Mechanical Vibration Ⅰ (2)

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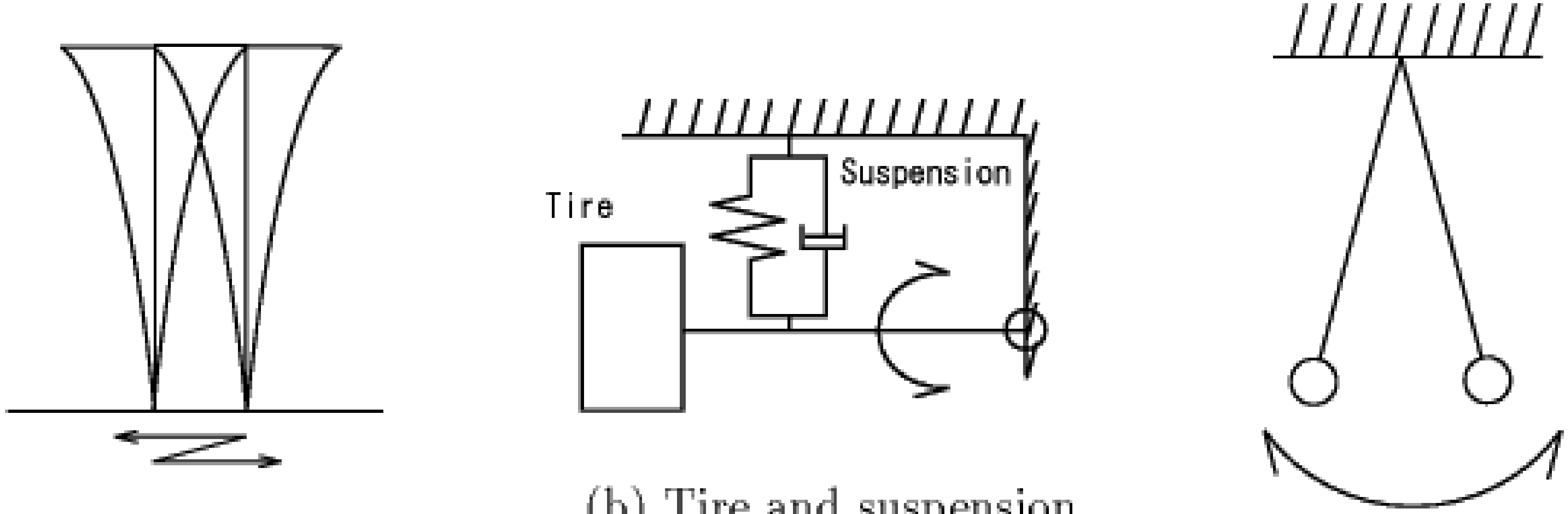
One-degree-of-freedom Vibration System

Actual vibration system (1)



(a) Tall Building

(b) Tire and suspension

(c) Simple pendulum

Fig.1 Actual vibration systems

Actual vibration system (2)

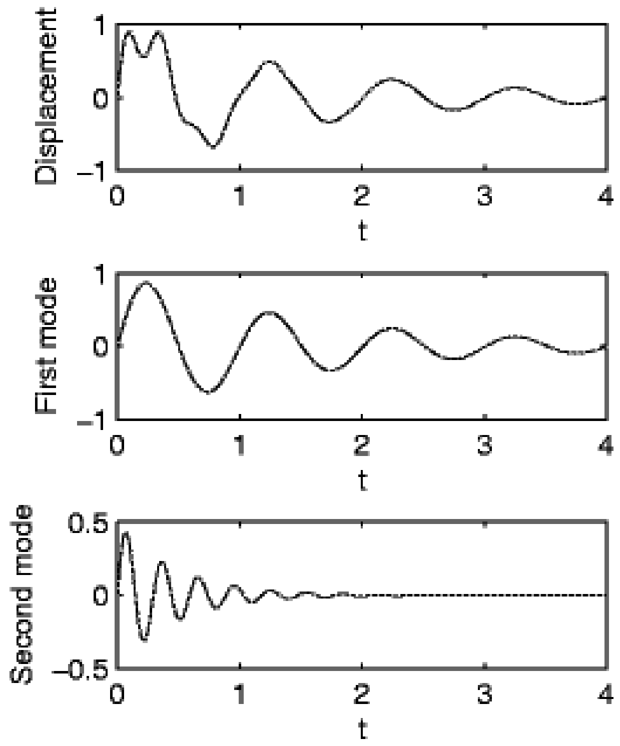


Fig.2 Example of free vibration

In many cases, the first mode is dominant in the free vibration.

Most vibration systems can be modeled as one degree-of-freedom vibration system.

Multi-degree-of-freedom system can be represented as a superposition of one-degree-of-freedom vibration systems.

Analytical model

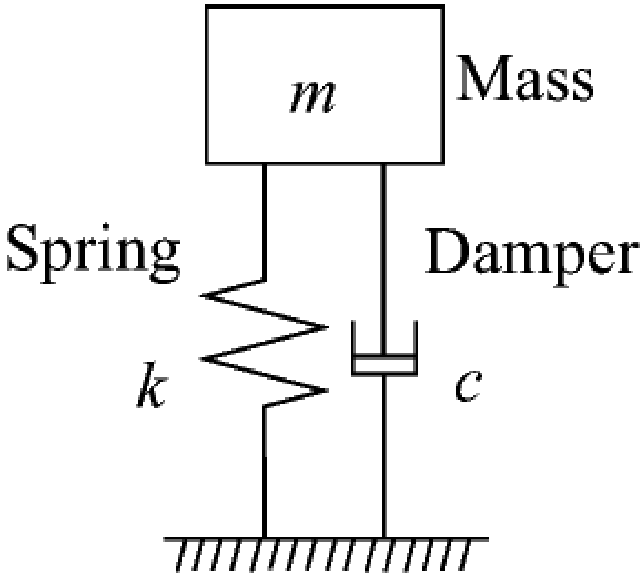


Fig.3 Analytical model of
one-degree-of-freedom vibration system

SI Unit

m : kg

k : N/m   Stiffness constant

c : Ns/m   Damping constant

Deriving the equation of motion (1)

- Coordinate system
  - The origin of the displacement should be placed on the equilibrium point.
  - The direction of the force should agree with that of the displacement. Both of the Case 1 and 2 in Fig.4 are acceptable.

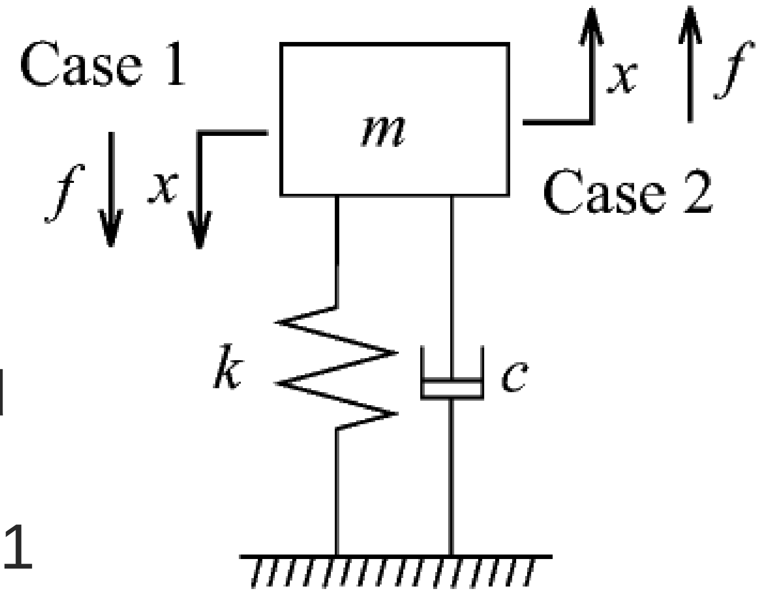


Fig.4 Coordinate system

Deriving the equation of motion (2)

- Reaction Force
  - Reaction force of the damper is proportional to the velocity of the mass and its sign is minus.
  - Reaction force of the spring is proportional to the displacement of the mass and its sign is minus.

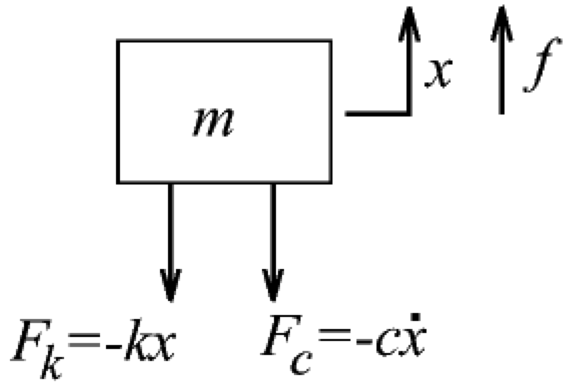


Fig.5 Free body and acting force

Deriving the equation of motion (3)

- d'Alembert's principle (1)

The sum of the differences between the generalized forces acting on a system and the time derivative of the generalized momentum of the system itself along an infinitesimal displacement compatible with the constraints of the system (a virtual displacement), is zero.

Deriving the equation of motion (4)

- d'Alembert's principle (2)

$$\left\{ F - \frac{d(mv)}{dt} \right\} \delta x = 0$$

where $$F = -c\dot{x} - kx + f$$

Thus, the equation of motion is represented as the following.

$$m\ddot{x} + c\dot{x} + kx = f$$