

2. Derivation and Interpretation of solutions for Wave Equations

波動方程式の解釈

$$\nabla^2 \phi + k^2 \phi = -q(x, y, z) \quad \text{Observer } \phi(x', y', z')$$

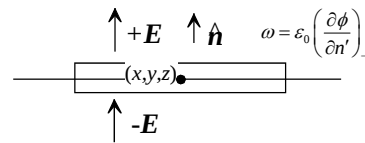
$$\phi = \frac{1}{4\pi} \int_v q \frac{e^{-jkr}}{r} dv + \frac{1}{4\pi} \int_s \left(\frac{\partial \phi}{\partial n} \right)_- \frac{e^{-jkr}}{r} ds + \frac{1}{4\pi} \int_s (-\phi)_- \frac{\partial}{\partial n} \frac{e^{-jkr}}{r} ds$$

$$\equiv \frac{1}{4\pi} \int_v q \frac{e^{-jkr}}{r} dv + \phi_A + \phi_B \quad \frac{\partial}{\partial n}: \text{いずれも積分系 integ.coord.}$$

Physical interpretation

$\varepsilon_0 \left(\frac{\partial \phi}{\partial n} \right)_-$ なる 1 重層 [2-2] = ω

- $\phi_A = \frac{1}{4\pi} \int_s \left(\frac{\partial \phi}{\partial n} \right)_- \frac{e^{-jkr}}{r} ds$
A
at the discontinuity (不連続の点 $e^{-jkr} = 1$)



$$\phi_A|_+ = 0 \quad \dots (A)$$

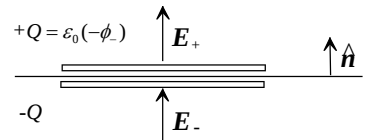
$$\left. \frac{\partial \phi_A}{\partial n} \right|_- = -\frac{\omega}{\varepsilon} = -\left(\frac{\partial \phi}{\partial n} \right)_-$$

$\varepsilon_0 (-\phi_-)$ なる 2 重層 [2-3]

- $\phi_B = \frac{1}{4\pi} \int_s \left(-\phi \right)_- \frac{\partial}{\partial n} \frac{e^{-jkr}}{r} ds$
B

$$\phi_B|_+ = \left(-\phi \right)_- \quad \dots (B)$$

$$\left. \frac{\partial \phi_B}{\partial n'} \right|_- = 0$$



Then したがって

$$\phi|_+ = \phi_A|_+ + \phi_B|_+ = (-\phi)_- \quad \leftarrow B$$

$$\phi_+ - \phi_- = -\phi_- \quad \therefore \phi_+ = 0$$

$$\left. \frac{\partial}{\partial n'} \phi \right|_- = \left. \frac{\partial \phi_A}{\partial n'} \right|_- + \left. \frac{\partial \phi_B}{\partial n'} \right|_- = \left(-\frac{\partial \phi}{\partial n} \right)_- \quad \leftarrow -A$$

Obs. 観測系 Obs. 観測系

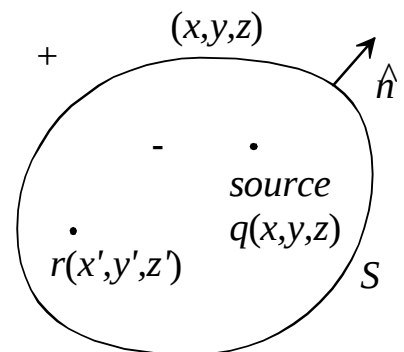
$$\left(\frac{\partial \phi}{\partial n'} \right)_+ - \left(\frac{\partial \phi}{\partial n'} \right)_- = \left(-\frac{\partial \phi}{\partial n} \right)_- \quad \therefore \frac{\partial \phi_+}{\partial n} = 0$$

同じ意味として良い

$\therefore S$ の外の領域 $\tilde{V}$ は $\tilde{V}$ に波源がなければすべて零になる。

$$\therefore \phi = 0 + \frac{1}{4\pi} \int_s \left(\frac{\partial \phi}{\partial n} \right)_+ \frac{e^{-jkr}}{r} dS + \frac{1}{4\pi} \int_s \left(-\phi \right)_+ \frac{\partial}{\partial n} \frac{e^{-jkr}}{r} dS$$

↓ ↓
0 0



$\therefore$ Fields outside of $S(\tilde{V})$ is null if there is no source in $\tilde{V}$.

Explain what happens if the sources exist outside of the surface S ??

◆ Single Layer

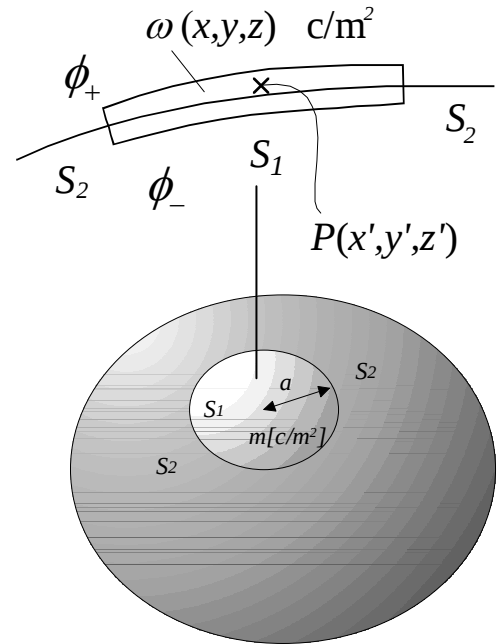
$$\begin{aligned}\phi(x', y', z') &= \frac{1}{4\pi\epsilon_0} \int_s \frac{\omega(x, y, z)}{r} ds \\ &= \frac{1}{4\pi\epsilon_0} \left(\int_{s_1} + \int_{s_2} \right) \\ &= \phi_1 + \phi_2 \quad \phi_2 : \text{continuous}\end{aligned}$$

$$|\omega| \leq m$$

$$|\phi_1| = \frac{1}{4\pi\epsilon_0} \left| \int_{s_1} \frac{\omega}{r} ds \right| < \frac{1}{4\pi\epsilon_0} \int_0^a \frac{m}{r} 2\pi r dr = \frac{m}{4\pi\epsilon_0} 2\pi a \rightarrow 0$$

$\lim_{a \rightarrow 0} s$

$$\phi_{-}^{+} = \phi_{+} - \phi_{-} = 0 \quad \text{continuous}$$

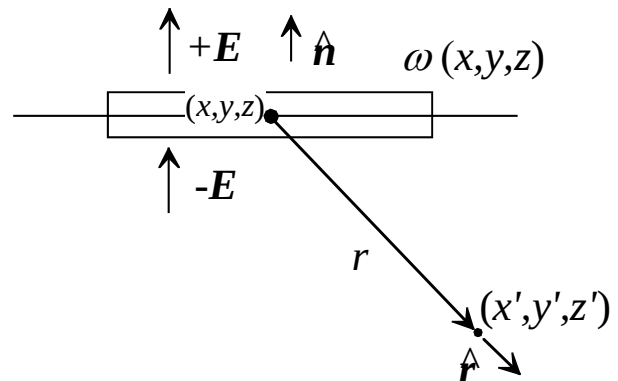


From divergence theorem,

ところで、発散定理より

$$(E_{+} - E_{-}) \cdot \hat{n} = \frac{\omega}{\epsilon_0}$$

$$(-\nabla' \phi_{+} + \nabla' \phi_{-}) \cdot \hat{n} = \left. \frac{\partial \phi}{\partial n'} \right|_{+} = \frac{\omega}{\epsilon_0}$$



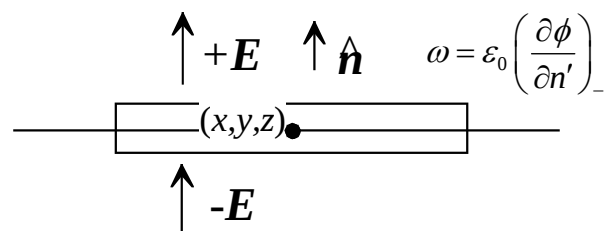
◆ Physical interpretation(ϕ_A) $\rightarrow$ Single layer

$$\phi_A(x', y', z') \equiv \frac{1}{4\pi} \int_s A(x, y, z) \frac{1}{r} ds$$

$$\phi_A|_{-}^{+} = 0 \quad \epsilon_0 A [c/m^2] \text{ の電気 1 重層 single-layer e charge}$$

$$\left. \frac{\partial \phi_A}{\partial n'} \right|_{+} = A \quad \dots (A)$$

観測点系



Double Layer

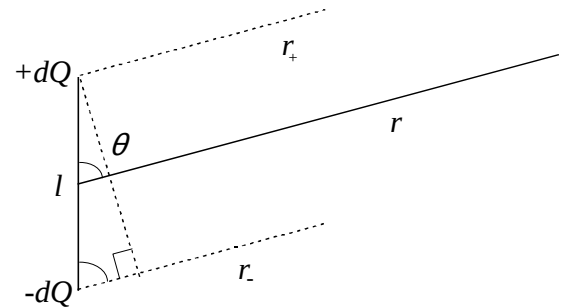
$$d\phi = \frac{dQ}{4\pi\epsilon_0} \left(\frac{1}{r_+} - \frac{1}{r_-} \right) \quad \text{電気モーメント}/m^2 \quad dQl \equiv \tau ds \quad \tau[c/m]$$

$$\cong \frac{dQ}{4\pi\epsilon_0} \frac{r_- - r_+}{r^2} = \frac{dQ}{4\pi\epsilon_0} \frac{l \cos \theta}{r^2} = \frac{\tau \cos \theta}{4\pi\epsilon_0 r^2} ds$$

$$= \frac{-\tau}{4\pi\epsilon_0} d\Omega = \frac{\tau}{4\pi\epsilon_0} \frac{\partial}{\partial n} \left(\frac{1}{r} \right) ds$$

$$\therefore d\Omega = -\mathbf{n} \times \hat{\mathbf{r}} \frac{1}{r^2} ds = -\mathbf{n} \cdot \left(\nabla \frac{1}{r} \right) ds = -\frac{\partial}{\partial n} \left(\frac{1}{r} \right) ds$$

積分系に関する微分



$$\text{Obs. } \nabla' \frac{1}{r} = -\frac{\hat{\mathbf{r}}}{r^2}$$

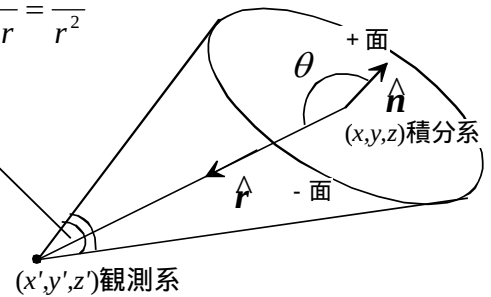
$$\text{Int. } \nabla \frac{1}{r} = \frac{\hat{\mathbf{r}}}{r^2}$$

Discontinuity across the surface from + to - is

観測点が+から-に横切る時の不連続量は

$$d\phi|_{-}^{+} = \frac{-\tau}{4\pi\epsilon_0} \Omega \Big|_{-}^{+} = \frac{-\tau}{4\pi\epsilon_0} \cdot -4\pi = \frac{\tau}{\epsilon_0}$$

$$d\phi|_{-}^{+} = \frac{\tau}{4\pi\epsilon_0} \frac{\partial}{\partial n} \left(\frac{1}{r} \right) ds \Big|_{-}^{+} = \phi|_{-}^{+} = \frac{\tau}{\epsilon_0} \quad (\text{上式より})$$

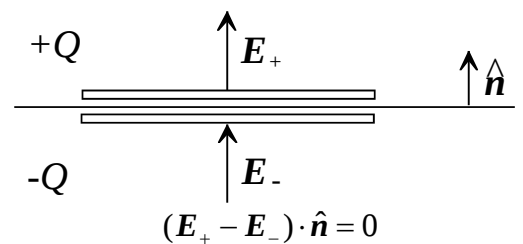


また、

From Gauss's Law, 電荷打消 (ガウス) より $(\mathbf{E}_+ - \mathbf{E}_-) \cdot \hat{\mathbf{n}} = 0$

$$(-\nabla' \phi_+ + \nabla' \phi_-) \cdot \mathbf{n} = \frac{\partial \phi}{\partial n'} \Big|_{+}^{-} = 0$$

$$(\mathbf{E}_+ - \mathbf{E}_-) \cdot \hat{\mathbf{n}} = 0 \quad \text{観測系}$$



Physical interpretation(ϕ_B) $\rightarrow$ Double layer

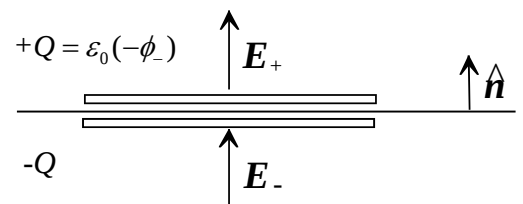
$$\phi_B = \frac{1}{4\pi} \int B \frac{\partial}{\partial n} \left(\frac{1}{r} \right) ds'$$

積分系

$$\phi_B|_{-}^{+} = B \quad \epsilon_0 B \text{ なる電気 2 重層 Double-layer}$$

$$\frac{\partial \phi_B}{\partial n'} \Big|_{-}^{+} = 0 \quad (\text{B})$$

観測点系



Fields produced by Surface Integrations of Single and Double Layer (Example: static fields for a point source in Volume V)

○If we have only the point source q inside V (V 内に点波源 q のみ存在する場合)。

$$\begin{aligned}\nabla^2 \phi + k^2 \phi &= -q(x, y, z) & \phi(x', y', z') \\ \phi &= \frac{1}{4\pi} \int_V q \frac{e^{-jkr}}{r} dv + \frac{1}{4\pi} \int_S \left(\frac{\partial \phi}{\partial n} \right)_- \frac{e^{-jkr}}{r} ds + \frac{1}{4\pi} \int_S (-\phi)_- \frac{\partial}{\partial n} \frac{e^{-jkr}}{r} ds \\ &\equiv \frac{1}{4\pi} \int_V q \frac{e^{-jkr}}{r} dv + \phi_A + \phi_B & \frac{\partial}{\partial n}: \text{いずれも積分系 integ. coord.} \\ & & \varepsilon_0 \left(\frac{\partial \phi}{\partial n} \right)_- \text{なる 1 重層} & \varepsilon_0 (-\phi)_- \text{なる 2 重層}\end{aligned}$$

◆ Fields and potentials produced by the original source q (Original な波源の作る Field とポテンシャル)

$$\begin{aligned}\phi_V &= \frac{q}{4\pi r} \\ \mathbf{E}_V &= \frac{q}{4\pi r^2} \hat{\mathbf{r}}\end{aligned}$$

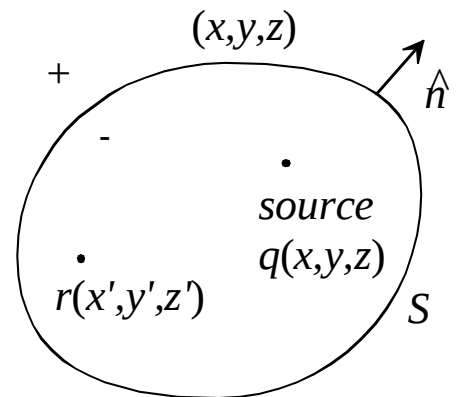
◆ Huygens sources assumed on the surface S ($r = R$) 上に仮定される Huygens 波源)

• Fields/potentials produced by the **single layer** at $r = R$ (in ; $r < R$, out ; $r > R$) ($r = R$ の位置に想定される 1 重層が作る Field とポテンシャル)

$$\begin{aligned}\varepsilon_0 \left(\frac{\partial \phi}{\partial n} \right) &= \varepsilon_0 \frac{-q}{4\pi R^2} \\ \phi_A &= \frac{-q}{4\pi R} (\text{Const.}) & \text{in, } \frac{-q}{4\pi r} & \text{out} \\ \mathbf{E}_{A-} &= \mathbf{0} & \text{in, } \frac{-q\hat{\mathbf{r}}}{4\pi r^2} & \text{out}\end{aligned}$$

• Fields/potentials produced by the **double layer** at $r = R$ (in ; $r < R$, out ; $r > R$) $r = R$ の位置に想定される 2 重層が作る Field とポテンシャル

$$\begin{aligned}\varepsilon_0 \frac{-q}{4\pi R} \\ \phi_B &= \frac{+q}{4\pi R} (\text{Const.}) & \text{in, } 0 & \text{out} \\ \mathbf{E}_{B-} &= \mathbf{0} & \text{in, } 0 & \text{out}\end{aligned}$$



◆ Fields/potentials in V ($r < R$) or outside ($r > R$) produced by single and double layers on S (S 上の 1 重層と 2 重層の波源が、内 ($r < R$)、外 ($r > R$) に作る Field とポテンシャル)

$$\begin{aligned}\phi_S &= 0 (\text{Const.}) & \text{in, } \frac{-q}{4\pi r} & \text{out} \\ \mathbf{E}_S &= \mathbf{0} & \text{in, } \frac{-q\hat{\mathbf{r}}}{4\pi r^2} & \text{out}\end{aligned}$$

Consequently, these are 0 in S , while it is identical with the fields produced by the original source inside with negative sign (x -1). (つまり、 S 内では完全に零、 S 外では Original な点波源の作る Field とポテンシャルの符号を逆にしたものである。)

- ◆ If the contributions from 3 sources (original, single and double layers) are summed up, the fields/potentials outside and inside of V are expressed as. (Original な波源、1 重層、2 重層を 3 つの波源と見なして、その 3 波源の寄与の合計として、 S 内、 S 外の Field とポテンシャルを計算すると、)

$$\phi = \frac{q}{4\pi r} \quad S \text{ 内 } (R \text{ に依らぬ})、 \quad 0 \quad S \text{ 外}$$

$$\mathbf{E} = \frac{q}{4\pi r^2} \hat{\mathbf{r}} \quad S \text{ 内 } (R \text{ に依らぬ})、 \quad 0 \quad S \text{ 外}$$

○ From alternative point of view, the fields outside of S is considered. Since the sources are confined in V (outside of V'), and the definition of $\mathbf{n}$ is toward the inner directions, we have the following results. (視点を換え、観測点が S の外にある場合の解を表現する。波源は S の外から見ると V 外 (V') にしかない。 $\mathbf{n}$ は逆に内側を向くこと に注意すると、)

$$\nabla^2 \phi = -q(x, y, z) \quad q = 0$$

$$\phi = \frac{1}{4\pi} \int_{V'} q \frac{1}{r} dv + \frac{1}{4\pi} \int_S \left(\frac{\partial \phi}{\partial n} \right)_+ \frac{1}{r} ds + \frac{1}{4\pi} \int_S (-\phi)_+ \frac{\partial}{\partial n} \frac{1}{r} ds$$

$$\equiv \frac{1}{4\pi} \int_{V'} 0 \frac{1}{r} dv + \phi_A + \phi_B \quad \frac{\partial}{\partial n}: \text{いずれも積分系 integ. coord.}$$

$\varepsilon_0 \left(\frac{\partial \phi}{\partial n} \right)_+$ なる 1 重層 $\varepsilon_0 (-\phi_+)$ なる 2 重層

- ◆ Fields/potentials due to the original sources. (Original な波源の作る Field とポテンシャル)

$$\phi_{V'} = 0$$

$$\mathbf{E}_{V'} = \mathbf{0}$$

- ◆ Huygens sources assumed on the surface S ($r = R$) 上に仮定される Huygens 波源

• Fields/potentials produced by the **single layer** at $r = R$ (in ; $r < R$, out ; $r > R$) ($r = R$ の位置に想定される 1 重層が作る Field とポテンシャル)

$$\varepsilon_0 \left(\frac{\partial \phi}{\partial n} \right) = \varepsilon_0 \frac{+q}{4\pi R^2}$$

$$\phi_A = \frac{+q}{4\pi R} (\text{Const.}) \quad \text{in,} \quad \frac{+q}{4\pi r} \quad \text{out}$$

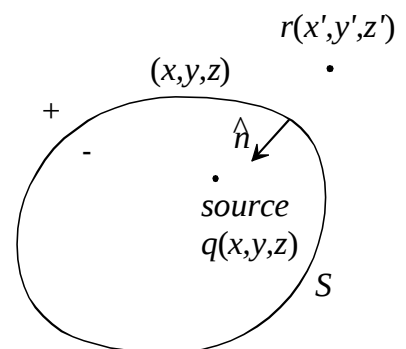
$$\mathbf{E}_{A-} = \mathbf{0} \quad \text{in,} \quad \frac{+q\hat{\mathbf{r}}}{4\pi r^2} \quad \text{out}$$

• Fields/potentials produced by the **double layer** at $r = R$ (in ; $r < R$, out ; $r > R$) $r = R$ の位置に想定される 2 重層が作る Field とポテンシャル

$$\varepsilon_0 \frac{-q}{4\pi R}$$

$$\phi_B = \frac{-q}{4\pi R} (\text{Const.}) \quad \text{in,} \quad 0 \quad \text{out}$$

$$\mathbf{E}_{B-} = \mathbf{0} \quad \text{in,} \quad \mathbf{0} \quad \text{out}$$



- ◆ Fields/potentials in V ($r < R$) or outside ($r > R$) produced by single and double layers on S (S

上の1重層と2重層の波源が、内 ($r < R$)、外 ($r > R$) に作る *Field* とポテンシャル)

$$\phi_s = 0(\text{Const.}) \quad \text{in,} \quad \frac{+q}{4\pi r} \quad \text{out}$$

$$\mathbf{E}_s = \mathbf{0} \quad \text{in,} \quad \frac{+q\hat{\mathbf{r}}}{4\pi r^2} \quad \text{out}$$

Consequently, these are 0 in S, while it is identical with the fields produced by the original source inside with negative sign (x -1).(つまり、S 内では完全に零、S 外では *Original* な点波源の作る *Field* とポテンシャル (同じ符号) である。)

○Then, if we have the point source q' outside V only (次に、 V 内の波源 q が消滅し、 V 外にのみ新しい波源 q' がある場合を考える。) $\mathbf{n}$ is outward normal unit vector and r' is the distance between q' and the observer. $\mathbf{n}$ の向きは V 外へ向いている とする。また、 q' から観測点まではかった距離を r とする。

◆ Fields and potentials produced by the original source q' (Original な波源の作る Field とポテンシャル)

$$\phi_V = \frac{q'}{4\pi r'}$$

$$\mathbf{E}_V = \frac{q'}{4\pi r'^2} \hat{\mathbf{r}}'$$

◆ Huygens sources assumed on the surface S ($r' = R$) (S 上に仮定される Huygens 波源)

• Fields/potentials produced by the **single layer** at $r' = R'$ (1 重層が作る Field とポテンシャル(注意!! out; $r' < R$, in; $r' > R$))

$$\phi_A = \frac{+q'}{4\pi R'} (\text{Const.}) \quad \text{out,} \quad \frac{+q'}{4\pi r'} \quad \text{in}$$

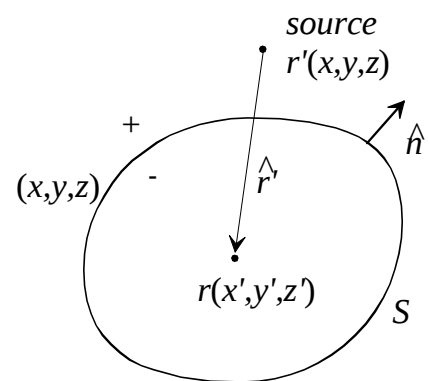
$$\mathbf{E}_{A-} = \mathbf{0} \quad \text{out,} \quad \frac{+q' \hat{\mathbf{r}}'}{4\pi r'^2} \quad \text{in}$$

• Fields/potentials produced by the **double layer** at $r' = R'$ ($r' = R'$ の位置に想定される 2 重層が作る Field とポテンシャル)

$$\epsilon_0 \frac{+q'}{4\pi R'}$$

$$\phi_B = \frac{-q'}{4\pi R'} (\text{Const.}) \quad \text{out,} \quad 0 \quad \text{in}$$

$$\mathbf{E}_{B-} = \mathbf{0} \quad \text{out,} \quad \mathbf{0} \quad \text{in}$$



◆ Fields/potentials in V out ($r' < R$), in ($r' > R$) produced by single and double layers on S (S 上の 1 重層と 2 重層の波源が、に作る Field とポテンシャル)

$$\phi_S = 0 (\text{Const.}) \quad \text{out,} \quad \frac{+q'}{4\pi r} \quad \text{in}$$

$$\mathbf{E}_S = \mathbf{0} \quad \text{out,} \quad \frac{+q' \hat{\mathbf{r}}'}{4\pi r'^2} \quad \text{in}$$

Consequently, these are 0 outside of S , while it is identical with the fields produced by the original source q' outside with the same sign (つまり、 S 外では完全に零、 S 内では V 外にある点波源 q' の作る Field とポテンシャルと同じ符号である。)

◆ If the contributions from 3 sources (original, single and double layers) are summed up, the fields/potentials outside and inside of V are expressed as. (V 外にある波源 q' 、1 重層、2 重層を 3 つの波源と見なして、その 3 波源の寄与の合計として、 S 内、 S 外の Field とポテンシャルを計算すると、)

$\phi = \frac{q'}{4\pi r'}$	S 外 (R に依らぬ) 、	0	S 内
$E = \frac{q'}{4\pi r'^2} \hat{\mathbf{r}}'$	S 外 (R に依らぬ) 、	0	S 内

○In general cases, we have sources q in V and q' outside of V . The equivalent sources assumed on S are the sum of those assumed for q and q' independently. (一般に、 V 内に q 、 V 外に q' の波源があるとする。 S 上の等価波源の定義は、 q と q' とがそれぞれ存在する時の波源の和があると仮定する。(n は V 外 out を向く。))

◆ Fields/potentials due to equivalent sources on S are (S 上の波源が作る Field、ポテンシャルは、)

- contribution associated with q (q から S 上に作られる波源の寄与)

$$\phi_S = 0(\text{Const.}) \quad \text{in,} \quad \frac{-q}{4\pi r} \quad \text{out}$$

$$\mathbf{E}_S = \mathbf{0} \quad \text{in,} \quad \frac{-q\hat{\mathbf{r}}}{4\pi r^2} \quad \text{out}$$

- contribution associated with q' (q' から S 上に作られる波源の寄与)

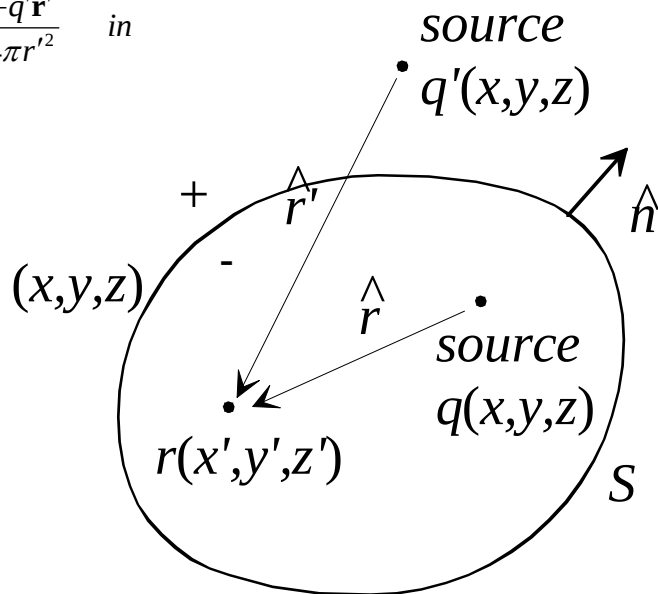
$$\phi_S = 0(\text{Const.}) \quad \text{out,} \quad \frac{+q'}{4\pi r'} \quad \text{in}$$

$$\mathbf{E}_S = \mathbf{0} \quad \text{out,} \quad \frac{+q'\hat{\mathbf{r}}'}{4\pi r'^2} \quad \text{in}$$

- Sum of these above are (両方の寄与)

$$\phi_S = \frac{-q}{4\pi r} \quad \text{out,} \quad \frac{+q'}{4\pi r'} \quad \text{in}$$

$$\mathbf{E}_S = \frac{-q\hat{\mathbf{r}}}{4\pi r^2} \quad \text{out,} \quad \frac{+q'\hat{\mathbf{r}}'}{4\pi r'^2} \quad \text{in}$$



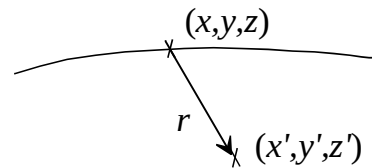
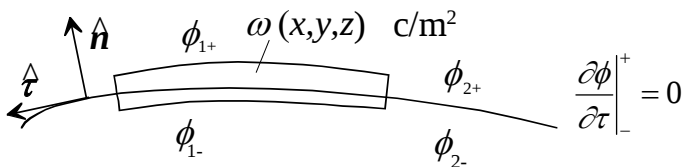
ベクトルの不連続性 *Jump relation of Vectors*

$$\mathbf{A} \equiv \frac{1}{4\pi\epsilon} \int \omega \nabla \phi \, ds = \frac{1}{4\pi\epsilon} \int \omega (-\nabla' \phi) \, ds = -\nabla' \left\{ \frac{1}{4\pi\epsilon} \int \omega \phi \, ds \right\}$$

$$\text{観測系} \quad \phi = \frac{e^{-jkr}}{r} \Rightarrow \frac{1}{r}$$

不連続に着目 As for the discontinuity

$$\left. \begin{aligned} n \cdot (\mathbf{A}_+ - \mathbf{A}_-) &= -\frac{\partial}{\partial n'} \left\{ \frac{1}{4\pi\epsilon} \int \omega \phi \, ds \right\} \Big|_-^+ = \frac{\omega}{\epsilon} \\ \phi \text{ は連続だから } \hat{\mathbf{n}} \times (\mathbf{A}_+ - \mathbf{A}_-) &= 0 \end{aligned} \right\} \quad (\text{A}) \text{より}$$



$$\mathbf{A}_+ = \mathbf{A}_+ \cdot \hat{\mathbf{n}} \hat{\mathbf{n}} + \frac{(\mathbf{A}_+ - \mathbf{A}_+) \cdot \hat{\mathbf{n}}}{C \times \hat{\mathbf{n}}} \hat{\mathbf{n}}$$

Finally we get

$$(\text{A}) \text{より} \quad \therefore \mathbf{A}_+ - \mathbf{A}_- = \frac{\omega}{\epsilon} \hat{\mathbf{n}} \quad \dots (C)$$

Q.E.D

$$(2) \quad \mathbf{B}(x', y', z') = \frac{\mu_0}{4\pi} \int_s \mathbf{K}(x, y, z) \times \nabla \left(\frac{1}{r} \right) da \text{ を考える。} \mathbf{K} \cdot \hat{\mathbf{n}} = 0 \text{ とする。}$$

$$(3) \quad \mathbf{K} = \mathbf{i}K_x + \mathbf{j}K_y + \mathbf{k}K_z$$

$$(4) \quad \mathbf{B}(x', y', z') = \frac{\mu_0}{4\pi} \left[\mathbf{i} \times \int_s K_x \nabla \left(\frac{1}{r} \right) da + \mathbf{j} \times \int_s K_y \nabla \left(\frac{1}{r} \right) da + \mathbf{k} \times \int_s K_z \nabla \left(\frac{1}{r} \right) da \right]$$

$$(5) \quad \mathbf{E}(x', y', z') = \frac{1}{4\pi} \int \omega(x, y, z) \nabla \left(\frac{1}{r} \right) da \text{ の性質を考察する}$$

$$(6) \quad \mathbf{E}_+ - \mathbf{E}_- = \omega \hat{\mathbf{n}} \quad (\text{C}) \text{より}$$

$$(7) \quad (4), (5), (6) \rightarrow \mathbf{B}_+ - \mathbf{B}_- = \mu_0 \mathbf{K} \times \hat{\mathbf{n}}$$

$$(8) \quad (7) \rightarrow \hat{\mathbf{n}} \cdot (\mathbf{B}_+ - \mathbf{B}_-) = 0$$

$$(9) \quad \hat{\mathbf{n}} \times (\mathbf{B}_+ - \mathbf{B}_-) = \mu_0 \hat{\mathbf{n}} \times (\mathbf{K} \times \hat{\mathbf{n}})$$

$$(10) \quad \hat{\mathbf{n}} \times (\mathbf{K} \times \hat{\mathbf{n}}) = (\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}) \mathbf{K} - (\hat{\mathbf{n}} \cdot \mathbf{K}) \hat{\mathbf{n}} \\ \downarrow \\ 0$$

Finally we get

$$(11) \quad (9) \text{より} \quad \hat{\mathbf{n}} \times (\mathbf{B}_+ - \mathbf{B}_-) = \mu_0 \mathbf{K} \quad \dots (D)$$

Q.E.D

These four relations are summarized as:

$$\phi \equiv \frac{1}{4\pi} \int_s A(x, y, z) \frac{1}{r} ds \quad \hat{n} \text{ (+ 向き directed)}$$

$$\phi|_{-}^{+} = 0$$

$$\left. \frac{\partial \phi}{\partial n'} \right|_{+}^{-} = -A(x', y', z') \quad (A)$$

観測点系

$$\phi \equiv \frac{1}{4\pi} \int B \frac{\partial}{\partial n} \left(\frac{1}{r} \right) ds \quad \hat{n} \text{ (+ 向き)}$$

積分系

$$\phi|_{-}^{+} = B(x', y', z')$$

$$\left. \frac{\partial \phi}{\partial n'} \right|_{-}^{+} = 0 \quad (B)$$

観測点系

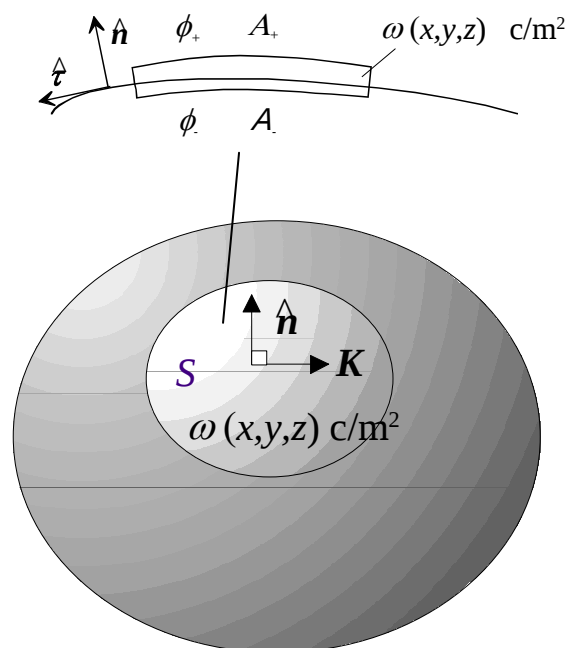
$$\mathbf{A} \equiv \frac{1}{4\pi} \int \omega \nabla \phi ds$$

$$\mathbf{A}|_{-}^{+} = \omega \hat{n} \quad (C)$$

$$\mathbf{A} \equiv \frac{1}{4\pi} \int \mathbf{K} \times \nabla \phi ds \quad \text{ただし } \mathbf{K} \cdot \hat{n} = 0$$

$$\mathbf{A}|_{-}^{+} = \mathbf{K} \times \hat{n}$$

$$\hat{n} \times \mathbf{A}|_{-}^{+} = \mathbf{K} \quad (\because \mathbf{K} \perp \hat{n}) \quad (D)$$



後に使う。(For later use)