

6. Dielectric Waveguide

$$\epsilon_r = n^2$$

6.1 Dielectric slab waveguide

EM waves are classified into two groups, TE and TM mode.

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x$$

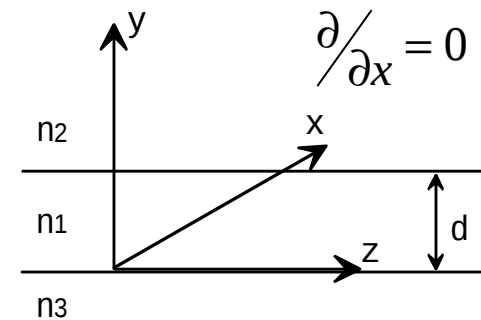
$$-j\beta H_x = j\omega\epsilon E_y$$

$$-\frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x$$

$$-j\beta E_x = -j\omega\mu_0 H_y$$

$$-\frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z$$



asymmetric slab waveguide

$$\frac{\partial^2 E_x}{\partial y^2} + (\omega^2 \epsilon \mu_0 - \beta^2) E_x = 0$$

Dielectric slab waveguide : TE mode

$$E_x = \begin{cases} (A \cos ud + B \sin ud)e^{-w_2(y-d)} & : \quad y > d \\ A \cos uy + B \sin uy & : \quad d > y > 0 \\ Ae^{w_3 y} & : \quad 0 > y \end{cases}$$

$$w_2 = \sqrt{\beta^2 - n_2^2 \omega^2 \epsilon_0 \mu_0} = \sqrt{\beta^2 - n_2^2 k_0^2}$$

$$u = \sqrt{n_1^2 \omega^2 \epsilon_0 \mu_0 - \beta^2} = \sqrt{n_1^2 k_0^2 - \beta^2}$$

$$w_3 = \sqrt{\beta^2 - n_3^2 \omega^2 \epsilon_0 \mu_0} = \sqrt{\beta^2 - n_3^2 k_0^2}$$

$$k_0 = \omega \sqrt{\epsilon_0 \mu_0} = \frac{2\pi}{\lambda}$$

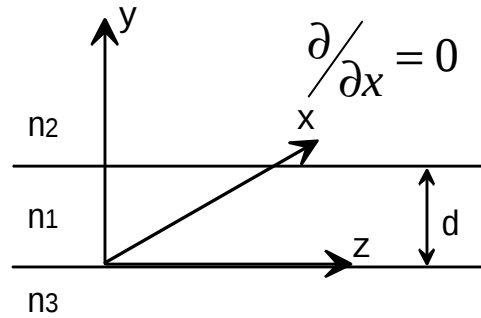
$$H_z = \frac{1}{j\omega\mu_0} \frac{\partial E_x}{\partial y} = \frac{1}{j\omega\mu_0} \times \begin{cases} -w_2 (A \cos ud + B \sin ud)e^{-w_2(y-d)} & : \quad y > d \\ u(-A \sin uy + B \cos uy) & : \quad d > y > 0 \\ w_3 A e^{w_3 y} & : \quad 0 > y \end{cases}$$

boundary condition : E_x and H_z must be continuous at $y=0, d$.

$$\tan ud = \frac{u(w_2 + w_3)}{u^2 - w_2 w_3}$$

characteristic equation

Dielectric slab waveguide : TM mode

$$\begin{aligned}
 &\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x \\
 &-j\beta H_x = j\omega\epsilon E_y \\
 &-\frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \\
 &\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x \\
 &-j\beta E_x = -j\omega\mu_0 H_y \\
 &-\frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z
 \end{aligned}$$


$$\tan ud = \frac{\frac{u}{n_1^2} \left(\frac{w_2}{n_2^2} + \frac{w_3}{n_3^2} \right)}{\left(\frac{u}{n_1^2} \right)^2 - \frac{w_2}{n_2^2} \frac{w_3}{n_3^2}}$$

symmetric slab waveguide : TE mode

symmetric slab waveguide.

$$\tan 2u \frac{d}{2} = \frac{2uw}{u^2 - w^2} = \frac{2\frac{w}{u}}{1 - \left(\frac{w}{u}\right)^2} \quad (\text{TE mode})$$

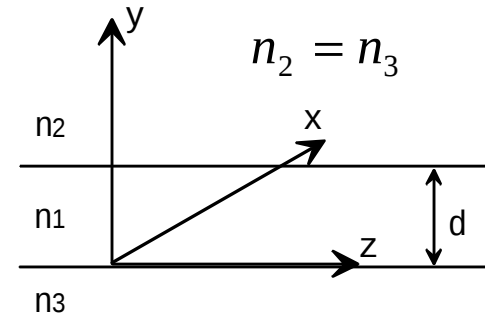
$$w = w_2 = w_3$$

$$\tan^2 \frac{ud}{2} + \left(\frac{u}{w} - \frac{w}{u} \right) \tan \frac{ud}{2} - 1 = 0$$

$$\left(\tan \frac{ud}{2} - \frac{w}{u} \right) \left(\tan \frac{ud}{2} + \frac{u}{w} \right) = 0$$

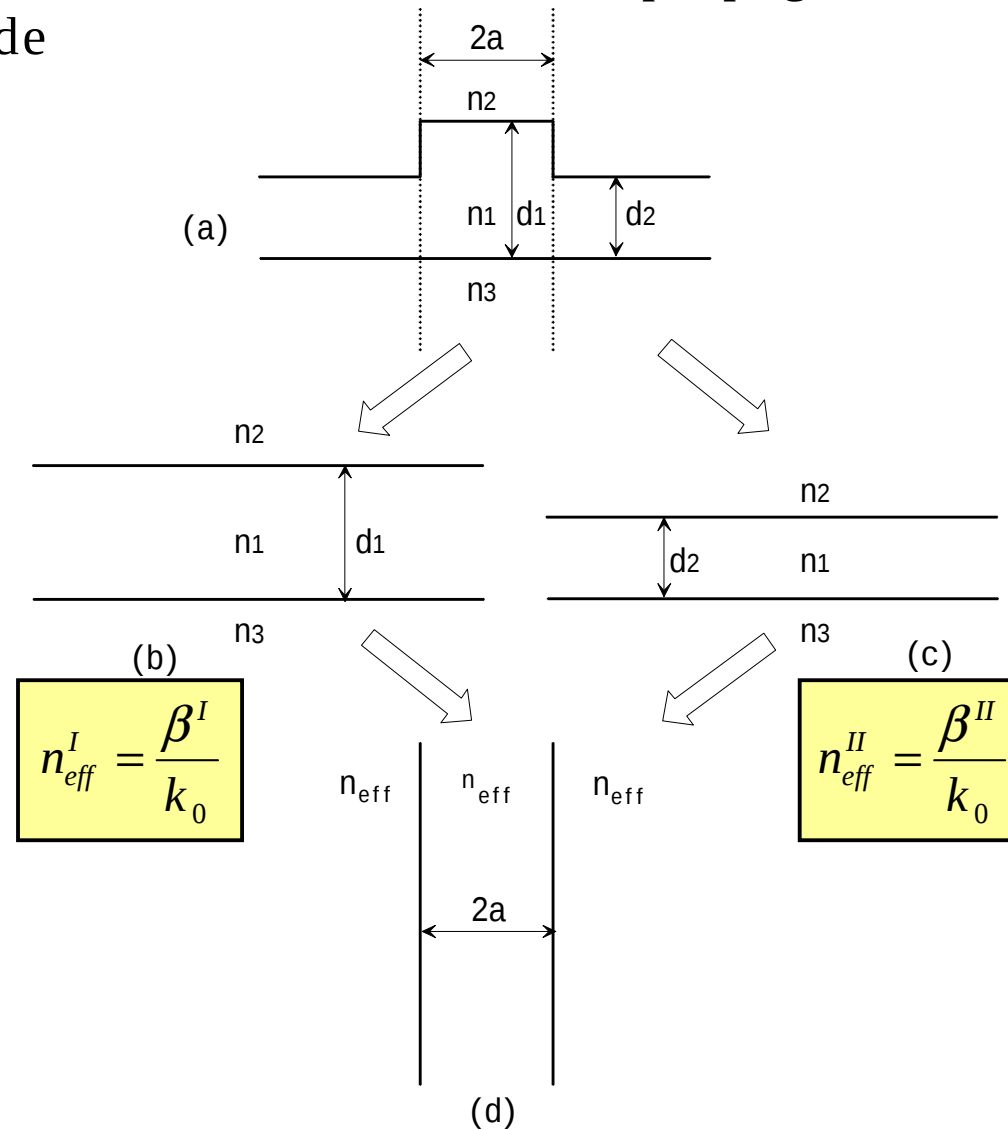
$$\tan \frac{ud}{2} = \frac{w}{u} \quad (\text{even mode})$$

$$\tan \frac{ud}{2} = -\frac{u}{w} \quad (\text{odd mode})$$



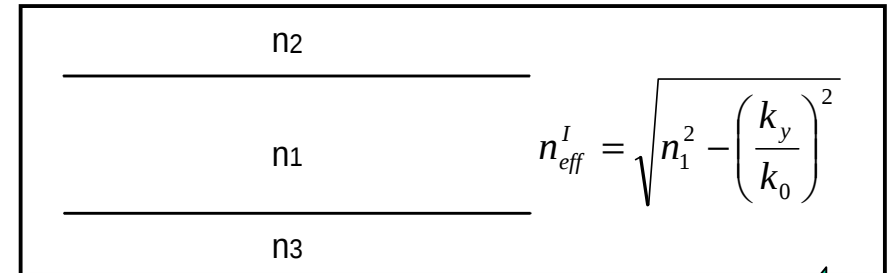
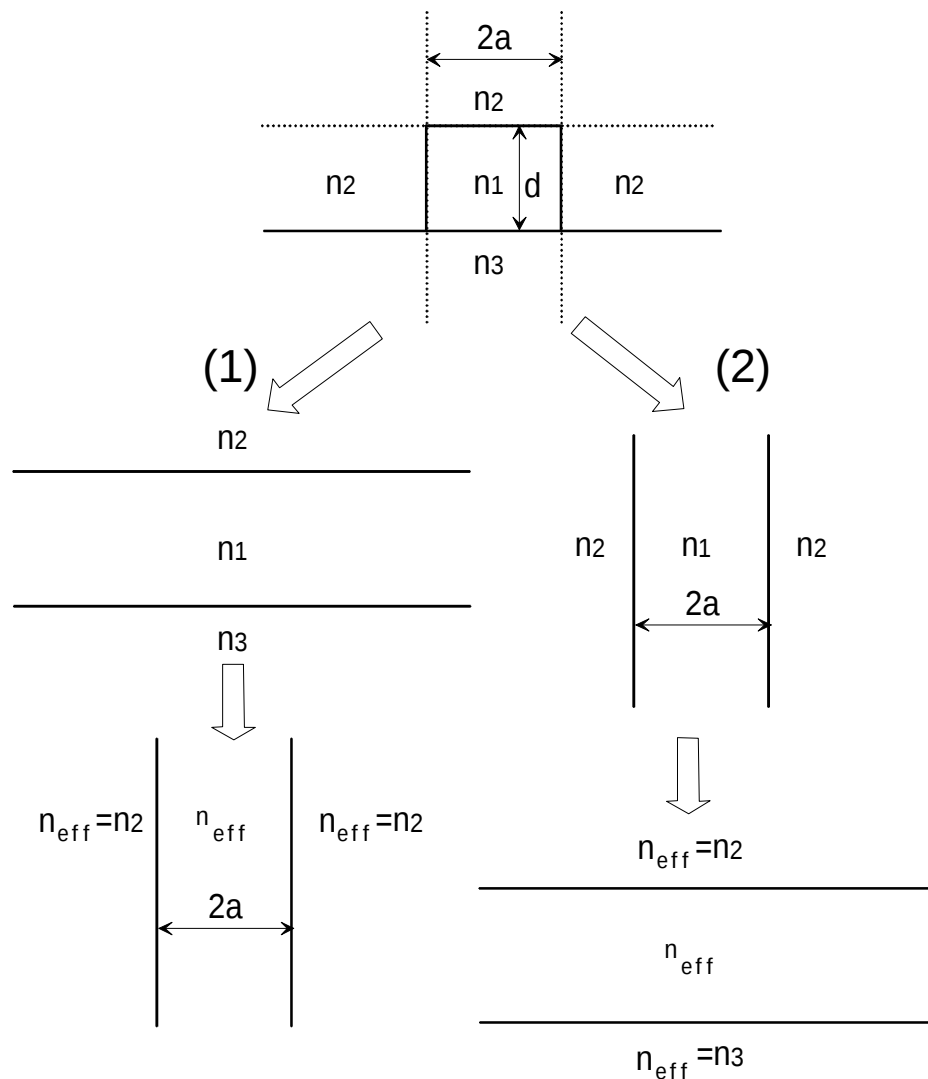
6.2 Effective Index Method (EIM)

Approximate method to determine the propagation constant of 3D- waveguide

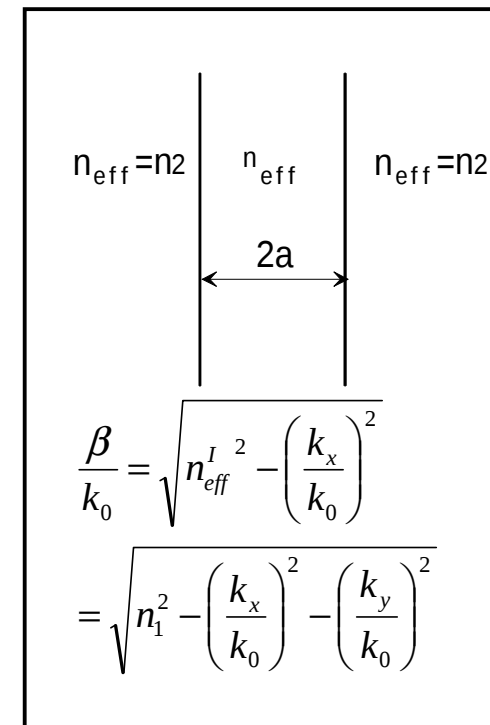


6.3 Generalized Effective Index Method (GEIM)

issue to be considered :



recursive procedure



6.4 Perturbation Feedback Method

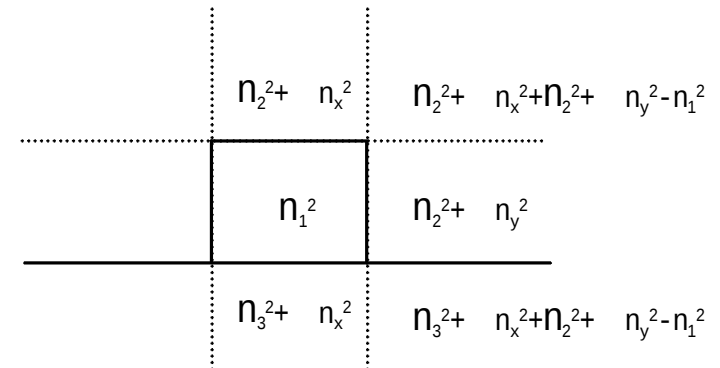
better approximation than EIM and GEIM

(i) Calculate index perturbation.

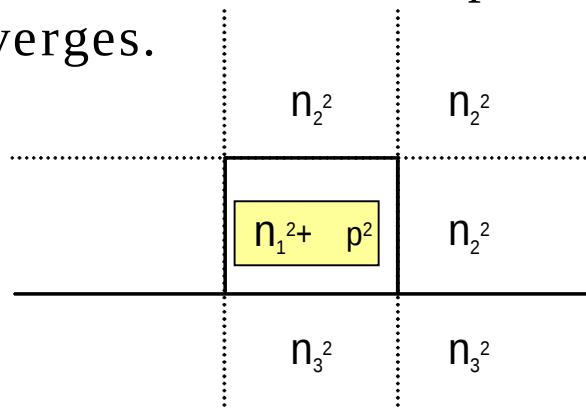
$$\Delta n_x^2 = \left(\frac{k_x}{k_0} \right)^2, \quad \Delta n_y^2 = \left(\frac{k_y}{k_0} \right)^2$$

(ii) Calculate perturbation weighted with a field distribution.

$$\Delta_p^2 = \frac{k_0^2 \int_S |f(x)g(y)|^2 \delta n^2 dS}{\int_S |f(x)g(y)|^2 dS}$$



(iii) Modify the waveguide structure. Then, repeat procedure until perturbation converges.

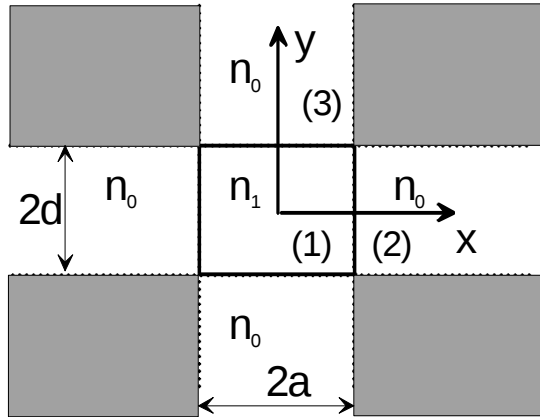


6.5 Marcatili's Method

$$E_{pq}^x \text{ mode } (H_x = 0)$$

$$\begin{aligned}
 & \frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x \\
 & -j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad H_z = \frac{1}{j\beta} \frac{\partial H_y}{\partial y} \\
 & \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \xrightarrow{H_x=0} E_z = \frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial x} \\
 & \frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x \xrightarrow{H_x=0} E_y = \frac{1}{\beta\omega\epsilon} \frac{\partial^2 H_y}{\partial x \partial y} \\
 & -j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y \longrightarrow E_x = \frac{\omega\mu_0}{\beta} H_y + \frac{1}{\beta\omega\epsilon} \frac{\partial^2 H_y}{\partial x^2} \\
 & \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z \\
 & \text{wave equation} \quad \frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (\omega^2 \epsilon \mu_0 - \beta^2) H_y = 0
 \end{aligned}$$

Marcatili's Method



ignore shaded region

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + (\omega^2 \epsilon \mu_0 - \beta^2) H_y = 0$$

$$H_y = \begin{cases} A \cos(k_x x - \phi) \cos(k_y y - \varphi) & : \text{in (1)} \\ A \cos(k_x a - \phi) \exp(-\gamma_x(x - a)) \cos(k_y y - \varphi) & : \text{in (2)} \\ A \cos(k_x x - \phi) \cos(k_y d - \varphi) \exp(-\gamma_y(y - d)) & : \text{in (3)} \end{cases}$$

$$-k_x^2 - k_y^2 + (n_1^2 k_0^2 - \beta^2) = 0$$

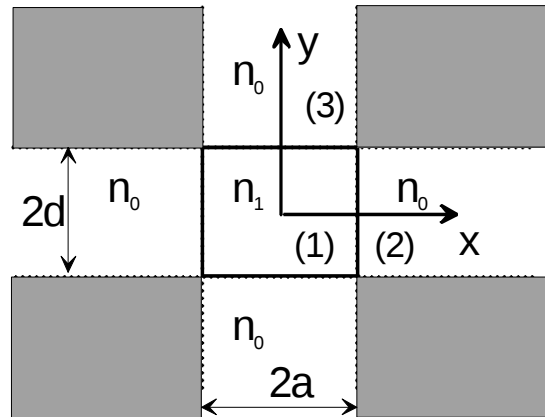
$$\gamma_x^2 - k_y^2 + (n_0^2 k_0^2 - \beta^2) = 0$$

$$-k_x^2 + \gamma_y^2 + (n_0^2 k_0^2 - \beta^2) = 0$$

$$\phi = (p-1)\pi/2$$

$$\varphi = (q-1)\pi/2$$

Marcatili's Method



ignore shaded region

interface between (1) and (2)

$$E_z: \frac{1}{n_1^2} \frac{\partial H_y}{\partial x} \Big|_{x=a} = \frac{1}{n_0^2} \frac{\partial H_y}{\partial x} \Big|_{x=a}$$

$$\tan(k_x a - \phi) = \frac{n_1^2 \gamma_x}{n_0^2 k_x}$$

$$\therefore k_x a - \phi = \tan^{-1} \left(\frac{n_1^2 \gamma_x}{n_0^2 k_x} \right)$$

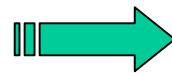
interface between (1) and (3)

$$\tan(k_y d - \phi) = \frac{\gamma_y}{k_y}$$

$$\therefore k_y d - \phi = \tan^{-1} \left(\frac{\gamma_y}{k_y} \right)$$

$$\gamma_x^2 = -k_x^2 + (n_1^2 - n_0^2)k_0^2$$

$$\gamma_y^2 = -k_y^2 + (n_1^2 - n_0^2)k_0^2$$



$$\beta^2 = n_1^2 k_0^2 - (k_x^2 + k_y^2)$$

Marcatili's Method

E_{pq}^y mode ($H_y = 0$)

$$\left\{ \begin{array}{l} E_x = -\frac{1}{\beta\omega^2\epsilon} \frac{\partial^2 H_x}{\partial x \partial y} \\ E_y = -\frac{\omega\mu_0}{\beta} H_x - \frac{1}{\beta\omega\epsilon} \frac{\partial^2 H_x}{\partial y^2} \\ E_z = -\frac{1}{j\omega\epsilon} \frac{\partial H_x}{\partial y} \\ H_y = 0 \\ H_z = \frac{1}{j\beta} \frac{\partial H_x}{\partial x} \end{array} \right.$$

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + (\omega^2\epsilon\mu_0 - \beta^2)H_x = 0$$