Incompressible Flow around a body

GSIC

Useful CAE tool for aerodynamics studies

- Aerodynamics design for cars.
- Air flow analysis for heat problems inside PC.
- Environmental studies of the wind around buildings.

We are interested in the interaction between flow and bodies (materials)

Boundary Condition at the body surface

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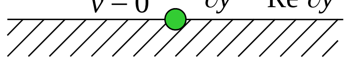
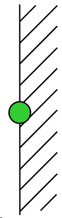
Non-slip velocity condition:

$u = 0$, $v = 0$



$$\frac{\partial p}{\partial y} = \frac{1}{\mathrm{Re}} \frac{\partial^2 v}{\partial y^2} \approx 0$$ High Reynolds number

$u = 0$, $v = 0$



$$\frac{\partial p}{\partial x} = \frac{1}{\mathrm{Re}} \frac{\partial^2 u}{\partial x^2} \approx 0$$ High Reynolds number

SMAC Algorithm :

$$\frac{\partial p}{\partial y} = 0 \quad (x\text{-Wall Boundary})$$

$$\frac{\partial p}{\partial x} = 0 \quad (y\text{-Wall Boundary})$$

} Neumann Boundary condition

Discretization for u, v and p at the *x*-wall

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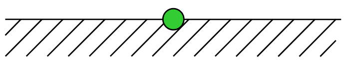
Arakawa-A:

$u_{i,j} = 0 \qquad v_{i,j} = 0 \qquad \frac{p_{i,j+1} - p_{i,j-1}}{2\Delta y} = 0$

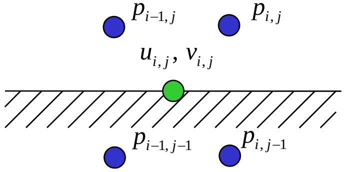


Arakawa-B:

$u_{i,j} = 0 \qquad v_{i,j} = 0$

$$\frac{p_{i,j} - p_{i,j-1}}{\Delta y} = 0$$

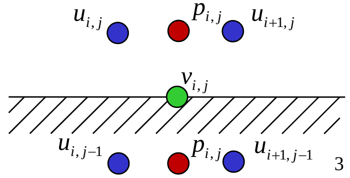
$$\frac{p_{i-1,j} - p_{i-1,j-1}}{\Delta y} = 0$$



Arakawa-C:

$u_{i,j} = -u_{i,j-1} \qquad v_{i,j} = 0 \qquad \frac{p_{i,j} - p_{i,j-1}}{\Delta y} = 0$

$u_{i+1,j} = -u_{i+1,j-1}$



Discretization for u, v and p at the *y*-wall

GSIC

Arakawa-A:

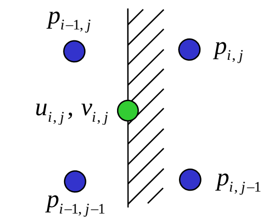
$u_{i,j} = 0 \qquad v_{i,j} = 0 \qquad \frac{p_{i+1,j} - p_{i-1,j}}{2\Delta x} = 0$

Arakawa-B:

$u_{i,j} = 0 \qquad v_{i,j} = 0$

$$\frac{p_{i,j} - p_{i,j-1}}{\Delta x} = 0$$

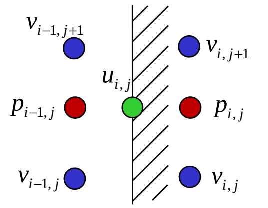
$$\frac{p_{i-1,j} - p_{i-1,j-1}}{\Delta x} = 0$$



Arakawa-C:

$u_{i,j} = 0 \qquad v_{i,j+1} = -v_{i-1,j+1} \qquad \frac{p_{i,j} - p_{i-1,j}}{\Delta x} = 0$

$v_{i,j} = -v_{i-1,j}$



Boundary Treatment for the pressure

Indicator: iw (at the same point as the pressure)

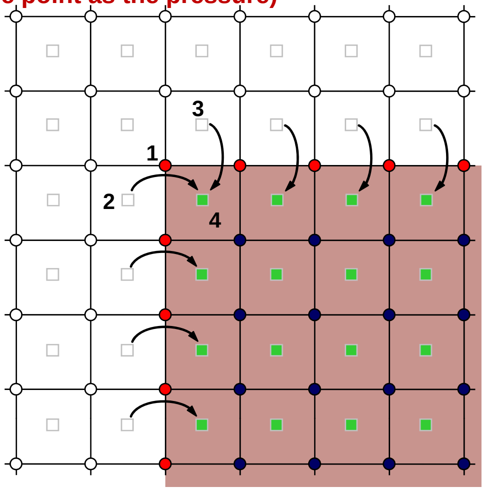
iw = 1 inside the body.
iw = 0 outside the body.

$if\ \ iw_{i+1,j} = 0$

when we solve $p_{i,j}$,

$$\frac{p_{i-1,j} - 2p_{i,j} + p_{i+1,j}}{\Delta x^2}$$

$$\frac{p_{i-1,j} - 2p_{i,j} + p_{i,j}}{\Delta x^2}$$



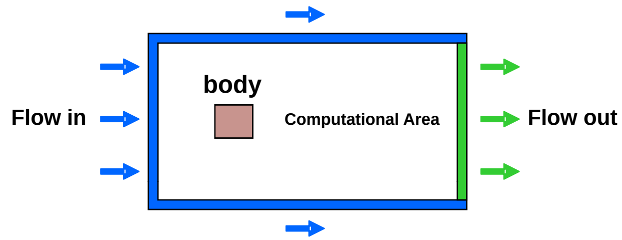
Boundary Condition for Computational Domain

Computational domain : $0 \le x \le L_x \quad 0 \le y \le L_y$

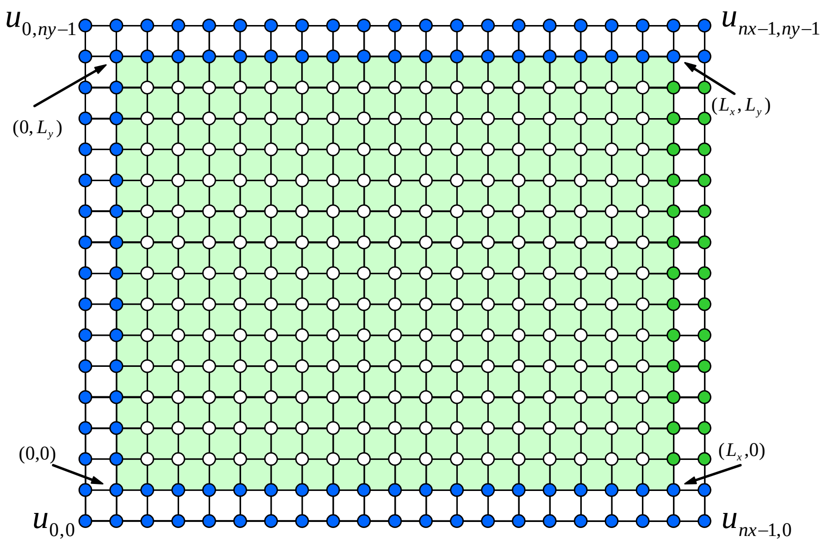
Grid size : $nx, \quad ny$

Grid spacing : $\Delta x = L_x/(nx-3), \quad \Delta y = L_y/(ny-3)$

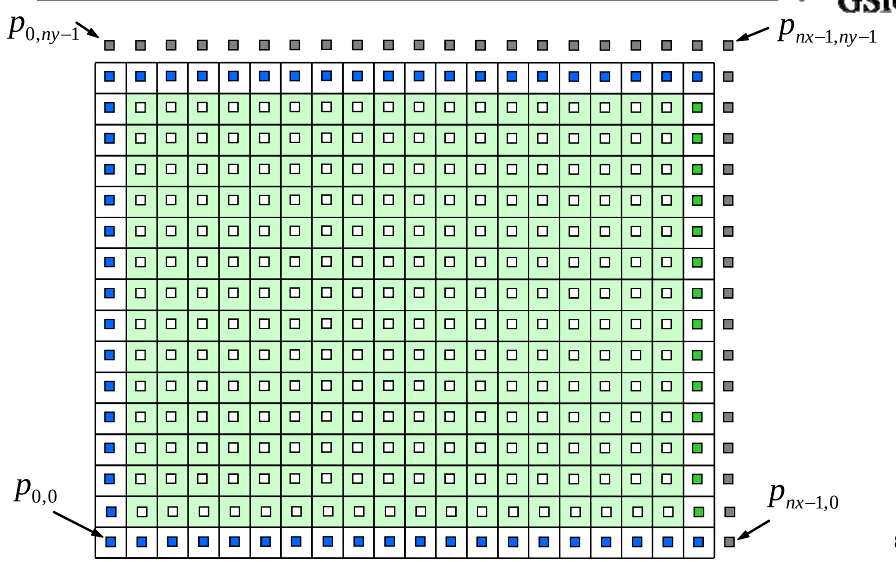
Flow in / Flow out :



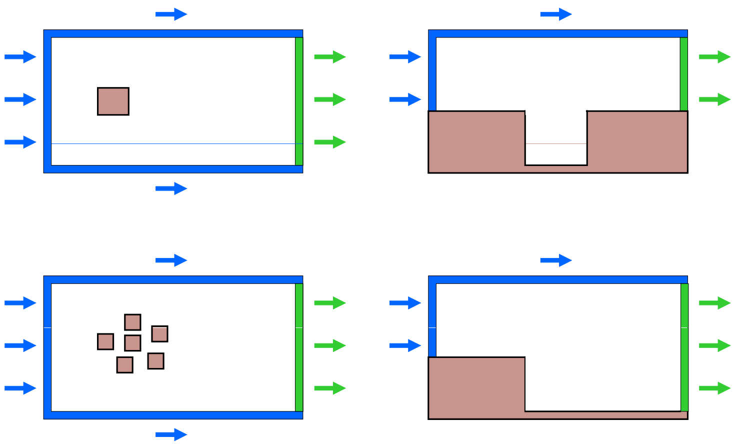
Boundary Condition for the Velocity



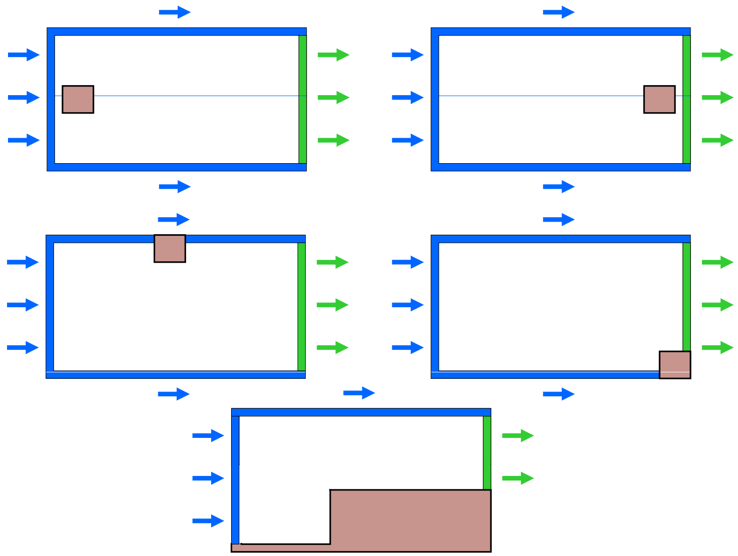
Boundary Condition for the Pressure



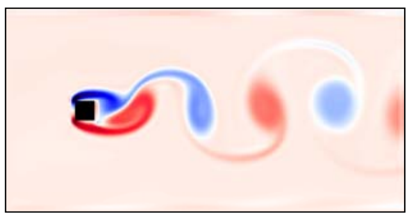
Proper Layout of the bodies

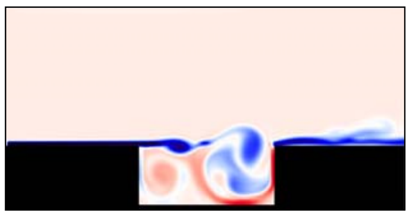


Bad Layout of the bodies



Typical Computational Results





Source Code

Submission of the Final Report

Modify the sample code and set the body of your favorite shape.

Show the numerical results (snap shots) and describe your consideration from the view points of both numerical calculation and fluid dynamics.

Deadline: 15th February, 2009