Explanation of teleportation

Before teleportation:

$$(\alpha|0\rangle + \beta|1\rangle)\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

After teleportation:

$$|??\rangle(\alpha|0\rangle + \beta|1\rangle),$$

where ? is 0 or 1.

Superdense coding

1 qubit can carry at most 1 bit of information.

$$\Uparrow$$

Since 2×2 matrix has at most 2 eigenvalues, the number of measurement outcomes of measuring 1 qubit is at most 2.

Superdense coding sends 2 bits of information by sending 1 qubit.

- The sender and receiver are spatially apart.
- They share
$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

Orthogonal states can be distinguished

$\{|\varphi_1\rangle, \ldots, |\varphi_n\rangle\}$: orthonormal basis of a linear space $\mathcal{H}$.
If we know the state of a system is in one of $\{|\varphi_1\rangle, \ldots, |\varphi_n\rangle\}$, then we can distinguish them.

$$A = \sum_{k=1}^{n} k|\varphi_k\rangle\langle\varphi_k| \tag{2}$$

- A is a Hermitian matrix.
- If the state is one of $\{|\varphi_1\rangle, \ldots, |\varphi_n\rangle\}$, then we can distinguish them by measuring A.

More formally, the state before measurement is $|\varphi_k\rangle$ **if** and **only if** the measurement outcome is k.

Superdense coding 2

The sender applies either I, X, Z, or XZ to his physical system. This color represents the sender and this color represents the receiver.

$$(X \otimes I)\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|10\rangle + |01\rangle}{\sqrt{2}}, \tag{3}$$

$$(Z \otimes I)\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|00\rangle - |11\rangle}{\sqrt{2}}, \tag{4}$$

$$(XZ \otimes I)\frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{|10\rangle - |01\rangle}{\sqrt{2}}. \tag{5}$$

The above three states and $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ form an orthonormal basis of the state space of 2 qubits.

Superdense coding 3

The sender sends his physical system to the receiver.
The receiver has 2 qubits, and the state of 2 qubits is either (3), (4), (5), or $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$.
Since they are orthogonal, they can be distinguished by measuring an appropriate observable.
The receiver can distinguish 4 states, and thus he/she can obtain two bits of information.

Exercise

1. Show that the matrix (2) is Hermitian.
2. Derive the identity (5) in detail. The lecturer explain the derivation of (3) in detail upon request. (等式 (5) の導出を詳細に書け。講師は要請があれば (3) の導出を詳細に書く)
3. Show that the inner product of the vectors (3) and (5) is zero. The lecturer shows the computation of the inner product of $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ and the vector (4) upon request.
4. Prove that the measurement outcome is k **if** the state **before measurement** is $|\varphi_k\rangle$ in page 2009-5-10. (日本語訳：もし測定前の状態が $|\varphi_k\rangle$ ならば測定結果が k であることを証明せよ)
5. Prove that the state **before measurement** is $|\varphi_k\rangle$ **if** the measurement outcome is k in page 2009-5-10. (日本語訳：もし測定結果が k ならば測定前の状態は $|\varphi_k\rangle$ であることを証明せよ)
6. If your answers to the previous exercises were evaluated as incorrect, please indicate whether or not you agree to that evaluation. What topic do you want to be included in this lecture? Do you understand the superdense coding? If not, write which part was dif-

ficult for your understanding.

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