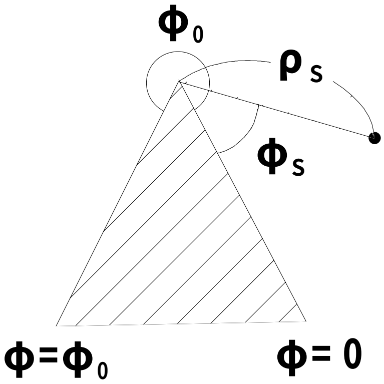
Electric Line Current Scattering by a Conductor Wedge

$$E_z = -\frac{\omega\mu\pi I}{\phi_0}\sum_{n=1}^{\infty} H_{\upsilon_n}^{(2)}(k\rho_>)\, J_{\upsilon_n}(k\rho_<)\sin\upsilon_n\phi\ \sin\phi_S$$

$source\ (\rho_S\ ,\phi_S)\ ,\ observation(\rho\ ,\phi)$

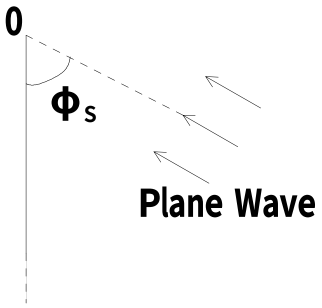
$\rho_> \cdots \rho > \rho_S$, $\quad \upsilon_n = \dfrac{n\pi}{\phi_0}$

$\rho_< \cdots \rho < \rho_S$



$\rho_S \to \infty\ ,\ \phi_0 \to 2\pi \quad \left(\upsilon_n = \dfrac{n}{2}\right)$

Plane Wave Scattering by a Semi - Infinite Plane



$$E_z = -\frac{\omega\mu\pi I}{2\pi}\sum_{n=1}^{\infty} H_{\frac{n}{2}}^{(2)}(k\rho_S)\, J_{\frac{n}{2}}(k\rho)\sin\frac{n}{2}\phi\ \sin\frac{n}{2}\phi_S$$

$$\rho_S \to \infty\ \ H_{\frac{n}{2}}^{(2)}(k\rho_S) \to \sqrt{\frac{2}{\pi k\rho_S}}\ e^{-j\left(k\rho_S - \frac{\pi}{4}\right)}\ e^{j\frac{\pi}{2}\frac{n}{2}}$$

$$E_z = -\frac{\omega\mu I}{2}\sqrt{\frac{2}{\pi k\rho_S}}\; e^{-j\left(k\rho_S-\frac{\pi}{4}\right)} \sum_{n=1}^{\infty} J_{\frac{n}{2}}(k\rho)\sin\frac{n\phi}{2}\sin\frac{n\phi_S}{2}\; e^{j\frac{n}{4}}$$

$$= 2E_0 \sum_{n=1}^{\infty} J_{\frac{n}{2}}(k\rho)\sin\frac{n\phi}{2}\sin\frac{n\phi_S}{2}\; e^{j\frac{n}{4}}$$

$$\left[\begin{array}{l} E_0 : \textit{Plane wave at the origin (without a scatter)} \\ E_z^i(\rho=0) = -\frac{\omega\mu}{4} I\, H_0^{(2)}(k\rho_S) \qquad \textit{line source} \\ \qquad\qquad \downarrow \quad \rho_S \to \infty \\ E_z^i(\rho=0) = -\frac{\omega\mu I}{4}\sqrt{\frac{2}{\pi k\rho_S}}\; e^{-j\left(k\rho_S-\frac{\pi}{4}\right)} \triangleq E_0 \end{array}\right]$$

$$\sin\frac{n\phi}{2}\sin\frac{n\phi_S}{2} = \frac{1}{2}\left\{\cos\frac{n}{2}(\phi-\phi_S) - \cos\frac{n}{2}(\phi+\phi_S)\right\}$$

$$E_z = E_0\left\{\sum_{n=0}^{\infty}\cos\frac{n}{2}(\phi-\phi_S)\, J_{\frac{n}{2}}(k\rho)\, e^{j\frac{n}{4}} - \sum_{n=0}^{\infty}\cos\frac{n}{2}(\phi+\phi_S)\, J_{\frac{n}{2}}(k\rho)\, e^{j\frac{n}{4}}\right\}$$

$$= E_0\left\{\sum_{n=0}^{\infty}\cos\frac{n}{2}(\phi-\phi_S)\, J_{\frac{n}{2}}(k\rho)\, e^{j\frac{n}{4}} - \langle \phi_S \to -\phi_S \rangle\right\}$$

(slow convergence for large k)

$$E_z = E_0\left\{\sum_{n=0}^{\infty}\cos\frac{n}{2}(\phi-\phi_S)\, J_{\frac{n}{2}}(k\rho)\, e^{j\frac{n}{4}} - \langle \phi_S \to -\phi_S \rangle\right\}$$

$$= E_0\left\{\sqrt{\frac{j}{\pi}}\; e^{jk\rho\cos(\phi-\phi_S)} \int_{-\infty}^{\sqrt{2k\rho}\cos\frac{\phi-\phi_S}{2}} e^{-jt^2}\,dt - \langle \phi_S \to -\phi_S \rangle\right\}$$

Incidence (E_z^i) Reflection $(E_z^{\vee})$

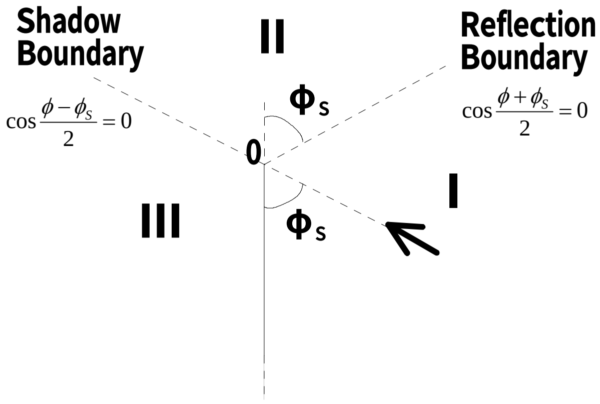
Reflection Coefficient = -1

$$\left(\because \sum_{n=0}^{\infty} 2\varepsilon_n\, j^{\frac{n}{2}} \cos n\phi\; J_{\frac{n}{2}}(\rho) = \sqrt{\frac{j}{\pi}}\; e^{j\rho\cos\phi}\int_{-\infty}^{\sqrt{2\rho}\cos\frac{\phi}{2}} e^{-jt^2}\,dt \;,\; \varepsilon_n = \left\{\begin{array}{l} 1\,(n=0) \\ 2\,(n\neq 0)\end{array}\right.\right)$$

$$E_z = E_0 \left\{ \sqrt{\frac{j}{\pi}}\, e^{\,j\xi^i}\, e^{-jk\rho} \int_{-\infty}^{\sqrt{\xi^i}} e^{-jt^2} dt - \langle \phi_S \to -\phi_S \rangle \right\}$$

$$\xi^i \triangleq 2k\rho \cos^2 \frac{\phi - \phi_S}{2} = k\rho\{1 + \cos(\phi - \phi_S)\}$$

$$\xi^r \triangleq 2k\rho \cos^2 \frac{\phi + \phi_S}{2} = k\rho\{1 + \cos(\phi + \phi_S)\}$$



	I	II	III
$\cos\frac{\phi-\phi_S}{2}=0$	+	+	-
$\cos\frac{\phi+\phi_S}{2}=0$	+	-	-

i) $\cos\frac{\phi - \phi_S}{2} > 0$ （Ⅰ&Ⅱ）

$$E_z^i = E_0 \sqrt{\frac{j}{\pi}}\, e^{\,j\xi^i}\, e^{-jk\rho} \int_{-\infty}^{\sqrt{\xi^i}} e^{-jt^2} dt$$

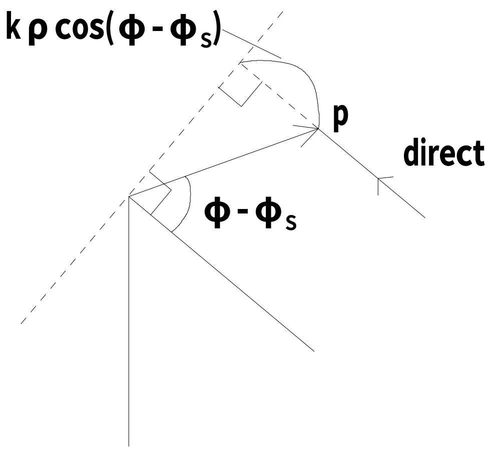
$$= E_0 \left\{ \sqrt{\frac{j}{\pi}}\, e^{\,j\xi^i}\, e^{-jk\rho} \left(\int_{-\infty}^{\infty} e^{-jt^2} dt - \int_{\sqrt{\xi^i}}^{\infty} e^{-jt^2} dt \right) \right\}$$

$$= E_0 \left\{ e^{\,j\xi^i}\, e^{-jk\rho} - \sqrt{\frac{j}{\pi}}\, e^{\,j\xi^i}\, e^{-jk\rho} \int_{\sqrt{\xi^i}}^{\infty} e^{-jt^2} dt \right\} \qquad \left(\because \int_{-\infty}^{\infty} e^{-jt^2} dt = \sqrt{\pi}\, e^{-\frac{j\pi}{4}} \right)$$

$$= E_0 \left\{ \underline{e^{jk\rho\cos(\phi - \phi_S)}} - \underline{\frac{\sqrt{j}\, e^{-\frac{j\pi}{2}}}{2\sqrt{\pi}\sqrt{\xi^i}}\, e^{-jk\rho}\, \mathbb{F}(\xi^i)} \right\}$$

direct　　　diffraction

$\mathbb{F}(x) \triangleq 2\sqrt{x}\, e^{\,jx + j\frac{\pi}{2}} \int_{\sqrt{x}}^{\infty} e^{-jt^2} dt$: *Fresnel Integral*



ii） $\cos\frac{\phi-\phi_S}{2}<0$ （III）

$$E_z^i = E_0\sqrt{\frac{j}{\pi}}\, e^{j\xi^i}\, e^{-jk\rho}\int_{-\infty}^{-\sqrt{\xi^i}} e^{-jt^2}dt = E_0\,\frac{\sqrt{j}\, e^{-j\frac{\pi}{2}}}{2\sqrt{\pi}\sqrt{\xi^i}}\, e^{-jk\rho}\ \mathbb{F}(\xi^i)$$

iii) $\cos\frac{\phi+\phi_S}{2}>0$ （I）

$$E_z^r = E_0\left\{\underline{e^{-jk\rho}\, e^{j\xi^r}} - \frac{\sqrt{j}\, e^{-\frac{j\pi}{2}}}{2\sqrt{\pi}\sqrt{\xi^r}}\, e^{-jk\rho}\ \mathbb{F}(\xi^r)\right\}$$

reflection

iv) $\cos\frac{\phi+\phi_S}{2}<0$ （II&III）

$$E_z^r = E_0\,\frac{\sqrt{j}\, e^{-\frac{j\pi}{2}}}{2\sqrt{\pi}\sqrt{\xi^r}}\, e^{-jk\rho}\ \mathbb{F}(\xi^r)$$

$\xi \gg 1 \quad \mathbb{F}(\xi) \to 1$ (k ρ ≫ 1 , far from SB & RB)

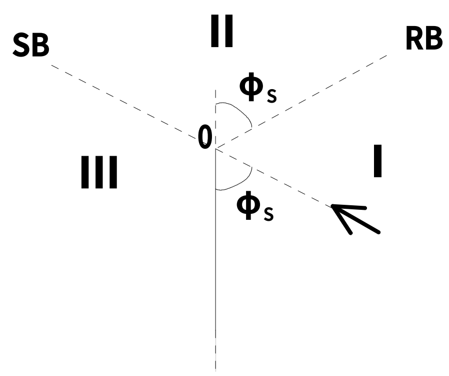
$$E_z = E_z^i - E_r^i$$

$$\begin{cases} = E_d^i - E_d^r + E_0 \dfrac{e^{-j\frac{\pi}{4}}}{2\sqrt{\pi}} \dfrac{e^{-jk\rho}}{\sqrt{2k}\sqrt{\rho}} \left(\dfrac{-1}{\cos\frac{\phi-\phi_S}{2}} - \dfrac{-1}{\cos\frac{\phi+\phi_S}{2}} \right) & \cdots \mathrm{I} \\ = E_d^i \qquad + E_0 \dfrac{e^{-j\frac{\pi}{4}}}{2\sqrt{\pi}} \dfrac{e^{-jk\rho}}{\sqrt{2k}\sqrt{\rho}} \left(\dfrac{-1}{\cos\frac{\phi-\phi_S}{2}} - \dfrac{+1}{\cos\frac{\phi+\phi_S}{2}} \right) & \cdots \mathrm{II} \\ = \qquad\qquad + E_0 \dfrac{e^{-j\frac{\pi}{4}}}{2\sqrt{\pi}} \dfrac{e^{-jk\rho}}{\sqrt{2k}\sqrt{\rho}} \left(\dfrac{+1}{\cos\frac{\phi-\phi_S}{2}} - \dfrac{+1}{\cos\frac{\phi+\phi_S}{2}} \right) & \cdots \mathrm{III} \end{cases}$$

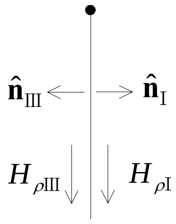
Incidence Reflection diffraction

Plane Wave Cylindrical wave

$O[(k\rho)^0]$ $O[(k\rho)^{-1/2}]$



Current on the semi - infinite plane



$\mathbf{J} = \hat{\mathbf{n}} \times \mathbf{H}$

$$H_\rho = \frac{j}{\omega\mu\rho}\frac{\partial}{\partial\phi}(E_z)$$

$$H_{\rho\mathrm{I}}(\phi = 0) = -\frac{2kE_0}{\omega\mu}\sin\phi_S \; e^{jk\rho\cos\phi_S} \qquad \leftarrow O[(k\rho)^0]$$

$$+\frac{jE_0 e^{-j\frac{\pi}{4}}}{2\omega\mu\sqrt{\pi}} e^{-jk\rho}\frac{1}{\rho\sqrt{2k\rho}}\frac{\sin^2\frac{\phi_S}{2}}{\cos^2\frac{\phi_S}{2}} \qquad \leftarrow O[(k\rho)^{-3/2}]$$

$$H_{\rho\mathrm{III}}(\phi = 2\pi) = -\frac{jE_0 e^{-j\frac{\pi}{4}}}{2\omega\mu\sqrt{\pi}} e^{-jk\rho}\frac{1}{\rho\sqrt{2k\rho}}\frac{\sin^2\frac{\phi_S}{2}}{\cos^2\frac{\phi_S}{2}} \qquad \leftarrow O[(k\rho)^{-3/2}]$$