Equation of Motion of an undamped free vibration system

$$m\ddot{x} + kx = 0$$

The characteristic equation

$$m\lambda^2 + k = 0 \quad \Rightarrow \quad \lambda = \pm i\,\omega_n, \quad \omega_n = \sqrt{\frac{k}{m}}$$

# Free vibration – Initial value response (1)

General solution

$$x(t) = C_1 e^{i\omega_n t} + C_2 e^{-i\omega_n t}$$

$$= D_1 \cos \omega_\mathrm{n} t + D_2 \sin \omega_\mathrm{n} t$$

$$= A \cos (\omega_\mathrm{n} t - \phi)$$

# Free vibration – Initial value response (2)

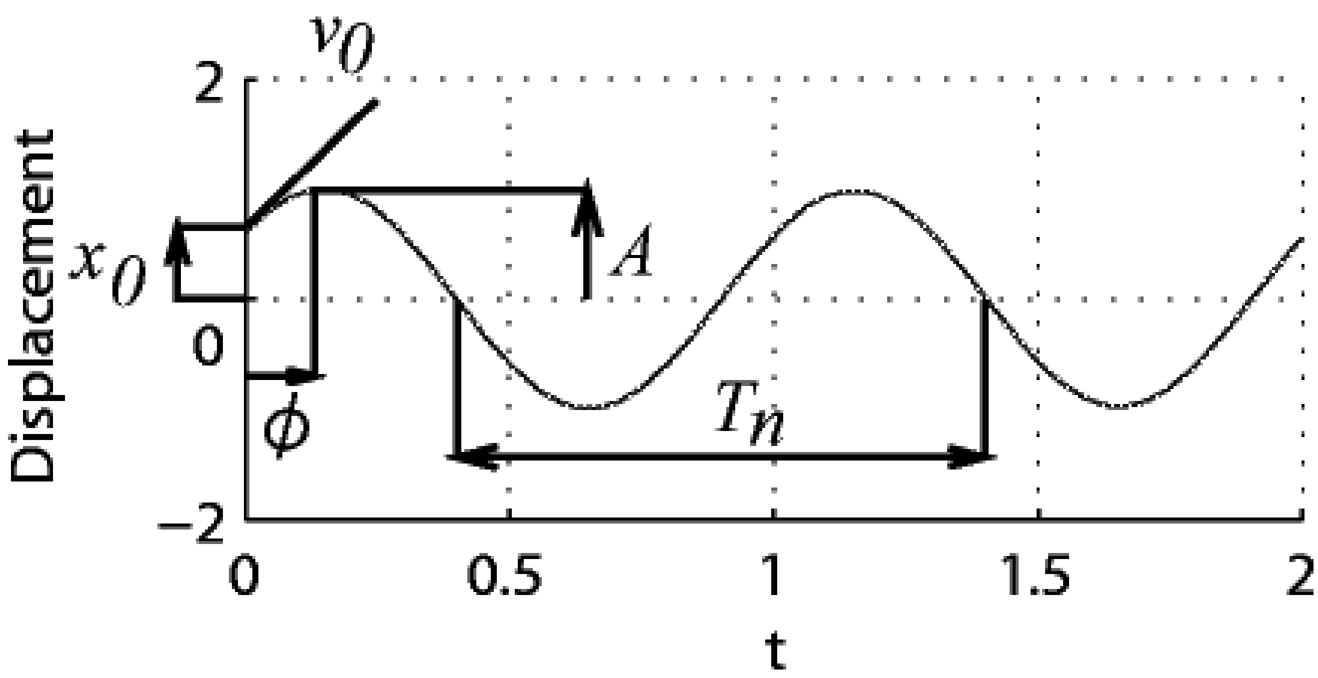


Fig.6 Example of initial value response

# Natural angular frequency, natural frequency and natural period

Table 2 Important parameters

| Notation | Unit | Meaning | |
|---|---|---|---|
| $\omega_n$ | rad/s | Natural angular frequency | $=\sqrt{\frac{k}{m}}$ |
| $f_n$ | Hz (=1/s) | Natural frequency | $=\frac{\omega_n}{2\pi}$ |
| $T_n$ | s | Natural period | $=\frac{1}{f_n}=\frac{2\pi}{\omega_n}$ |