2. Derivation and Interpretation of solutions for Wave Equations

波動方程式の解釈

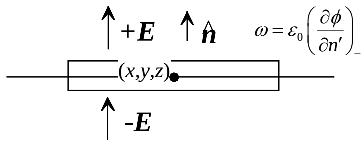
$\nabla^2\phi + k^2\phi = -q(x,y,z)$ Observer $\phi(x',y',z')$

$$\phi = \frac{1}{4\pi}\int_v q\frac{e^{-jkr}}{r}dv + \underline{\frac{1}{4\pi}\int_s\left(\frac{\partial\phi}{\partial n}\right)_- \frac{e^{-jkr}}{r}ds} + \underline{\frac{1}{4\pi}\int_s(-\phi)_-\frac{\partial}{\partial n}\frac{e^{-jkr}}{r}ds}$$

$$\equiv \frac{1}{4\pi}\int_v q\frac{e^{-jkr}}{r}dv + \phi_A + \phi_B$$

$\frac{\partial}{\partial n}$：いずれも積分系 $integ.coord.$

◈ *Physical interpretation*



$\varepsilon_0\left(\frac{\partial\phi}{\partial n}\right)_-$ なる１重層[2-2]=ω

- $\phi_A = \frac{1}{4\pi}\int_s\underset{A}{\left(\frac{\partial\phi}{\partial n}\right)_-}\frac{e^{-jkr}}{r}ds$

at the discontinuity （不連続の点 $e^{-jkr}=1$）

$\phi_A\Big|_-^+ = 0 \qquad \cdots(A)$

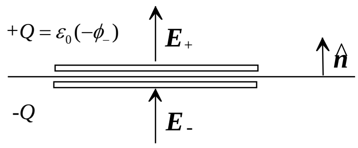
$\frac{\partial\phi_A}{\partial n}\Big|_-^+ = -\frac{\omega}{\varepsilon} = -\underset{A}{\left(\frac{\partial\phi}{\partial n}\right)_-}$

$\varepsilon_0(-\phi_-)$なる２重層[2-3]

- $\phi_B = \frac{1}{4\pi}\int_s\underset{B}{(-\phi)_-}\frac{\partial}{\partial n}\frac{e^{-jkr}}{r}ds$

$\phi_B\Big|_-^+ = \underset{B}{(-\phi)_-} \qquad \cdots(B)$

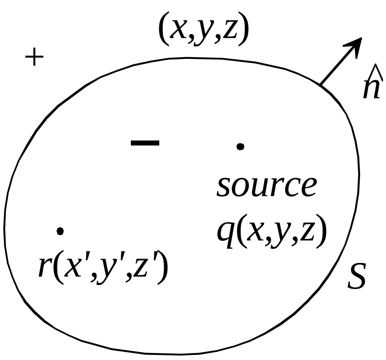
$\frac{\partial\phi_B}{\partial n'}\Big|_-^+ = 0$



Then したがって

$\phi\Big|_-^+ = \phi_A\Big|_-^+ + \phi_B\Big|_-^+ = (-\phi)_- \qquad \leftarrow B$

$\phi_+ - \phi_- = -\phi_- \qquad \therefore \phi_+ = 0$



$\frac{\partial}{\partial n'}\phi\Big|_-^+ = \frac{\partial\phi_A}{\partial n'}\Big|_-^+ + \frac{\partial\phi_B}{\partial n'}\Big|_-^+ = \left(-\frac{\partial\phi}{\partial n}\right)_- \qquad \leftarrow -A$

Obs. 観測系 *Obs.* 観測系

$\left(\frac{\partial\phi}{\partial n'}\right)_+ - \left(\frac{\partial\phi}{\partial n'}\right)_- = \left(-\frac{\partial\phi}{\partial n}\right)_- \qquad \therefore \frac{\partial\phi_+}{\partial n} = 0$

同じ意味として良い

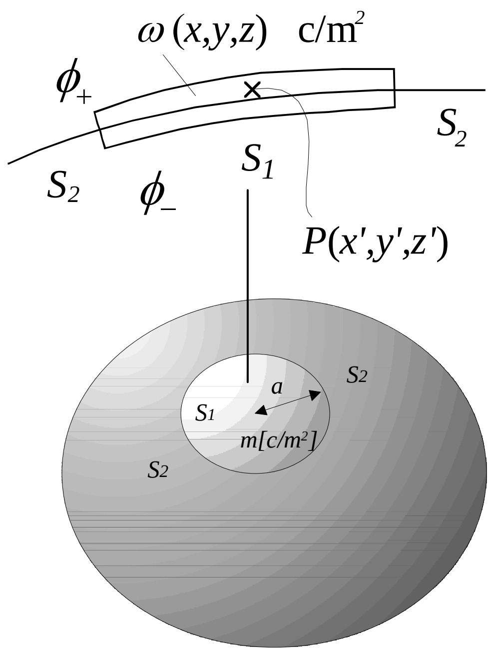
$\therefore$ Sの外の領域$\tilde{V}$は$\tilde{V}$に波源が
なければすべて零になる。

$$\because \phi = 0 + \frac{1}{4\pi}\int_S\underset{\downarrow\ 0}{\left(\frac{\partial\phi}{\partial n}\right)_+}\frac{e^{-jkr}}{r}dS + \frac{1}{4\pi}\int_S\underset{\downarrow\ 0}{-\phi_+}\frac{\partial}{\partial n}\frac{e^{-jkr}}{r}dS$$

$\therefore$ *Fields outside of* $S(\tilde{V})$ *is null if there is no source in* $\tilde{V}$.

Explain what happens if the sources exist outside of the surface S ??

◆ *Single Layer*



$$\phi(x',y',z') = \frac{1}{4\pi\varepsilon_0}\int_s \frac{\omega(x,y,z)}{r}ds$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\int_{s1} + \int_{s2}\right)$$

$$= \phi_1 + \phi_2 \qquad \phi_2 : continuous$$

$$|\omega| \le m$$

$$|\phi_1| = \frac{1}{4\pi\varepsilon_0}\left|\int_{S_1}\frac{\omega}{r}ds\right| < \frac{1}{4\pi\varepsilon_0}\int_0^a \frac{m}{r}2\pi r\, dr = \frac{m}{4\pi\varepsilon_0}2\pi a \to 0$$

$$\lim_{a\to 0} s$$

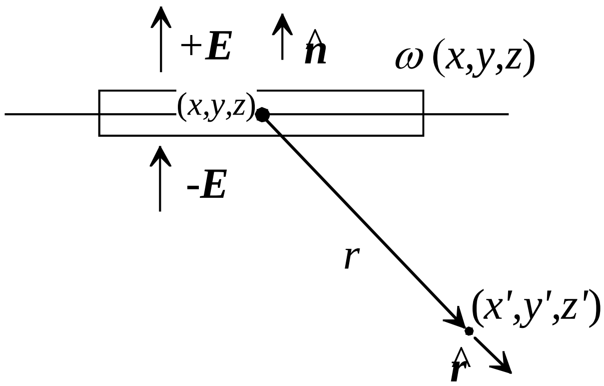
$$\phi\big|_-^+ = \phi_+ - \phi_- = 0 \qquad continuous$$

From divergence theorem,

ところで、発散定理より

$$(E_+ - E_-)\cdot\hat{\boldsymbol{n}} = \frac{\omega}{\varepsilon_0}$$

$$(-\nabla'\phi_+ + \nabla'\phi_-)\cdot\hat{\boldsymbol{n}} = \frac{\partial\phi}{\partial n'}\bigg|_+^- = \frac{\omega}{\varepsilon_0}$$



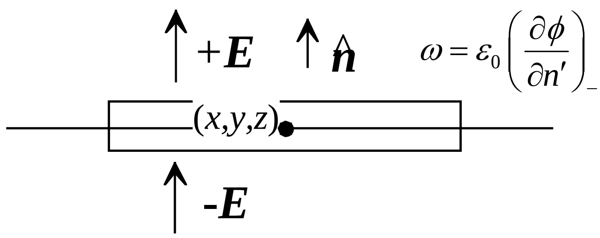
◆ *Physical interpretation(ϕ_A) → Single layer*

$$\phi_A(x',y',z') \equiv \frac{1}{4\pi}\int_s A(x,y,z)\frac{1}{r}ds$$

$$\phi_A\big|_-^+ = 0 \qquad \varepsilon_0 A\ [c/m^2]\text{の電気１重層}\quad single\text{-}layer\quad e\,charge$$

$$\frac{\partial\phi_A}{\partial n'}\bigg|_+^- = A \qquad \cdots(A)$$

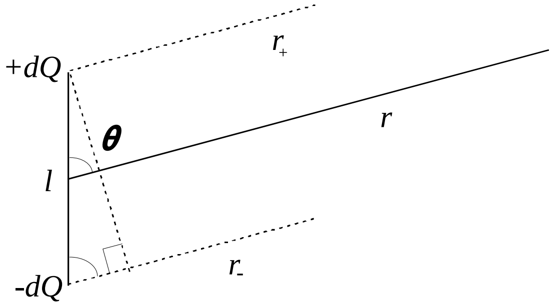
観測点系



◈ Double Layer

$$d\phi = \frac{dQ}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) \quad \text{電気モーメント}/m^2 \quad dQl \equiv \tau ds \quad \tau[c/m]$$

$$\cong \frac{dQ}{4\pi\varepsilon_0}\frac{r_- - r_+}{r^2} = \frac{dQ}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2} = \frac{\tau\cos\theta}{4\pi\varepsilon_0 r^2}ds$$



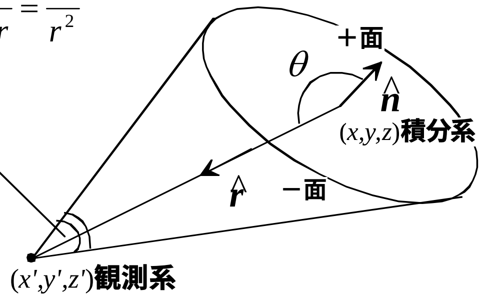
$$= \frac{-\tau}{4\pi\varepsilon_0}d\Omega = \frac{\tau}{4\pi\varepsilon_0}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)ds$$

$$\because d\Omega = -\mathbf{n}\times\hat{\mathbf{r}}\frac{1}{r^2}ds = -\mathbf{n}\cdot\left(\nabla\frac{1}{r}\right)ds = -\frac{\partial}{\partial n}\left(\frac{1}{r}\right)ds$$

積分系に関する微分

$Obs.\ \nabla'\frac{1}{r} = -\frac{\hat{r}}{r^2}$

$Int.\ \nabla\frac{1}{r} = \frac{\hat{r}}{r^2}$



Discontinuity across the surface from ＋ to － is

観測点が＋から－に横切る時の不連続量は

$$d\phi\Big|_-^+ = \frac{-\tau}{4\pi\varepsilon_0}\Omega\Big|_-^+ = \frac{-\tau}{4\pi\varepsilon_0}\cdot -4\pi = \frac{\tau}{\varepsilon_0}$$

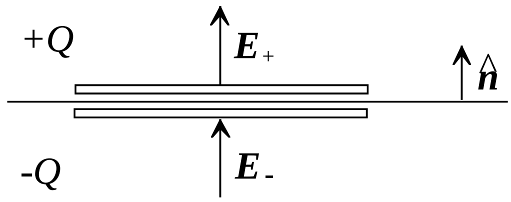
$$d\,\phi\Big|_-^+ = \frac{\tau}{4\pi\varepsilon_0}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)ds\Big|_-^+ = \phi\Big|_-^+ = \frac{\tau}{\varepsilon_0} \quad \text{(上式より)}$$

また、

From Gauss's Law, 電荷打消（ガウス）より　$(\boldsymbol{E}_+ - \boldsymbol{E}_-)\cdot\hat{\boldsymbol{n}} = 0$

$$\left(-\nabla'\phi_+ + \nabla'\phi_-\right)\cdot\boldsymbol{n} = \frac{\partial\phi}{\partial n'}\Big(_+^- = 0$$

$(\boldsymbol{E}_+ - \boldsymbol{E}_-)\cdot\hat{\boldsymbol{n}} = 0$　観測系

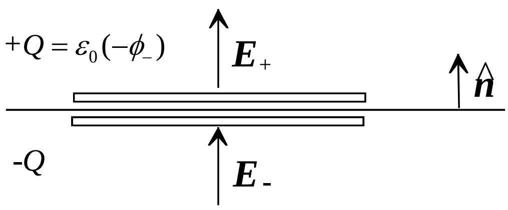


$(\boldsymbol{E}_+ - \boldsymbol{E}_-)\cdot\hat{\boldsymbol{n}} = 0$

◈ *Physical interpretation(ϕ_B) → Double layer*

$$\phi_B = \frac{1}{4\pi}\int B\frac{\partial}{\partial n}\left(\frac{1}{r}\right)ds'$$

積分系



$\phi_B\Big|_-^+ = B \quad \varepsilon_0 B$ なる電気２重層 $Double-layer$

$$\frac{\partial\phi_B}{\partial n'}\Big|_-^+ = 0$$

（B）

観測点系

Fields produced by Surface Integrations of Single and Double Layer (Example: static fields for a point source in Volume V)

〇If we have only the point source q inside V (*V* 内に点波源 *q* のみ存在する場合)。

$$\nabla^2\phi + k^2\phi = -q(x,y,z) \qquad \phi(x',y',z')$$

$$\phi = \frac{1}{4\pi}\int_v q\frac{e^{-jkr}}{r}dv + \frac{1}{4\pi}\int_s\left(\frac{\partial\phi}{\partial n}\right)_- \frac{e^{-jkr}}{r}ds + \frac{1}{4\pi}\int_s(-\phi)_- \frac{\partial}{\partial n}\frac{e^{-jkr}}{r}ds$$

$$\equiv \frac{1}{4\pi}\int_v q\frac{e^{-jkr}}{r}dv + \phi_A + \phi_B$$

$\frac{\partial}{\partial n}$: いずれも積分系 *integ. coord.*

$\varepsilon_0\left(\frac{\partial\phi}{\partial n}\right)_-$ なる 1 重層　　$\varepsilon_0(-\phi_-)$なる 2 重層

- *Fields and potentials produced by the original source q (Original な波源の作る Field とポテンシャル)*

$$\phi_V = \frac{q}{4\pi r}$$

$$\mathbf{E}_V = \frac{q}{4\pi r^2}\hat{\mathbf{r}}$$

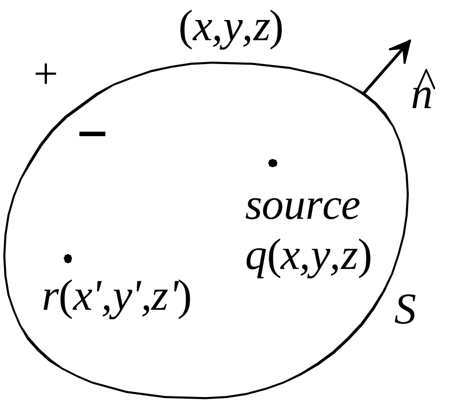
- *Huygens sources assumed on the surface S (S（ r =R）上に仮定される Huygens 波源)*

・Fields/potentials produced by the **single layer** at $r = R$（*in* ; $r < R$, *out* ; $r > R$）($r = R$ の位置に想定される 1 重層が作る *Field* とポテンシャル）

$$\varepsilon_0\left(\frac{\partial\phi}{\partial n}\right) = \varepsilon_0\frac{-q}{4\pi R^2}$$

$$\phi_A = \frac{-q}{4\pi R}(Const.) \quad in, \quad \frac{-q}{4\pi r} \quad out$$

$$\mathbf{E}_{\mathbf{A}-} = \mathbf{0} \quad in, \quad \frac{-q\hat{\mathbf{r}}}{4\pi r^2} \quad out$$



・Fields/potentials produced by the **double layer** at $r = R$（*in* ; $r < R$, *out* ; $r > R$）$r = R$ の位置に想定される 2 重層が作る *Field* とポテンシャル

$$\varepsilon_0\frac{-q}{4\pi R}$$

$$\phi_B = \frac{+q}{4\pi R}(Const.) \quad in, \quad 0 \quad out$$

$$\mathbf{E}_{B-} = \mathbf{0} \quad in, \quad \mathbf{0} \quad out$$

- *Fields/potentials in V*（$r < R$）or outside （$r > R$）*produced by single and double layers on S (S 上の 1 重層と 2 重層の波源が、内*（$r < R$）*、外*（$r > R$）*に作る Field とポテンシャル)*

$$\phi_S = 0(Const.) \quad in, \quad \frac{-q}{4\pi r} \quad out$$

$$\mathbf{E}_S = \mathbf{0} \quad in, \quad \frac{-q\hat{\mathbf{r}}}{4\pi r^2} \quad out$$

Consequently, these are 0 in S, while it is identical with the fileds produced by the original source inside with negative sign (x -1).(つまり、*S* 内では完全に零、*S* 外では *Original* な点波源の作る *Field* とポテンシャルの符号を逆にしたものである。)

◆ *If the contributions from 3 sources (original, single and double layers) are summed up,the fields/potentials outside and inside of V are expressed as.* (*Original* な波源、１重層、２重層を３つの波源と見なして、その３波源の寄与の合計として、*S* 内、*S* 外の *Field* とポテンシャルを計算すると、)

$\phi = \frac{q}{4\pi r}$　　*S* 内（*R* に依らぬ）、　0　*S* 外

$\mathbf{E} = \frac{q}{4\pi r^2}\hat{\mathbf{r}}$　　*S* 内（*R* に依らぬ）、　0　*S* 外

〇From alternative point of view, the fields outside of S is considered. Since the sources are confined inV(outside of V'), and the definition of ***n*** is toward the inner directions, we have the following results. (視点を変え、観測点が *S* の外にある場合の解を表現する。波源は *S* の外から見ると *V* 外（*V'*）にしかない。***n*** は逆に内側を向くことに注意すると、)

$$\nabla^2\phi = -q(x,y,z) \qquad q=0$$

$$\phi = \frac{1}{4\pi}\int_{v'} q\frac{1}{r}dv + \frac{1}{4\pi}\int_s\left(\frac{\partial\phi}{\partial n}\right)_+\frac{1}{r}ds + \frac{1}{4\pi}\int_s(-\phi)_+\frac{\partial}{\partial n}\frac{1}{r}ds$$

$$\equiv \frac{1}{4\pi}\int_{v'} 0\frac{1}{r}dv + \qquad \phi_A \qquad + \phi_B$$

$\frac{\partial}{\partial n}$: いずれも積分系　*integ. coord.*

$\varepsilon_0\left(\frac{\partial\phi}{\partial n}\right)_+$ なる１重層　　$\varepsilon_0(-\phi_+)$なる２重層

◆ *Fields/potentials due to the original sources.(Original* な波源の作る *Field* とポテンシャル)

$\phi_{V'} = 0$

$\mathbf{E}_{V'} = \mathbf{0}$

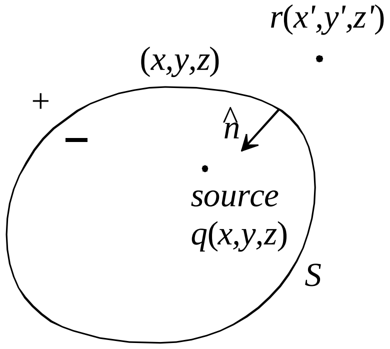
◆ *Huygens sources assumed on the surface S (S*（*r* =*R*）上に仮定される *Huygens* 波源

・Fields/potentials produced by the **single layer** at $r = R$（*in* ; $r < R$, *out* ; $r > R$）($r = R$ の位置に想定される１重層が作る *Field* とポテンシャル)

$$\varepsilon_0\left(\frac{\partial\phi}{\partial n}\right) = \varepsilon_0\frac{+q}{4\pi R^2}$$

$\phi_A = \frac{+q}{4\pi R}(Const.)$　　*in*,　　$\frac{+q}{4\pi r}$　　*out*

$\mathbf{E}_{A-} = \mathbf{0}$　　*in*,　　$\frac{+q\hat{\mathbf{r}}}{4\pi r^2}$　　*out*



・Fields/potentials produced by the **double layer** at $r = R$（*in* ; $r < R$, *out* ; $r > R$）$r = R$ の位置に想定される２重層が作る *Field* とポテンシャル

$$\varepsilon_0\frac{-q}{4\pi R}$$

$\phi_B = \frac{-q}{4\pi R}(Const.)$　　*in*,　　0　　*out*

$\mathbf{E}_{B-} = \mathbf{0}$　　*in*,　　$\mathbf{0}$　　*out*

◆ *Fields/potentials in V*（$r < R$）or outside （$r > R$）*produced by single and double layers on S(S*

上の 1 重層と 2 重層の波源が、内（$r < R$）、外（$r > R$）に作る *Field* とポテンシャル)

$$\phi_S = 0(Const.) \quad in, \quad \frac{+q}{4\pi r} \quad out$$

$$\mathbf{E}_S = \mathbf{0} \quad in, \quad \frac{+q\hat{\mathbf{r}}}{4\pi r^2} \quad out$$

Consequently, these are 0 in S, while it is identical with the fileds produced by the original source inside with negative sign (x -1).(つまり、*S* 内では完全に零、*S* 外では *Original* な点波源の作る *Field* とポテンシャル（同じ符号）である。)

○Then, if we have the point source q' outside V only (次に、V 内の波源 q が消滅し、V 外にのみ新しい波源 q' がある場合を考える。)***n*** is outward normal unit vector and r' is the distance between q' and the observer. nの向きは V 外へ向いているとする。また、q' から観測点まではかった距離を r' とする。

- *Fields and potentials produced by the original source q' (Original な波源の作る Field とポテンシャル)*

$$\phi_V = \frac{q'}{4\pi r'}$$

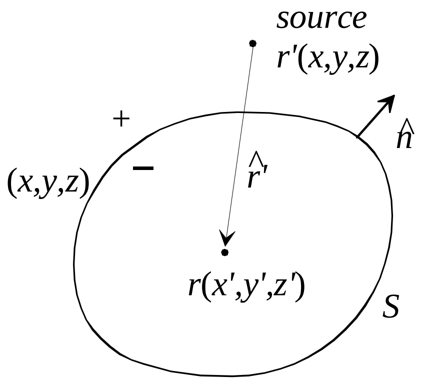
$$\mathbf{E}_V = \frac{q'}{4\pi r'^2}\hat{\mathbf{r}}'$$

- *Huygens sources assumed on the surface S　(r =R)　(S 上に仮定される Huygens 波源)*

・Fields/potentials produced by the **single layer** at $r' = R'$ (1 重層が作る *Field* とポテンシャル（注意!! *out* ; $r' < R'$, *in* ; $r' > R'$）

$$\phi_A = \frac{+q'}{4\pi R'}(Const.) \quad out, \quad \frac{+q'}{4\pi r'} \quad in$$

$$\mathbf{E}_{A-} = \mathbf{0} \quad out, \quad \frac{+q'\hat{\mathbf{r}}'}{4\pi r'^2} \quad in$$



・Fields/potentials produced by the **double layer** at $r' = R'$ ($r' = R'$ の位置に想定される 2 重層が作る *Field* とポテンシャル)

$$\varepsilon_0 \frac{+q'}{4\pi R'}$$

$$\phi_B = \frac{-q'}{4\pi R'}(Const.) \quad out, \quad 0 \quad in$$

$$\mathbf{E}_{B-} = \mathbf{0} \quad out, \quad \mathbf{0} \quad in$$

- *Fields/potentials in V out（$r' < R'$）, in（$r' > R'$）produced by single and double layers on S (S 上の 1 重層と 2 重層の波源が、に作る Field とポテンシャル)*

$$\phi_S = 0(Const.) \quad out, \quad \frac{+q'}{4\pi r} \quad in$$

$$\mathbf{E}_S = \mathbf{0} \quad out, \quad \frac{+q'\hat{\mathbf{r}}'}{4\pi r'^2} \quad in$$

Consequently, these are 0 outside of S, while it is identical with the fields produced by the original source q' outside with the same sign (つまり、S 外では完全に零、S 内では V 外にある点波源 q'の作る *Field* とポテンシャルと同じ符号である。)

- *If the contributions from 3 sources (original, single and double layers) are summed up,the fields/potentials outside and inside of V are expressed as. (V 外にある波源 q'、1 重層、 2 重層*を３つの波源と見なして、その３波源の寄与の合計として、S 内、S 外の Field とポテンシャルを計算すると、)

$\phi = \dfrac{q'}{4\pi r'}$ S 外（R に依らぬ）、 0 S 内

$\mathbf{E} = \dfrac{q'}{4\pi r'^2}\hat{\mathbf{r}}'$ S 外 (R に依らぬ) 、 0 S 内

〇In general cases, we have sources q in V and q’ outside of V. The equivalent sources assumed on S are the sum of those assumed for q and q’ independently. (一般に、*V* 内に *q*、*V* 外に *q’* の波源があるとする。*S* 上の等価波源の定義は、*q* と *q’* とがそれぞれ存在する時の波源の和があると仮定する。（nは *V* 外 *out* を向く。））

◈ *Fields/potentials due to equivalent sources on S are (S* 上の波源が作る *Field*、ポテンシャルは、)

・contribution *associated with q (q* から S 上に作られる波源の寄与)

$$\phi_S = 0(Const.) \quad in, \qquad \frac{-q}{4\pi r} \quad out$$

$$\mathbf{E}_S = \mathbf{0} \quad in, \qquad \frac{-q\hat{\mathbf{r}}}{4\pi r^2} \quad out$$

・contribution *associated with q’(q’* から *S* 上に作られる波源の寄与)

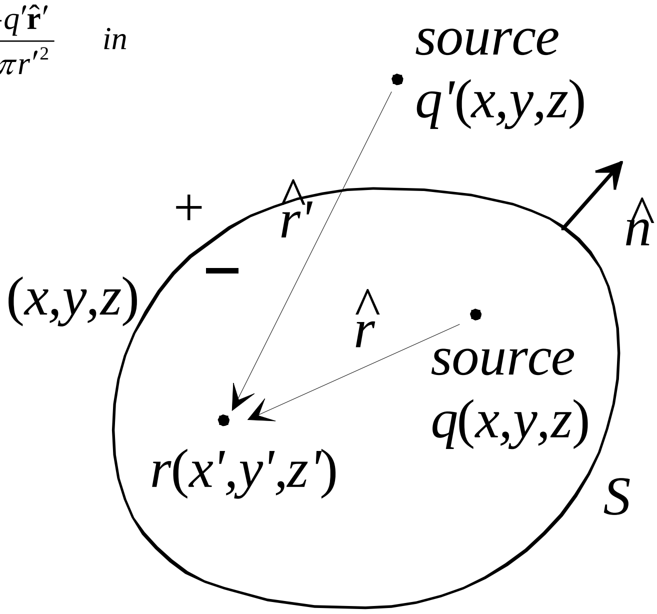
$$\phi_S = 0(Const.) \quad out, \qquad \frac{+q'}{4\pi r'} \quad in$$

$$\mathbf{E}_S = \mathbf{0} \quad out, \qquad \frac{+q'\hat{\mathbf{r}}'}{4\pi r'^2} \quad in$$

・Sum of these above are (両方の寄与)

$$\phi_S = \frac{-q}{4\pi r} \quad out, \qquad \frac{+q'}{4\pi r'} \quad in$$

$$\mathbf{E}_S = \frac{-q\hat{\mathbf{r}}}{4\pi r^2} \quad out, \qquad \frac{+q'\hat{\mathbf{r}}'}{4\pi r'^2} \quad in$$



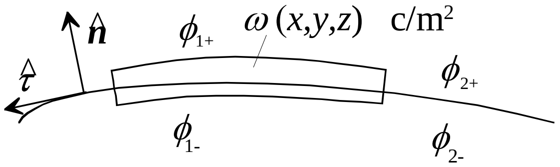
ベクトルの不連続性 Jump relation of Vectors

① $$\mathbf{A} \equiv \frac{1}{4\pi\varepsilon}\int \omega \nabla \phi \, ds = \frac{1}{4\pi\varepsilon}\int \omega\left(-\nabla'\phi\right) ds = -\nabla'\left\{\frac{1}{4\pi\varepsilon}\int \omega\phi \, ds\right\}$$

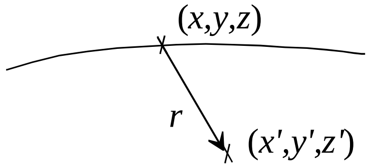
観測系 $\phi = \dfrac{e^{-jkr}}{r} \Rightarrow \dfrac{1}{r}$

不連続に着目 *As for the discontinuity*

$$\left.\begin{aligned} &n\cdot\left(\mathbf{A}_+ - \mathbf{A}_-\right) = -\frac{\partial}{\partial n'}\left\{\frac{1}{4\pi\varepsilon}\int \omega\phi \, ds\right\}\Bigg|_-^+ = \frac{\omega}{\varepsilon} \\ &\phi \text{は連続だから} \quad \hat{\mathbf{n}}\times\left(\mathbf{A}_+ - \mathbf{A}_-\right) = 0 \end{aligned}\right\} \quad \text{(A)より}$$



$$\left.\frac{\partial\phi}{\partial\tau}\right|_-^+ = 0$$



$$\mathbf{A}_+ = \mathbf{A}_+\cdot\hat{\mathbf{n}}\,\hat{\mathbf{n}} + \underbrace{(\mathbf{A}_+ - \mathbf{A}_+\cdot\hat{\mathbf{n}})}_{\boldsymbol{C}\times\hat{\boldsymbol{n}}}\,\hat{\mathbf{n}}$$

Finally we get

（A）より $\quad \underline{\therefore \mathbf{A}_+ - \mathbf{A}_- = \dfrac{\omega}{\varepsilon}\hat{\mathbf{n}}} \qquad \cdots(C)$

Q.E.D

(2) $\mathbf{B}\left(x',y',z'\right) = \dfrac{\mu_0}{4\pi}\displaystyle\int_s \mathbf{K}\left(x,y,z\right)\times\nabla\left(\frac{1}{r}\right) da$ を考える。$\mathbf{K}\cdot\hat{\mathbf{n}} = 0$ とする。

(3) $\mathbf{K} = \mathbf{i}K_x + \mathbf{j}K_y + \mathbf{k}K_z$

(4) $\mathbf{B}\left(x',y',z'\right) = \dfrac{\mu_0}{4\pi}\left[\mathbf{i}\times\displaystyle\int_s K_x\nabla\left(\frac{1}{r}\right)da + \mathbf{j}\times\int_s K_y\nabla\left(\frac{1}{r}\right)da + \mathbf{k}\times\int_s K_z\nabla\left(\frac{1}{r}\right)da\right]$

(5) $\mathbf{E}\left(x',y',z'\right) = \dfrac{1}{4\pi}\displaystyle\int \omega\left(x,y,z\right)\nabla\left(\frac{1}{r}\right)da$ の性質を考察する

(6) $\mathbf{E}_+ - \mathbf{E}_- = \omega\ \hat{\mathbf{n}}$ （C）より

(7) $(4),(5),(6) \rightarrow \mathbf{B}_+ - \mathbf{B}_- = \mu_0\mathbf{K}\times\hat{\mathbf{n}}$

(8) $(7)\rightarrow \quad \hat{\mathbf{n}}\cdot\left(\mathbf{B}_+ - \mathbf{B}_-\right) = 0$

(9) $\quad \hat{\mathbf{n}}\times\left(\mathbf{B}_+ - \mathbf{B}_-\right) = \mu_0\hat{\mathbf{n}}\times\left(\mathbf{K}\times\hat{\mathbf{n}}\right)$

(10) $\quad \hat{\mathbf{n}}\times\left(\mathbf{K}\times\hat{\mathbf{n}}\right) = \left(\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}\right)\mathbf{K} - \underbrace{\left(\hat{\mathbf{n}}\cdot\mathbf{K}\right)}_{\downarrow\,0}\hat{\mathbf{n}}$

Finally we get

(11) (9)より $\quad \underline{\hat{\mathbf{n}}\times\left(\mathbf{B}_+ - \mathbf{B}_-\right) = \mu_0\mathbf{K}} \qquad \cdots$(D)

Q.E.D

These four relations are summarized as:

$$\phi \equiv \frac{1}{4\pi}\int_s A(x,y,z)\frac{1}{r}ds \qquad \hat{n}:(+向き\ directed)$$

$$\phi\Big|_-^+ = 0$$

$$\left.\frac{\partial\phi}{\partial n'}\right|_+^- = -A(x',y',z') \qquad \text{(A)}$$

観測点系

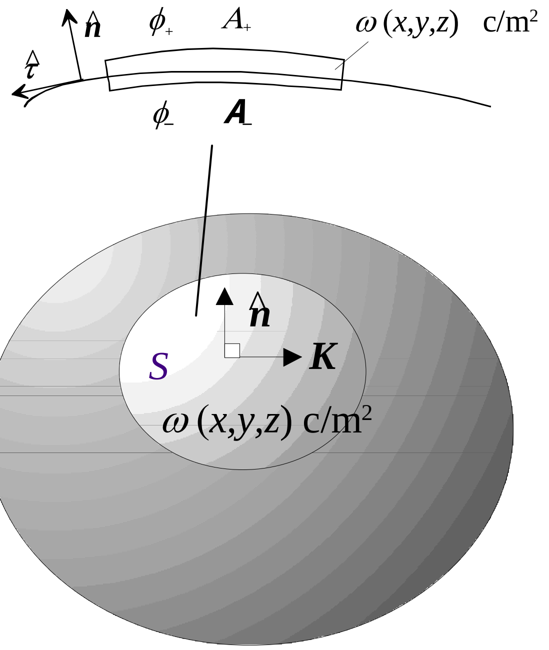
$$\phi \equiv \frac{1}{4\pi}\int B\frac{\partial}{\partial n}\left(\frac{1}{r}\right)ds \qquad \hat{n}:(+向き)$$

積分系

$$\phi\Big|_-^+ = B(x',y',z')$$

$$\left.\frac{\partial\phi}{\partial n'}\right|_-^+ = 0 \qquad \text{(B)}$$

観測点系



$$\mathbf{A} \equiv \frac{1}{4\pi}\int \omega\nabla\phi ds$$

$$\mathbf{A}\Big|_-^+ = \omega\ \ \hat{\mathbf{n}} \qquad \text{(C)}$$

$$\mathbf{A} \equiv \frac{1}{4\pi}\int \mathbf{K}\times\nabla\phi\, ds \qquad ただし\ \ \mathbf{K}\cdot\hat{\mathbf{n}} = 0$$

$$\mathbf{A}\Big|_-^+ = \mathbf{K}\times\hat{\mathbf{n}}$$

$$\hat{\mathbf{n}}\times\mathbf{A}\Big|_-^+ = K \qquad (\because K\perp\hat{n}) \qquad \text{(D)}$$

後に使う。(*For later use*)