Digital Modulation & Demodulation

Agenda

- Channel Capacity
- Modulation and Coding
- Digital Modulation
- Degradation
- AMC
- Non-binary Modulation

Channel Capacity of Discrete-time memory-less Gaussian Channel with Bandwidth *W*

$$C = \frac{1}{2}\log_2\left(1+\frac{p}{\sigma^2}\right)\times 2W\ [\mathrm{bps}]$$

$$p = Signal - Power$$

$$\sigma^2 = Noise - Power$$

AWGN Channel

$$Y = X + N$$

X : Transmitted Signal

N : Additive Noise

Y : Received Signal

$\overline{X^2} = P$: Signal Power

$\overline{N^2} = \sigma^2$: Noise Power

$$\overline{Y^2} = \overline{X^2} + \overline{N^2} = P + \sigma^2$$

Mutual Information between X and Y

$$I(X:Y) = H(Y) - H(Y|X)$$
$$= H(X) - H(X|Y)$$

$H(\quad)$: Entropy

$H(\quad|\quad)$: Conditional Entropy

When X, N : Gaussian

$$I(X:Y) \rightarrow \text{Max}$$

$$\text{Max } I(X:Y) = \frac{1}{2}\log_2(\overline{Y^2}) - \frac{1}{2}\log_2(\overline{N^2})$$
$$= \frac{1}{2}\log_2((P+\sigma^2)/\sigma^2)$$
$$= \frac{1}{2}\log_2(1+(P/\sigma^2))$$

Sampling Theorem

If signal has a bandwidth of W [Hz],

2W samples in sec are maximum number of independent data

Channel Capacity

$$C = \frac{1}{2}\log_2\left(1+\frac{P}{\sigma^2}\right)\times 2W \qquad \text{[bps]}$$

Capacity when Interference exists

- Y=X+I+N
- Both TX and RX know I : C does not change
- Both TX and RX do not know I : C decreases
- TX knows but RX does not know : C does not change !? ⇒ "Dirty Paper Coding"

Review of Digital Modulation

- Criterion on Modulation Scheme

$$\frac{C}{W} = \log_2\left[1 + \frac{E_b}{N_0} \times \frac{C}{W}\right] \quad \text{Band Efficiency (Shannon, 1949)}$$

C : Channel Capacity [bit / s]

W : Bandwidth [Hz]

E_b : Required Energy per bit [Joule]

N_0 : Noise Power Spectrum per Hz [Watt / Hz]

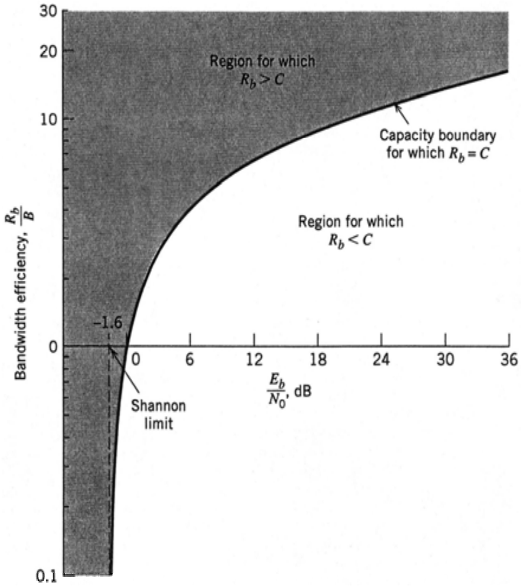
Reliable (Error-free) Communication

Data Transmission Rate, R

$$R < C$$

Inverse Coding Theorem

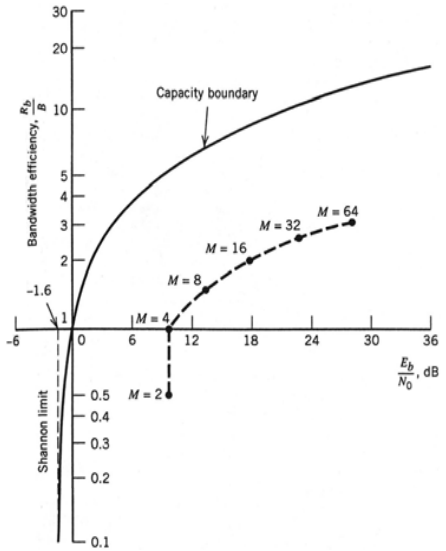
- If R > C , error probability of code word becomes 1
- No reliable communication !



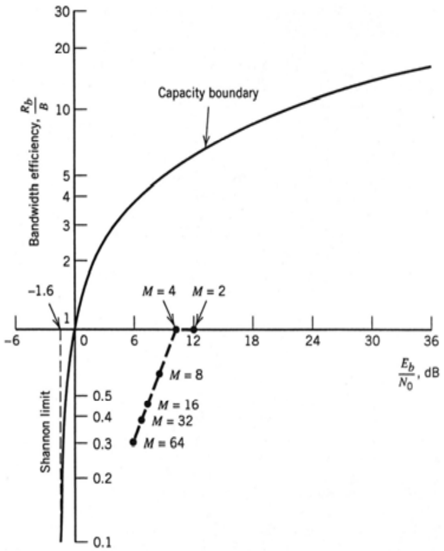
$C/W \to 0,\ E_b/N_0 = \ln 2\ (-1.6\ \text{dB})$: Shannon Limit

$C/W > 1$: Band-limited Region, → Multi-level QAM

$C/W < 1$: Power-limited Region, → Multi-level PSK



(a) QAM



(b) PSK

Channel Coding

- Introduction of Adequate Redundancy
- Reduction of bit error rate
- FEC (Forward Error Correction)

Rate, BER and SNR in BPSK

For BPSK $M(\sigma^2) = \sum_{a_i=\pm 1} \int_{y_i} p(a_i, y_i) \log \frac{p(a_i, y_i)}{p(a_i)p(y_i)} dy_i$

$$= \underbrace{\sum_{a_i=\pm 1} \int_{y_i} p(a_i, y_i) \log p(y_i | a_i) da_i dy_i}_{\text{Entropy of Gaussian noise}} - \underbrace{\int_{y_i} p(y_i) \log p(y_i) dy_i}_{\text{Approximated using Monte Carlo}}$$

$$R < M(\sigma^2) = M\left(\frac{1}{2R\, E_b/N_o}\right) \Rightarrow E_b/N_o > \frac{1}{2RM^{-1}(R)}$$

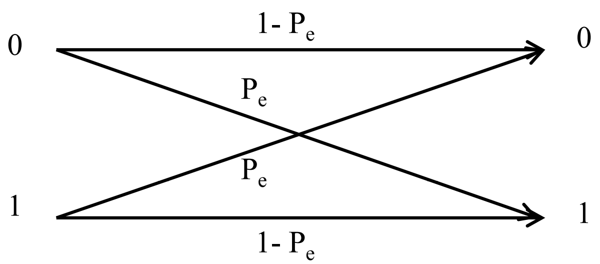
For rate R and given BER, what is the minimum SNR???

With given BER, mutual information is $1 + BER \log(BER) + (1 - BER)\log(1 - BER)$

New code-rate is $R' = R\big(1 + BER \log(BER) + (1 - BER)\log(1 - BER)\big)$

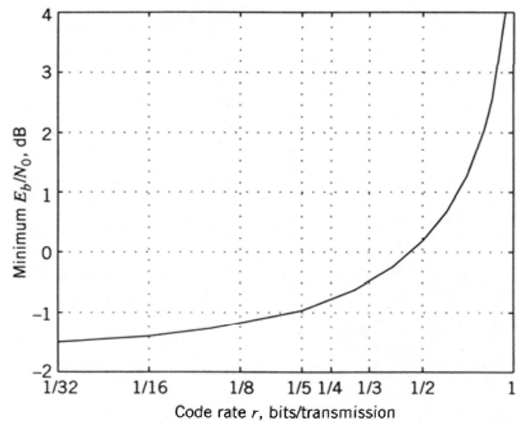
Then we have $\sigma^2 = M^{-1}(R') \Rightarrow E_b/N_o = \frac{1}{2\sigma^2 R}$

BER : P_e vs. Entropy H

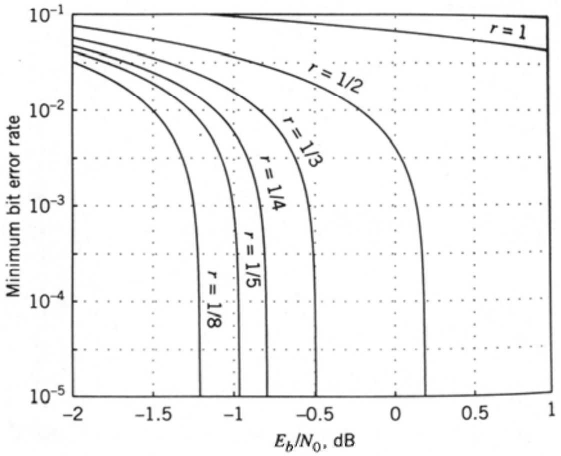


$$H = 1 + (1 - P_e)\log_2(1 - P_e) + P_e \log_2 P_e$$

Error-free Min E_b / N_0 vs. Code Rate (r) BPSK over AWGN Channel



BER vs. E_b / N_0



Basics of Analog & Digital Modulation

Baseband signal : $g(t)$

$\downarrow$

Modulated signal : $s(t) = A(t)\cos[2\pi f_c t + \phi(t)]$

Amplitude Modulation : $A(t) \leftarrow g(t)$, ASK (**Amplitude** Shift Keying)

Phase Modulation : $\phi(t) \leftarrow g(t)$, PSK (**Phase** Shift Keying)

Frequency Modulation : $\partial\phi(t)/\partial t \leftarrow g(t)$, FSK (**Frequency** Shift Keying)

Fundamentals of Demodulation

Incoherent Scheme	Envelope Detection Frequency Discrimination	ASK, FSK FSK
Coherent Scheme	Coherent Detection Delayed Detection	PSK, FSK, ASK PSK, FSK

Incoherent Scheme

→ (Envelop)Detector + Filter

Coherent Scheme

→ Mixer (Multiplier) + LO (Local Oscillator)

Optimum Detection Scheme

Quality of demodulated signal is BER (Bit Error Rate).

BER is mainly determined by SNR (Signal-to-Noise Ratio).

SNR should be maximized.

– **Matched Filter**

← Radar Signal Detection, Maximizing SNR, but not good signal waveform recovery

Matched Filter : $H(f)$

Noise : $n(f) = N_0/2$, White Gauss Noise

(Input) Signal : $S_i(f)$ fixed uniquely

Output noise power, $P_n = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 \,\mathrm{df}$

Output signal power at T_s,

$$P_s = |s_o(T_s)|^2 = \left| \int_{-\infty}^{\infty} S_i(f) H(f) \exp(j2\pi T_s) \mathrm{df} \right|^2$$

By Schwarz' Inequality, Maximum SNR,

$\gamma_{\max} = P_s / P_n$ can be obtained at

$$H(f) = S_i^*(f) \exp(-j2\pi fT_s)$$
$$h(t) = S_i^*(T_s - t)$$

$$\gamma_{\max} = \frac{2}{N_0}\int_{-\infty}^{\infty} |S_i(f)|^2 \,\mathrm{df}$$

$=$ Signal Energy/Noise Power Spectrum Density

– Correlation Detection:

– Output signal from Matched filter sampled at T_s is a correlation between received signal $r(t)$ and input signal $s_i(t)$.

$$s_o(T_s) = \int_0^{T_s} r(u)\, s_i(u)\, \mathrm{du}$$

– Maximum Likelihood Detection:
Minimizing BER
MAP (Maximum a posteriori probability) estimation

Maximum Likelihood sequence estimation

$\text{Max Prob}\left(\mathbf{s}_i | \mathbf{r}\right)$

$\mathbf{s}_i$: input sequence

$\mathbf{r}$: received sequence $= \mathbf{s}_i + \mathbf{n}$

$\mathbf{n}$: noise sequence

$\rightarrow \text{Min}\left(|\mathbf{s}_i - \mathbf{r}|^2\right) \rightarrow \text{Max}\left(\mathbf{r}\cdot\mathbf{s}_i - \frac{1}{2}|\mathbf{s}_i|^2\right)$

$\rightarrow$ Correlation detection $\text{Max}\left(\mathbf{r}\cdot\mathbf{s}_i\right)$

MSK: Power Efficiency Oriented

- MSK (Minimum Shift Keying): Constant Envelope Modulation

Mark-signal and space-signal $(0 \le t \le T)$

(T : Symbol Duration Time)

$$s_{\text{mark}}(t) = \cos\left(2\pi f_c t + \pi \Delta f t\right)$$
$$s_{\text{space}}(t) = \cos\left(2\pi f_c t - \pi \Delta f t\right)$$

Correlation ρ between $s_{\text{mark}}(t)$ and $s_{\text{space}}(t)$

$$\rho = \int_0^T s_{\text{mark}}(t)\, s_{\text{space}}(t)\,\mathrm{dt} \approx \frac{\sin\left(2\pi\Delta f T\right)}{4\pi\Delta f} \rightarrow 0$$

$\Delta f = 1/2T$ is a **minimum frequency shift**.

$$s_{\text{mark}} = \cos\left(2\pi f_c t\right)\cos\left(\pi\Delta f t\right) - \sin\left(2\pi f_c t\right)\sin\left(\pi\Delta f t\right)$$
$$s_{\text{space}} = \cos\left(2\pi f_c t\right)\cos\left(\pi\Delta f t\right) + \sin\left(2\pi f_c t\right)\sin\left(\pi\Delta f t\right)$$

Similar to OQPSK (Offset QPSK)

MSK : cosine modulation : Spectrum $\left[\frac{\cos 2\pi fT}{1-16 f^2 T^2}\right]^2$

OQPSK : rectangular modulation : Spectrum $\left[\frac{\sin 2\pi fT}{2\pi fT}\right]^2$

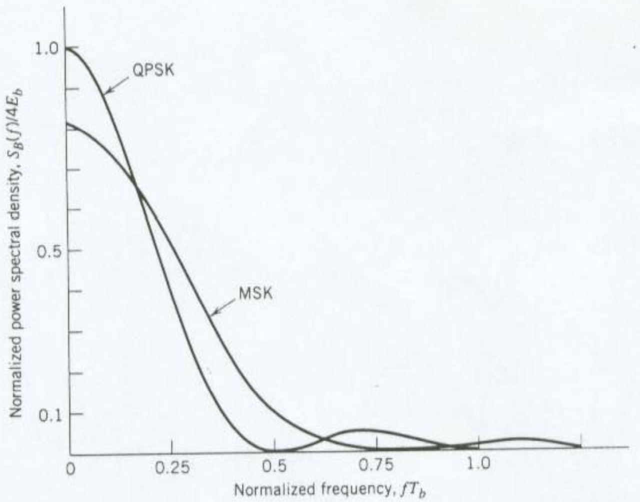


FIGURE Power spectra of QPSK and MSK signals.

- Narrowing Band of MSK : Main-lobe of MSK is wider than those of QPSK, OQPSK.
  → Partial response technique for narrowing band
  - TFM (Tamed FM): similar to 8 PSK
    Phase shift by digital data $(a_k = \pm 1)$

$$\text{MSK}: \phi_{k+1} - \phi_k = \frac{\pi}{2}(a_k)$$

$$\text{TFM}: \phi_{k+1} - \phi_k = \frac{\pi}{2}\left(\frac{a_{k-1}}{4} + \frac{a_k}{2} + \frac{a_{k+1}}{4}\right)$$

- GMSK (Gaussian-filtered MSK): European countries standard, GSM
  - Narrow Main-Lobe Spectrum
  - Good off-band Spectrum f^{-4}
  - Almost Constant Envelope → High Efficient Power Amplifiers are available
  - Good Eye Pattern → Low BER

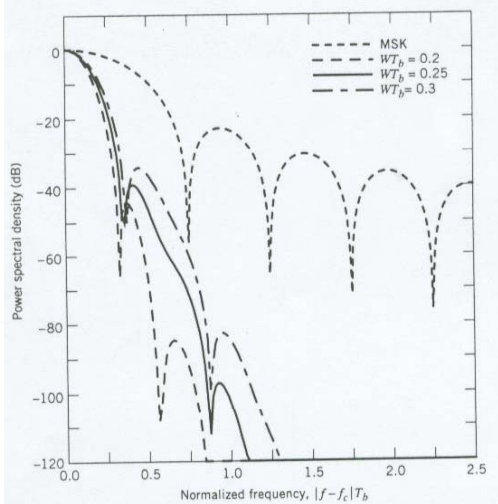


FIGURE Power spectra of MSK and GMSK signals for varying time-bandwidth product. (Reproduced with permission from Dr. Gordon Stüber, Georgia Tech.)

– Multi-level MSK:

4-valued FSK $\sim$ $\pi/4$ shift QPSK
Frequency Discrimination Detection is available

Demodulation Characteristics

- CNR vs. E_b/N_0

$$\frac{C}{N} = \frac{E_b}{N_0} \times \frac{1}{\beta BT}$$

β : Ratio of Equivalent Noise Bandwidth to 3dB Bandwidth
(e.g. $\sqrt{\frac{\pi}{\ln 2}}/2 \approx 1.06$ for Gaussian Filter)
B : 3dB Bandwidth
T : 1 bit Duration Time

- BER of Coherent Detection:

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{E_b}{N_0}}\right]$$

$$\mathrm{erfc}\left[x\right] = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}\,dt : \text{complementary error function}$$

$$\cong \frac{1}{\sqrt{\pi}x}e^{-x^2} \quad (x \gg 1)$$

- BER of Delayed Detection (Differential Detection):
Carrier Regeneration is not necessary.

$$P_e = \frac{1}{2}\exp\left[-\frac{E_b}{N_0}\right]$$

- Frequency Discriminator:
outputs an instantaneous frequency
No Carrier Regeneration

Linear Modulation: Bandwidth Efficiency Oriented

Recently, a highly efficient class-F power amplifier is available.
Cell size becomes small.

- PSK
  - QPSK (Quadri PSK) and $\pi/4-$shift QPSK: PDC, PHS in Japan
  - 1 symbol = 2 bits
    Merit of $\pi/4-$shift QPSK
    - Small Envelope Fluctuation
    - Easy Timing Recovery.

- OPSK (Offset QPSK), SQPSK (Staggered QPSK)
  $T/2$ offset between I-channel baseband signal and Q-channel baseband signal
  Power spectrum of OQPSK is the same as those of QPSK and $\pi/4-$shift QPSK.

- Demodulation characteristics

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{\gamma}{2}}\right]$$

$\gamma = E_s/N$

- QAM (Quadrature AM)
  - QPSK → 16QAM, 256QAM
  - Demodulation Characteristics

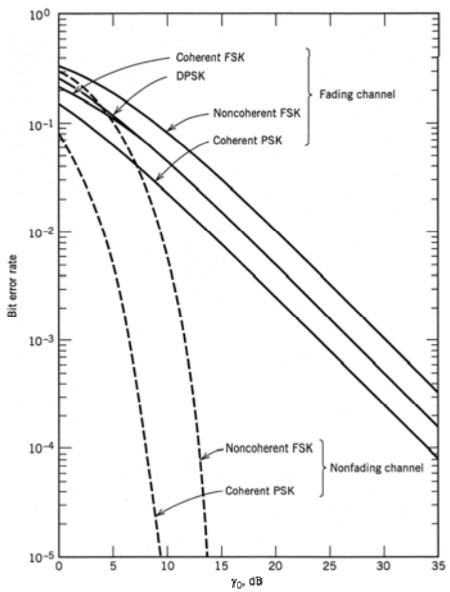
$$P_{e,16\mathrm{QAM}} = \frac{3}{8}\,\mathrm{erfc}\left[\sqrt{\frac{\gamma}{10}}\right]$$

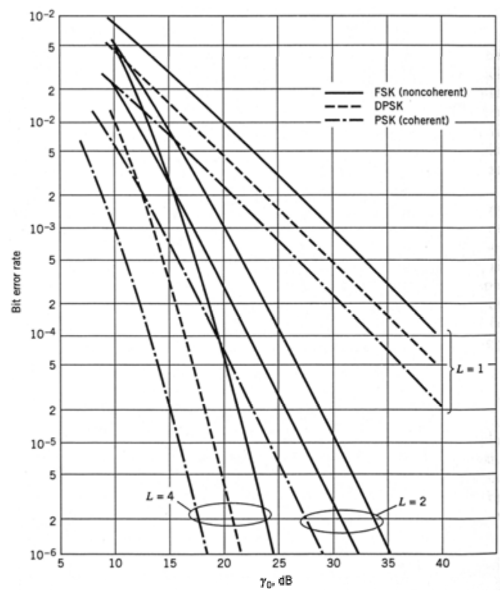
$$P_{e,64\mathrm{QAM}} = \frac{7}{24}\,\mathrm{erfc}\left[\sqrt{\frac{\gamma}{42}}\right]$$

$$P_{e,256\mathrm{QAM}} = \frac{15}{64}\,\mathrm{erfc}\left[\sqrt{\frac{\gamma}{170}}\right]$$

- Useful FEC for Multi-level QAM
  BCH Code, RS Code, Goppa Code,
  Algebraic-Geometry Code

- TCM (Trellis Coded Modulation, Ungerboeck)
  → 14.4kbps MODEM





- Degradation due to Linear / Nonlinear Distortion
  - Linear Distortion
    - MODEM: Phase error, Amplitude error
    - Filter: Amplitude / Delay-Frequency Characteristics
    - Coherent Detection: Carrier Phase Jitter
    - Clock Synchronization: Timing Phase Jitter
    - Others: Quantization error, Gain Fluctuation, DC Drift
  - Nonlinear Distortion
    AM-AM and AM-PM conversion in power amplifier

Capacity Bound

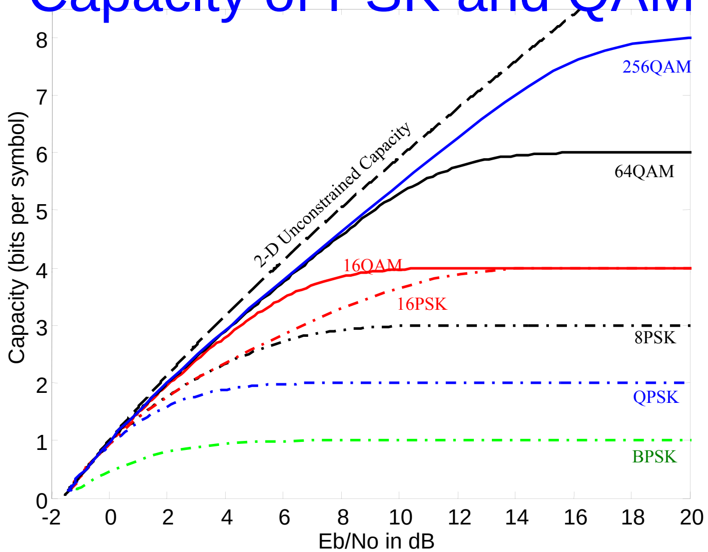
- For Analog

$$C = B\log(1+\frac{P}{N_o B}) \Rightarrow \frac{C}{B} = \log(1+\frac{E_b}{N_o}\frac{C}{B})$$

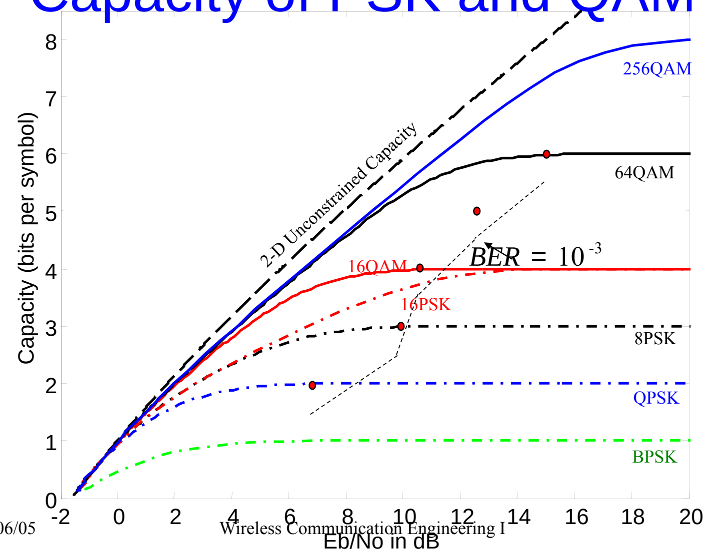
Let $R = \frac{C}{B}$ then $R = \log(1+\frac{E_b}{N_o}R)$

- For Digital: with M-ary constellation, the distribution of received signals become mixture of multiple Gaussian distributions. We must use some method such as Monte Carlo simulation to evaluate C

Capacity of PSK and QAM



Capacity of PSK and QAM



Rate, BER and SNR in BPSK

For BPSK $M(\sigma^2) = \sum_{a_i=\pm 1} \int_{y_i} p(a_i, y_i) \log \frac{p(a_i, y_i)}{p(a_i)p(y_i)} dy_i$

$$= \underbrace{\sum_{a_i=\pm 1} \int_{y_i} p(a_i, y_i) \log p(y_i|a_i) da_i dy_i}_{\text{Entropy of Gaussian noise}} - \underbrace{\int_{y_i} p(y_i) \log p(y_i) dy_i}_{\text{Approximated using Monte Carlo}}$$

$$R < M(\sigma^2) = M\left(\frac{1}{2R E_b/N_o}\right) \Rightarrow E_b/N_o > \frac{1}{2RM^{-1}(R)}$$

For rate R and given BER, what is the minimum SNR???

With given BER, mutual information is $1 + BER \log(BER) + (1 - BER)\log(1 - BER)$

New code-rate is $R' = R(1 + BER \log(BER) + (1 - BER)\log(1 - BER))$

Then we have $\sigma^2 = M^{-1}(R') \Rightarrow E_b/N_o = \frac{1}{2\sigma^2 R}$

Capacity for M-ary constellation

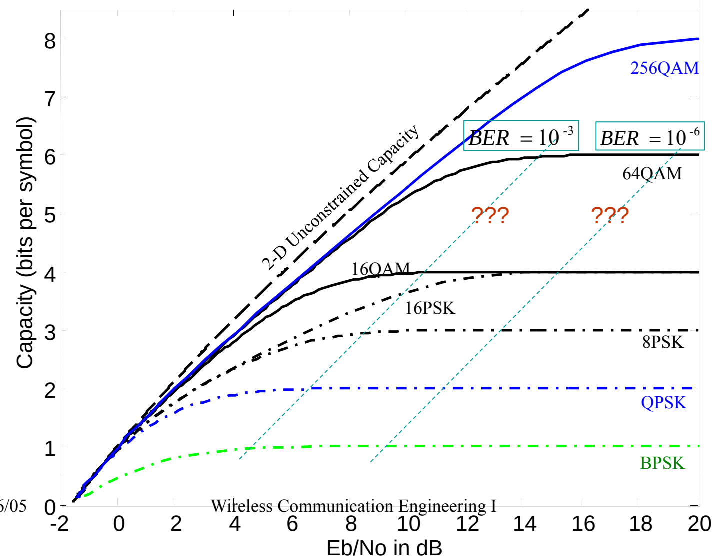
For discrete input, continuous output, memory-less AWGN channel.
Assuming equally likely M-ary constellation

$$C = \log(M) - \frac{1}{M\pi} \sum_{m=1}^{M} \int_{-\infty}^{+\infty} \exp(-|t|^2) * \log\left[\sum_{j=1}^{M} \exp\left(-\frac{2\operatorname{Re}\{t(x_m - x_j)^*\}}{\sqrt{N_0}} - \frac{|x_m - x_j|^2}{N_0}\right)\right]$$

x_m, x_j are the constellation points. $N_0/2$ is noise variance per dimension

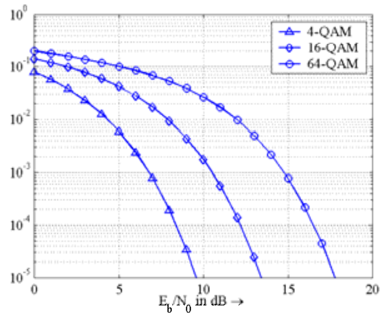
Average SNR is $\gamma = \frac{1}{MN_0} \sum_{i=1}^{M} |x_i|^2$ $\quad E_b/N_0 = \frac{1}{MN_0 \log(M)} \sum_{i=1}^{M} |x_i|^2$

Relationship of Rate, BER and SNR

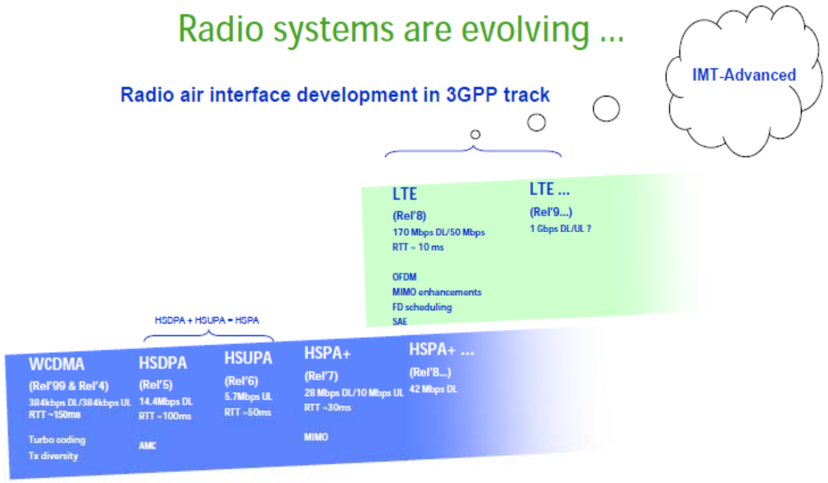


Brief review of AMC

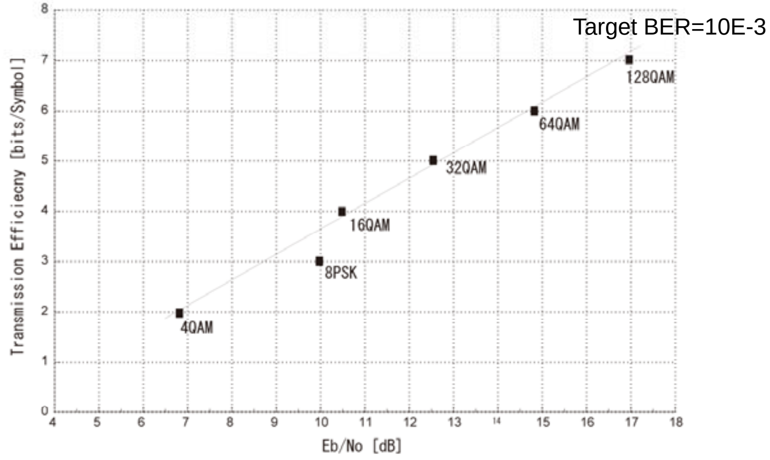
1. AMC: Adaptive Modulation and Coding
   Depending on the condition of the channel, the transmitter could be adapting some of the following: constellation size, code rate, and power.



Modulation Format	Bandwidth efficiency R/B (log2(M))	Eb/No to get BER=10E-3
64QAM	6	14.7
32QAM	5	12.5
16 QAM	4	10.5dB
8 PSK	3	10dB
4 QAM	2	6.8dB



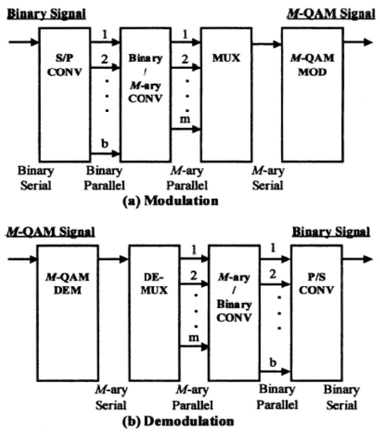
Brief review of AMC



Need 6QAM, 8QAM, 12QAM, 24QAM … etc

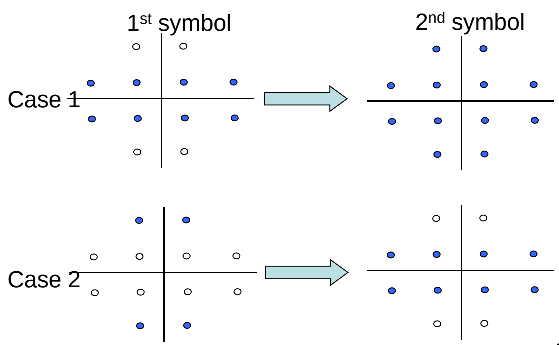
M-QAM with M is not power of 2

1. Use m M-QAM symbols to transmit b bits.
2. Transmission efficiency: b/m [bit/symbol]



M-QAM with M is $3 \times 2^{p-1}$

Consider M is $3x2^{p-1}$ such as 12, 24, 48
2p+1 bits are transmitted using 2 symbols

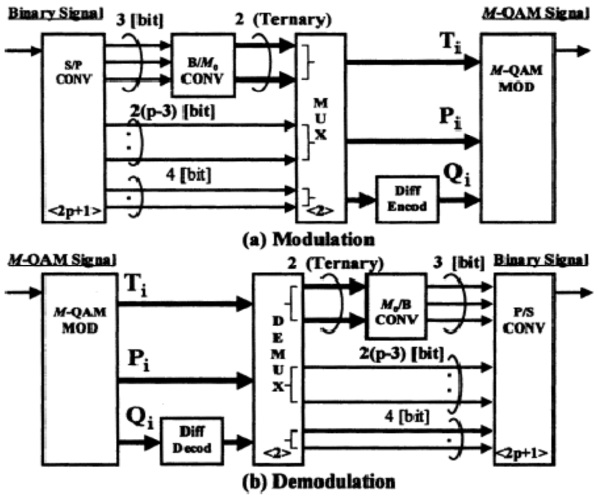


8x12=96

4x8=32

96+32=128 ~ 7bits

Configuration



Coding Scheme

Coding for $3 \times 2^{p-1}$ QAM scheme

$(b_{2p}, \cdots)$	P_1
$(0, \cdots 0, 0)$	0
$(0, \cdots 0, 1)$	1
$(0, \cdots 1, 1)$	2
$(0, \cdots 1, 0)$	3
·	·
·	·
·	·
$(1, \cdots 0, 0)$	$2^{p-3}-1$

$(\cdots, b_7)$	P_2
$(0, \cdots 0, 0)$	0
$(0, \cdots 0, 1)$	1
$(0, \cdots 1, 1)$	2
$(0, \cdots 1, 0)$	3
·	·
·	·
·	·
$(1, \cdots 0, 0)$	$2^{p-3}-1$

(b_6, b_5)	Q_1
(0, 0)	0
(0, 1)	1
(1, 1)	2
(1, 0)	3

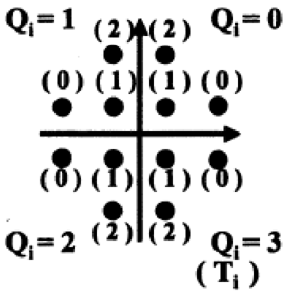
(b_4, b_3)	Q_2
(0, 0)	0
(0, 1)	1
(1, 1)	2
(1, 0)	3

(b_2, b_1, b_0)	(T_1, T_2)
(0, 0, 0)	(0, 0)
(0, 0, 1)	(0, 1)
(0, 1, 0)	(0, 2)
(1, 1, 0)	(1, 0)
(1, 1, 1)	(1, 1)
(0, 1, 1)	(1, 2)
(1, 0, 0)	(2, 0)
(1, 0, 1)	(2, 1)

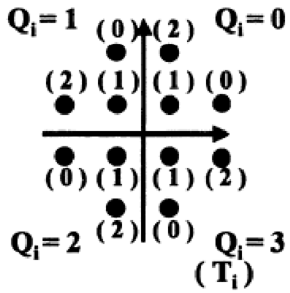
For T1 and T2, we can not use Hamming distance,
but use Lee distance.
Above combination is one of the best combinations
where average Hamming distance is minimum at Lee distance = 1
(Min Hamming distance is 21/16)

0,0 (0,0,0)	0,1 (0,0,1)	2,1 (1,0,1)
0,2 (0,1,0)		2,0 (1,0,0)
1,2 (0,1,1)	1,1 (1,1,1)	1,0 (1,1,0)

Mapping on 12-QAM

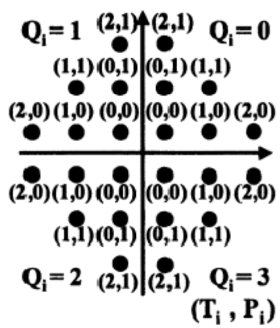


(a-1) 12QAM, axially symmetric locations, 180° transparent

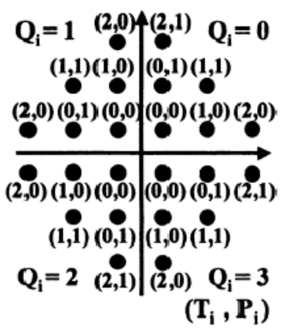


(a-2) 12QAM, radially symmetric locations, 90° transparent

Mapping on 24-QAM

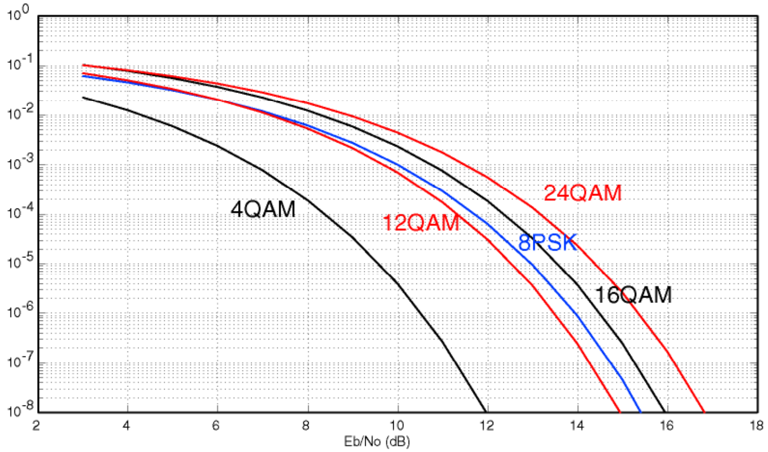


(b-1) 24QAM, axially symmetric locations, 180° transparent

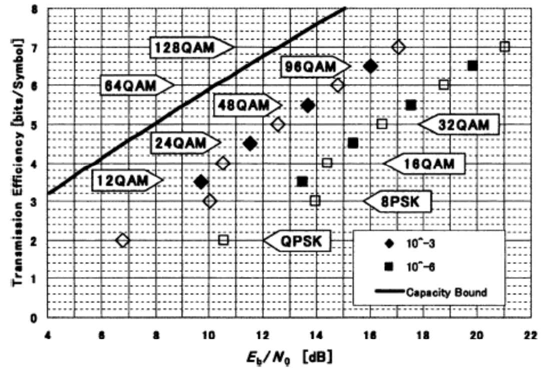


(b-2) 24QAM, radially symmetric locations, 90° transparent

BER Performance

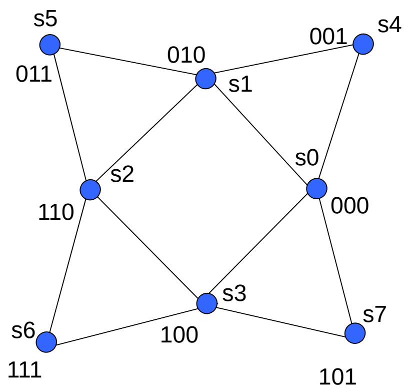


Comparison



Transmission efficiency versus required E_b/N_0 of $3 \times 2^{p-1}$ QAM.

8QAM Star type



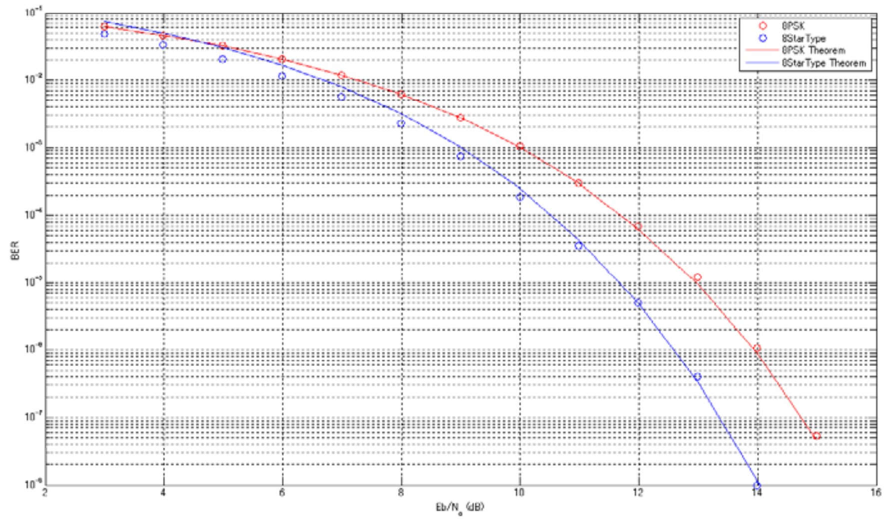
If Average Power = 1 then Minimum Euclidean is 0.9194 (for 8PSK is 0.7654)

$$P_b(8QAM\_Star) = \frac{2}{3} erfc\left(\sqrt{\frac{3\gamma_b}{(3+\sqrt{3})}}\right)$$

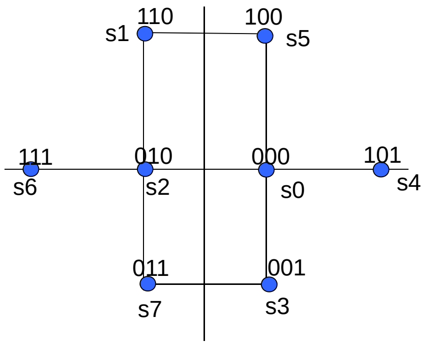
$$= \frac{4}{3} Q\left(\sqrt{\frac{6\gamma_b}{3+\sqrt{3}}}\right)$$

Minimum Euclidean Distance（Ex：s1 and s4）：12 cases
1 bit error：8 cases, 2 bit error：4 cases

8PSK-8Star BER Performance



8QAM Square Type



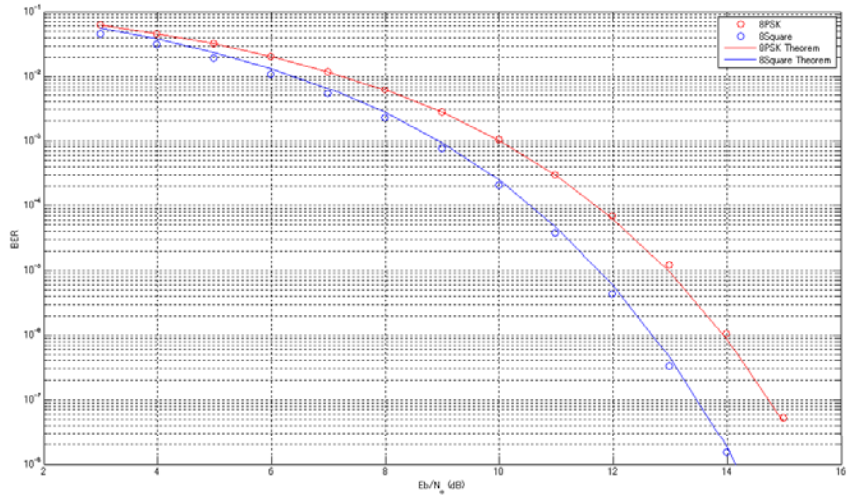
If Average Power = 1 then
Minimum Euclidean is 0.8944
(for 8PSK is 0.7654)

$$P_b(8QAM\_Square) = \frac{3}{8} erfc\left(\sqrt{\frac{3\gamma_b}{5}}\right)$$

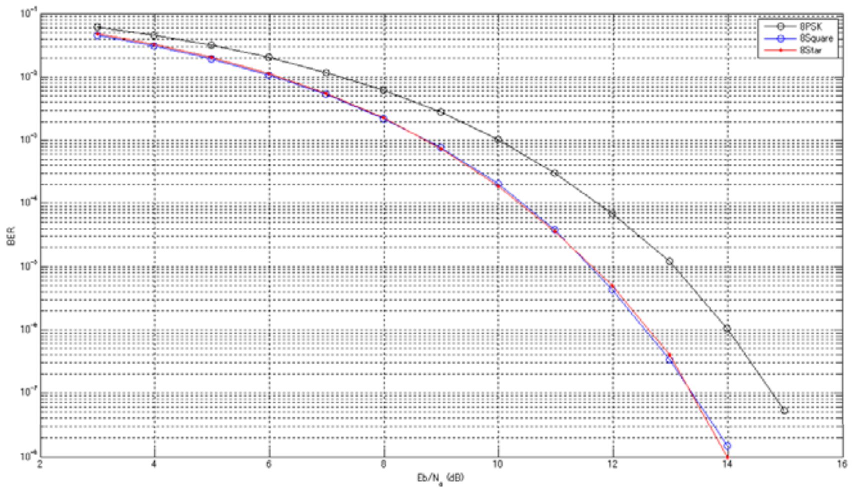
$$= \frac{3}{4} Q\left(\sqrt{6\gamma_b/5}\right)$$

Minimum Euclidean Distance（Ex：s0 and s4）：9 cases
1 bit error：7 cases, 2 bit error：2 cases

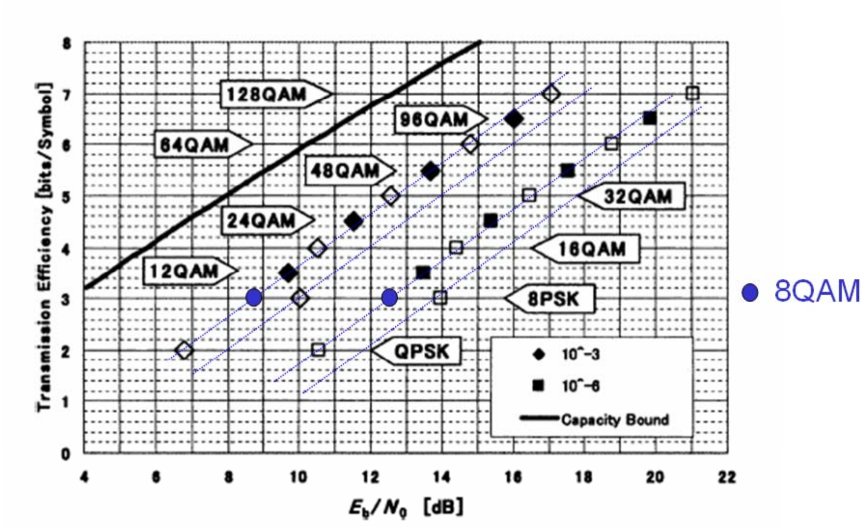
8PSK-8Square BER Performance



8-ary BER Performance

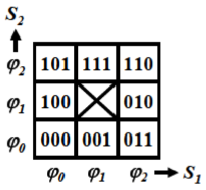


Comparison with other mod. schemes

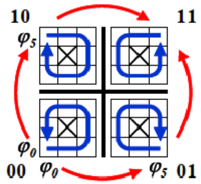


6-PSK: Review

- Use 2 symbols to send 5 bits
- 3bit ($b2$, $b1$, $b0$) is assigned to 8 cells for first 3 phases ($\varphi0$, $\varphi1$, $\varphi2$) of symbols $S1$ and $S2$,
- This "frame" of cells is "folded-out" twice along the horizontal and vertical axes
- The other bits ($b4$, $b3$) are assigned to the 4 frames



(a) Circular Gray Code

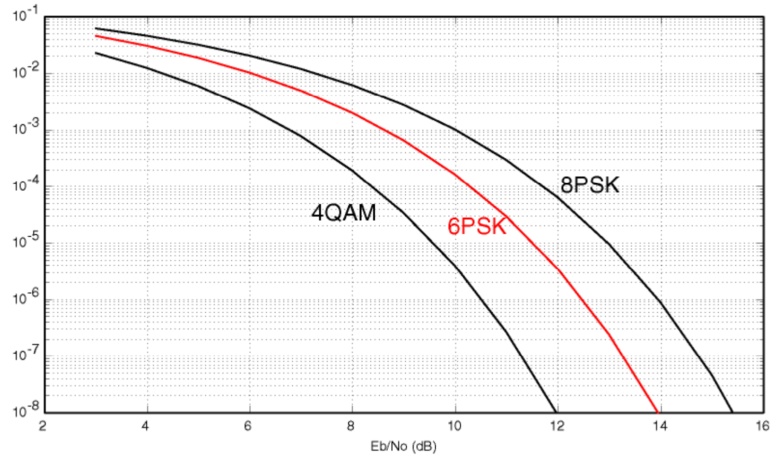


(b) Mirror Symmetry

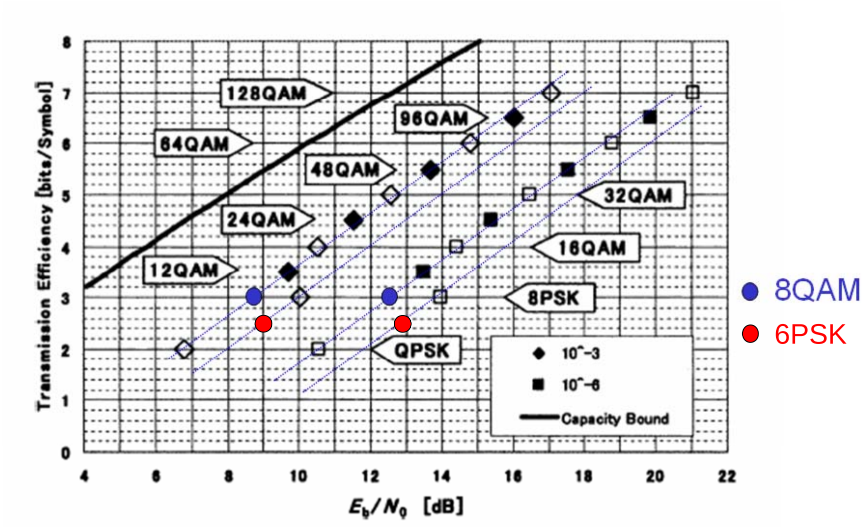
The Second Symbol S_2						
φ_5	10 / 000	10 / 001	10 / 011	11 / 011	11 / 001	11 / 000
φ_4	10 / 100	X	10 / 010	11 / 010	X	11 / 100
φ_3	10 / 101	10 / 111	10 / 110	11 / 110	11 / 111	11 / 101
φ_2	00 / 101	00 / 111	00 / 110	01 / 110	01 / 111	01 / 101
φ_1	00 / 100	X	00 / 010	01 / 010	X	01 / 100
φ_0	00 / 000	00 / 001	00 / 011	01 / 011	01 / 001	01 / 000
	φ_0	φ_1	φ_2	φ_3	φ_4	φ_5
	The First Symbol S_1					

Note: The two bits of the upper subcell in each cell are b_4 and b_3. And three bits of the lower subcell in each cell are b_2, b_1 and b_0. The cells indicated "X" are not used in the transmitted signal. However, if these cells are received as a result of symbol error, they are decoded (b_4, b_3, 1, 1, 1), for example.

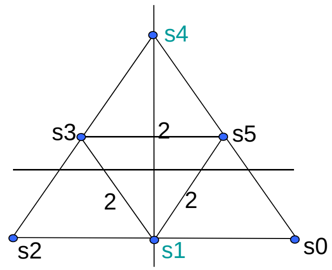
6-PSK BER Performance



Comparison with other mod. schemes



6-ary Triangle Type 1



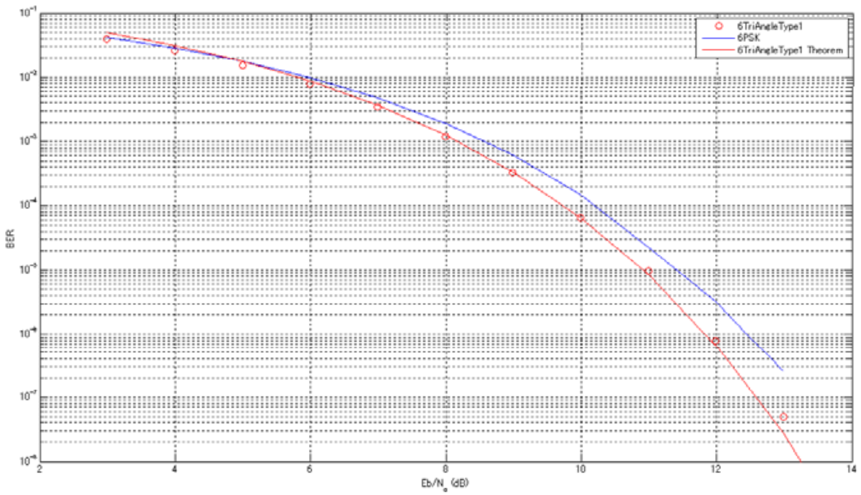
s0	10 000	10 001	10 011	11 011	11 001	11 000
s1	10 100		10 010	11 010		11 100
s2	10 101	10 111	10 110	11 110	11 111	11 101
s3	00 101	00 111	00 110	01 110	01 111	01 101
s4	00 100		00 010	01 010		01 100
s5	00 000	00 001	00 011	01 011	01 001	01 000
	s0	s1	s2	s3	s4	s5

Minimum Euclidean Distance : 9 cases
1 bit error : 6 cases, 2 bit error : 3 cases

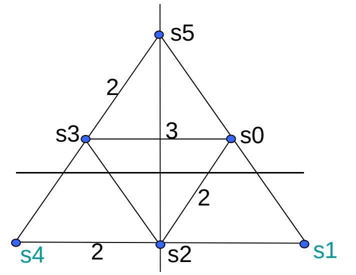
If Average Power = 1 then Minimum Euclidean is 1.0954 (for 6PSK is 1)

$$P_b(6TriAngleType1) = \frac{3}{5} erfc\left(\sqrt{\frac{3\gamma_b}{4}}\right) = \frac{6}{5} Q\left(\sqrt{\frac{6\gamma_b}{4}}\right)$$

6PSK-6TriAngleType1



6-ary Triangle Type 2



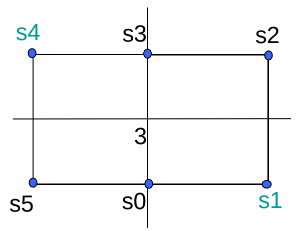
s0	10 000	10 001	10 011	11 011	11 001	11 000
s1	10 100		10 010	11 010		11 100
s2	10 101	10 111	10 110	11 110	11 111	11 101
s3	00 000	10 001	10 011	01 110	01 111	01 101
s4	10 100		10 010	01 010		01 100
s5	10 101	10 111	10 110	01 011	01 001	01 000
	s0	s1	s2	s3	s4	s5

Minimum Euclidean Distance：9 cases
1 bit error：5 cases, 2 bit error：3 cases, 3 bit error: 1

If Average Power = 1 then Minimum Euclidean is 1.139 (for 6PSK is 1)

$$P_b(6TriAngleType2) = \frac{3}{5} erfc(\sqrt{\frac{30\gamma_b}{37}}) = \frac{6}{5} Q(\sqrt{\frac{60\gamma_b}{37}})$$

6-ary Square Type



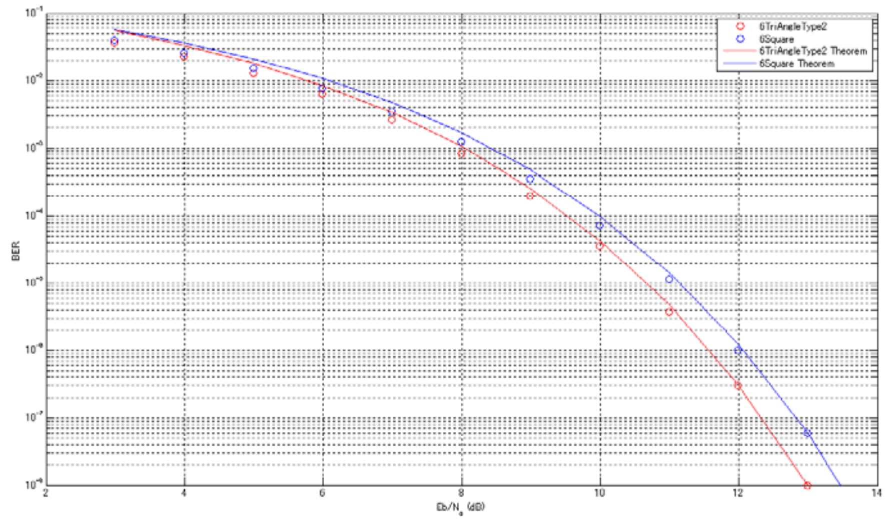
s0	10 000	10 001	10 011	11 011	11 001	11 000
s1	10 100		10 010	11 010		11 100
s2	10 101	10 111	10 110	11 110	11 111	11 101
s3	00 000	10 001	10 011	01 110	01 111	01 101
s4	10 100		10 010	01 010		01 100
s5	10 101	10 111	10 110	01 011	01 001	01 000
	s0	s1	s2	s3	s4	s5

Minimum Euclidean Distance：7 cases
1 bit error：6 cases, 2 bit error：1 case

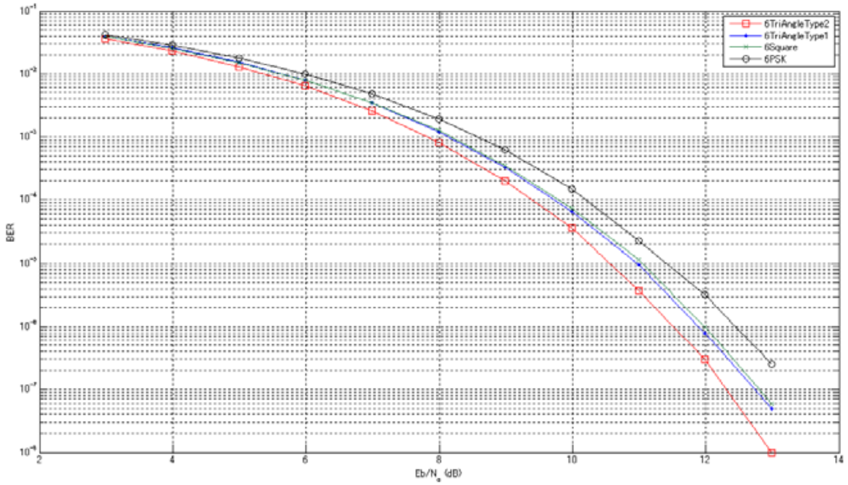
If Average Power = 1 then Minimum Euclidean is 1.0690 (for 6PSK is 1)

$$P_b(6SquareType) = \frac{5}{8} erfc(\sqrt{\frac{5\gamma_b}{7}}) = \frac{5}{4} Q(\sqrt{\frac{10\gamma_b}{7}})$$

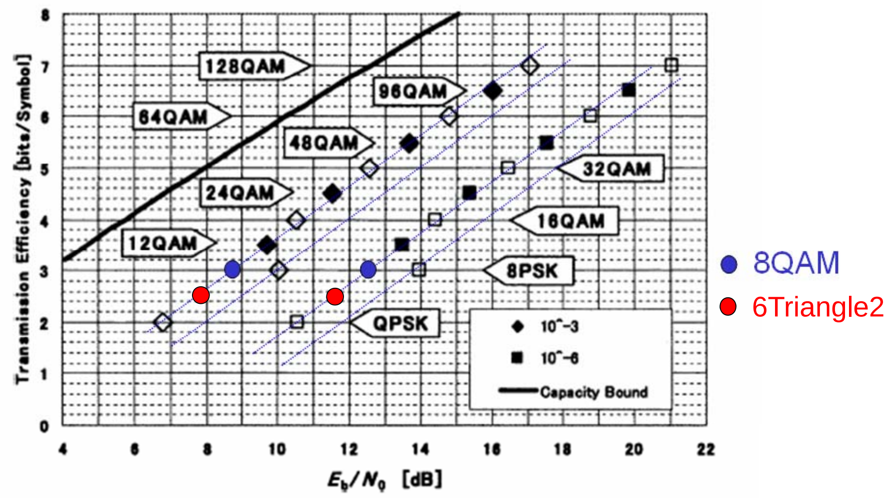
6Square-6TriAngleType2



6PSK-6Square-6TriAngle



Comparison with other mod. schemes



Relationship between rate, SNR, BER

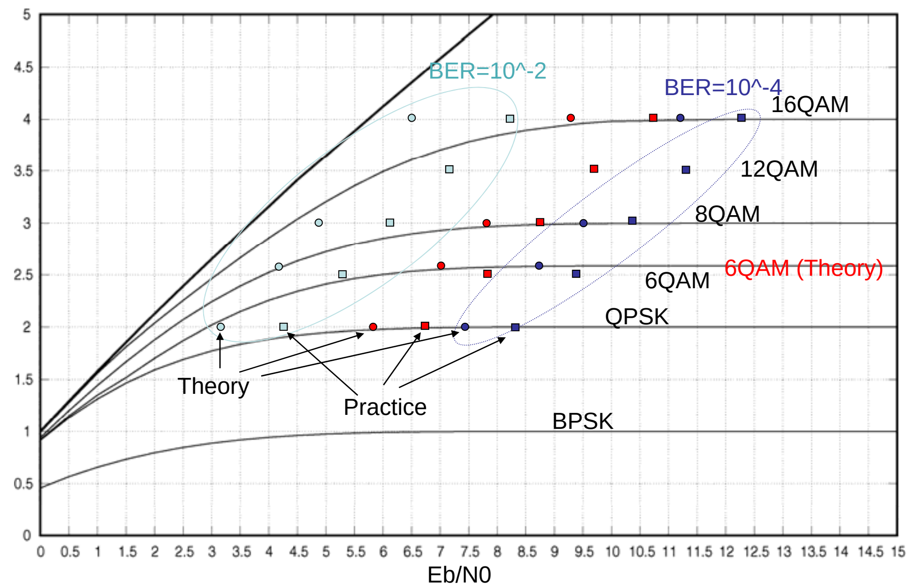
- The scenario is: for rate R and given BER, what is the minimum required SNR ?

With given BER, mutual information is $1 + BER\log(BER) + (1-BER)\log(1-BER)$

New code-rate is $R' = R\left(1 + BER\log(BER) + (1-BER)\log(1-BER)\right)$

Then we have $\sigma^2 = M^{-1}(R') \Longrightarrow E_b/N_o = \dfrac{1}{2\sigma^2 R}$

Relationship between rate, SNR, BER



Others

- Approximations of erfc() $erfc(x) = 1 - \sqrt{1 - \exp\left(-x^2 \frac{4 + \pi 0.14 x^2}{\pi + \pi 0.14 x^2}\right)}$
- Appro. BER for 4-QAM $P_b = Q\left(\sqrt{2\gamma_b}\right)$
- Appro. BER for M-QAM $P_b = \frac{2}{\log(M)} erfc\left(\sqrt{\frac{3\gamma_b \log(M)}{2(M-1)}}\right) = \frac{4}{\log(M)} Q\left(\sqrt{\frac{3\gamma_b \log(M)}{(M-1)}}\right)$
- Appro. BER for 6-QAM $P_b = \frac{3}{5} erfc\left(\sqrt{\frac{30\gamma_b}{37}}\right) = \frac{6}{5} A\left(\sqrt{\frac{60\gamma_b}{37}}\right)$
- Appro. BER for 8-QAM $P_b = \frac{3}{8} erfc\left(\sqrt{\frac{3\gamma_b}{5}}\right) = \frac{3}{4} Q\left(\sqrt{\frac{6\gamma_b}{5s}}\right)$
- Appro. BER for 12-QAM $P_b = \frac{25}{26} erfc\left(\sqrt{\frac{\gamma_b}{2}}\right) = \frac{25}{13} Q\left(\sqrt{\gamma_b}\right)$
- Appro. BER for 24-QAM $P_b = \frac{57}{144} erfc\left(\sqrt{\frac{9\gamma_b}{28}}\right) = \frac{57}{72} erfc\left(\sqrt{\frac{9\gamma_b}{14}}\right)$