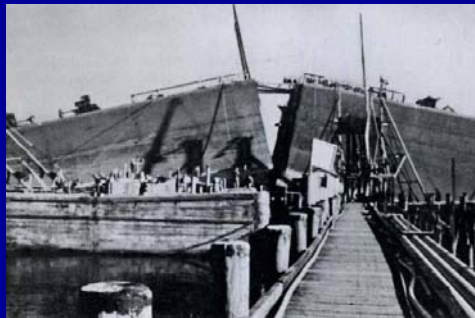


4. Fundamental Concepts of Fracture Mechanics

1. Introduction

During World War II
5000 ships built



over 1000 ships: cracks by 1946

200 ships: serious damage

9 T-2 tankers

7 Liberty ships

} broken in two

What is Fracture Mechanics?

Strength of Materials

Yield $\sigma = \sigma_y$

Failure $\sigma = \sigma_B$

Fatigue $S - N$ Diagram, *Fatigue Limit*

Fracture Mechanics

Failure $K = K_c$ Stress Intensity Factor K
Fracture Toughness K_c

Fatigue $\frac{da}{dN} = C((\Delta K)^m - (\Delta K_{th})^m)K_c$

3

Fracture Mechanics is

Mechanics of Members with Cracks

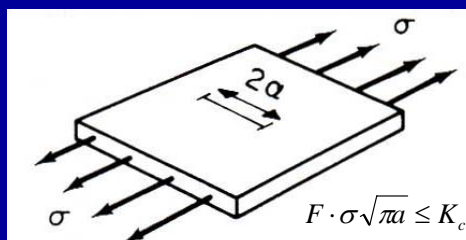
Stress Intensity Factor, K

Stress, σ

Crack Length, a

Fracture Toughness, K_c

Material Property



4

Primary Factors controlling Brittle Fracture : 1

- 1) Material toughness (K_{Ic} , K_{Ic} , $K_{Id} = C \sigma \sqrt{a}$)
 - the ability to carry load or deform plastically in the presence of notch
 - for slow loading and linear -elastic behavior.
 - K_{Ic} : under conditions of plane stress
 - K_{Ic} : plane strain
 - impact or dynamic loading
 - K_{Id} : under condition of maximum constraint (plane strain)
 - For elastic-plastic behavior
 - R-curve resistance, J_{Ic} , and CTOD

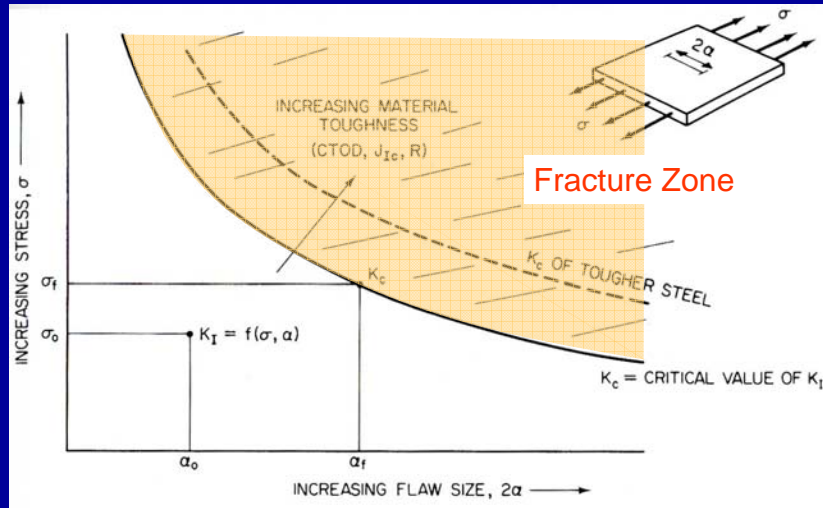
5

Primary Factors controlling Brittle Fracture : 2

- 2) Crack size (a)
 - Brittle fracture initiate from discontinuities varying from small cracks within a weld ark strike (case in a T-2 tanker) to much larger fatigue cracks.
- 3) Stress level (σ): Tensile stress are necessary for brittle failure to occur.
- Brittle failure can occur without all three factors being present if the other factors are sufficiently severe.
- Other factors such as temperature, loading rate, stress concentration, residual stress, and so on will affect the three primary factors

6

Fundamental Concepts of Fracture Mechanics



Relationship among a stress condition, a crack size and fracture toughness

7

Example

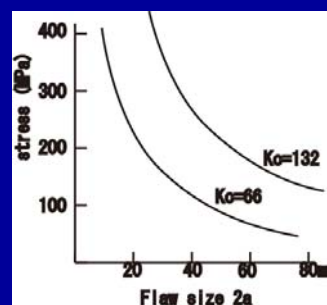
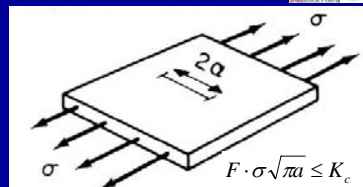
Consider a through thickness crack in a wide plate,
design stress is 38MPa ,310MPa

$$K_I = \sigma \sqrt{\pi a}$$

$$K_c = 66,132 \text{ MPa}\sqrt{\text{m}}$$

what is the tolerable flaw size?

If residual stresses due to welding present so that the total stress is 552MPa at the crack.

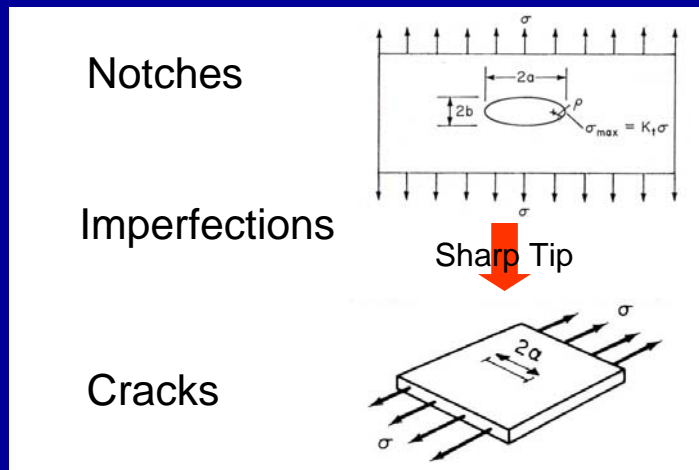


	Kc=66	Kc=132
$\sigma = 138 \text{ MPa}$	$2a=145\text{mm}$	291
$\sigma = 310$	27.9	57.7
$\sigma = 552$	4.6	18.2

8

2. Meaning of Stress Analysis of Members with Notches

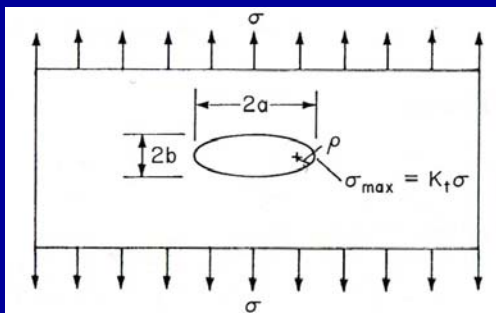
Types of Discontinuities



9

Stress Concentration Factor :K_t

K_t For An Elliptical Hole



$$K_T = \frac{\sigma_{\max}}{\sigma} = 1 + \frac{2a}{b}$$

$$\rho = \frac{b^2}{a}$$

$$\sigma_{\max} \cong 2\sigma \sqrt{\frac{a}{\rho}} \quad (\rho \ll a)$$

$$\sqrt{\frac{a}{\rho}} \rightarrow \infty \quad (\rho \rightarrow 0)$$

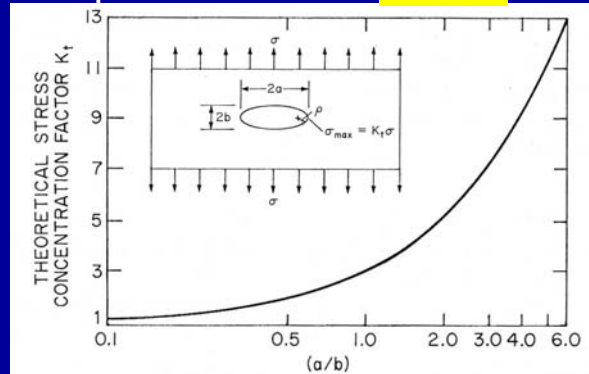
$$K_t \rightarrow \infty \quad ???$$

10

Stress Analysis of Members with Notches

Stress Concentration Factor, K_t

For An Elliptical Hole $K_t \rightarrow \infty$ Meaningless



A Different Approach is needed to analyze the behavior of structures containing cracks or sharp imperfections

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3. History of Fracture Mechanics

1921 Griffith

Griffith's Formula

1948 Irwin

1957 Irwin

Stress Intensity Factor, K

.....

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Griffith Theory

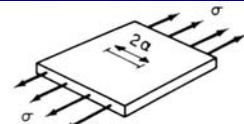
Consider an plate contains a through-thickness crack of length $2a$ and that is subjected to uniform tensile stress σ
The total potential energy of the system, U is written as

$$U = U_o - U_a + U_\gamma$$

U_o : elastic energy of the unclacked plate

$$U_a = \frac{\pi \sigma^2 a^2}{E} : \text{decrease in elastic energy caused by the crack}$$

$$U_\gamma : 2(2a\gamma_e) : \text{elasticsurface energy by the formation of the crack surface}$$



$$\frac{\partial U}{\partial a} = 0 \longrightarrow \text{const.}$$

$$\sigma \sqrt{a} = \left(\frac{2\gamma_e E}{\pi} \right)^{1/2}$$

crack extension is governed by the crack length and the material property

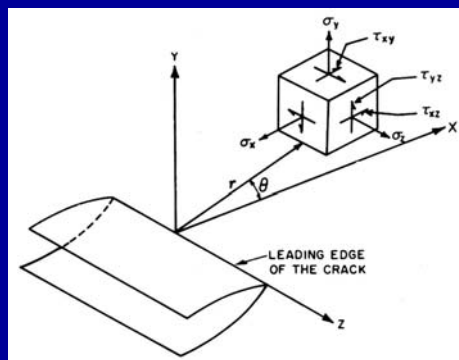
13

4. Stress Analysis of Members with Notches

Stress Distribution
along X-axis
near the notch root

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}}$$

$\rho \rightarrow 0$



$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}} \left(1 + \frac{\rho}{2r+\rho} \right) + \left(\frac{\rho}{2r+\rho} \right)$$

$0 \leq r \ll a, \rho \ll a$

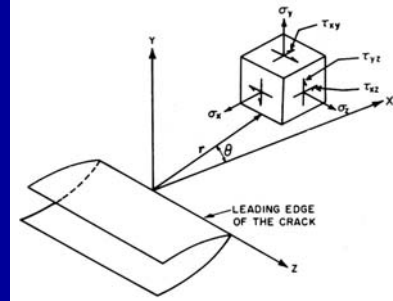
$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}} \left(1 - \frac{\rho}{2r+\rho} \right)$$

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Stress Analysis of Members with Cracks

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r + \rho}}$$

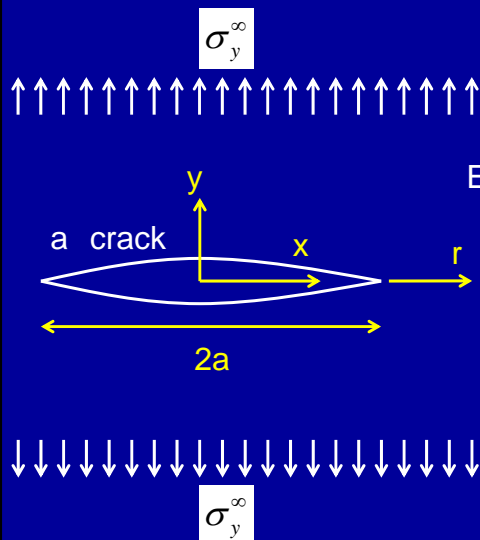
$$\rho \rightarrow 0$$



Stress near a crack tip

1. Singularity about $\frac{1}{\sqrt{r}}$
2. Intensity of Stress Singularity is Proportional to
far field stress σ_∞
square root of crack length a

Stress Analysis of Members with Cracks



Exact solution of stress near crack

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} = \frac{a + r}{\sqrt{r(2a + r)}}$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} - 1 = \frac{a + r}{\sqrt{r(2a + r)}} - 1$$

Stress Analysis of Members with Cracks

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} = \frac{a+r}{\sqrt{r(2a+r)}}$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2 - a^2}} - 1 = \frac{a+r}{\sqrt{r(2a+r)}} - 1$$

$r/a < 1$
Series Expansion

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} + \frac{3}{4}\sqrt{\frac{r}{2a}} - \frac{5}{32}\left(\sqrt{\frac{r}{2a}}\right)^3 + \dots$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} - 1 + \frac{3}{4}\sqrt{\frac{r}{2a}} - \frac{5}{32}\left(\sqrt{\frac{r}{2a}}\right)^3 + \dots$$

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}} = \frac{K}{\sqrt{2\pi r}}$$

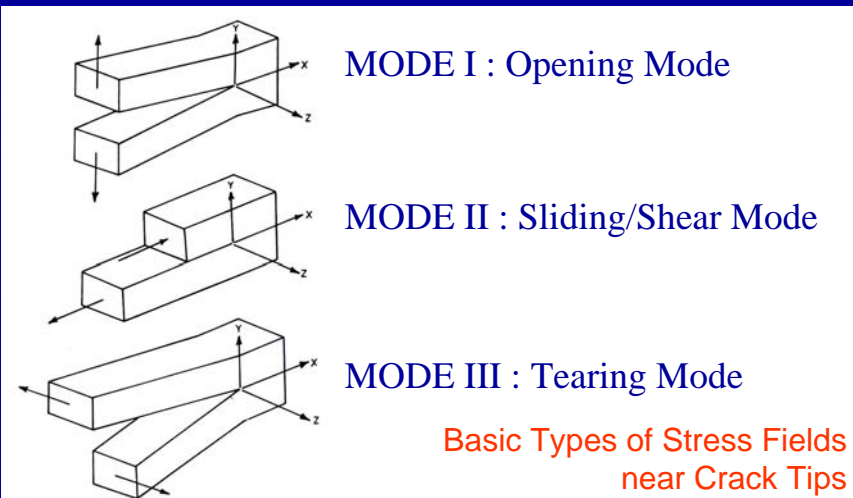
$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} - 1$$

$$K = \sigma\sqrt{\pi a}$$

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5. Stress Analysis for Cracks in Elastic Solids

Three Types of Relative Movements of Two Crack Surfaces



Important Basic Mode I

Most engineering situations corresponding to Mode I

$$\sigma_x = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2} \right]$$

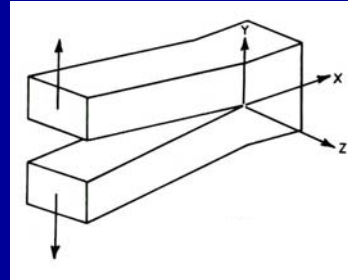
$$\sigma_y = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos \frac{\theta}{2} \left[1 + \sin \frac{\theta}{2} \cdot \sin \frac{3\theta}{2} \right]$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \sin \frac{\theta}{2} \cdot \cos \frac{\theta}{2} \cdot \cos \frac{3\theta}{2}$$

$$u = \frac{K_I}{G} \left[\frac{r}{2\pi} \right]^{\frac{1}{2}} \cos \frac{\theta}{2} \left[1 - 2\nu + \sin^2 \frac{\theta}{2} \right]$$

$$v = \frac{K_I}{G} \left[\frac{r}{2\pi} \right]^{\frac{1}{2}} \sin \frac{\theta}{2} \left[1 - 2\nu - \cos^2 \frac{\theta}{2} \right]$$

$$w = 0$$



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6. Stress Intensity Factor

The Applied Stress
The Crack Shape and Size
The Structural Configuration



The Value of The Stress Intensity Factor, K

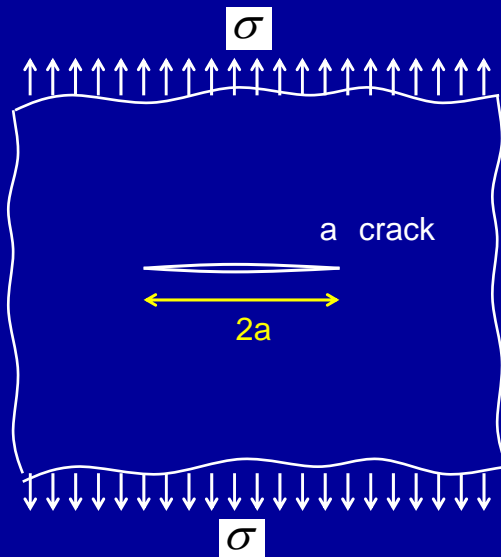
$$K = \sigma \sqrt{\pi a} \cdot F$$

F : Correction factor

Local Stress Field \longleftrightarrow Global Stress Field

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Stress Intensity Factor Equations (1)

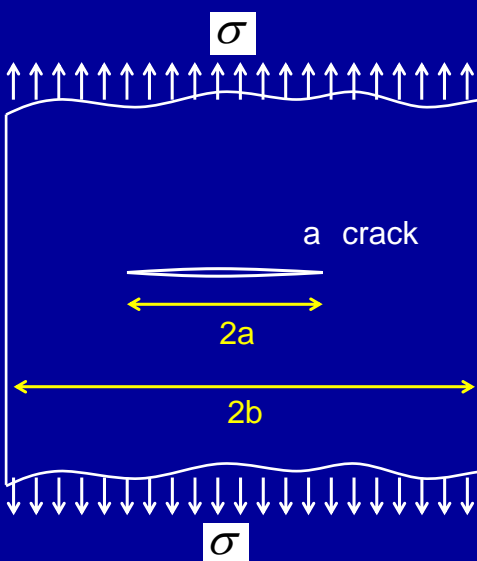


A Through Thickness Crack
In an **Infinite Plate** subject
to Uniform Tensile Stress

$$K = \sigma \sqrt{\pi a}$$

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Stress Intensity Factor Equations (2)



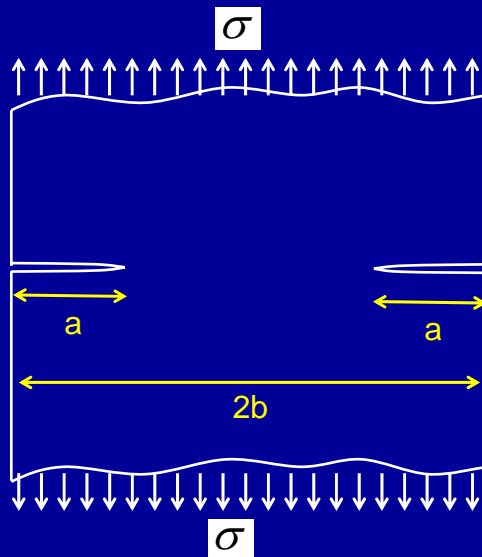
A Through Thickness Crack
In a Plate with **Finite Width**
Subject to
Uniform Tensile Stress

$$K = \sigma \sqrt{\pi a} \left(\frac{2b}{\pi a} \tan \frac{\pi a}{2b} \right)^{\frac{1}{2}}$$

$$= \sigma \sqrt{\pi a} \sqrt{\sec \left(\frac{\pi a}{2b} \right)}$$

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Stress Intensity Factor Equations (3)



Double-Edge Cracks
In a plate with finite width

$$K = \sigma \sqrt{\pi a} \cdot 1.12$$

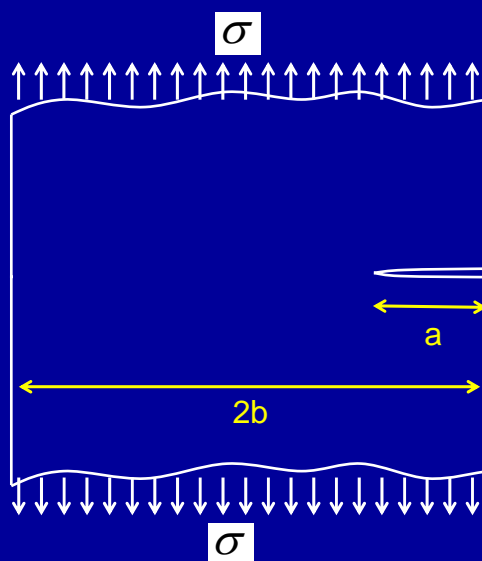
original form

$$K = \sigma \sqrt{\pi a} \cdot F(\alpha), \alpha = \frac{a}{b}$$

$$F(\alpha) \cong \left(1 + 0.122 \cos^2 \frac{\pi \alpha}{2} \right) \sqrt{\frac{2}{\pi \alpha} \tan \frac{\pi \alpha}{2}}$$

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Stress Intensity Factor Equations (4)



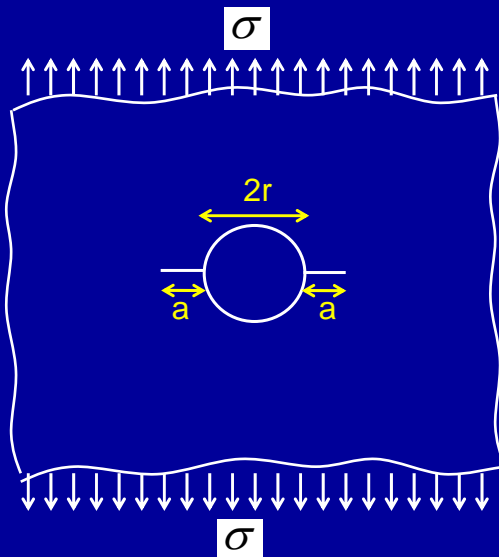
Single-Edge Crack
In a plate with finite width

$$K = 1.12 \cdot \sigma \sqrt{\pi a} \cdot f\left(\frac{a}{b}\right)$$

a/b	f
0.1	1.03
0.2	1.07
0.3	1.15
0.5	1.35
0.7	1.69
0.9	2.20

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Stress Intensity Factor Equations (5)



Cracks growing
from a round hole

$$K = \sigma \sqrt{\pi a} \cdot f\left(\frac{a}{r}\right)$$

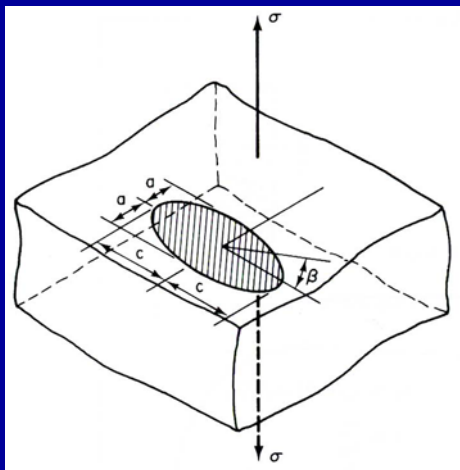
$$a \rightarrow 0 \quad f\left(\frac{a}{r}\right) \rightarrow 3$$

For short cracks

$$K = K_t \sigma \sqrt{\pi a}$$

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Stress Intensity Factor Equations (6)



An Embedded elliptical crack
or a circular crack
In an Infinite Plate

$$K = \frac{\sigma \sqrt{\pi a}}{\Phi_0} \left(\sin^2 \beta + \frac{a^2}{c^2} \cos^2 \beta \right)^{\frac{1}{4}}$$

$$\Phi_0 = \int_0^{\frac{\pi}{2}} \left[1 - \left(\frac{c^2 - a^2}{c^2} \right) \sin^2 \theta \right]^{\frac{1}{2}} d\theta$$

$$K = \sigma \sqrt{\frac{\pi a}{Q}} \quad \text{for } \beta = \frac{\pi}{2}$$

$$Q = \Phi_0^2$$

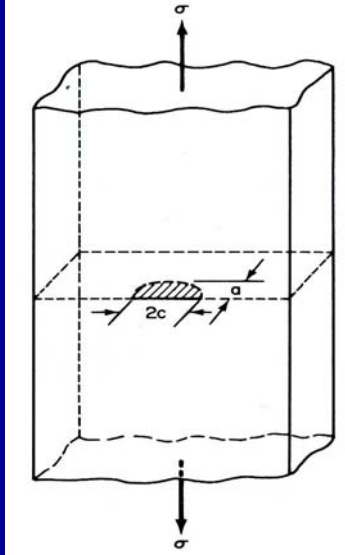
$$K = 0.65 \sigma \sqrt{\pi a} = 1.15 \sigma \sqrt{a}$$

for $\beta = \frac{\pi}{2}, a = c$ (a circle)

$$K = \frac{2}{\sqrt{\pi}} \sigma \sqrt{a} = 1.13 \sigma \sqrt{a}$$

Exact expression for circular crack

Stress Intensity Factor Equations (6)



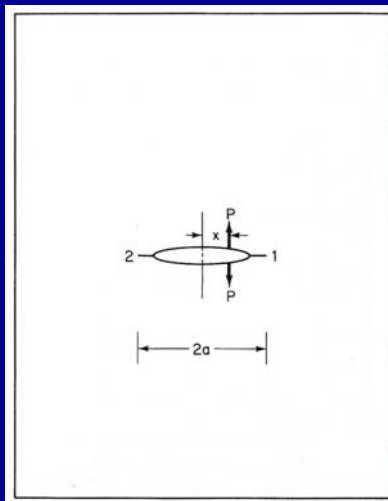
A Surface crack
or a circular crack
In an Infinite Plate

$$K = 1.12\sigma \sqrt{\pi \frac{a}{Q}} \cdot M_k$$

$$M_k = 1.0 + 1.2 \left(\frac{a}{t} - 0.5 \right) \quad t/a > 0.5$$

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Stress Intensity Factor Equations (7)



Cracks with wedge forces
and internal pressure

Eccentric line forces, P,
per unit thickness

$$K_{@1} = \frac{P}{\sqrt{\pi a}} \left(\frac{a+x}{a-x} \right)^{\frac{1}{2}}$$

$$K_{@2} = \frac{P}{\sqrt{\pi a}} \left(\frac{a-x}{a+x} \right)^{\frac{1}{2}}$$

$$K_{@1} = K_{@2} = \frac{P}{\sqrt{\pi a}} \quad \text{at } x = 0$$

$$K = p\sqrt{\pi a} \quad : \text{ internal pressure}$$

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Engineering Calculation of Stress Intensity Factor

Pedro Albrecht/ K. Yamada, ASCE, STR, Feb.1977

$$K = F(a)\sigma\sqrt{\pi a}$$

$$F(a) = F_e \cdot F_s \cdot F_w \cdot F_g$$

F_e: The Shape of Crack Front,
which is often assumed as an elliptical crack, correction

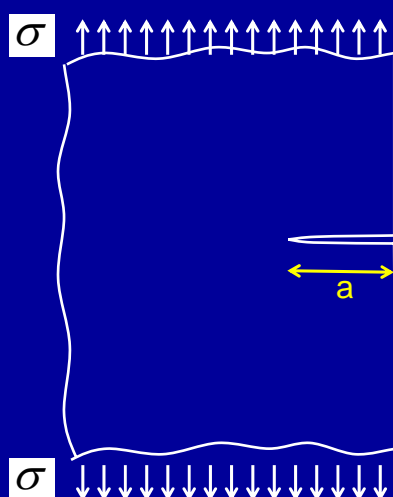
F_s: Effects of Free Surface, Front Free Surface Correction

F_w: Finite Width, The Back Surface Correction

F_g: Non-uniform Opening Stresses, Stress Gradient Correction

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2 Dimensional Crack Problems(1)



The Edge Crack
in a semi-infinite plate

$$K = F(a)\sigma\sqrt{\pi a}$$

$$F(a) = F_s = 1.12$$

Front Free Surface Correction

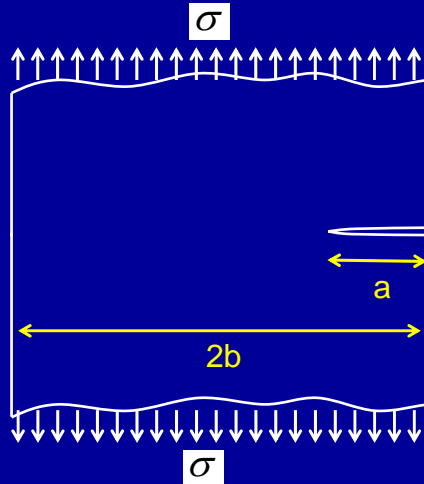


$$K = 1.12\sigma\sqrt{\pi a}$$

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2 Dimensional Crack Problems(2)

Single-Edge Crack
In a plate with finite width



$$K = F(a)\sigma\sqrt{\pi a}$$

$$F(a) = F_s \cdot F_w$$

$$K = 1.12 \cdot \sigma\sqrt{\pi a} \cdot f\left(\frac{a}{b}\right)$$

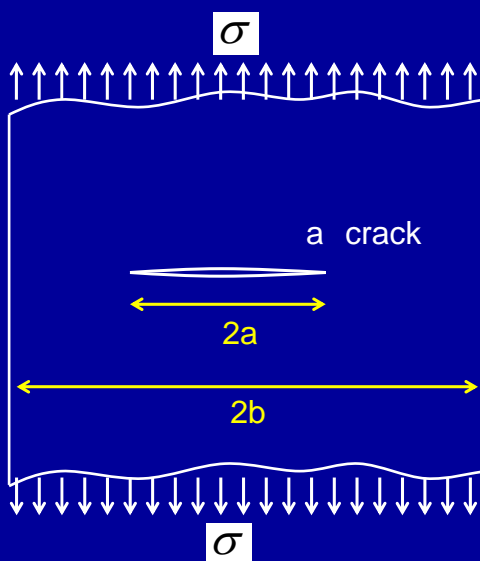
Front Free
Surface Correction

Finite Width

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2 Dimensional Crack Problems (3)

A Through Thickness Crack
In a Plate with **Finite Width**
Subject to
Uniform Tensile Stress



$$K = \sigma\sqrt{\pi a} \left(\frac{2b}{\pi a} \tan \frac{\pi a}{2b} \right)^{1/2}$$

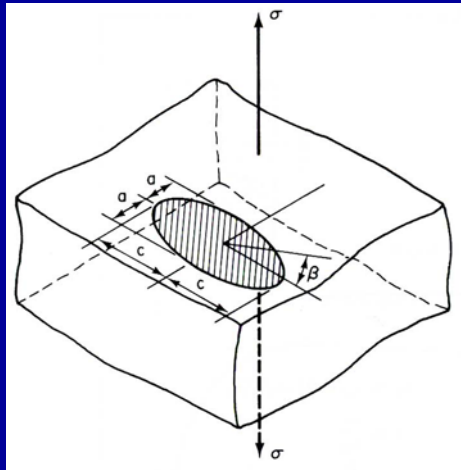
$$= \sigma\sqrt{\pi a} \sqrt{\sec\left(\frac{\pi a}{2b}\right)}$$

Finite Width

$$F(a) = F_w$$

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3 Dimensional Crack Problems (1)



An Embedded elliptical crack
or a circular crack
In an Infinite Plate

$$K = \frac{1}{\Phi_0} \left(\sin^2 \beta + \frac{a^2}{c^2} \cos^2 \beta \right)^{\frac{1}{4}} \cdot \sigma \sqrt{\pi a}$$

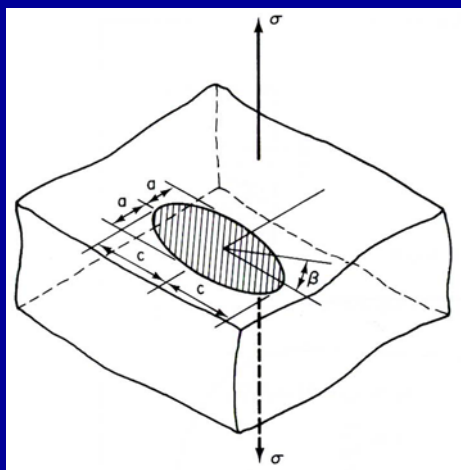
$$\Phi_0 = \int_0^{\frac{\pi}{2}} \left[1 - \left(\frac{c^2 - a^2}{c^2} \right) \sin^2 \theta \right]^{\frac{1}{2}} d\theta$$

$$K = \frac{1}{\Phi_0} \sigma \sqrt{\pi a} \quad \text{for } \beta = \frac{\pi}{2}$$

$$F_e \quad F(a) = F_e = \frac{1}{\Phi_0}$$

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3 Dimensional Crack Problems (1)



An Embedded elliptical crack
or a circular crack
In an Infinite Plate

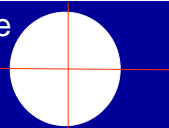
$$K = \frac{1}{\Phi_0} \left(\sin^2 \beta + \frac{a^2}{c^2} \cos^2 \beta \right)^{\frac{1}{4}} \cdot \sigma \sqrt{\pi a}$$

$$\Phi_0 = \int_0^{\frac{\pi}{2}} \left[1 - \left(\frac{c^2 - a^2}{c^2} \right) \sin^2 \theta \right]^{\frac{1}{2}} d\theta$$

$$K = \frac{1}{\Phi_0} \sigma \sqrt{\pi a} \quad \text{for } \beta = \frac{\pi}{2}$$

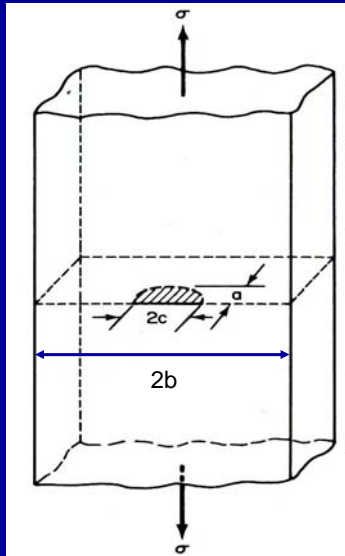
$$F_e \quad F(a) = F_e = \frac{1}{\Phi_0} = \frac{2}{\pi} \quad \text{for } a = c$$

Circle



$$K = \frac{2}{\pi} \sigma \sqrt{\pi a}$$

3 Dimensional Crack Problems (2)



A Surface crack
or a circular crack
In a Plate with Finite Width

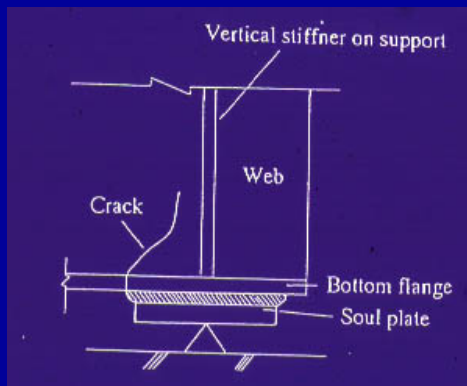
$$K = \underbrace{\alpha \cdot M_k}_{F_s} \underbrace{\frac{1}{\Phi_0}}_{F_e} \underbrace{F_g}_{F_w} \cdot \sigma \sqrt{\pi a} \cdot \sqrt{\frac{2b}{\pi} \tan \frac{\pi}{2b}}$$

$$M_k = 1.0 + 1.2 \left(\frac{a}{t} - 0.5 \right)$$

Stress Gradient Correction
Correction for Stress Concentration

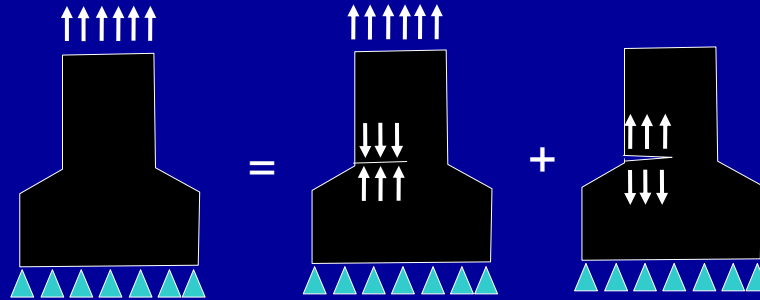
Geometry Correction Factor F_g

Cracks always occur at geometrical discontinuities
as cover plate ends



Calculation of F_g

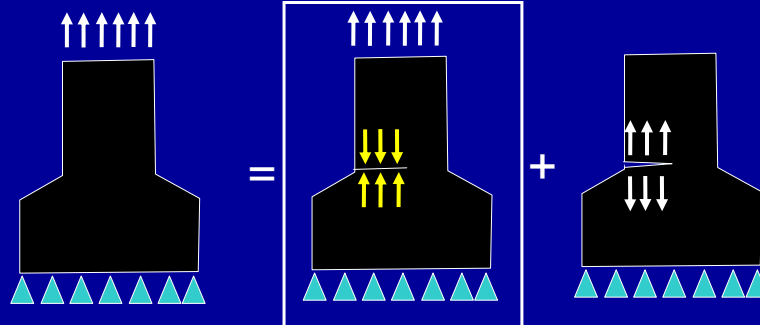
Superimpose



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Calculation of F_g

Superimpose



Step 1

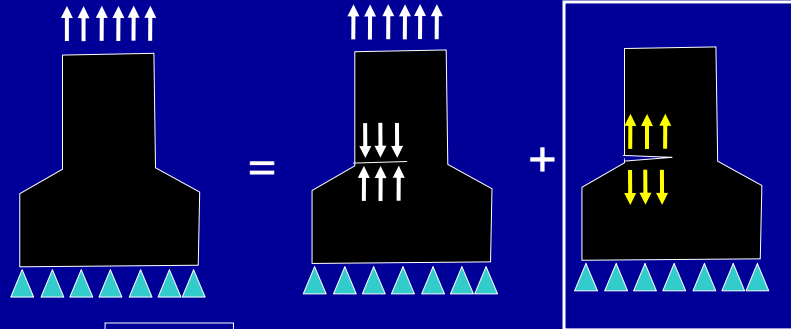
Compute the actual stress
Along the line where the crack
would be inserted
By any suitable methods.

FEM

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Calculation of F_g

Superimpose

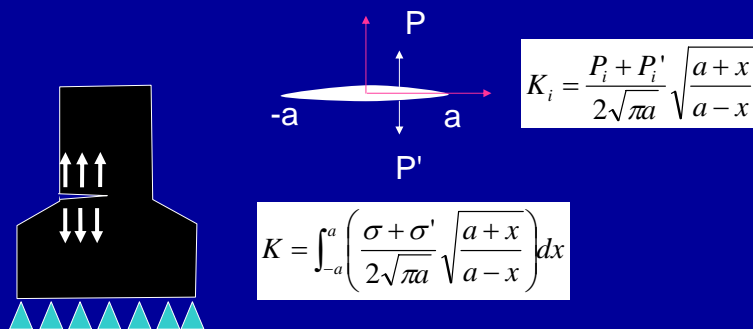


Step 2

Insert a crack of given length
Along the same line and apply
the stress determined in Step 1

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Calculation of F_g



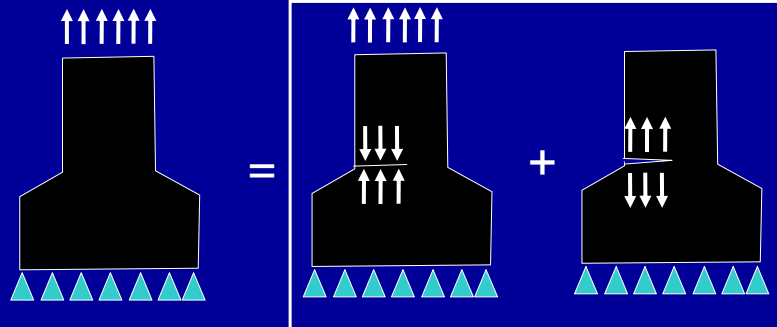
Step 3

Compute K by integrating K_i over the
length of the crack with the stress
determined in Step 1 applied

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Calculation of F_g

Superimpose



Step 4

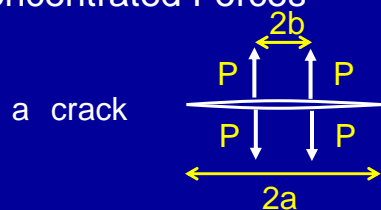
Repeat steps 2 and 3
for any desired crack size

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Calculation of F_g

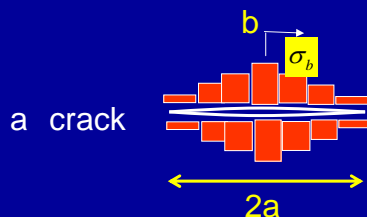
Calculation of K in Step 3

Concentrated Forces



$$K = \frac{2P}{\sqrt{\pi a}} \frac{a}{\sqrt{a^2 - b^2}}$$

Distributed Forces



$$K = \sqrt{\pi a} \frac{2}{\pi} \int_0^a \frac{\sigma_b}{\sqrt{a^2 - b^2}} db$$

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Calculation of F_g

Stress Distribution in Step 1

Closed form expressions defining the stress distribution
In the uncracked body are usually not available.

From FEM

$$K = \sqrt{\pi a} \frac{2}{\pi} \sum_{i=1}^n \sigma_{b_i} \int_{b_i}^{b_{i-1}} \frac{1}{\sqrt{a^2 - b^2}} db$$

In which the discrete stress σ_{b_i} is applied
over the element b_i and b_{i+1}

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Calculation of F_g

Stress Distribution in Step 1

After factoring out the mean stress

$$K = \sqrt{\pi a} \frac{2}{\pi} \sum_{i=1}^n \sigma_{b_i} \int_{b_i}^{b_{i-1}} \frac{1}{\sqrt{a^2 - b^2}} db$$

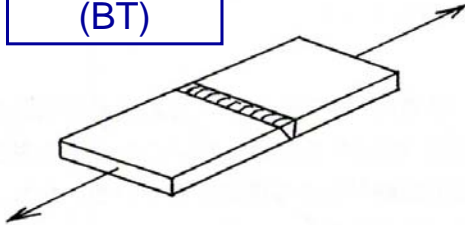


$$K = \sigma \sqrt{\pi a} \frac{2}{\pi} \sum_{i=1}^n \frac{\sigma_{b_i}}{\sigma} \left(\arcsin \frac{b_{i-1}}{a} - \arcsin \frac{b_i}{a} \right)$$

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Examples of F_g

Butt Joint
(BT)

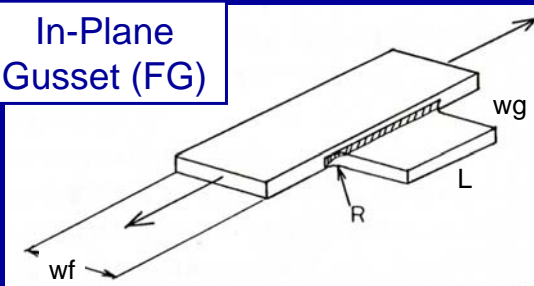


$$F_g = 1$$

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Examples of F_g

In-Plane
Gusset (FG)



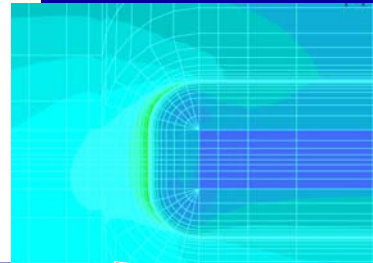
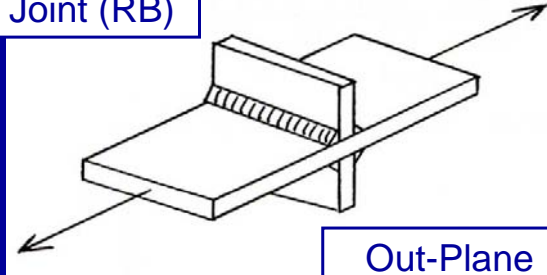
$$F_g = \frac{-1.115 \log\left(\frac{R}{w_f}\right) + 0.537 \log\left(\frac{L}{w_f}\right) + 0.1384 \log\left(\frac{w_g}{w_f}\right) + 0.6801}{1 + \frac{1}{1.158} \cdot \left(\frac{a}{w_f}\right)^{0.6051}}$$

By Zettlemoyer and Fisher

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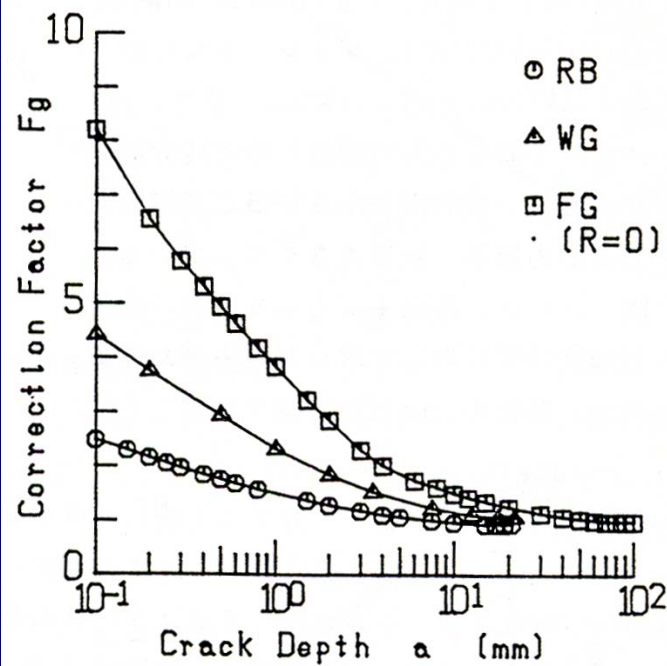
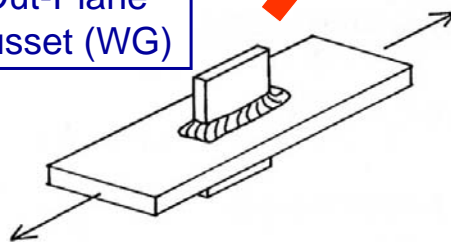
Examples of F_g

Cruciform Joint (RB)



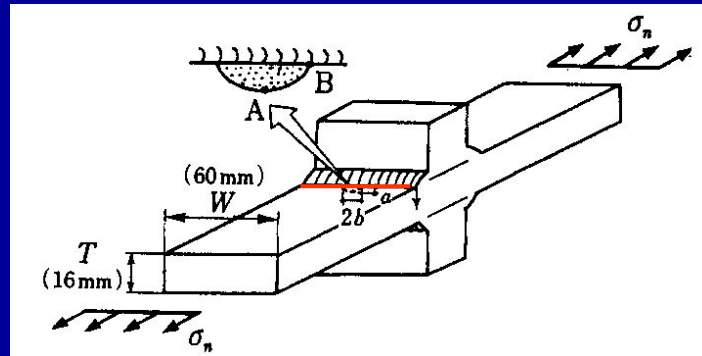
Stress Distribution From FEM

Out-Plane Gusset (WG)



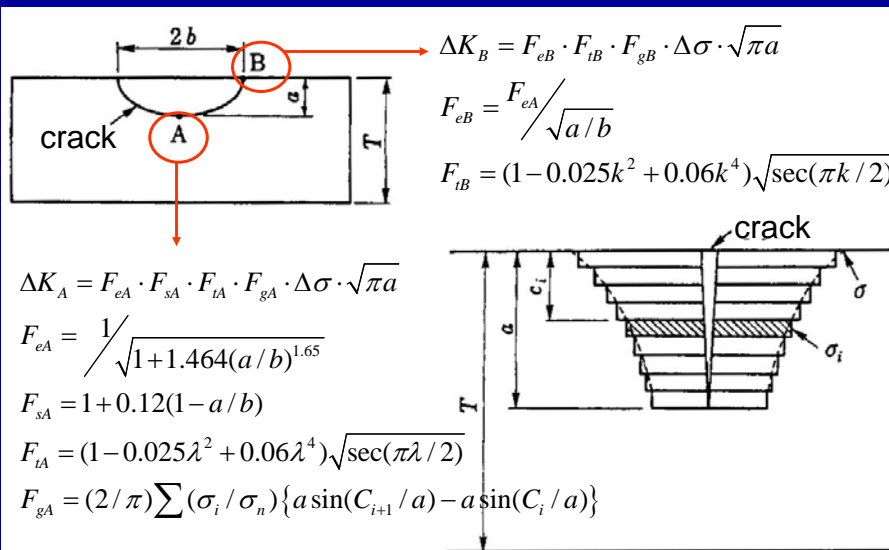
Examples of F_g

Cracks from Incomplete Penetration



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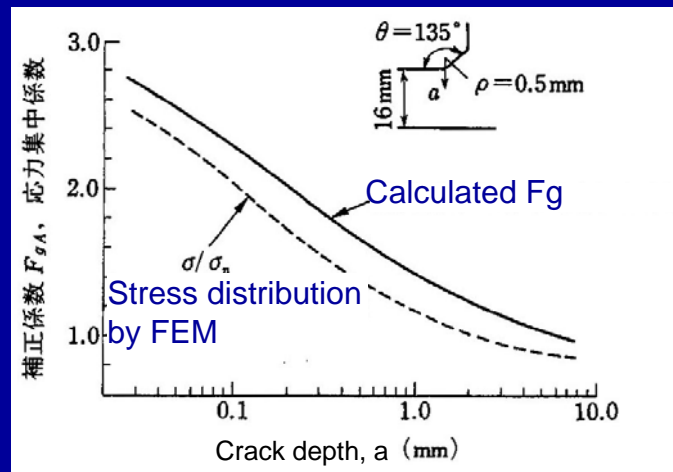
Model of Cracks



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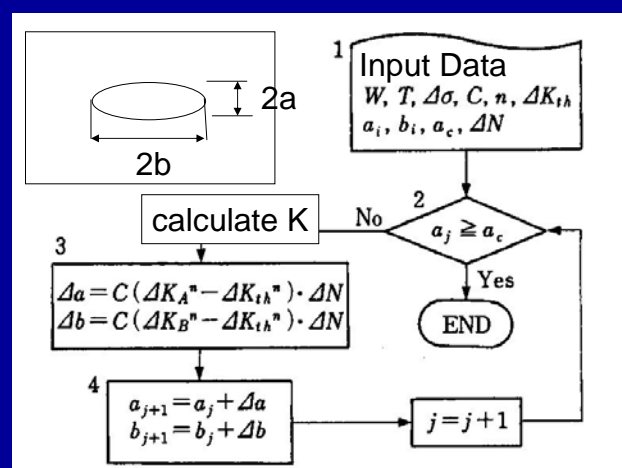
Calculation of Fg

Relationships of stress distribution and Fg



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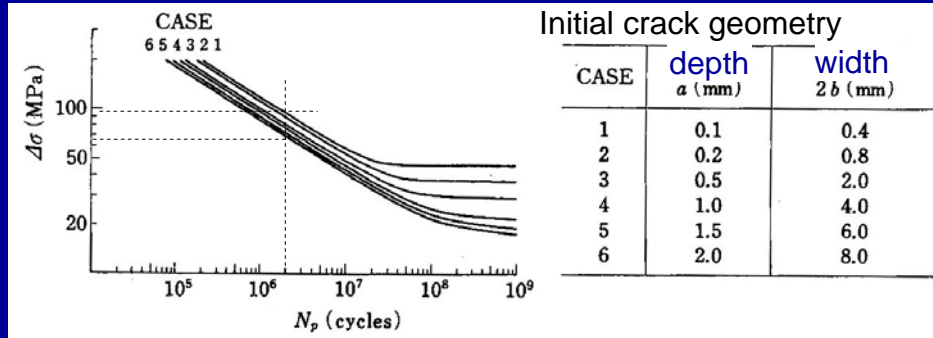
Calculation strategies



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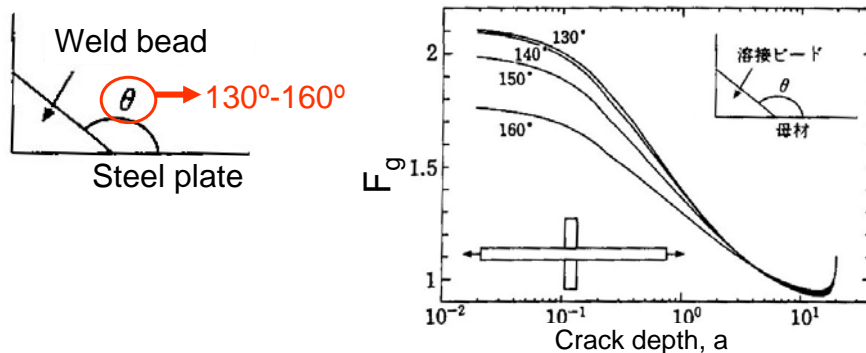
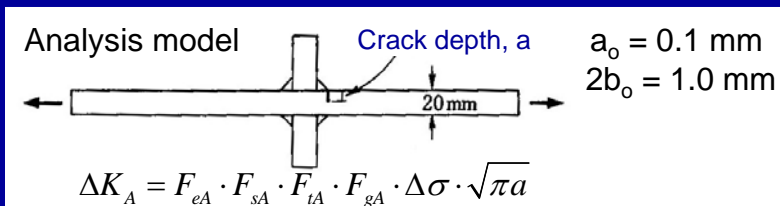
Results

Variation of initial crack width and height

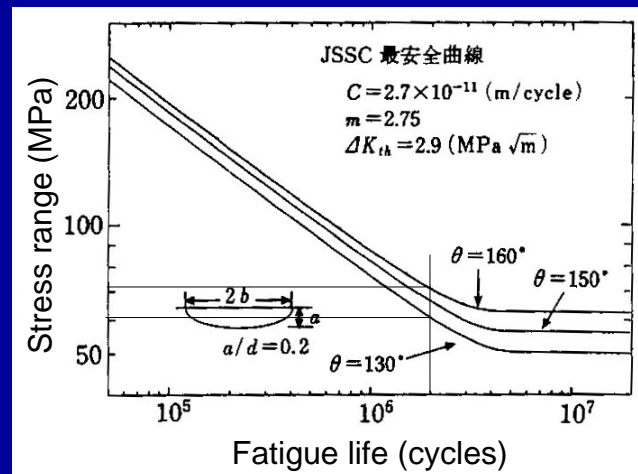


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Effects of weld shape

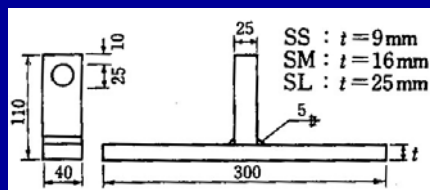


Effects of weld shape

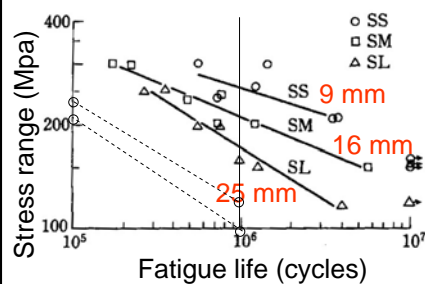


55

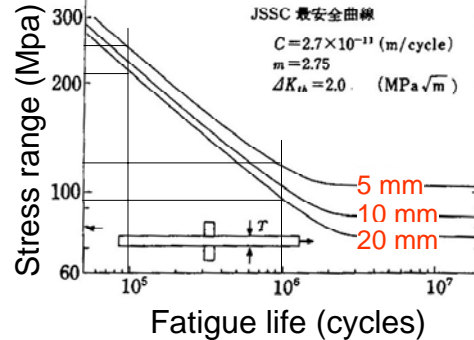
Effects of plate thickness



Experimental results



Computational results



Fracture Toughness

The Condition for Unstable Crack Growth

$$K = K_c$$

Fracture Toughness

The Critical Stress Intensity Factor
for Unstable Crack Growth

Mode I

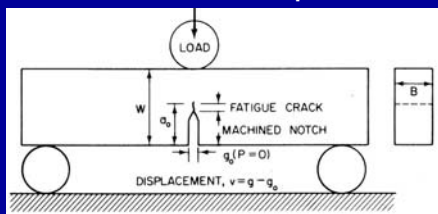
$$K_I = K_{Ic}$$

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Test Methods to Evaluate K_{Ic}

ASTM E-399

A Bend Specimen

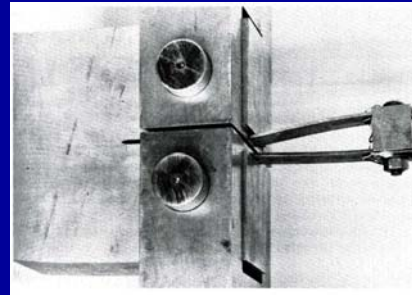
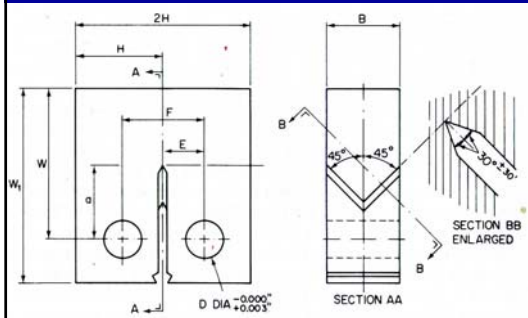


three point bend test

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Test Methods to Evaluate K_{Ic}

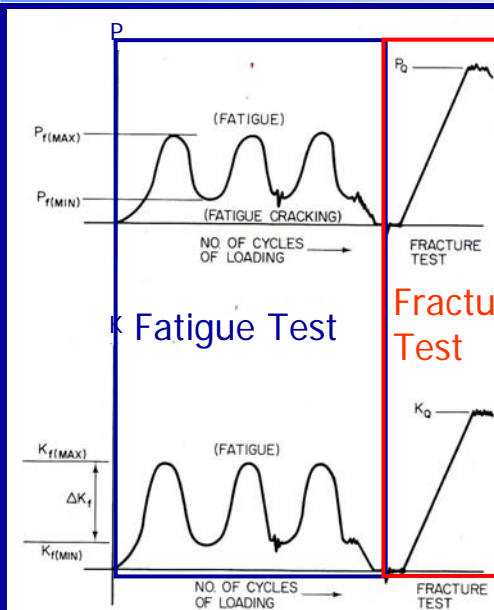
ASTM E-399 Compact Tension Specimen



Chevron notch : to keep the crack in plane and to ensure that it extended beyond the machined notch root ($0.05W$)

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Test Methods to Evaluate K_{Ic}



The Testing Program

Fatigue Cracking

Small Scale Yielding Condition



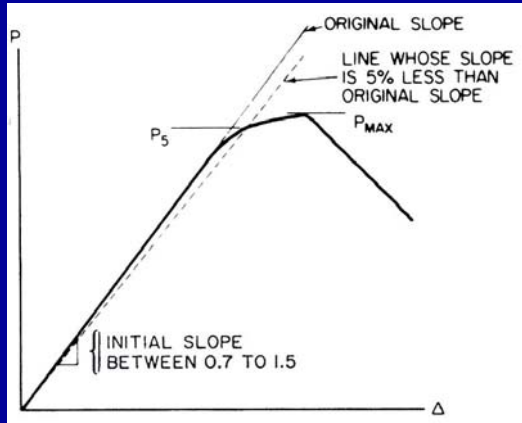
Fracture Test

Plane Strain Condition

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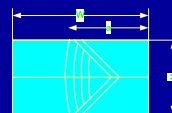
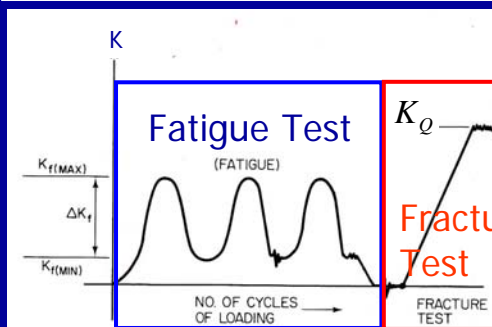
Test Methods to Evaluate K_{Ic}

in Plane Fracture Toughness Testing



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Test Methods to Evaluate K_{Ic}



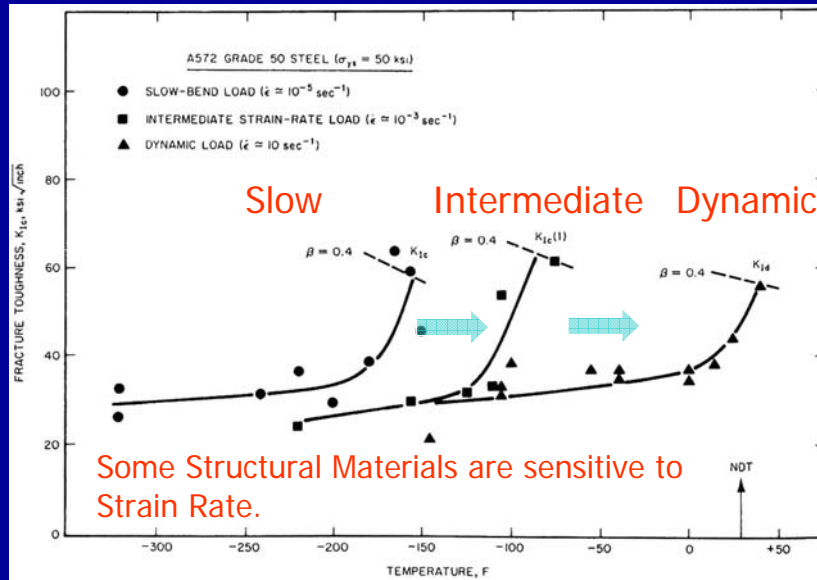
specimen minimum size requirement
a: crack length,
B: thickness
W: specimen depth

$$a \geq 2.5 \left(\frac{K_Q}{\sigma_{Yield}} \right)^2, B \geq 2.5 \left(\frac{K_Q}{\sigma_{Yield}} \right)^2, W \geq 5.0 \left(\frac{K_Q}{\sigma_{Yield}} \right)^2$$

$$K_Q \Rightarrow K_{Ic}$$

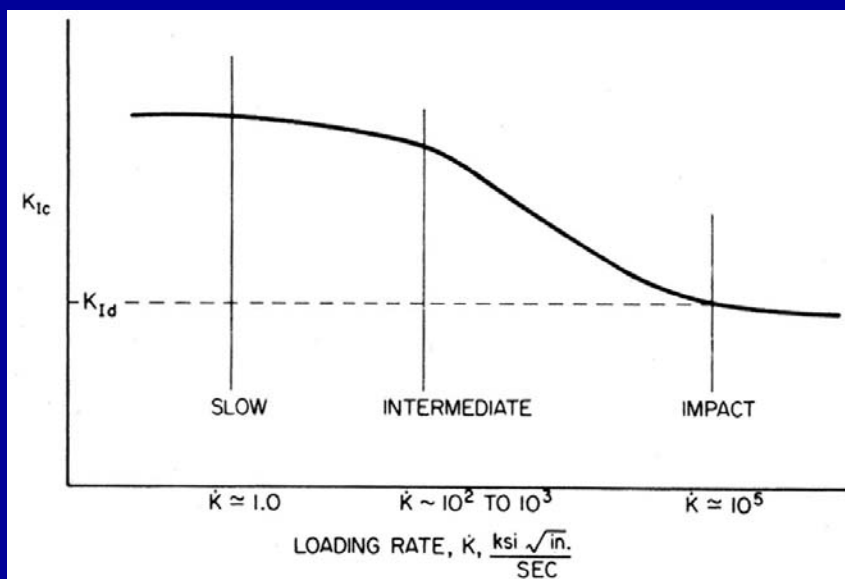
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Effects of Loading Rate on Fracture Toughness

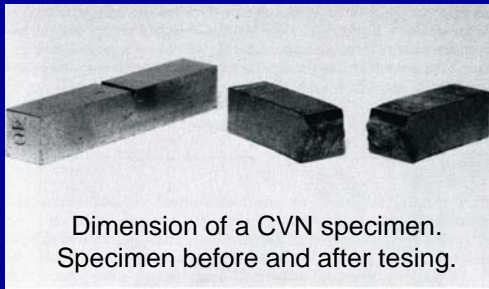
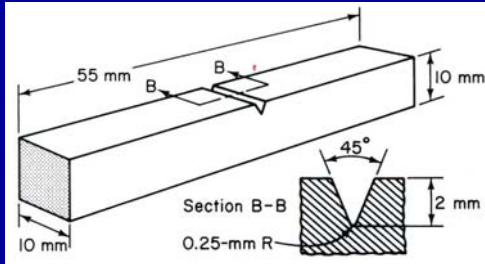


63

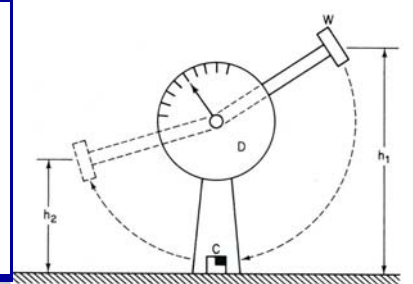
Effects of Loading Rate on Fracture Toughness



Charpy V-Notch (CVN) Impact Tests



Dimension of a CVN specimen.
Specimen before and after testing.

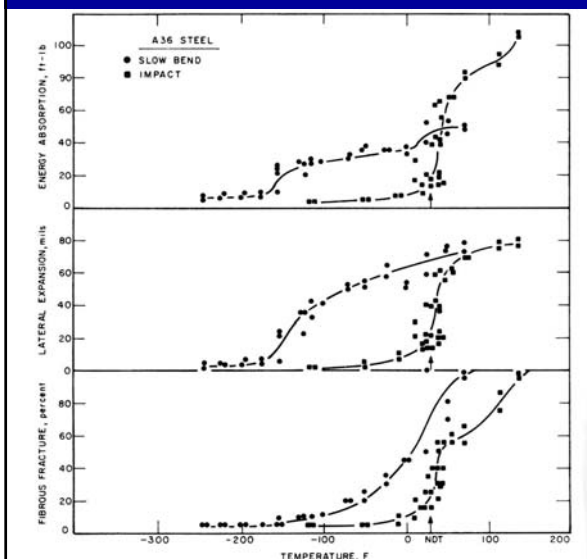


the energy absorbed by the specimen is related to the height differential $h_1 - h_2$, is recorded on dial D.

The Traditional Evaluation
Method of Fracture
Toughness

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Charpy Impact Tests



Absorbed Energy-Temperature

To Prevent
Brittle Fracture

Absorbed Energy: E_v
Transition
Temperature: T_{tr}

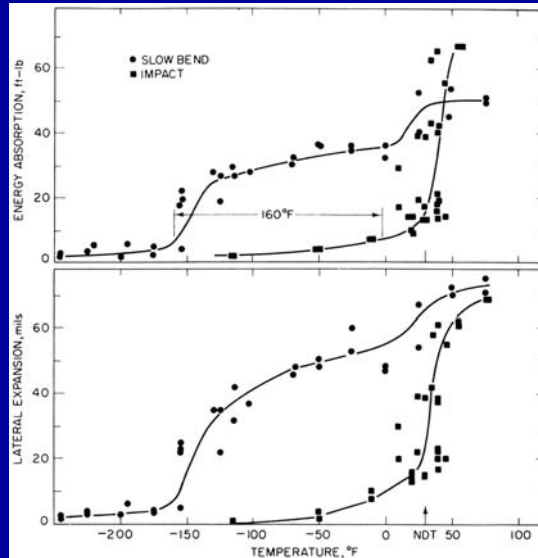
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CVN Tests vs K_{Ic} Tests

CVN Tests:
Dynamic Condition
Crack Initiation
+ Propagation

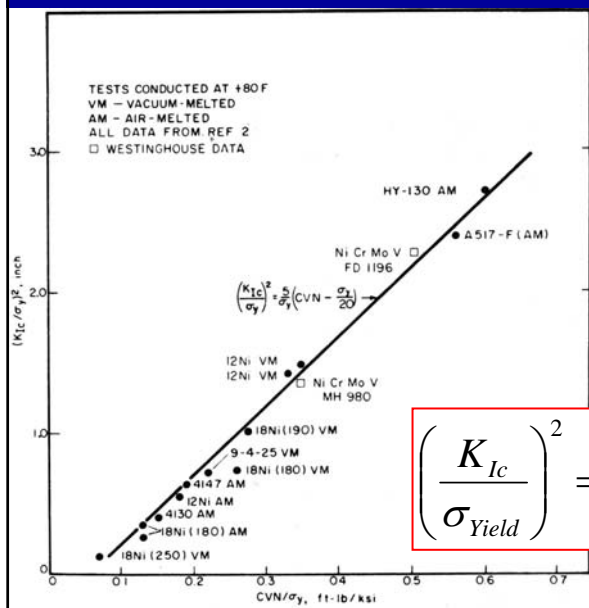


K_{Ic} Tests:
Slow Bending
Crack Initiation



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K_{Ic}- CVN Upper Shelf Correlation



Proposed by
Barsom and Rolfe

$$\left(\frac{K_{Ic}}{\sigma_{Yield}} \right)^2 = 5 \left(\frac{CVN}{\sigma_{Yield}} - 0.05 \right)$$

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